Persistent Data Structures

Dnepropetrovsk Lambda Club

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Immutable

Persistent

Functional

Data Structures

Data Structures

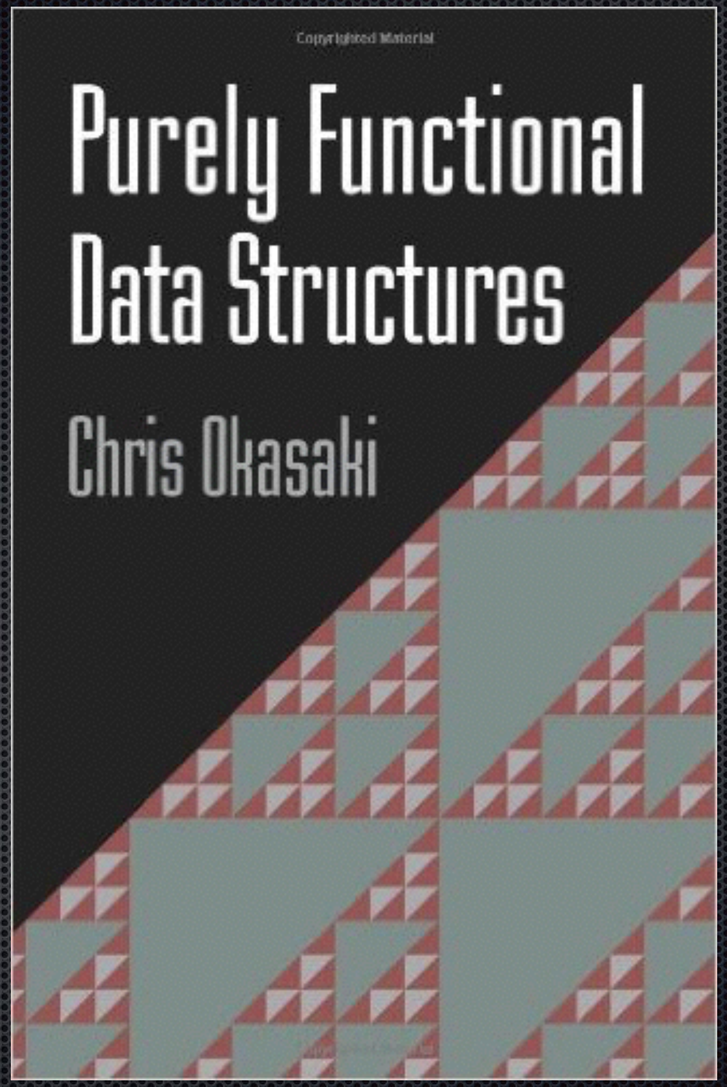
Ephemeral

Partially Persistent

Fully Persistent

Why?

Reasoning about code
Concurrency
History (structural sharing)
Performance (sometimes)



http://www.amazon.com/Purely-Functional-Structures-Chris-Okasaki/dp/0521663504

Structures

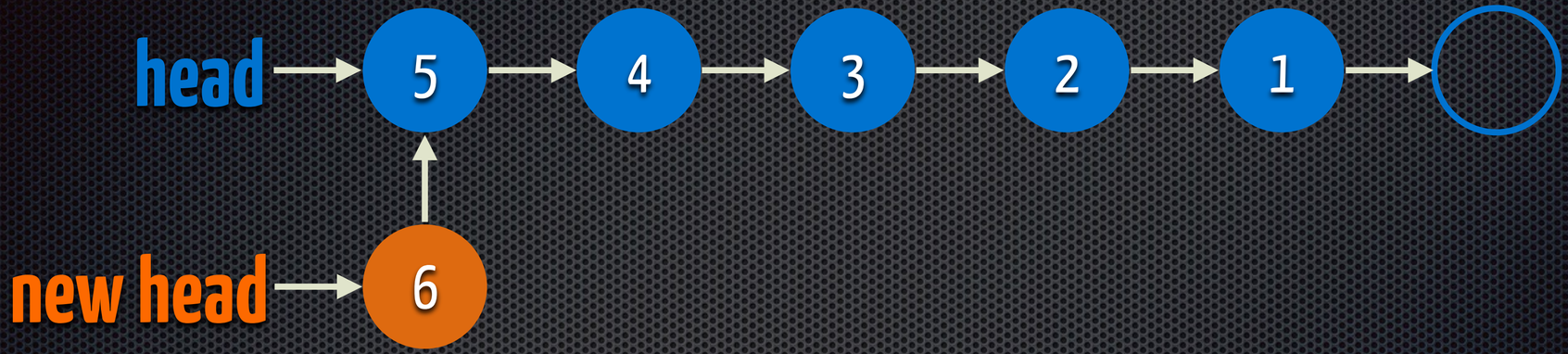
1. Linked List / Stack
2. Queue
3. TreeMap
4. Vector / Array / ArrayList
5. HashMap
6. Set

Linked List

Singly Linked List

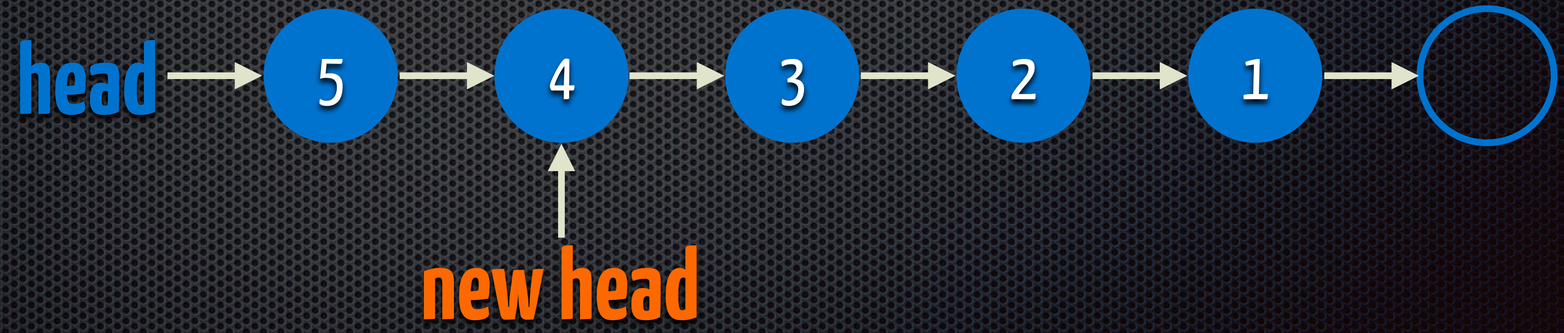
Linked List

Push



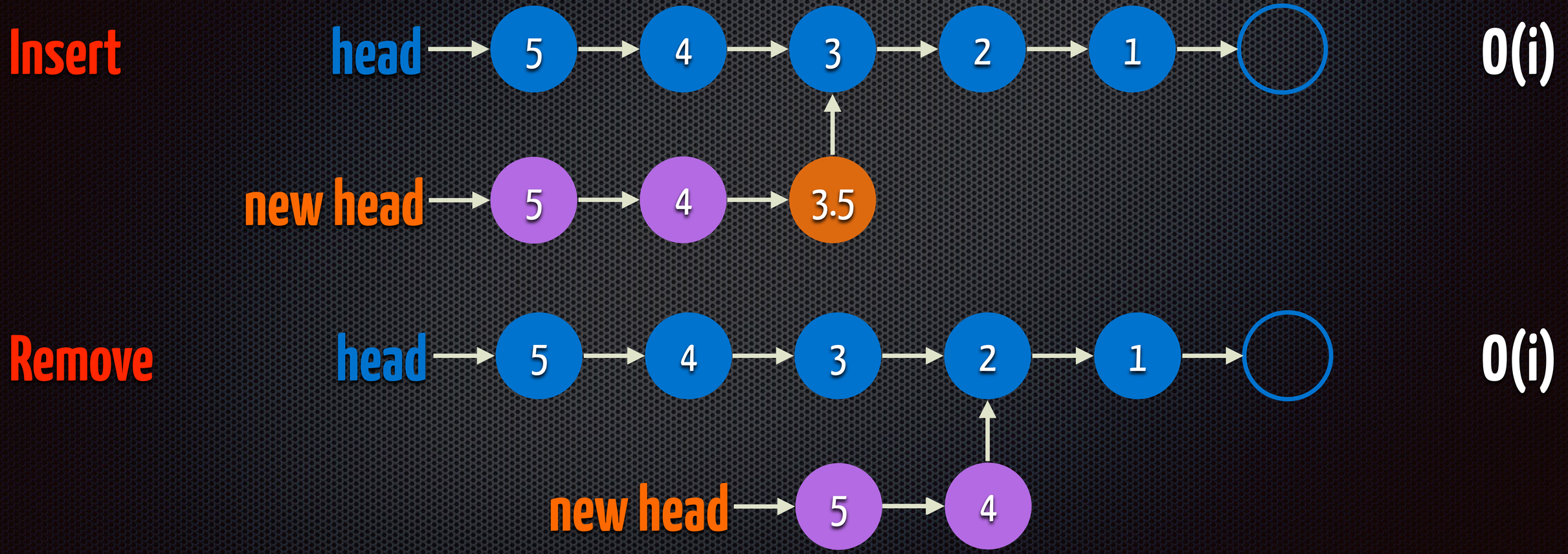
O(1)

Pop



O(1)

Linked List


Insert
head
5
4
3
2
1
O(i)
new head
5
4
3.5
Remove
head
5
4
3
2
1
O(i)
new head
5
4

Stack

Same as Linked List

Queue

Linked List + Linked List or Linked List + Vector

```
class Queue {
  constructor(dequeueList, enqueueList) {
    this.dequeueList = dequeueList;
    this.enqueueList = enqueueList;
  }

  enqueue(value) { // O(1)
    if (this.dequeueList == null) {
      return new Queue(List.empty().push(value), this.enqueueList);
    } else {
      return new Queue(this.dequeueList, (this.enqueueList || List.empty()).push(value));
    }
  }

  peek() { // O(1)
    if (this.dequeueList == null) {
      throw EMPTY;
    } else {
      return this.dequeueList.head();
    }
  }

  dequeue() { // effectively O(1) time (O(n) sometimes)
    if (this.dequeueList == null) {
      throw EMPTY;
    } else {
      let tail = this.dequeueList.tail();
      if (tail.isEmpty()) {
        return new Queue(this.enqueueList ? this.enqueueList.reverse() : null, null);
      } else {
        return new Queue(tail, this.enqueueList);
      }
    }
  }
}
```

TreeMap

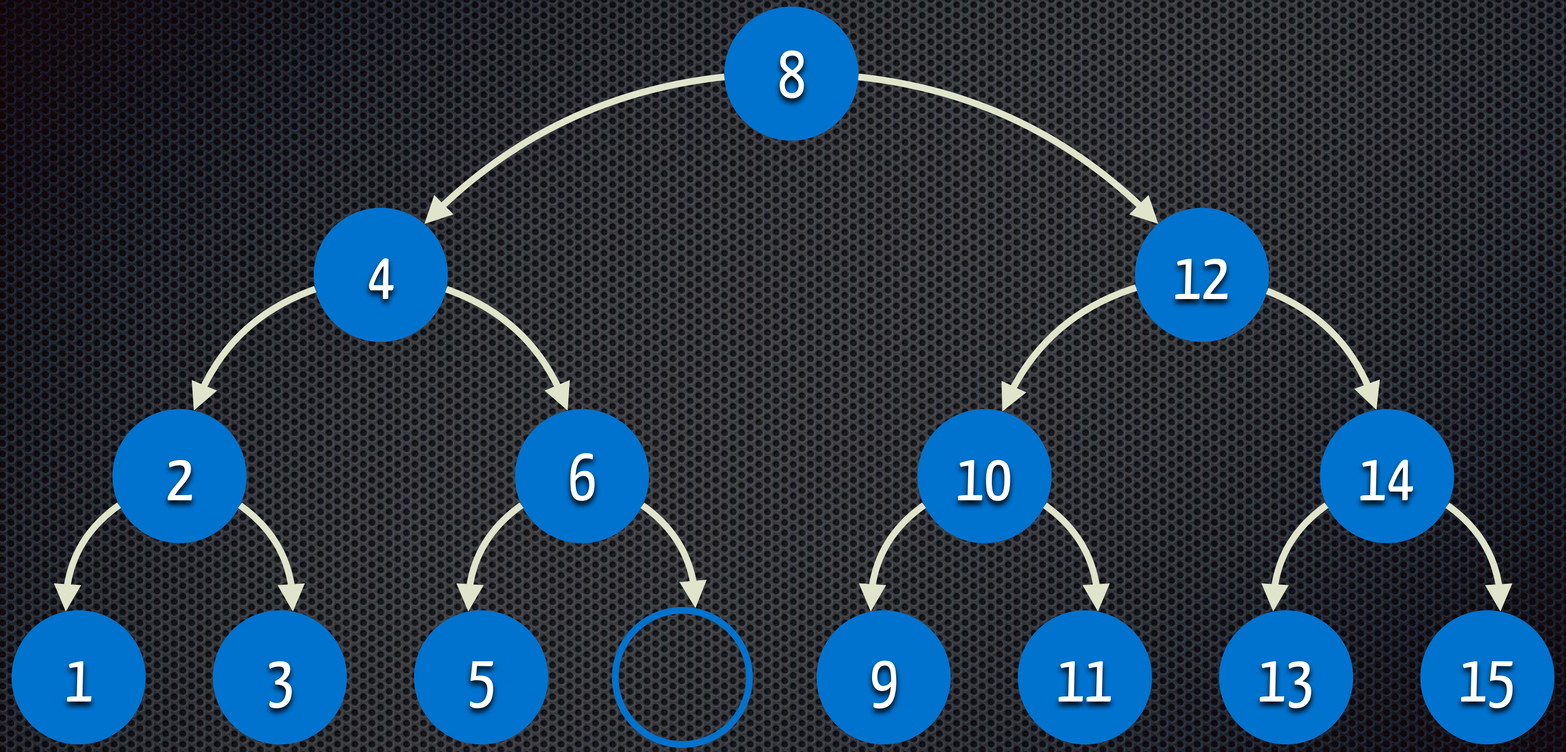
Map: `(key -> value)` relationship

Tree: implementation detail

TreeMap

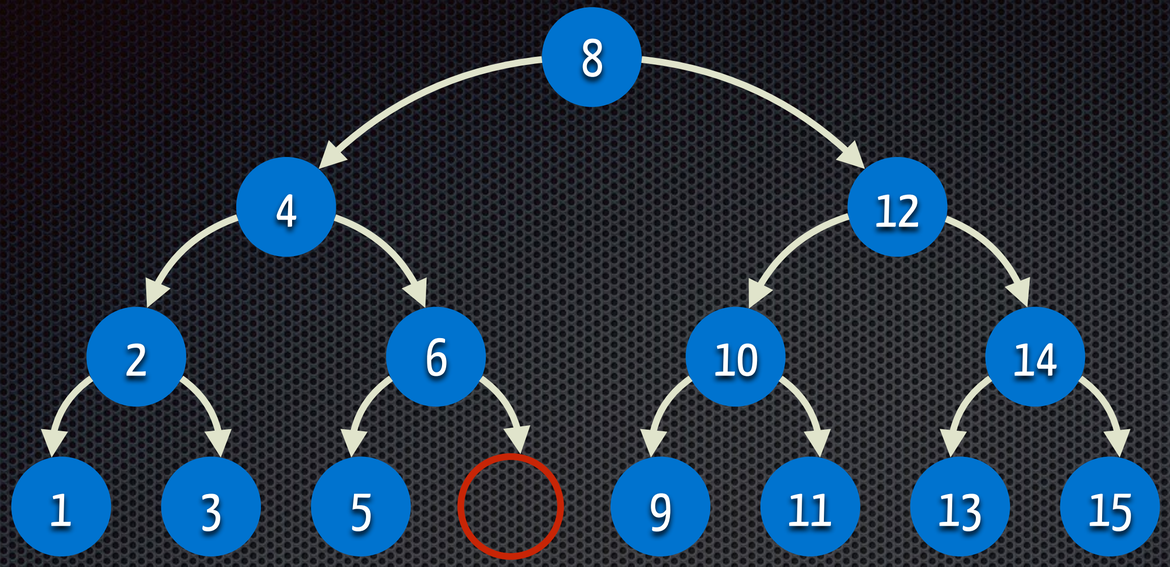
Binary Trees
Red-Black Trees

Binary Trees

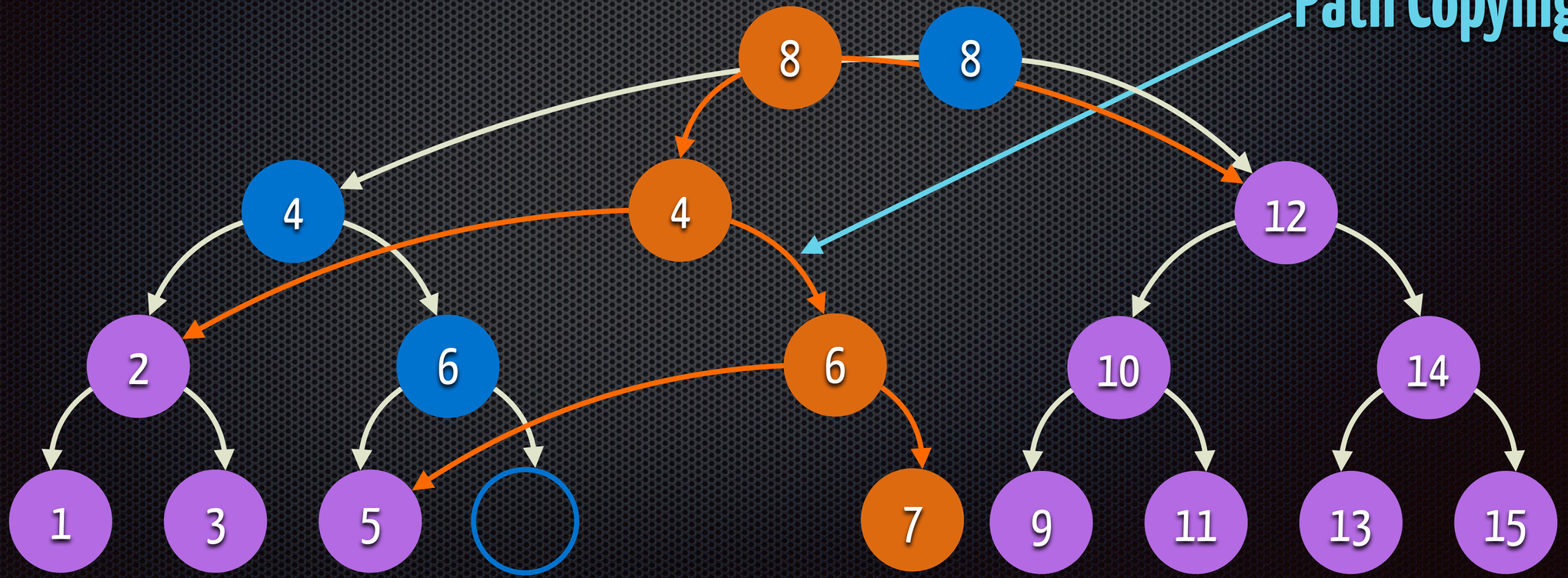


Add

$O(\log_2 n)$

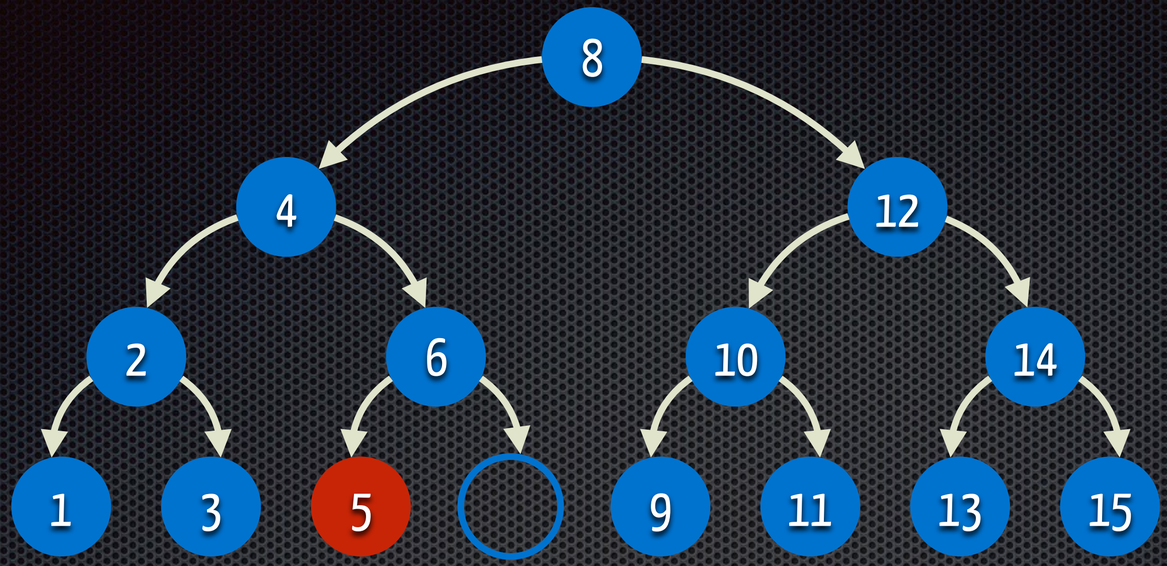


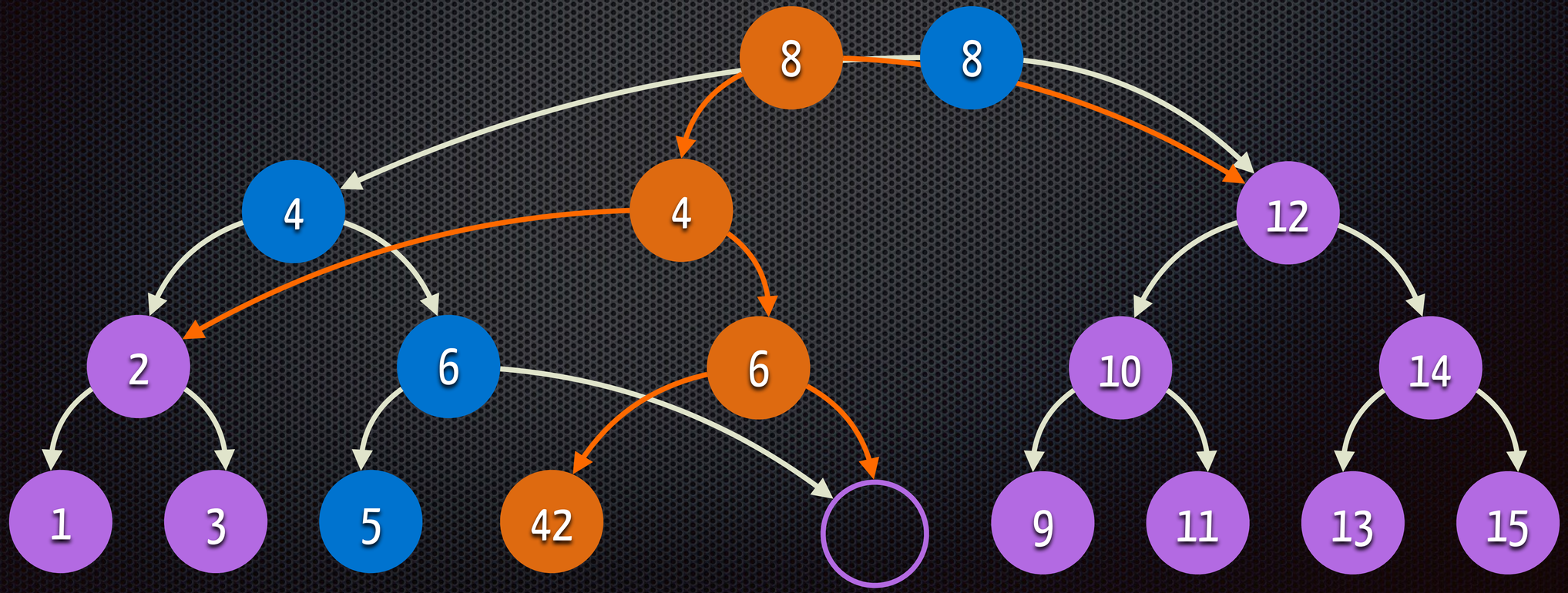
Path Copying



Update

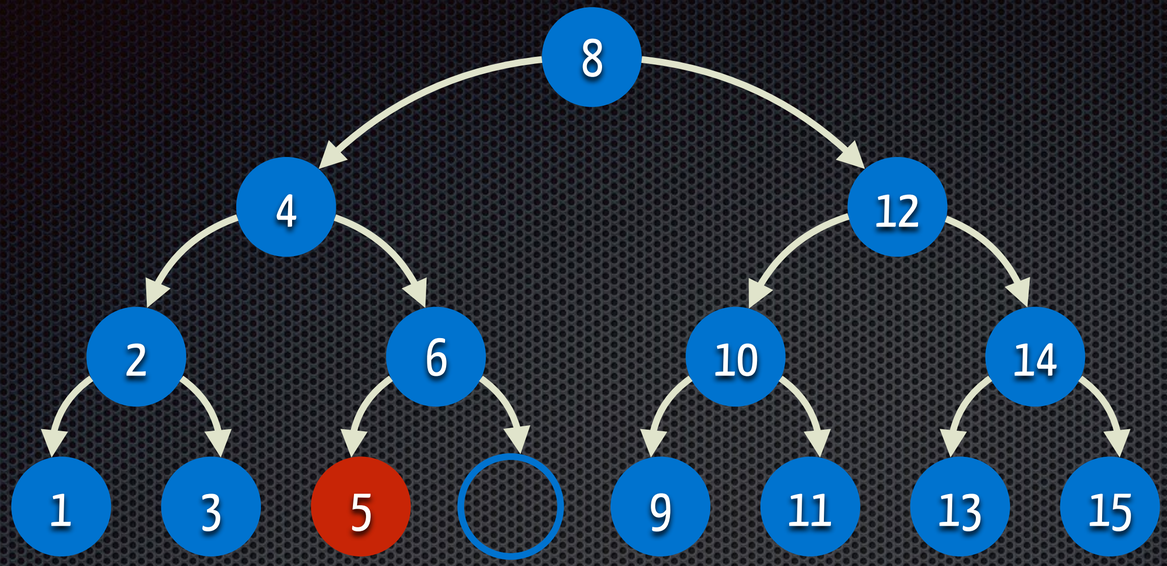
$O(\log_2 n)$

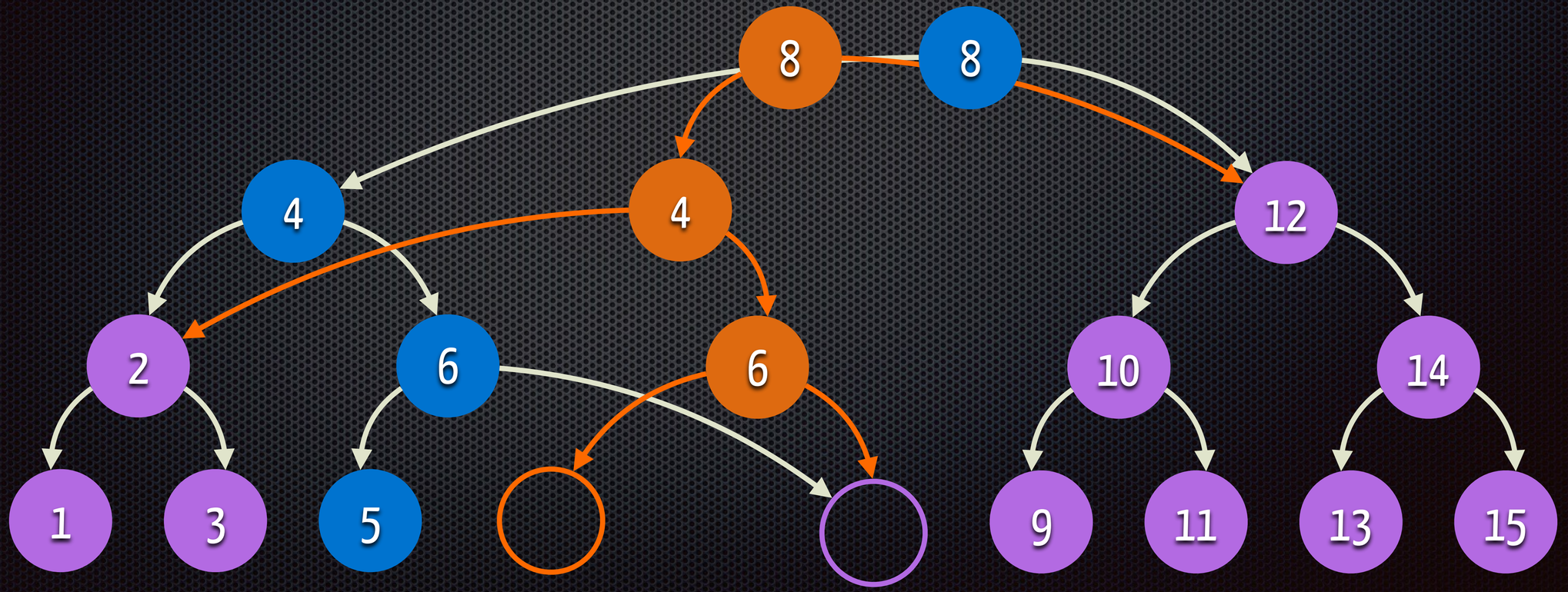




Remove

$O(\log_2 n)$



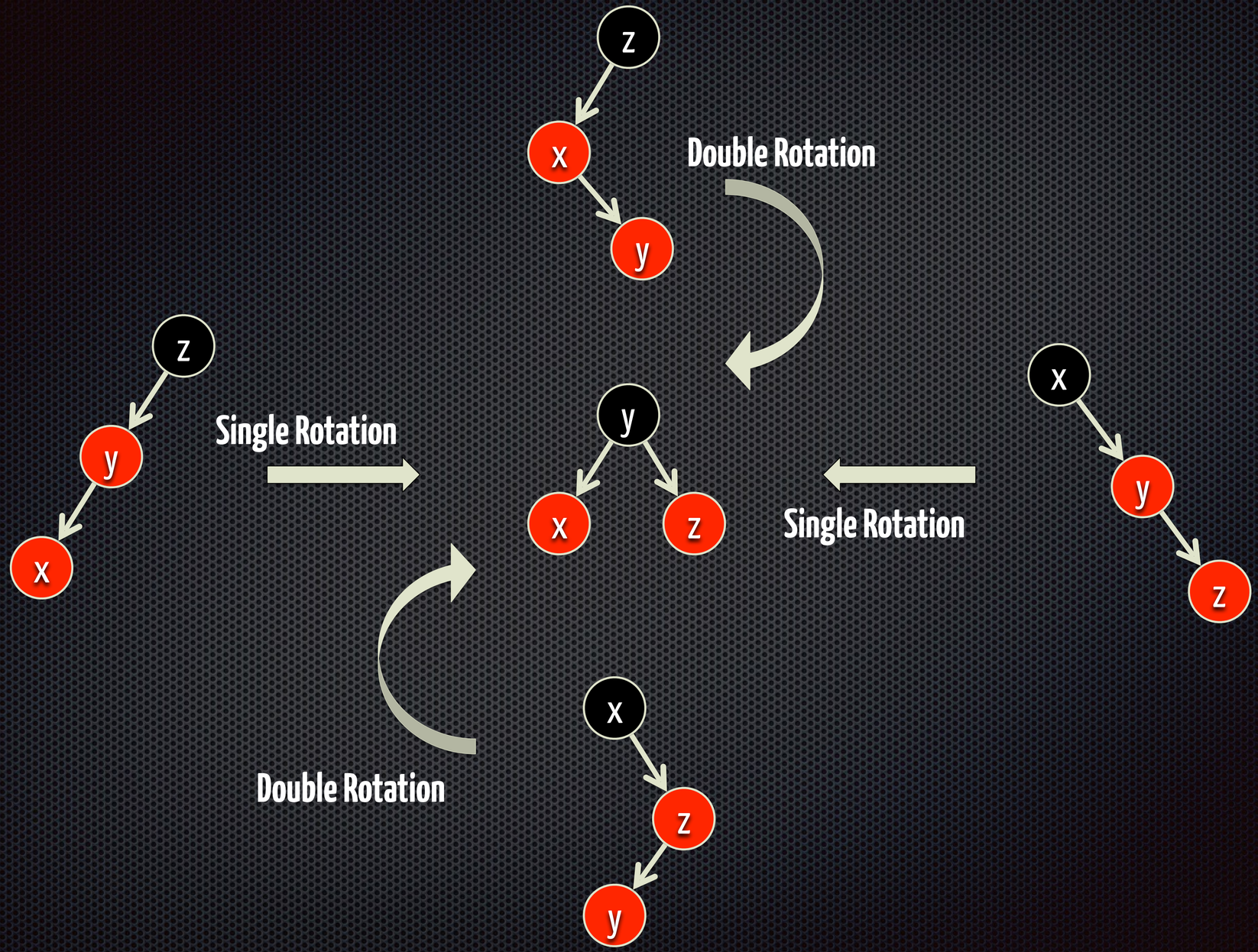


Red Black Trees

Red invariant: No red node has red parent

Black invariant: Every root-to-leaf path contains the same number of black nodes

When new node is inserted: it becomes Red



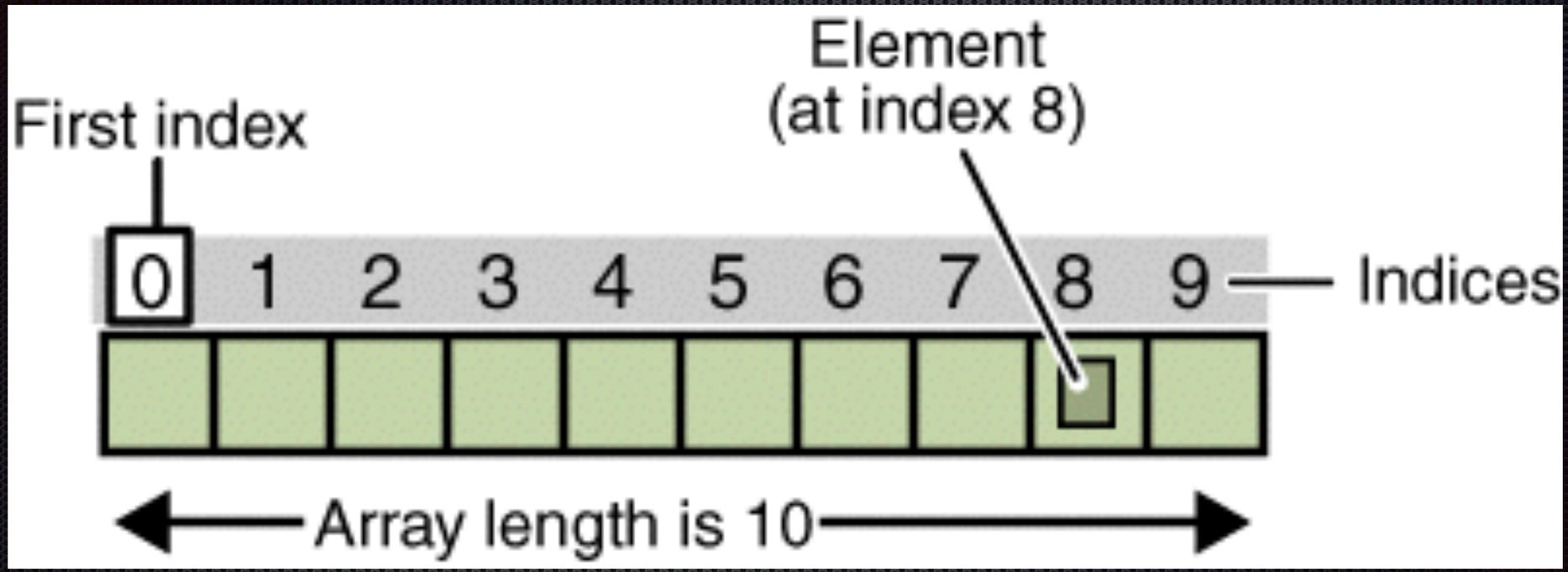
http://www.slideshare.net/vkostyukov/purely-functional-data-structures-in-scala-26175521

Warning!
Scala Ahead!

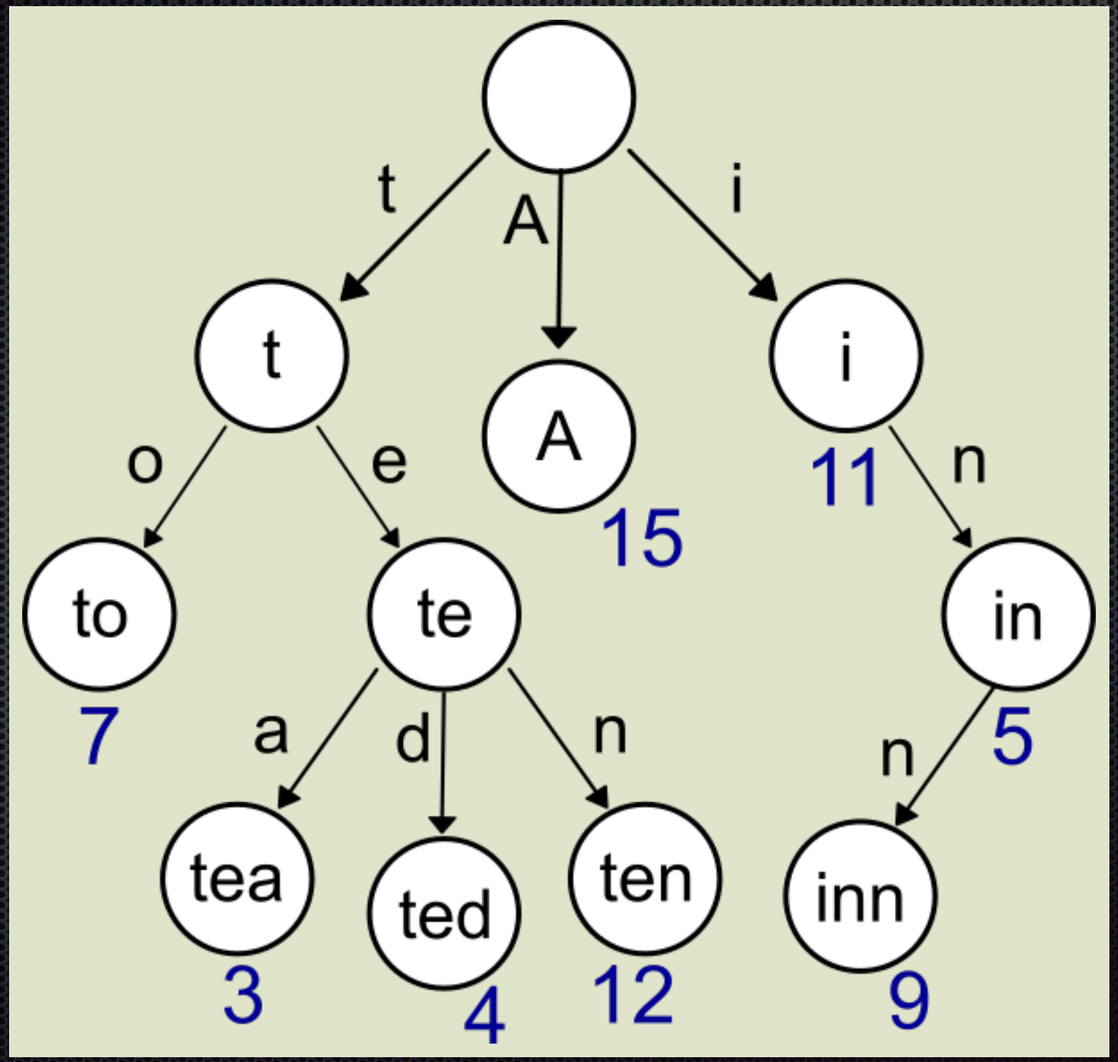
```
def balancedAdd(x: Int): Tree =
  if (isEmpty) RedBranch(x)
  else if (x < value) balance(isBlack, value, left.balancedAdd(x), right)
  else if (x > value) balance(isBlack, value, left, right.balancedAdd(x))
  else this

def balance(b: Boolean, x: Int, left: Tree, right: Tree): Tree =
  (b, left, right) match {
    case (true, RedBranch(y, RedBranch(z, a, b), c), d) =>
      BlackBranch(y, RedBranch(z, a, b), RedBranch(x, c, d))
    case (true, a, RedBranch(y, b, RedBranch(z, c, d))) =>
      BlackBranch(y, RedBranch(x, a, b), RedBranch(z, c, d))
    case (true, RedBranch(z, a, RedBranch(y, b, c)), d) =>
      BlackBranch(y, RedBranch(z, a, b), RedBranch(x, c, d))
    case (true, a, RedBranch(z, RedBranch(y, b, c), d)) =>
      BlackBranch(y, RedBranch(x, a, b), RedBranch(z, c, d))
    case (true, _, _) => BlackBranch(x, left, right)
    case (false, _, _) => RedBranch(x, left, right)
  }
```

Vector


Element
(at index 8)
First index
0 1 2 3 4 5 6 7 8 9
Indices
Array length is 10

Trie

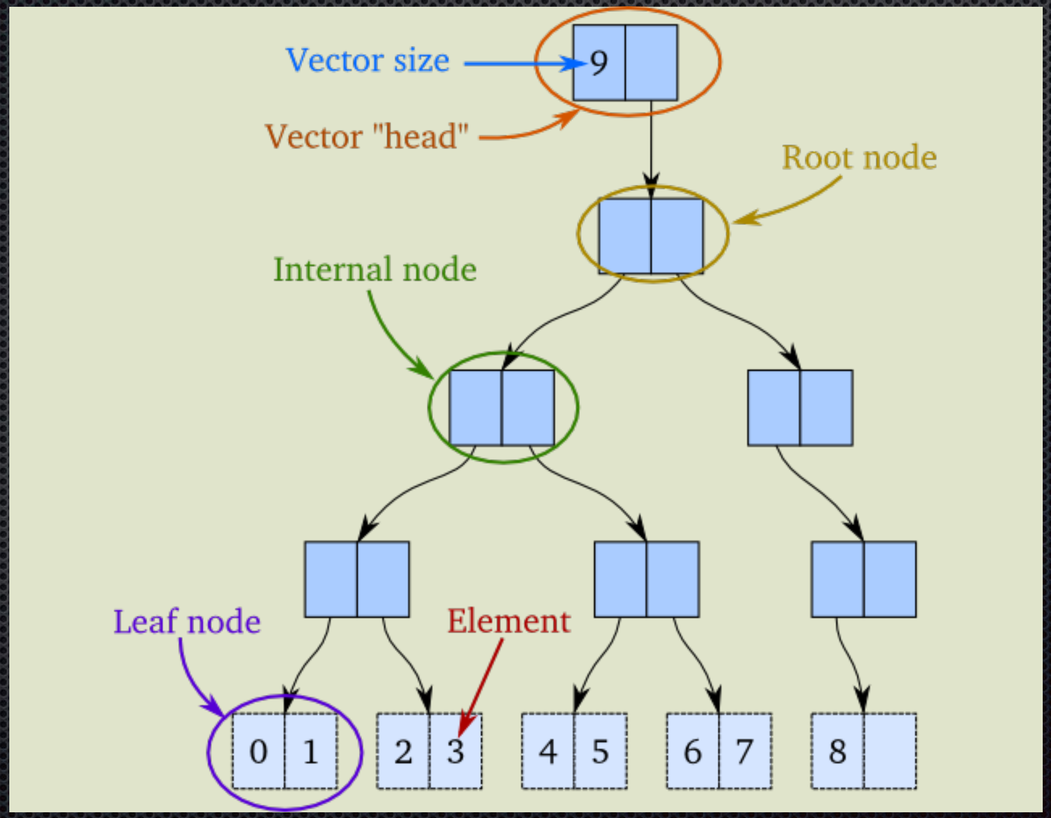


Vector

Persistent Bit-Partitioned Vector Trie

Vector

Persistent Bit-Partitioned Vector Trie



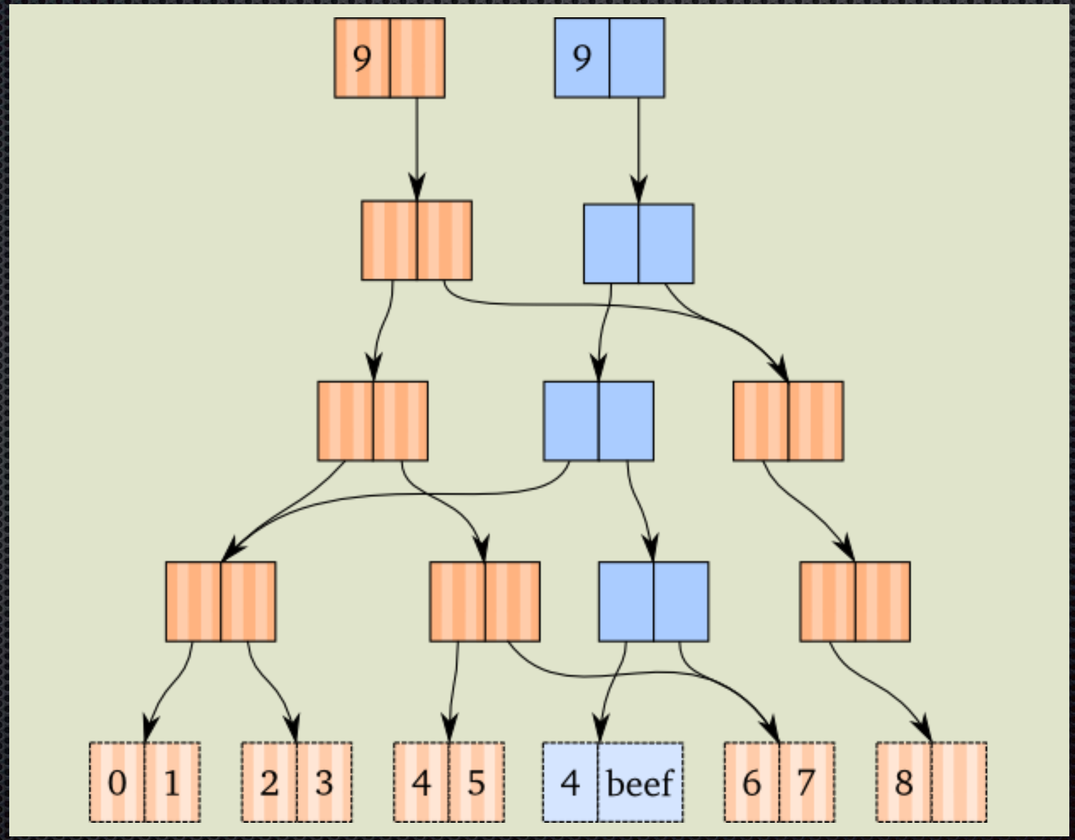
http://hypirion.com/musings/understanding-persistent-vector-pt-1

Update

$O(\log_2 n)$

Vector

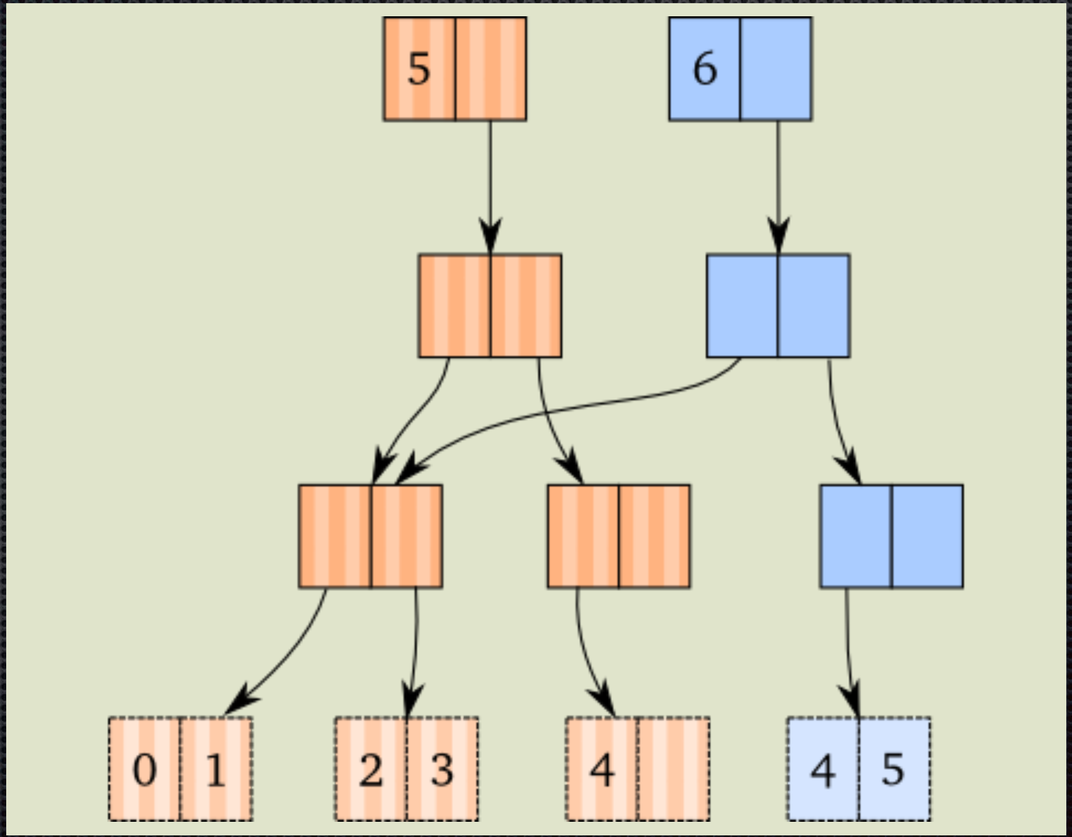
Update



Vector

Push (1)

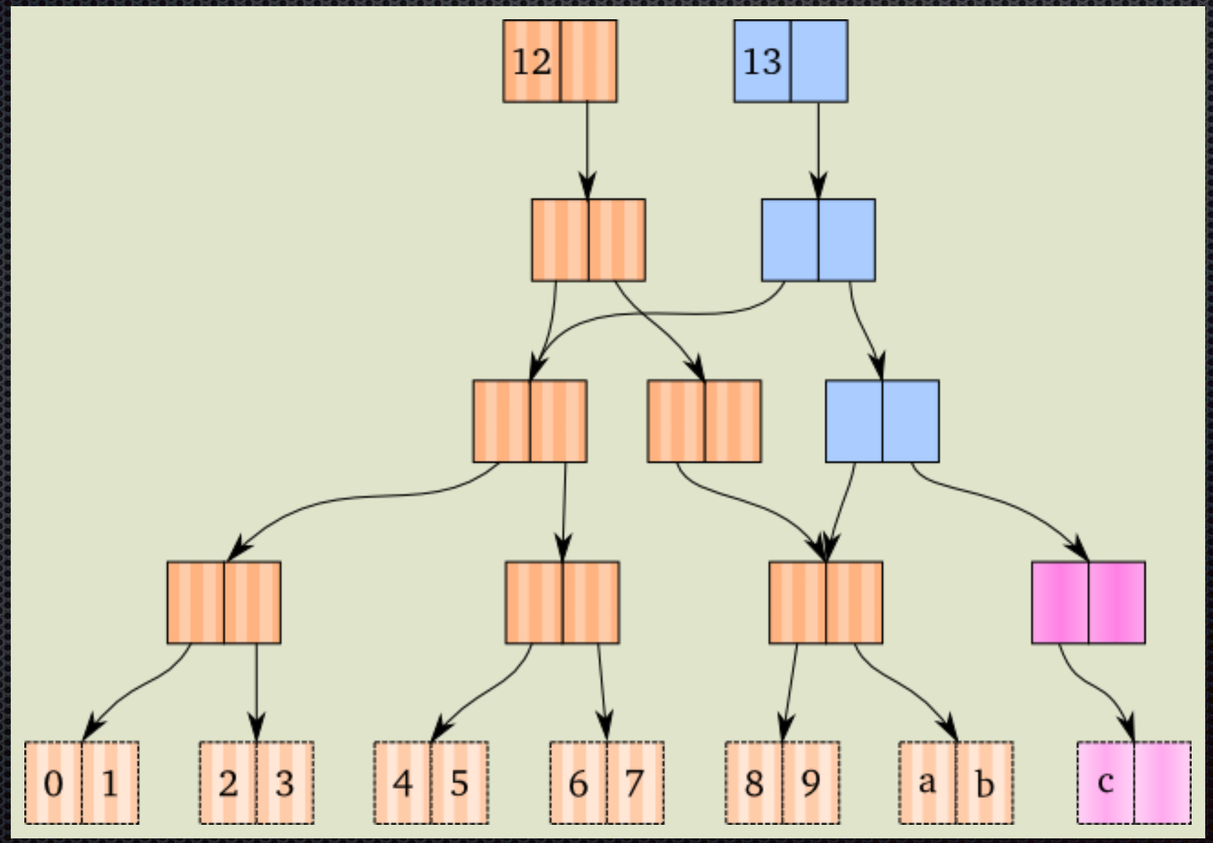
$O(\log_2 n)$



Vector

Push (2)

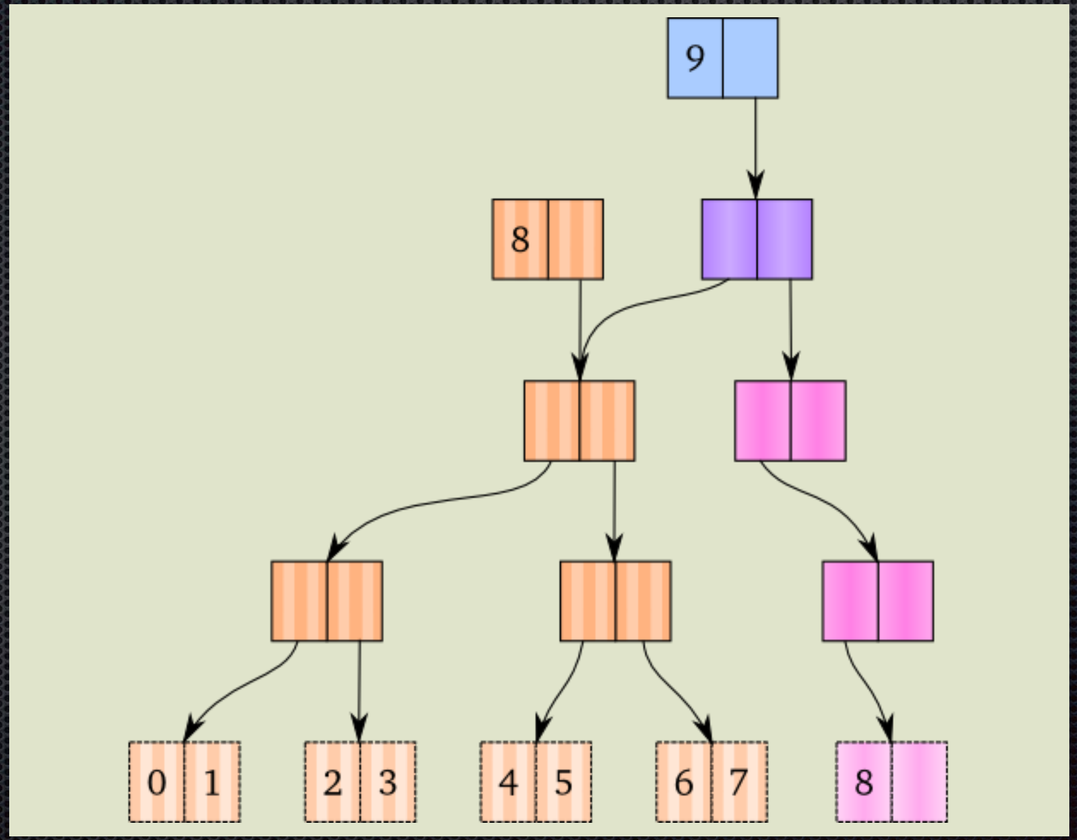
$O(\log_2 n)$



Vector

Push (3)

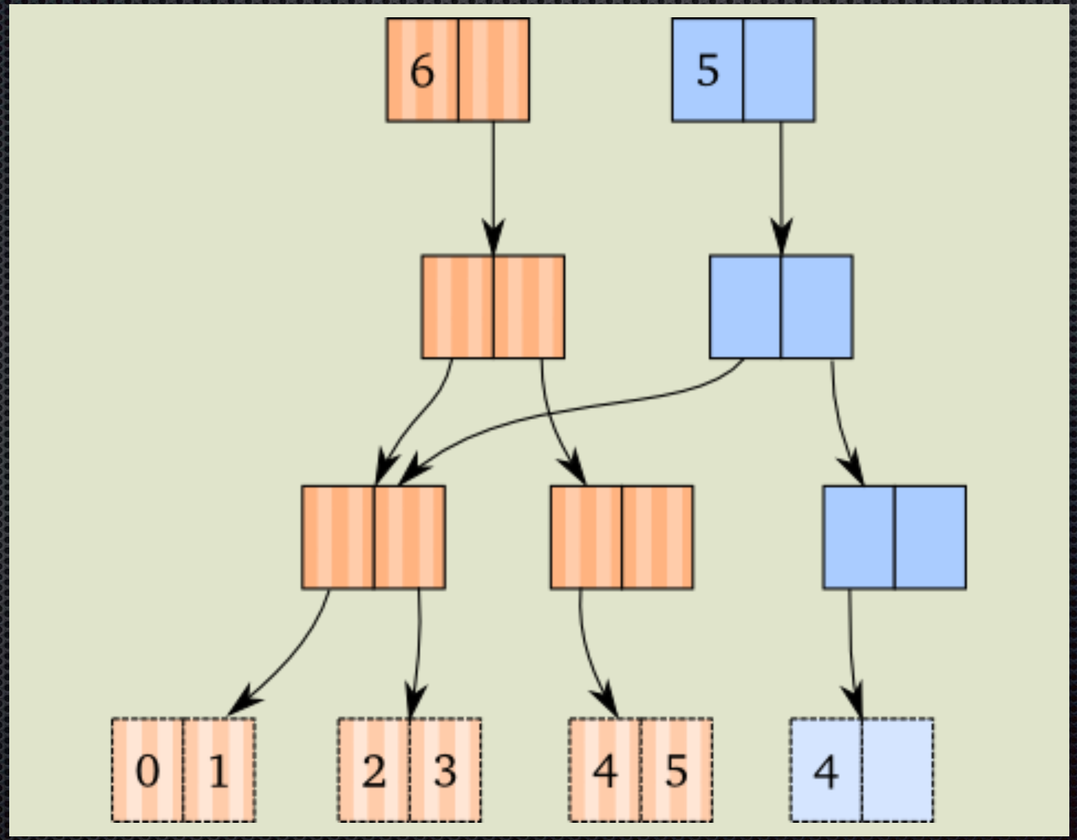
$O(\log_2 n)$



Vector

Remove (1)

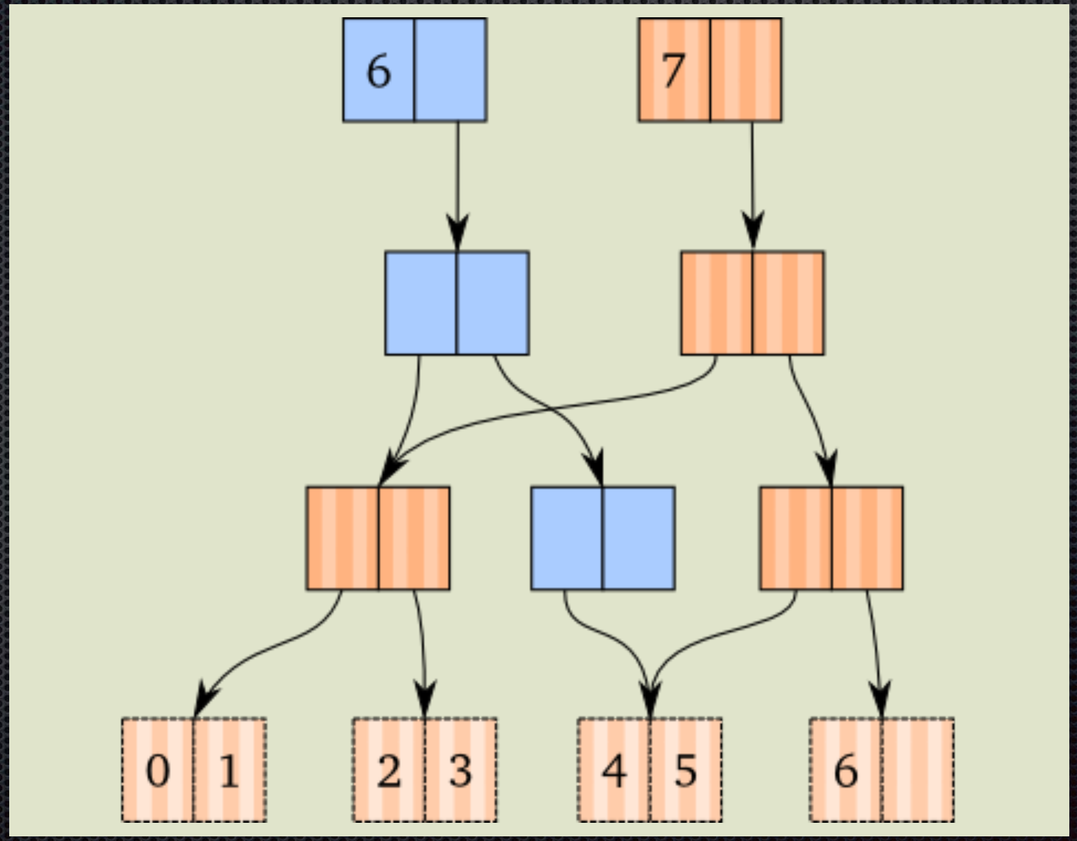
$O(\log_2 n)$



Vector

Remove (2)

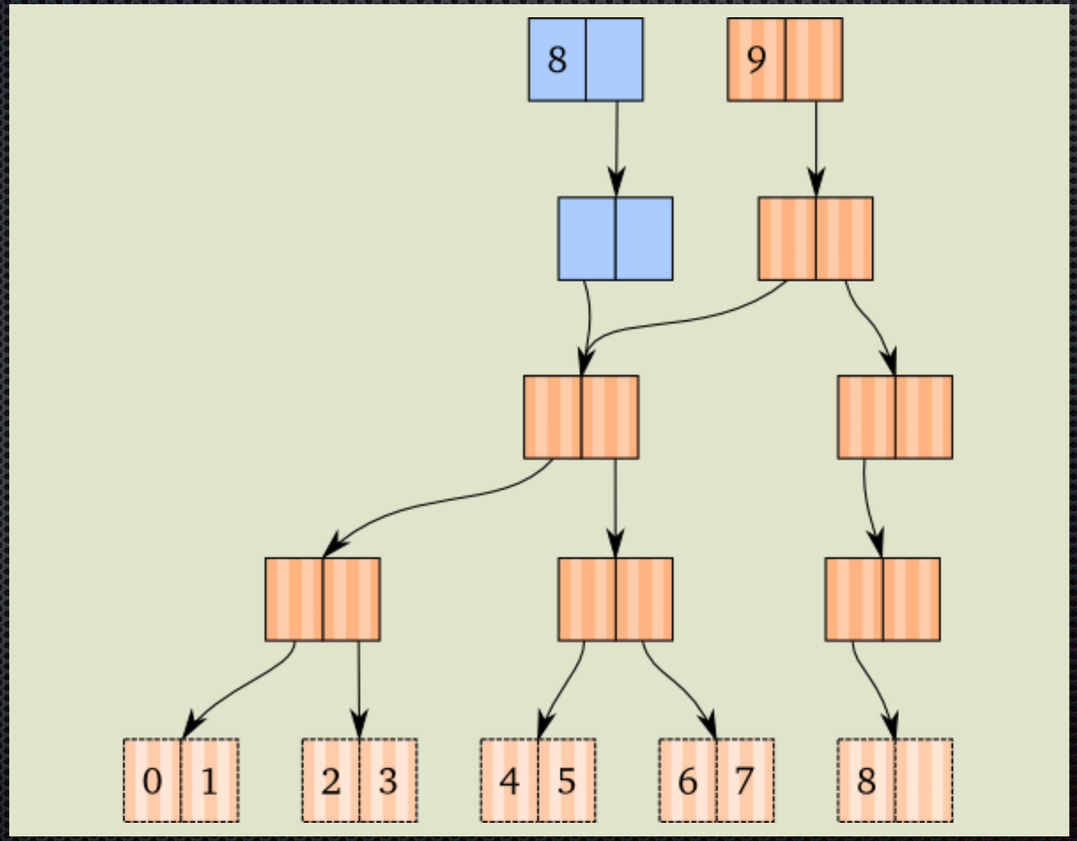
$O(\log_2 n)$



Vector

Remove (3)

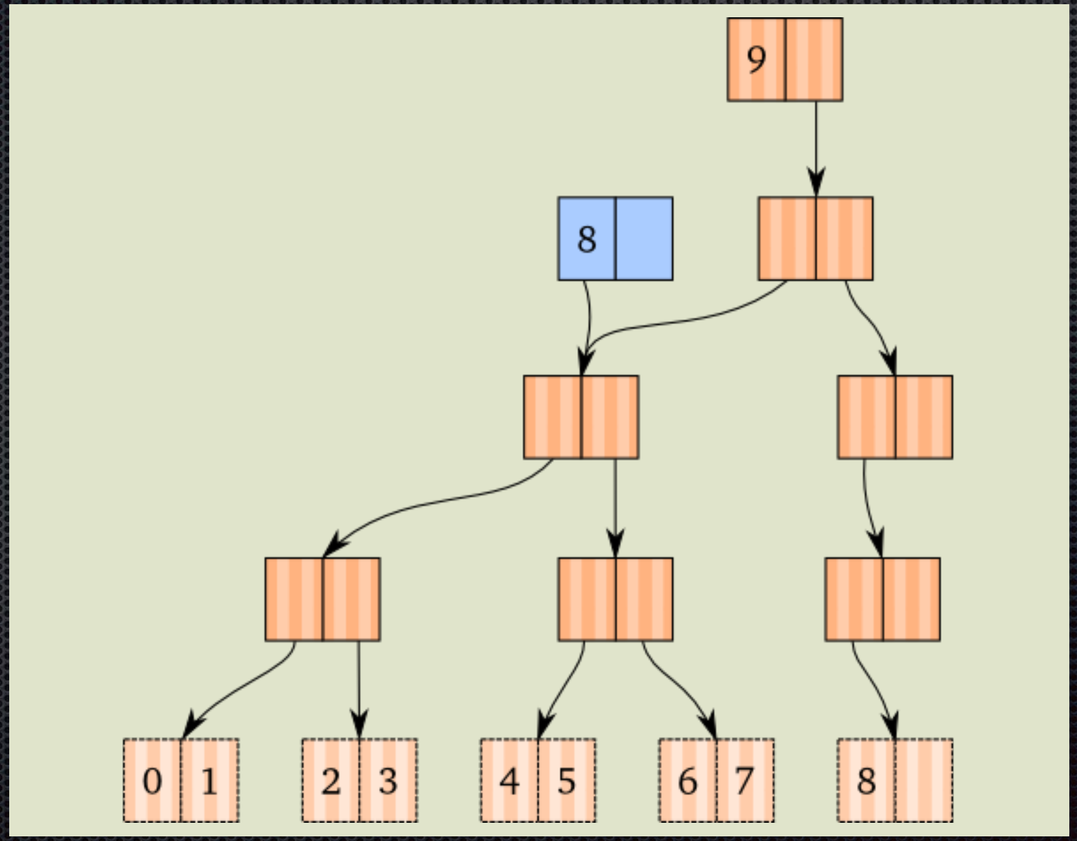
$O(\log_2 n)$



Vector

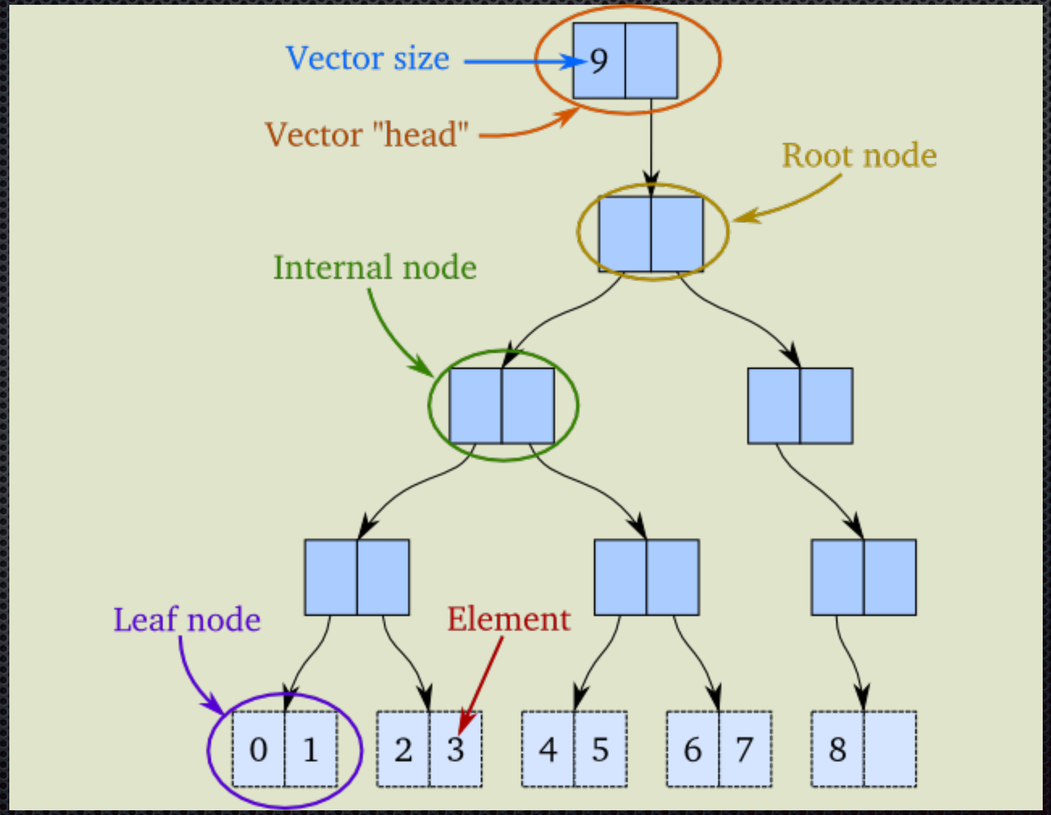
Remove (3.2)

$O(\log_2 n)$



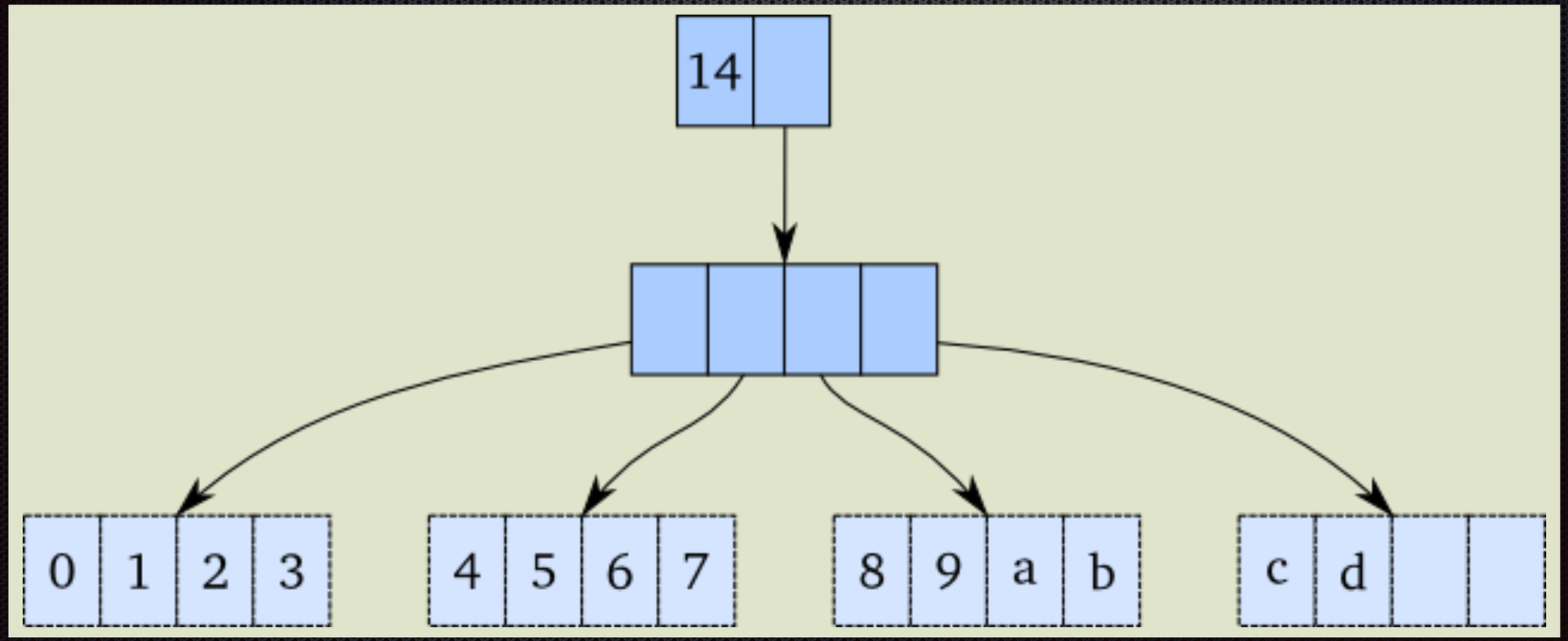
Vector

Branching Factor: 2



Vector

Branching Factor: 4



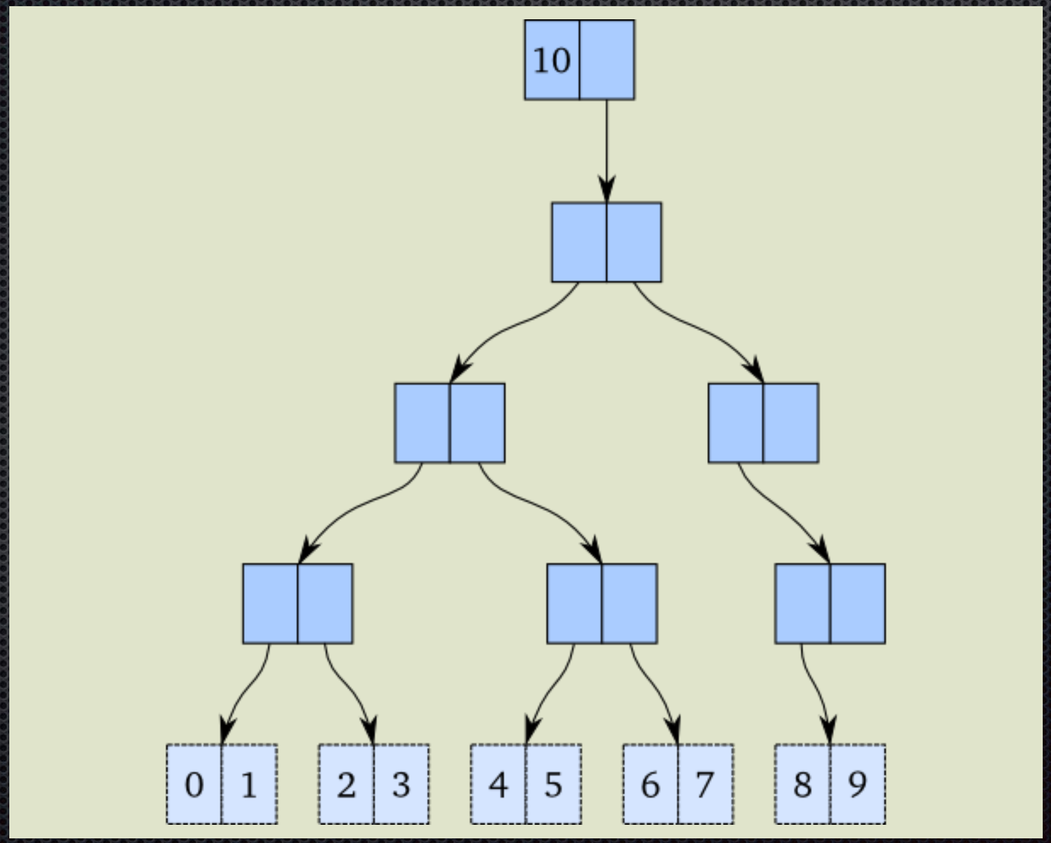
Vector

In reality branching factor is 32
And all mentioned operations are $O(\log_{32} n)$

Which sometimes being promoted as $O(1)$

Vector

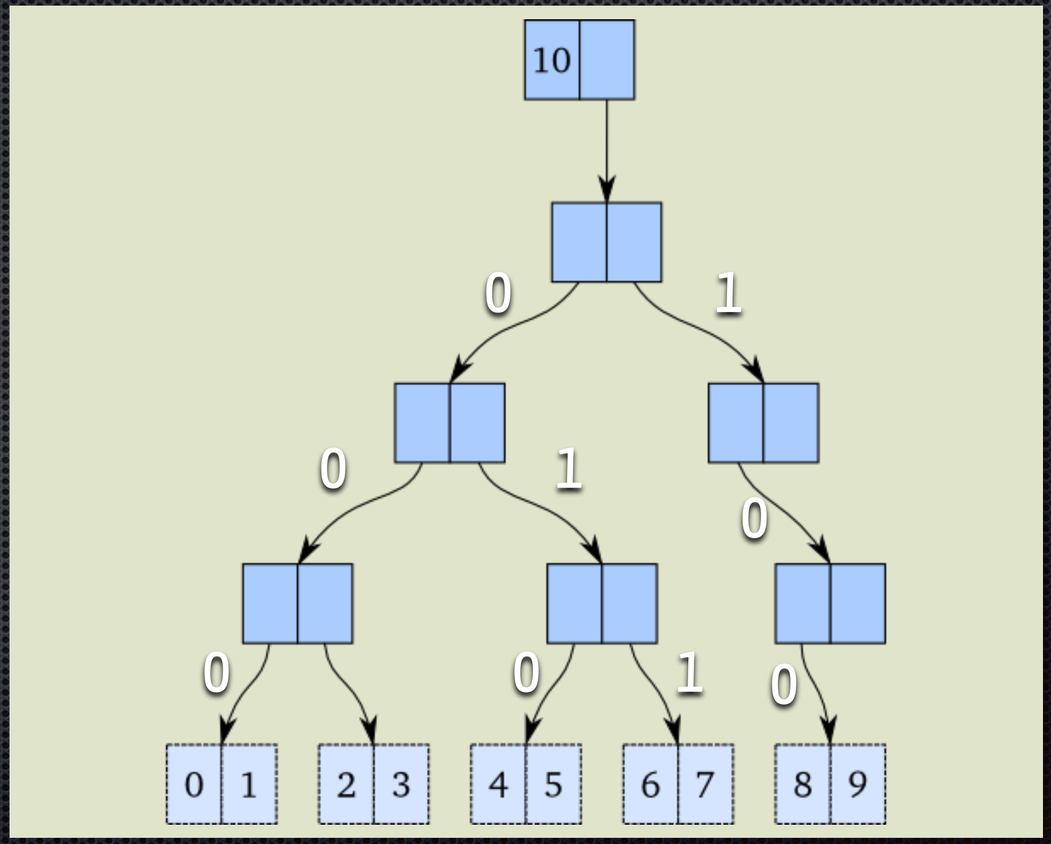
Indexing



Vector

Indexing: bit-partitioned

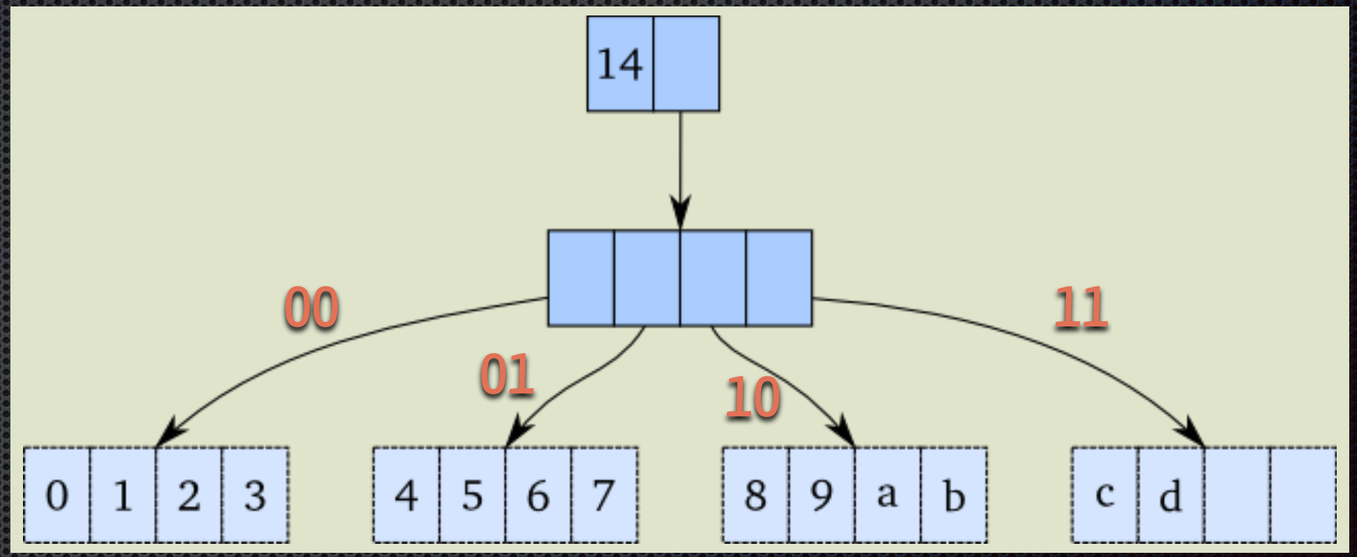
```
0:0000
1:0001
2:0010
3:0011
4:0100
5:0101
6:0110
7:0111
8:1000
9:1001
```



Vector

Indexing: bit-partitioned

0:0000
1:0001
2:0010
3:0011
4:0100
5:0101
6:0110
7:0111
8:1000
9:1001
a:1010
b:1011
c:1100
d:1101



Vector

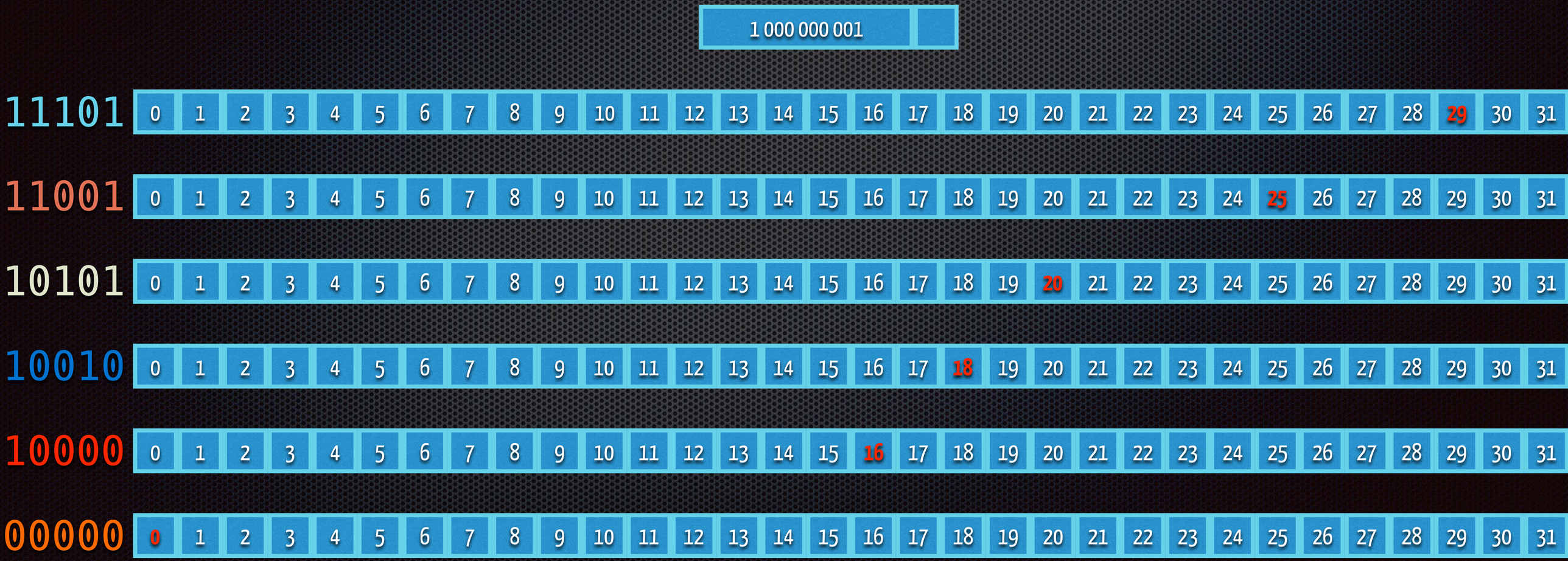
Indexing: bit-partitioned

Branching factor: 32

```
1 000 000 000: 0011 1011 1001 1010 1100 1010 0000 0000
 1 000 000 000: 00 11101 11001 10101 10010 10000 00000
```

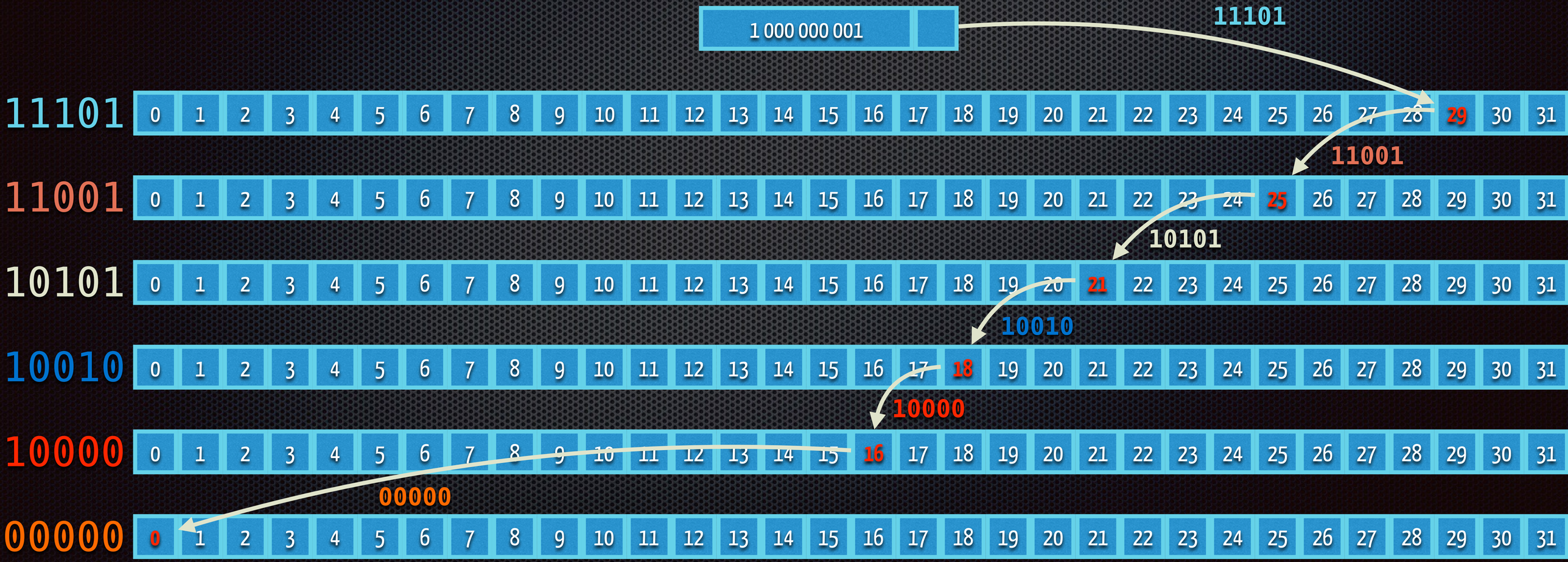
Vector

1 000 000 000: 00 11101 11001 10101 10010 10000 00000



Vector

1 000 000 000: 00 11101 11001 10101 10010 10000 00000



HashMap

Map: `( key -> value )` relationship

Hash: implementation detail

HashMap

Persistent Hash Array Mapped Trie

HashMap

Fast And Space Efficient Trie Searches

Phil Bagwell

Searching and traversing m-way trees using tries is a well known and broadly used technique. Three algorithms are presented, two have constant insert, search or delete cost, are faster than Hash Trees and can be searched twice as quickly as Ternary Search Trees (TST). The third has a lgN byte compare cost, like a TST, but is faster. All require 60% less memory space per node than TST and, unlike Hash Trees, are smoothly extensible and support sorted order functions. The new algorithms defining Array Compacted Trees (ACT), Array Mapped Trees (AMT), Unary Search Trees (UST) and their variants are discussed. These new search trees are applicable in many diverse areas such as high performance IP routers, symbol tables, lexicon scanners and FSA state tables.

Categories and Subject Descriptors: H.4.m [**Information Systems**]: Miscellaneous

General Terms: Searching,Database

Additional Key Words and Phrases: Trie,Tries,Search,Trees,TST,ACT,AMT,UST,Symbol Table

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Ideal Hash Trees

Phil Bagwell

Hash Trees with nearly ideal characteristics are described. These Hash Trees require no initial root hash table yet are faster and use significantly less space than chained or double hash trees. Insert, search and delete times are small and constant, independent of key set size, operations are O(1). Small worst-case times for insert, search and removal operations can be guaranteed and misses cost less than successful searches. Array Mapped Tries(AMT), first described in Fast and Space Efficient Trie Searches, Bagwell [2000], form the underlying data structure. The concept is then applied to external disk or distributed storage to obtain an algorithm that achieves single access searches, close to single access inserts and greater than 80 percent disk block load factors. Comparisons are made with Linear Hashing, Litwin, Neimat, and Schneider [1993] and B-Trees, R.Bayer and E.M.McCreight [1972]. In addition two further applications of AMTs are briefly described, namely, Class/Selector dispatch tables and IP Routing tables. Each of the algorithms has a performance and space usage that is comparable to contemporary implementations but simpler.

Categories and Subject Descriptors: H.4.m [**Information Systems**]: Miscellaneous

General Terms: Hashing,Hash Tables,Row Displacement,Searching,Database,Routing,Routers

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HashMap

```
17
18  /*
19   A persistent rendition of Phil Bagwell's Hash Array Mapped Trie
20
21   Uses path copying for persistence
22   HashCollision leaves vs. extended hashing
23   Node polymorphism vs. conditionals
24   No sub-tree pools or root-resizing
25   Any errors are my own
26   */
27
```

https://github.com/clojure/clojure/blob/master/src/jvm/clojure/lang/PersistentHashMap.java#L18-L26

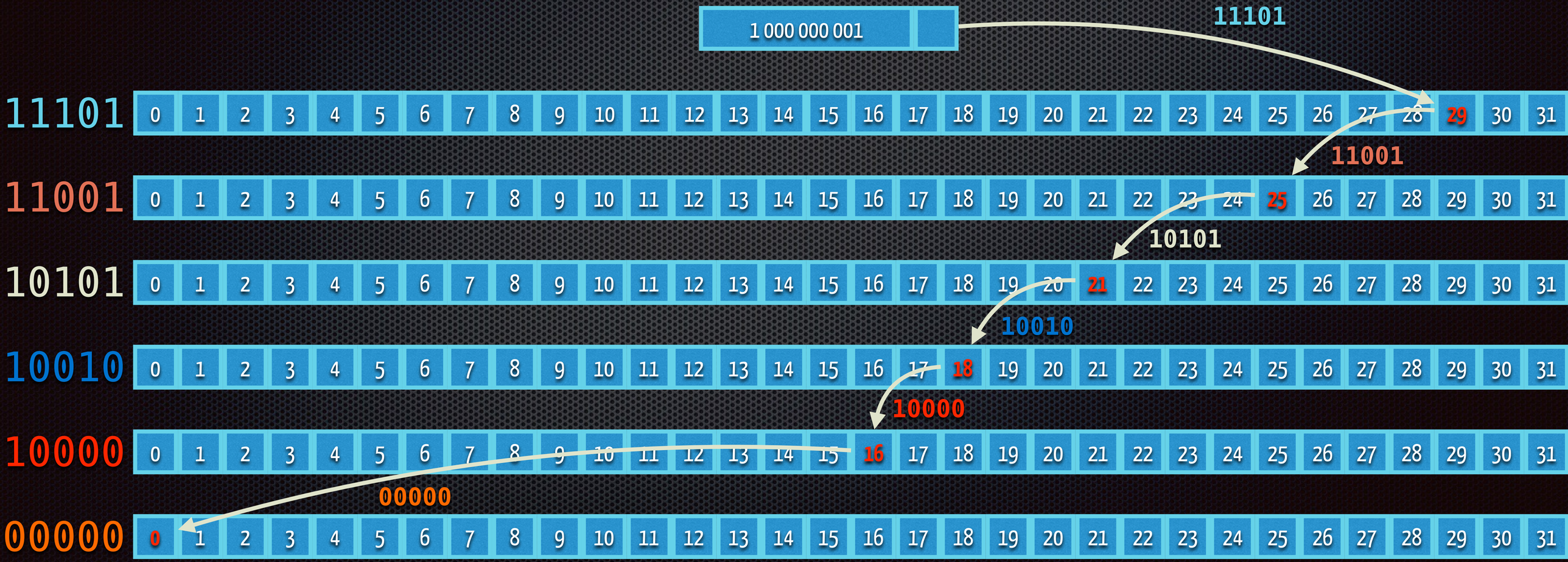
HashMap

```
int hashOfKey = hash(key)
```

use trie (remember vector?) to find key and value

Vector

1 000 000 000: 00 11101 11001 10101 10010 10000 00000



HashMap

We can't allocate that much memory from the beginning!

Solution: built trie incrementally!

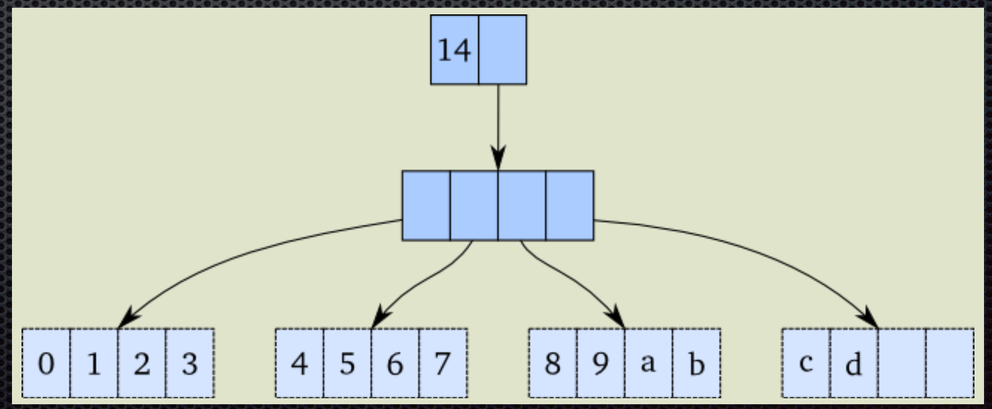
Path copying still works as expected

HashMap

BitmapIndexedNode

```
class BitmapIndexedNode implements INode {
  int bitmap;
  Object[] array;

  public INode assoc(int shift, int hash, Object key, Object val) {}
  public INode without(int shift, int hash, Object key) {}
  public IMapEntry find(int shift, int hash, Object key) {}
}
```



BitmapIndexedNode

add(k1, v1)
hash(k1)==1000000003

1 000 000 003: 00 11101 11001 10101 10010 10000 00011

bitmap: 0000 0000 0000 0000 0000 0000 0000 1000
00011

array: k1 v1

BitmapIndexedNode

add(k2, v2)
hash(k2)==1000000000

1 000 000 000: 00 11101 11001 10101 10010 10000 00000

bitmap: 0000 0000 0000 0000 0000 0000 0000 1001
00000

array: k2 v2 k1 v1

BitmapIndexedNode

```
add(k3, v3)
hash(k3)==1000000005
```

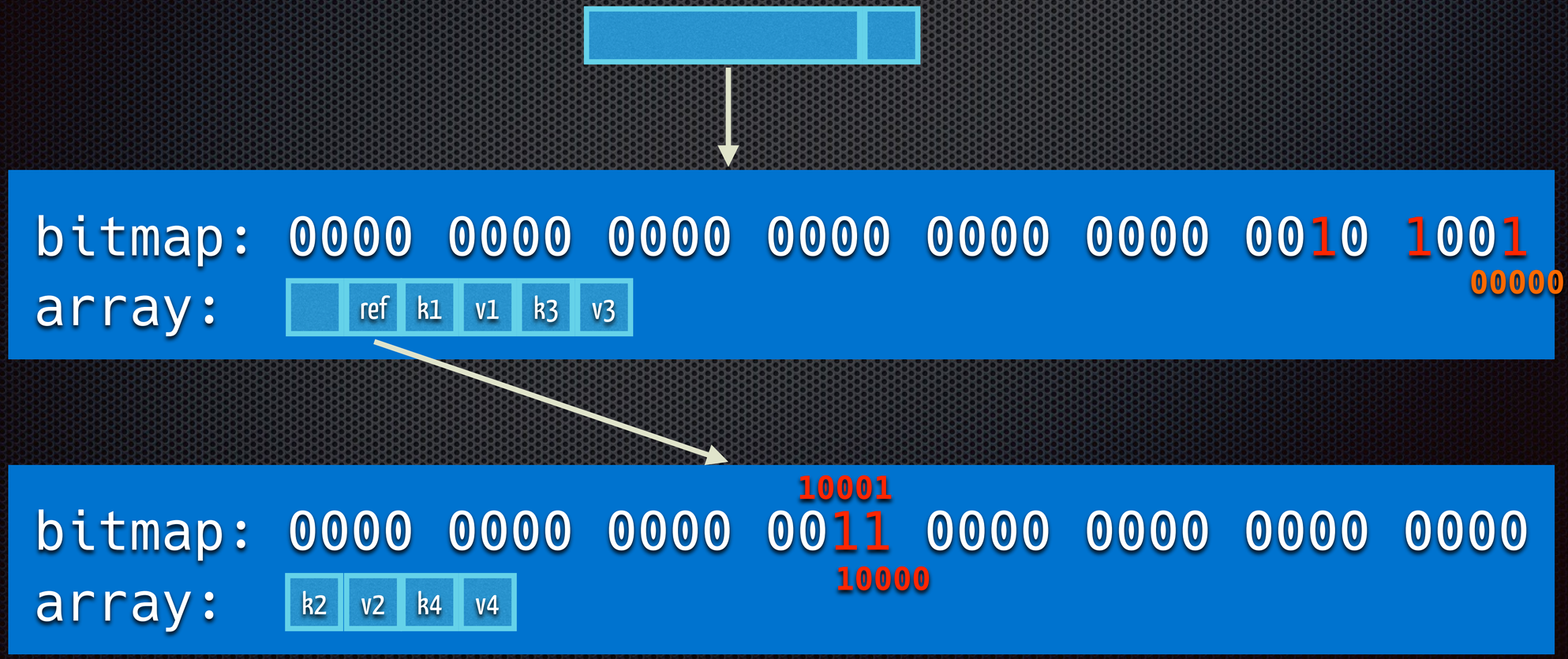
```
1 000 000 005: 00 11101 11001 10101 10010 10000 00101
```

```
bitmap: 0000 0000 0000 0000 0000 0000 0010 1001
                                          00101
array:  k2 v2 k1 v1 k3 v3
```

BitmapIndexedNode

add(k4, v4)
hash(k4)==1000000032

1 000 000 032: 00 11101 11001 10101 10010 10001 00000



HashMap

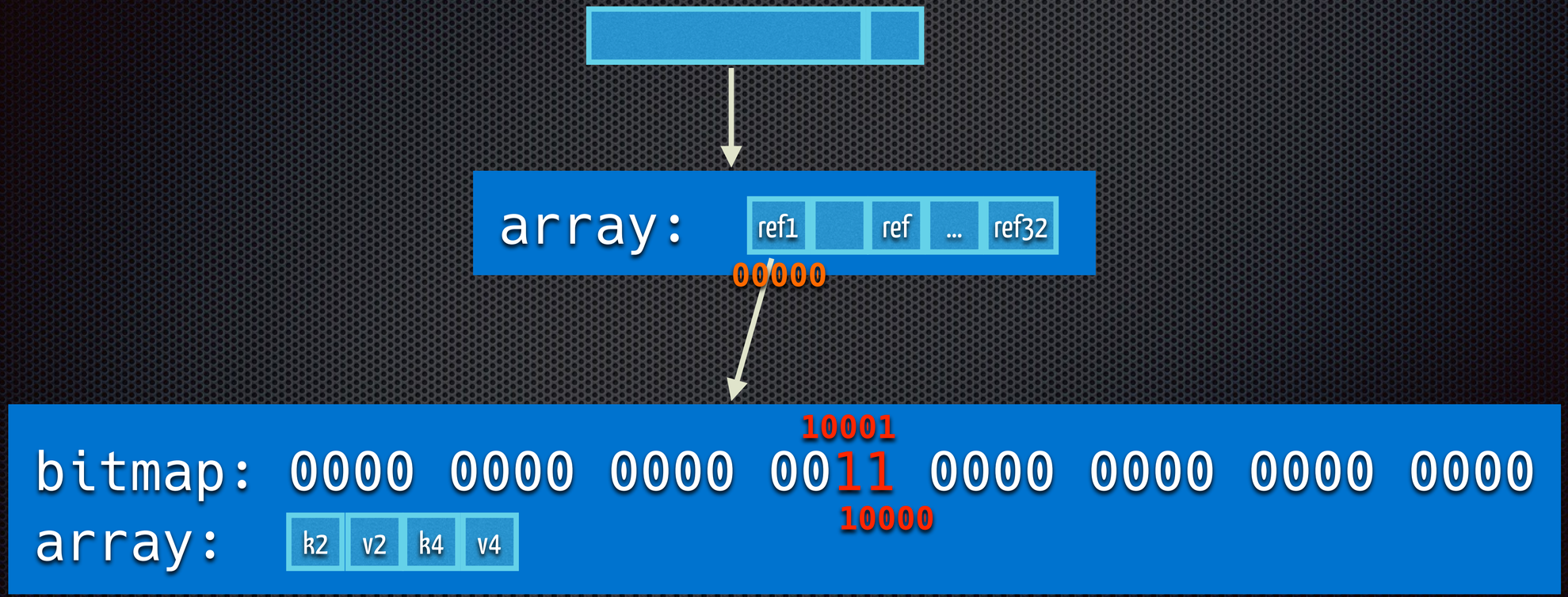
Optimisations

```
bitmap: 0000 0000 0000 0001 1111 1111 1111 1111
array:  k1 v1 k2 v2 k3 v3 ... k17 v17
```

When BitmapIndexedNode's size is greater than 16
(and array size is greater than 32),
then migrate to ArrayNode

HashMap

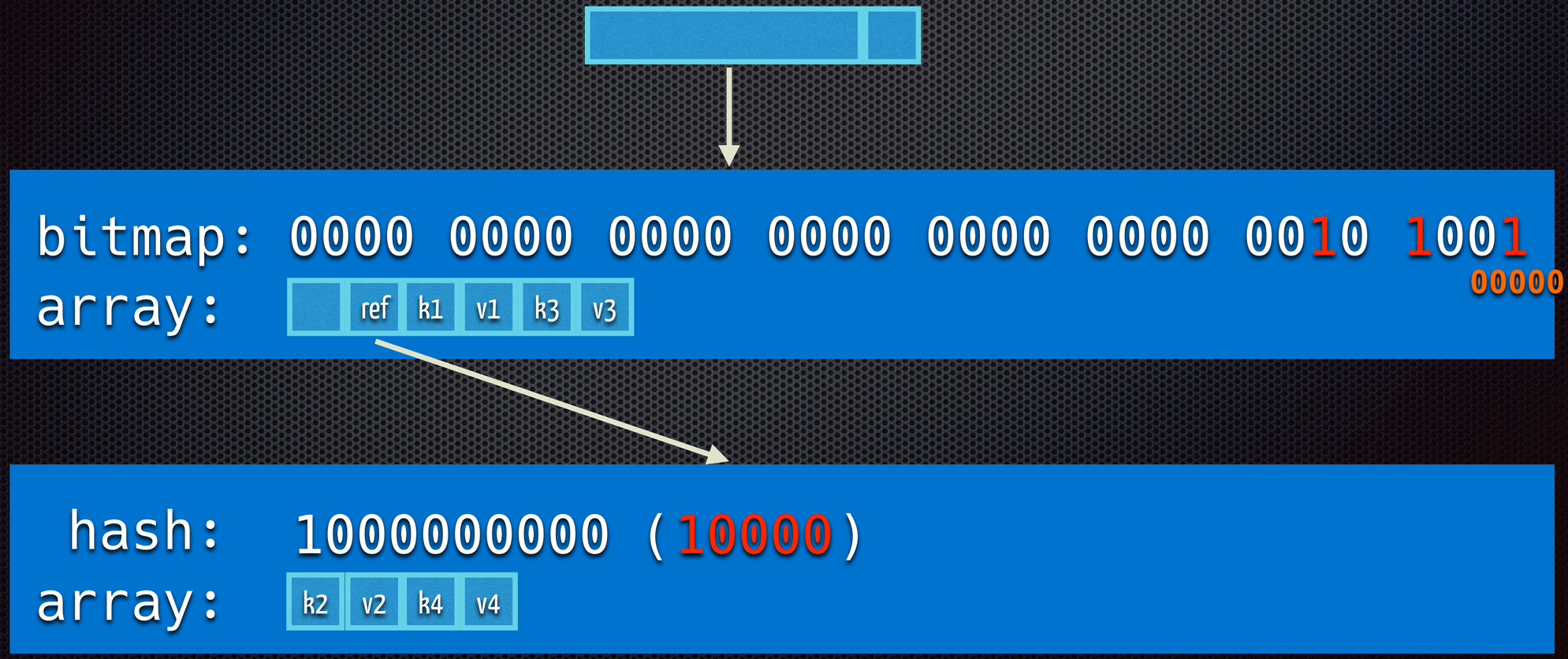
Optimisations: ArrayNode



Optimisations: HashCollisionNode

```
add(k4, v4)
hash(k4)==1000000000
```

```
1 000 000 000: 00 11101 11001 10101 10010 10000 00000
```



Set

TreeSet -> TreeMap

HashSet -> HashMap

value == key

1959	Birandais, Fredkin	Trie
1960	Windley,Booth, Colin,Hibbard	Binary Search Trees
1962	Adelson-Velsky, Landis	AVL Trees
1978	Guibas, Sedgwick	Red Black Trees
1985	Sleator, Tarjan	Splay Trees
1996	Okasaki	Purely Functional Data Structures
1998	Sedgwick	Ternary Search Trees
2000	Phil Bagwell	AMT
2001	Phil Bagwell	HAMT
2007	Rich Hickey	Clojure!

Frontend

Since immutable data never changes, subscribing to changes throughout the model is a dead-end and new data can only ever be passed from above.

This model of data flow aligns well with the architecture of React and especially well with an application designed using the ideas of Flux.

Frontend

https://facebook.github.io/immutable-js/

http://redux.js.org/

Thanks!

@fxposter