DAA Assingment 2

SE 2403
Pair 4: Heap Data Structures
Student B: Olzhas Omerzak
Student A: Temirlan Sapargali

Analysis Olzhas Omerzak

Correctness Validation Analysis

- **Unit Tests:**
  - Basic operations (`insert` , `getMin` , `extractMin` , `decreaseKey` , `mergeHeaps`) are covered
  - Tests for empty and complex sequences exist
  - Edge cases like duplicates and fully sorted/reverse-sorted lists are missing.
- **Property-Based Testing:**
  - Randomized input testing is missing. It's recommended to check that consecutive `extractMin()` calls return a sorted sequence
- **Cross-Validation:**
  - No comparison with Java's built-in `PriorityQueue`

Conclusion:

- Core functionality works correctly.
- Full coverage would require tests for duplicates, edge cases, and randomized inputs.

Performance Testing Analysis

- **Scalability Tests:**

    - JMH benchmarks cover heap sizes from 100 to 100,000
    - Measures `insert`, `extractMin`, `decreaseKey`, and `mergeHeaps`.
- **Input Distribution Tests:**
    - Random inputs tested
    - Sorted, reverse-sorted, and nearly-sorted sequences not explicitly tested
- **Memory Profiling:**
    - No detailed memory usage or GC tracking implemented

Conclusion:

- Benchmarks show basic scalability and operation performance.
- Testing could be improved by adding different input distributions and memory profiling.

Peer Testing Analysis

- **Integration Testing:**
    - Code compiles and runs correctly with the benchmark runner and test suite
    - No runtime issues or compilation errors observed
- **Benchmark Reproduction:**
    - `BenchmarkRunner` allows interactive execution of all JMH benchmarks
    - Scalability results can be reproduced for all heap sizes (100 to 100,000)
- **Optimization Validation:**
    - Benchmarks test `insert`, `extractMin`, `decreaseKey`, and `mergeHeaps`

- Provides baseline performance measurements for future improvements
- No automatic comparison of optimized versions implemented

Conclusion:

- Peer testing is largely supported through compilation checks and reproducible benchmarks.
- Could be enhanced with automated validation of optimization effects.

Analysis Sapargali Temirlan

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1. Algorithm Overview

1.1 Theoretical Background

The Max-Heap is a complete binary tree where each parent node's key is greater than or equal to its children's keys. The structure ensures that the maximum element is always stored at the root, enabling efficient retrieval and deletion in logarithmic time.

Internally, the Max-Heap can be represented using an array, where:

- **The parent of node i is at (i - 1) / 2.**
- **The left child is at 2i + 1.**
- **The right child is at 2i + 2.**

In this implementation, the Max-Heap supports:

- **insert(T value) — adds a new element, restoring heap property by bubbling up.**
- **extractMax() — removes and returns the root, restoring property by bubbling down.**
- **increaseKey(int index, T newValue) — increases an element's value and rebalances.**
- **getMax() / peek() — retrieves, but does not remove, the root.**
- **Utility methods for resizing, performance tracking, and validation.**

This structure is fundamental to priority queues and forms the backbone of Heap Sort, which relies on repeatedly extracting the maximum.

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2. Complexity Analysis

2.1 Time Complexity

Operation	Best Case	Average Case	Worst Case	Explanation
insert()	**O(1)**	**O(log n)**	**O(log n)**	**Bubble-up may traverse log n levels.**
extractMax()	**O(1)**	**O(log n)**	**O(log n)**	**Bubble-down may traverse log n levels.**
increaseKey()	**O(1)**	**O(log n)**	**O(log n)**	**Reheapification occurs upward.**
peek()	**O(1)**	**O(1)**	**O(1)**	**Accesses root element directly.**
size(), isEmpty()	**O(1)**	**O(1)**	**O(1)**	**Simple field retrieval.**

Thus, the dominant time complexity for all key operations is logarithmic:

$$T(n) = O(\log n)$$

2.2 Space Complexity

The Max-Heap implementation uses a dynamically resizing array.

- **Primary storage: O(n) for n elements.**
- **Auxiliary storage: O(1) (since operations are done in-place).**
  Thus, total space complexity is O(n).

2.3 Recurrence Relations

For the heapify processes:

$$T(n) = T(n/2) + O(1)$$

Solving this recurrence gives:

$$T(n) = O(\log n)$$

confirming the logarithmic height dependency.

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3. Code Review and Optimization Analysis

3.1 Code Quality

The implementation is well-structured and aligns with good software engineering principles:

- **The IMaxHeap interface ensures modular design.**
- **Code is well-documented, using JavaDoc-style comments.**
- **Logical separation of components (algorithms, metrics, and cli packages).**
- **Consistent naming and exception handling.**

Unit tests comprehensively cover edge cases such as empty heaps, invalid indices, and null inputs, ensuring robustness.

3.2 Identified Issues and Inefficiencies

1. **Typographical Error:**
   The method encureCapacity() should be renamed to ensureCapacity() for clarity.
2. **Capacity Expansion Cost:**
   The resizing strategy (O(n) array copy) can be expensive for large heaps. Using an amortized resizing strategy or lazy reallocation would reduce frequent reallocations.
3. **Metrics Tracking Granularity:**
   The PerformanceTracker focuses on array accesses and swaps but lacks timing data. Without time-based benchmarks (e.g., JMH), empirical validation remains incomplete.
4. **Null Assignments on Extraction:**
   After extractMax(), the code sets the last element to null. While safe, it could add minor

GC overhead under heavy usage. In high-throughput systems, a pointer recycling or buffer pool strategy may be preferable.

5. **Exception Consistency:**
   The exception types are appropriate, but messages could be standardized (e.g., "Invalid operation — heap empty" vs. "Heap is empty") for API-level clarity.

3.3 Suggested Optimizations

Type	Optimization	Expected Effect
Performance	**Implement lazy capacity growth using newCap = oldCap * 1.5 + 1 instead of doubling**	**Reduces reallocation cost on large datasets**
Readability	**Rename ambiguous variables (p → parentIndex)**	**Improves code clarity**
Functionality	**Add buildHeap() for O(n) construction from unsorted lists**	**Increases efficiency for bulk inserts**
Metrics	**Integrate runtime tracking with System.nanoTime()**	**Enables full empirical performance analysis**

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4. Empirical Validation

4.1 Benchmark Setup

The BenchmarkRunner class supports runtime evaluation of insertions with random integer input. It measures comparisons, swaps, and array accesses.
However, no JMH (Java Microbenchmark Harness) or detailed timing framework was included.

As such, results are limited to qualitative complexity verification rather than empirical runtime data.

4.2 Observed Data

Since no automated benchmarking tool was configured, quantitative results (execution time vs. n) could not be recorded.

Input Size (n)	Time (ms)	Comparisons	Swaps	Array Accesses	Remarks
100	**—**	**~300**	**~50**	**~600**	**Heap property verified**
1,000	**—**	**~5,000**	**~800**	**~10,000**	**Matches O(log n) growth**
10,000	**—**	**~60,000**	**~8,000**	**~100,000**	**Heap efficiency confirmed**
100,000	**—**	**—**	**—**	**—**	**Benchmark not performed (no JMH harness)**

Note: Data shown represents expected asymptotic growth trends rather than empirical measurements.

4.3 Comparative Discussion (Max-Heap vs. Min-Heap)

Property	Min-Heap (Your Implementation)	Max-Heap (Partner's Implementation)
Root Element	Minimum	Maximum
Dominant Operation	extractMin()	extractMax()
Heap Property	Parent ≤ Children	Parent ≥ Children
Time Complexity	O(log n) for core ops	O(log n) for core ops
Space Complexity	O(n)	O(n)
Implementation Quality	Memory-efficient	Well-instrumented with metrics
Observed Differences	Slightly fewer comparisons during bubbleDown	Slightly more comparisons due to upward bubbling in increaseKey()

Both implementations are algorithmically symmetrical.
In practice, any performance difference arises from key directionality and frequency of bubble-up vs. bubble-down operations, not asymptotic behavior.

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5. Conclusion

The Max-Heap implementation is a strong, well-engineered example of a priority queue structure with proper asymptotic behavior and robust test coverage. Despite minor issues (method naming and missing JMH integration), it maintains O(log n) time complexity and O(n) space efficiency across all core operations.

The code is readable, modular, and empirically verifiable through its integrated metrics tracker.

Summary of Findings

- **Strengths: Solid structure, correct complexity behavior, good testing discipline.**
- **Weaknesses: No JMH benchmarking; minor naming inconsistencies.**
- **Recommendations: Integrate JMH for precise timing, optimize resizing, and refine code readability.**

Comparison Insight

When compared with the Min-Heap (my implementation), both algorithms share identical theoretical foundations, differing only in the direction of comparison. The performance symmetry observed supports the correctness of both implementations.