Practical no. 1.

Topic : Limits and continuity.

① $$\lim_{x\to a}\left[\frac{\sqrt{a+3x}-\sqrt{3}\,x}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+\sqrt{2x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{(a+2x-3x)}{(3a+x-4x)}\times\frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}\right]$$

$$\lim_{x\to a}\frac{(a-x)}{3(a-x)}\frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\ \frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{2\sqrt{a}}{\sqrt{3a}}$$

$$\frac{2}{3\sqrt{3}}$$

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2. $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\dfrac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right)$

$\lim_{y\to 0}\dfrac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0}\dfrac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$\dfrac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$

$\dfrac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$

$\dfrac{1}{\sqrt{a}\cdot 2\sqrt{a}}$

$\dfrac{1}{2a}$

3. $\lim_{x\to\pi/6}\dfrac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

By substituting $x-\frac{\pi}{6}=h$

$x=h+\frac{\pi}{6}$

where $h\to 0$

$\lim_{h\to 0}\dfrac{\cos\left(h+\frac{\pi}{6}\right)-\sqrt{3}\sin\left(h+\pi/6\right)}{\pi-6\left(h+\pi/6\right)}$

$\lim_{h\to 0}\dfrac{\cos h.\cos\pi/6-\sin h\sin\pi/6-\sqrt{3}\sin h\cos\pi/6+\cos h\sin\pi/6}{\pi-\cancel{6}\left(\frac{6h+\pi}{\cancel{6}}\right)}$

$\lim_{h\to 0}\dfrac{\cos h\,\sqrt{3}/2-\sin h\,1/\sqrt{2}-\sqrt{3}\left(\sin h\frac{\sqrt{3}}{2}+\cos h\cdot\frac{1}{2}\right)}{-6h}$

$\lim_{h\to 0}\dfrac{\cos h\,\sqrt{3}/2-\frac{\sin h}{\sqrt{2}}-\sin h\,\frac{3}{2}+\cos h\,1/2}{-6h}$

$\lim_{h\to 0}\dfrac{+\sin\frac{4h}{2}}{+6h}$

$\frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h}=\frac{1}{3}\times 1=\frac{1}{3}$.

4. $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator and Denominator both :

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^3+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)\ (\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)\ (\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{8\left(\sqrt{x^2+3}+\sqrt{x^2+1}\right)}{2\left(\sqrt{x^2+5}+\sqrt{x^2-3}\right)}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit we get,

$= 4$.

5. Examine the continuity of the following function at given points.

(i)
$$f(x)=\begin{cases}\dfrac{\sin 2x}{\sqrt{1-\cos 2x}} & \text{for } 0<x\le \pi/2\\[2ex] \dfrac{\cos x}{\pi-2x} & \text{for } \dfrac{\pi}{2}<x<\pi\end{cases}\quad \text{at } x=\pi/2$$

a) $f(\pi/2)=\dfrac{\sin 2\pi/2}{\sqrt{1-\cos 2\pi/2}}$

$f(\pi/2)=0$

f at $x=\pi/2$ define.

b) $\lim_{x\to\pi/2} f(x)=\lim_{x\to\pi/2}\dfrac{\cos x}{\pi-2x}$

$x-\dfrac{\pi}{2}=h$

$x=h+\dfrac{\pi}{2}$

where $h\to 0$

$$\lim_{h\to 0}\frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi-2\left(h+\frac{\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h\to 0} \cos h \cdot 0 - \frac{\sin h}{-2h}$$

$$\lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$= \frac{1}{2}$$

(c) $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$$\lim_{x\to\pi/2} \frac{2\sin x\cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x\to\pi/2} \frac{2\sin x\cdot\cos x}{\sqrt{2}\sin x}$$

$$\lim_{x\to\pi/2} \sqrt{2}\cos x$$

L.H.S $\neq$ R.H.S $\therefore f$ is not continous at $x=\pi/2$.

ii) $f(x) = \frac{x^2-9}{x-3}$ $0<x<3$

$= x+3$ $3\le x<6$

$= \frac{x^2-9}{x+3}$ $6\le x<9$

at $x=3$ and at $x=6$

Sol^n:

at $x=3$

$$f(3) = \frac{x^2-9}{x-3} = 0$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$$
$$= 6$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3} \frac{x^2+9}{x-3}$$

$$= \lim_{x\to 3} \frac{(x+3)(x-3)}{(x-3)}$$

$$= x+3$$
$$= 6$$

$$f(3) = 3+3$$
$$= 6$$

$\therefore f$ is continous at 3.

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$x = 6$

$f(6) = \frac{x^2-9}{x+3}$

$= \frac{\cancel{(x+3)}\cancel{(x-3)}}{\cancel{(x+3)}} x-3$

$= 6+3$

$= 9$

$\lim_{x\to 6^-} = \lim_{x\to 6} \frac{x^2-9}{x-3}$

$= \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$

$= 9$

$\lim_{x\to 6^+} x+3$

$= 6+3$

$= 9$

$\therefore f$ is not continous at 6.

6. Find value of K, so that the function $f(x)$ is continous at the indicated point.

(i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\quad x < 0$
$= K$ $\quad x = 0$ } at $x = 0$

Solⁿ:-

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = K$

$2 \lim_{x\to 0} \frac{\sin^2 2x}{x^2} = K$

$2(2)^2 = K$

$K = 8$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$= K$ $\quad x = 0$ } at $x = 0$

Solⁿ:-

$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x} = K$

$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}} = K$

$e = K$

$\therefore K = e$

(iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$

$= k \quad x = \pi/3$ $\Big\}$ at $x = \pi/3$

$x - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$

where $h \to 0$

$f\left(\pi/3 + h\right) = \lim_{h\to0} \dfrac{\sqrt{3} - \tan\left(\pi/3 + h\right)}{\pi - 3\left(\pi/3 + h\right)}$

$\lim_{h\to0} \dfrac{\sqrt{3} - \tan\left(\pi/3 + h\right)}{\pi - 3\left(\pi/3 + h\right)}$

$\lim_{h\to0} \dfrac{\sqrt{3} - \dfrac{\tan\pi/3 + \tan h}{1 - \tan\pi/3 \cdot \tan h}}{\pi - \pi - 3h}$

$\lim_{h\to0} \dfrac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\pi/3 \times \tan h}}{-3h}$

$\lim_{h\to0} \dfrac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan\pi/3 \cdot \tan h}}{-3h}$

$\lim_{h\to0} \dfrac{\dfrac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$

$\lim_{h\to0} \dfrac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$

$\lim_{h\to0} \dfrac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$

$\frac{4}{3} \lim_{h\to0} \dfrac{\tan h}{h} \lim_{h\to0} \dfrac{1}{(1 - \sqrt{3}\tan h)}$

$\frac{4}{3} \dfrac{1}{(1 - \sqrt{3}(0))}$

$\frac{4}{3} = k$

7. Discuss the continuity of the following function ? which of these function have a removable discontinuity ? Redefine the function so as to remove the discontinuity.

(i) $f(x) = \frac{1-\cos 3x}{x \tan x}$ $x \neq 0$ } at $x=0$
$= 9$ $x=0$

$\lim_{x\to 0} \frac{1-\cos 3x}{x \tan x}$

$\lim_{x\to 0} \frac{2\sin^2 \frac{3}{2}x}{x \tan x}$

$2 \lim_{x\to 0} \frac{\sin^2 \frac{3}{2}x}{x^2}$

$\frac{2\times 9}{4}$

$= \frac{9}{2}$

$\lim_{x\to 0} f(x) = \frac{9}{2}$

$\therefore f$ is not Continous at $x=0$

Redefine function

$f(x) = \frac{1-\cos 3x}{x \tan x}$ $x \neq 0$
$= \frac{9}{2}$ $x=0$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

(ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$ $x \neq 0$ } at $x=0$
$= \pi/60$ $x=0$

$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim_{x\to 0} \frac{e^{3x}-1}{x}$ $\lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \cdot \log e \frac{\pi}{180} = \frac{\pi}{60}$

$\therefore f$ is Continuous at $x=0$

8. If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continous at $x = 0$ find $f(0)$.

Given, f is continous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$$

$$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\log e + \lim_{x \to 0} \frac{2 \sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x \to 0} \frac{\sin^2 x/2}{x^2}$$

$$1 + 2 \times \frac{1}{4} = \frac{3}{2}$$

$$\therefore f(0) = \frac{3}{2}$$

Practical no. 2.

Topic: Derivative.

Q1. Show that the following function defined from IR to IR are differentiable.

(i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x-a}$$

$$= \lim_{x\to a} \frac{\tan a - \tan x}{\tan x \tan a (x-a)}$$

put $x - a = h$

$x = a+h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h \tan(a+h) \tan a}$$

$$= \lim_{h\to 0} \frac{\tan(a-h) - (1 + \tan a \tan(a+h))}{h \tan(a+h) \tan a}$$

$$= \cancel{\lim_{h\to 0}} - \frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)(\tan a)}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$= \dfrac{-\sec^2 a}{\tan^2 a}$

$= \dfrac{-1}{\cos^2 a} \times \dfrac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\text{co}^2 a$

$\therefore$ f is differentiable $\forall a \in R$

(ii) cosec x

$f(x) = \operatorname{cosec} x$

$Df(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$= \lim\limits_{x \to a} \dfrac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$

$= \lim\limits_{x \to a} \dfrac{1/\sin x - 1/\sin a}{x - a}$

$= \lim\limits_{x \to a} \dfrac{\sin a - \sin x}{(x - a)\sin a \sin x}$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(h) = \lim\limits_{h \to 0} \dfrac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$

$= \lim\limits_{h \to 0} \dfrac{2\cos\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$

$= \lim\limits_{h \to 0} -\dfrac{\sin h/2}{h/2} \times \dfrac{1}{2} \times \dfrac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$

$= \dfrac{-1}{2} \times \dfrac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$

$= \dfrac{-\cos a}{\sin a}$

$= -\cot a \operatorname{cosec} a$.

(iii) sec x

$f(x) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$= \lim\limits_{x \to a} \dfrac{\sec x - \sec a}{x - a}$

$= \lim\limits_{x \to a} \dfrac{1/\cos x - 1/\cos a}{x - a}$

$= \lim\limits_{x \to a} \dfrac{\cos a - \cos x}{\cos x \cos a (x - a)}$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(h) = \lim\limits_{h \to 0} \dfrac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$

$= \lim\limits_{h \to 0} \dfrac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$

$= \lim\limits_{h \to 0} \dfrac{-2\sin\left(\frac{2a+h}{2}\right)\sin\frac{-h}{2} \times -1/2}{\cos a \cos(a+h) \times -h/2}$

$= -\frac{1}{2} \times \dfrac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$

$= -\frac{1}{2} \times -2 \dfrac{\sin a}{\cos a \times \cos a}$

$= \tan a \sec a$.

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Q2. If $f(x) = 4x+1, \; x \le 2$
$= x^2+5, \; x > 2$, at $x=2$, then

Find function is differentiable or not

Sol^n:-

L H D

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-1-4(2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2} \frac{4(x-2)}{(x-2)}$$

$$= 4$$

$Df(2^-) = 4$

R·H·D

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} x+2$$

$$= 4$$

$Df(2^+) = 4$

R·H·D = L·H·D

f is differentiable at $x = 2$.

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$= \lim_{x\to 3^+} \frac{4(x-3)}{(x-3)}$

$= 4.$

$Df(3^+) = 4$

R.H.D $\neq$ L.H.D

f is not differentiable at $x = 3$.

Q4. If $f(x) = 8x-5$, $x \le 2$
$= 3x^2-4x+7$, $x > 2$ at $x=2$,
then find f is differentiable or not.

Sol^n:-

$f(2) = 8\times2-5$
$= 16-5$
$= 11$

R.H.D:

$Df(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}$

$= \lim_{x\to 2^+} \frac{3x^2-4x-4}{x-2}$

$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}$

$= \lim_{x\to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$

$= \lim_{x\to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3\times2+2$
$= 8$

$Df(2^+) = 8$

L.H.D:

$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$

$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$

$= \lim_{x\to 2} \frac{8(x-2)}{(x-2)}$

$= 8$

$Df(2^-) = 8$

L.H.D = R.H.D

$\therefore$ f is differentiable at $x = 3$.

09/12/19

Practical 3

Topic: Application of derivative.

1. Find the intervals in which function is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$\therefore$ f is increasing if f $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \quad +\sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing if $f(f'(x)) < 0$

$\therefore 3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \quad +\sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

(ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing if $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing if $f'(x) < 0$

$\therefore 2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\because$ f is increasing if $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

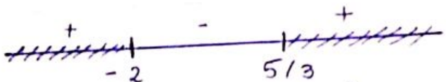
$2(3x^2 + x - 10) > 0$

$3x^2 + x - 10 > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$



$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

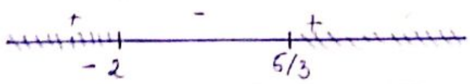
$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$



$x \in (-2, 5/3)$

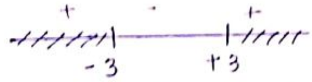
(iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because$ f is increasing if $f'(x) > 0$

$3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$



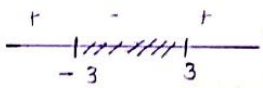
$x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing if $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$(x+3)(x-3) < 0$



$\therefore x \in (-3, 3)$

(v) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because$ f is increasing if $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

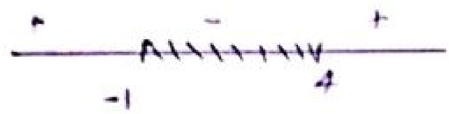
$\therefore 6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x-4)(x+1) < 0$



$x \in (-1, 4)$

2. Find the intervals in which function is concave upwards.

(i) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$(6 - 12x) > 0$

$12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\because f''(x) > 0$

$x \in (1/2, \infty)$

(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 2x - x + 2 > 0$

(v) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward if $f''(x) > 0$

$f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

$x + 1/6 > 0$

$x < -1/6$

~~$\therefore f''(x) \not> 0$~~

~~$\therefore$ There exist no interval~~.

16/12/19

$x \in (-1/6, \infty)$

Practical no.4.

Topic: Application of derivative and neutons method.

Q1. Find maximum and minimum values of following.

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/24$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2^2 + 16/22$

$= 4 + 16/4$

$= 4 + 4$

$= 8.$

$f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ Function reachs minimum value at $x = 2$, and $x = -2$

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(ii) $f(x) = 3-5x^3+3x^5$

$\therefore f'(x) = -15x^2+15x^4$

Consider, $f'(x)=0$

$\therefore -15x^2+15x^4=0$

$15x^4 = 15x^2$

$x^2=1$

$x = \pm 1$

$f''(x) = -30x+60x^3$

$f''(1) = -30+60$

$= 30>0$

$\therefore$ f has minimum value at $x=1$

$f(1) = 3-5(1)^3+3(1)^5$

$= 6-5$

$=1$

$f''(-1) = -30(-1)+60(-1)^3$

$= 30-60$

$= -30<0$

$\therefore$ f has maximum value at $x=-1$ and has the minimum value 1 at $x=1$

(iii) $f(x) = x^3-3x^2+1$

$\therefore f'(x) = 3x^2-6x$

Consider, $f'(x)=0$

$\therefore 3x^2-6x=0$

$3x(x-2)=0$

$3x=0$ or $x-2=0$

$x=0$ or $x=2$

$f''(x)=6x-6$

$f''(0) = 6(0)-6 = -6<0$

$\therefore$ f has maximum value at $x=0$

$\therefore f(0) = (0)^3-3(0)^2+1 = 1$

$f''(2) = 6(2)-6$

$= 12-6$

$= 6>0$

$\therefore$ f has minimum value at $x=2$

$f(2) = (2)^3-3(2)^2+1$

$= 8-3(4)+1$

$= 9-12$

$= -3$

$\therefore$ f has maximum value 1 at $x=0$ and f has minimum value -3 at $x=2$.

(iv) $f(x) = 2x^3-3x^2-12x+1$

$f'(x) = 6x^2-6x-12$

Consider, $f'(x)=0$

$\therefore 6x^2-6x-12=0$

$6(x^2-x-2)=0$

$x^2-x-2=0$

$x^2+x-2x-2=0$

$x(x+1)-2(x+1)=0$

$(x-2)(x+1)=0$

$x=2$ or $x=-1$

$f''(x) = 12x-6$

$f''(2) = 12(2)-6$

$= 24-6$

$= 18>0$

$\therefore$ f has minimum value at $x=2$

$\therefore f(2) = 2(2)^3-3(2)^2-12(2)+1$

$= 2(8)-3(4)-24+1$

$= 16-12-24+1$

$= 1-9$

$= -19$

$\therefore f''(-1) = 12(-1)-6$

$= -12-6$

$= -18<0$

$\therefore$ f has maximum value at $x=-1$

$\therefore f(-1) = 2(-1)^3-3(-1)^2-12(-1)+1$

$= -2-3+12+1$

$= 8$

$\therefore$ f has maximum value 8 at $x=-1$ and f has minimum value -19 at $x=2$.

Q2. Find the root of the following equation by Newton's (Take 4 iteration only) correct upto 4 decimal.

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 6.0895 - 1.0362 - 55$

$= -55.9467$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

∴ The root of the equation is 0.1712

(ii) $f(x) = x^3 - 4x - 9$ $[2, 3]$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

∴ By newton's method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

∴ $x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.5960$

$f'(x_1) = 23(2.7392)^2 - 4$

$= 18.6096$

$x_2 = x_1 - f(x_1)/f'(x_1) = 2.7392 - 0.696/18.5096 = 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4 = 17.9851$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$2.7071 = 0.0102/17.9851 = 2.7071 - 0.0006 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7065$

$$(iii)\quad f(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1, 2]$$

$$f'(x) = 3x^2 - 3.6x - 10$$

$$f(1) = (1)^3 - 1.8(7)^2 - 10(1) + 17$$

$$= 6.2$$

$$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$$

$$= 8 - 7.2 - 20 + 17 = -2.2$$

Let $x_0 = 2$ be initial approximation

By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)} = 2 - \frac{2.2}{5.2} = 2 - 0.4230$$

$$= 1.577$$

$$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$$

$$= 7.4608 - 5.6772 - 10$$

$$= -8.2164$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.577 + 0.3755 / 8.2164$$

$$= 1.577 + 0.0822 = 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$$

$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$

$$= 8.2588 - 5.97312 - 10$$

$$= -7.7143$$

$$x_3 = x_2 - f(x_2) / f'(x_2)$$

$$= 1.6592 + 0.0204 / 7.7143$$

$$= 1.6592 + 0.0026$$

$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$

$$= 4.5892 - 4.9708 - 16.618 + 17$$

$$= 0.0004$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$

$$= 8.2847 - 5.9824 - 10$$

$$= -7.6977$$

$$x_4 = x_3 - f(x_3) / f'(x_3)$$

$$= 1.8618 + \frac{0.0004}{7.6977}$$

$$= 1.6618$$

06/01/2020

Practical no. 5.

Topic : Integration

Q1. Solve the following integration

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

put $x+1 = t$

$dx = \frac{1}{t} \times dt$ where $t = 1, t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+C$

2) $\int(4e^{3x}+1)\,dx$

$= \int 4e^{3x}dx + \int 1\,dx$ $\quad$ # $\int e^{ax}dx = \frac{1}{a}\times e^{ax}$

$= 4\int e^{3x}dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}dx$

$= \int 2x^2 dx - \int 3\sin(x)dx + \int 5x^{1/2}dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}dx + \int 3x^{1/2}dx + \int \frac{4}{x^{1/2}}\,dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2\times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du$

$= \frac{t^4 \times \sin(2t^4)}{8}\,du$

$= \int \frac{u/2 \times \sin(u)}{8}\,du$

$= \frac{1}{16}\int u \times \sin(u)\,du$

$\int u\,dv = uv - \int v\,du$

where $u = v$

$dv = \sin u \times du$

$du = 1\,du \therefore v = -\cos(u)$

$= \frac{1}{16}(u\times(-\cos(u)) - \int -\cos(u)\,du$

$= \frac{1}{16}(u\times(-\cos u)) + \int \cos(u)\,du$

$\int \cos x\,dx = \sin(x)$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

Return the substitution on $u = 2t^4$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -t^4 \times \frac{\cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

6) $\int \sqrt{x}\,(x^2-1)\,dx$

$= \int \sqrt{x}\; x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$

$I_1 = \dfrac{x^{5/2+1}}{5/2+1} = \dfrac{x^{7/2}}{7/2} = \dfrac{2x^{7/2}}{7} = \dfrac{2\sqrt{x^7}}{7} = \dfrac{2x^3\sqrt{x}}{7}$

$I_2 = \dfrac{x^{1/2+1}}{1/2+1} = \dfrac{x^{3/2}}{3/2} = \dfrac{2x^{3/2}}{3/2} = \dfrac{2\sqrt{x^3}}{3}$

$= \dfrac{2x^3\sqrt{x}}{7} + \dfrac{2\sqrt{x^3}}{3} + C$

7) $\int \dfrac{\cos x}{\sqrt[3]{\sin^2 x}}$

$I = \int \dfrac{\cos x}{\sqrt[3]{\sin^2 x}}$

$I = \int \dfrac{dt}{\sqrt[3]{t^2}}$

$= \int \dfrac{dt}{t^{2/3}}$

$= \int t^{-2/3}\,dt$

$= 3t^{1/3} + C$

$= 3(\sin x)^{1/3} + C$

$= 3\cdot\sqrt[3]{\sin x} + C$

8) $\int \dfrac{1}{x^3}\sin\left(\dfrac{1}{x^2}\right)dx$

$I = \int \dfrac{1}{x^3}\sin\left(\dfrac{1}{x^2}\right)dx$

Let $\dfrac{1}{x^2} = t$

$x^{-2} = t$

$-\dfrac{2}{x^3}\,dx = dt$

$I = \dfrac{-1}{2}\int \dfrac{-2}{x^3}\sin\left(\dfrac{1}{x^2}\right)dx$

$= -\dfrac{1}{2}\int \sin t$

$= -\dfrac{1}{2}(-\cos t) + C$

$= \dfrac{1}{2}\cos t + C$

$\therefore\; I = \dfrac{1}{2}\cos\left(\dfrac{1}{x^2}\right) + C$

9) $\int e^{\cos^2 x} \cdot \sin 2x\, dx$

$I = \int e^{\cos^2 x} \sin 2x\, dx$

Let $\cos^2 x = t$

$-2\cos x \cdot \sin x\, dx = dt$

$-\sin 2x\, dx = dt$

$I = \int -\sin 2x\, e^{\cos^2 x} dx$

$= -\int e^t dt$

$= e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

10) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} dx$

Let $x^3 - 3x^2+1 = t$

$3(x^2-2x)dx = dt$

$(x^2-2x)dx = dt/3$

$I = \int \frac{1}{t} dt$

$= \frac{1}{3} \int dt/t$

$= \frac{1}{3} \log t + c$

$= 1/3 \log(x^3 - 3x^2+1) + c$

Practical no. 6.

Topic : Application of integration and numerical integration.

Q1. Find the length of the following curve

(i) $x = t \sin t, \quad y = 1 - \cos t \qquad t = [0, 2\pi]$

$$\text{arc length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2\left|\sin t/2\right| dt$$

$$= \int_0^{2\pi} 2\sin t/2\, dt$$

$$= \left(-4\cos\left(t/2\right)\right)_0^{2\pi}$$

$$= (-4\cos\pi) - (-4\cos 0)$$

$= 4 + 4$

$= 8$

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9 2. $y = \sqrt{4-x^2} \quad x \in [-2,2]$

$\frac{dy}{dx} = \frac{-2x}{2\sqrt{4-x^2}}$

$= \frac{-x}{\sqrt{4-x^2}}$

$L = 2 \int_0^2 \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$= 2\int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\, Dx$

$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$

10. $= 4\left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

3. $y = x^{3/2}$ in $[0,4]$

$f'(x) : 3/2\, x^{1/2}$

$[f'(x)]^2 = 9/4\, x$

$L = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$

$= \int_0^x \sqrt{1+9/4\, x}\, dx$

Put $u = 1+\frac{9}{4}x$, $du = \frac{9}{4}dx$

$L = \int_1^{1+\frac{9}{4}x} 4/9 \sqrt{u}\, du$

$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right)^{3/2} - 1\right]$

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4. $x = 3\sin t$, $y = 3\cos t$.

$x = 3\sin t$,
$y = 3\cos t$, this is a circle of radius r centered at the orgin.

we can use the formula,

$\frac{dx}{dt} = 3\cos t$, $\frac{dy}{dx} = -3\sin t$

length $L = \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} 3\sqrt{\sin^2 t + \cos^2 t}\, dt$

$= 3\int_0^{2\pi} dt = 6\pi$

5. $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = [1,2]$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2}\int^2 y^2 dy + \frac{1}{2}\int_1^2 y^{-2} dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= \frac{17}{12}$ units.

Q2. Using Simpson's rule, following:—

(1) $\int_0^2 e^{x^2}\,dx$ when $n = 4$

$\int_0^2 e^{x^2}\,dx = 16.4526$

In each curve the width of the sub interval be $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

and so the sub intervals will be $[0, 0.5]$ $[0.5, 1]$ $[1, 1.5]$ $[1.5, 2]$

By Simpson rule

$\int_0^2 e^{x^2}\,dx = \frac{1/2}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$

$= \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}\right)$

$= 17.3536$

(2) $\int_0^4 x^2\,dx$, $n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$

$= \frac{1}{3}\left[y(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right]$

$= \frac{1}{3}\left[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$

$= \frac{64}{3}$

$= 21.333$

(3) $\int_0^{\pi/3} \sqrt{\sin x}\,dx$, $n = 6$

$\Delta x = \frac{b-a}{h} = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
$y = \sqrt{\sin x}$	0	0.4167	0.584	0.707	0.801	0.875	0.930
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\int_0^{\pi/3} \sqrt{\sin x}\,dx = \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6\right)$

$= \frac{\pi/18}{3}\left(0 + 4(0.4167 + 0.707 + 0.875) + 2(0.584 + 0.801) + 0.930\right)$

$= 0.681$

Practical no 7.

Topic :- Differential Equation.

Q1. Solve the following differential equation.

(1) $x\frac{dy}{dx}+\frac{y}{x}=\frac{e^x}{x}$

$P(x)=\frac{1}{x}$

$Q(x)=\frac{e^x}{x}$

If $e^{\int 1/x\,dx}$

x

$y(IF)=\int Q(x)(IF)dx+C$

$=\int\frac{e^x}{x}x\,dx+C$

$=\int e^x dx+C$

$xy=e^x+C$

(2) $e^x\frac{dy}{dx}+2e^x y=1$

$\frac{dy}{dx}+\frac{2e^x}{e^x}=\frac{1}{e^x}$

$\frac{dy}{dx}+2y=\frac{1}{e^x}$

$\frac{dy}{dx}+2y=e^{-x}$

$P(x)=2\quad Q(x)=e^{-x}$

$\int P(x)=dx$

If $=e^{\int 2dx}$

$=e^{2x}$

$y(IF)=\int Q(x)(IF)dx+C$

$=\int e^x dx+C$

$=y.e^{2x}$

$=e^x+C$.

(5) $e^{2x}\dfrac{dy}{dx} + 2e^{2x}y = 2x$

$\dfrac{dy}{dx} + 2y = \dfrac{2x}{e^{2}x}$

$P(x) = 2$

$Q(x) = \dfrac{2x}{e} = 2xe^{-2x}$

$(IF) = e^{\int P(x)\,dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$y\,(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int 2xe^{-2x}\;e^{2x} + C$

$y\,e^{2x} = \int 2x + C$

$= x^{2} + C$

(6) $\sec^{2}x \cdot \tan y\,dx + \sec^{2}y\,\tan x\,dy = 0$

$\sec^{2}x \cdot \tan y\,dx = -\sec^{2}y\,\tan x\,dy$

$\dfrac{\sec^{2}x}{\tan x}\,dx = -\dfrac{\sec^{2}y}{\tan y}\,dy$

(7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 x$

$\frac{dv}{\cos^2 x} = dx$

$\int \sec^2 x \, dv = \int dx$

$\tan v = x + C$

$\tan(x-y+1) = x + C$

(8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y = v$

$2 + \frac{3dy}{dx} = \frac{du}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{du}{dx} - 2\right)$

$= \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+u}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{(v+2)}$

$= \int \frac{v+2}{v+1} dv$

$= 3dx$

$= \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = \int 3dx$

$v \log(v) = 3x + C$

$2x + 3y + \log|2x+3y+1| = 3x + C$

$3y = x - \log|2x+3y+1| + C.$

Practical no. 8.

Topic: Euler's method.

1. $\frac{dy}{dx} = y + e^x - 2$ $y(0)=2$ $h = 0.5$ find $y(2) = ?$

$f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

2. $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$, $h = 0.2$ find $y(1) = ?$

$y_0 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2934$

(3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0)=1$, $h=0.2$, find $y(1)=?$

$x_0=0$, $y(0)=1$, $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1.0899	0.4472	1.0894
2	0.4	1.2105	0.6059	1.2105
3	0.6	1.3513	0.7040	1.3513
4	0.8	1.5057	0.7696	1.5057
5	1			

$y(1) = 1.5051$

(4) $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$ $h=0.5$

$h=0.25$

$y_0=2$, $x_0=1$, $h=0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

$y_0=2$, $x_0=1$, $h=0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1123.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

(5) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1)=1$, $h=0.2$

$x_0=1$, $y_0=1$, $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

Practical no. 9.

Topic: Limits and partial order.

Evalute the following limits.

Q1.

① $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

$x \to -4$

$y \to -1$

$\frac{x^3-3y+y^2-1}{xy+5}$

$\frac{(-4)^3-3\times(-1)+(-1)^2-1}{(-4)(-1)+5}$

$\frac{-64+3+1-1}{4+5}$

$\frac{-64+3}{9}$

$= \frac{-61}{9}$

② $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

$x\to 2, \quad y \to 0$

$= \frac{(0+1)(2^2+0^2-4\times 2)}{2+3(0)}$

$= \frac{1(4-8)}{2}$

$= -4/2$

$= -2.$

③

(i) $f(x,y) = 2x/1+y^2$

$f(x) = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= 1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2) \over (1+y^2)^2$

$= \frac{2+2y^2-0}{(1+y^2)^2} = \frac{2}{(1+y^2)}$

At $(0,0) = 2/1 = 2$

$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right) = \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At $(0,0) = \frac{-4(0)(0)}{(1+0)^2} = 0$

$f(x,y) = \frac{y^2-xy}{x^2}$

$f_x = x^2\, \partial/\partial x\,(y^2-xy) - (y^2-xy)\,\partial/\partial x\,(x^2)$

$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$

$= \frac{-x^2y - 2x(y^2-xy)}{xy}$

$f_y = \frac{2y-x}{x^2}$

$f_{xx} = \frac{\partial}{\partial x}\left(-x^2y - \frac{2x(xy^2-xy)}{x^4}\right)$

$= x^4 \frac{(\partial/\partial x\,(-x^2y - 2xy^2 + 2x^2y) - (-x^2y - 2xy + 2x^2y)\partial/\partial x\, x^4}{(x^4)^2}$

$= x^4 \frac{(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6}$

$f_{yy} = \frac{d}{dy}\left(\frac{2y-x}{x^2}\right) = \frac{2-0}{x^2} = \frac{2}{x^2}$

$f_{xy} = \frac{d}{dx}\left(-\frac{x^2y - 2xy^2 + 2x^2y}{x^4}\right)$

$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$

$= \frac{x^2 \frac{\partial}{\partial x}(2y-x) - (2y-x)\frac{d}{\partial x}(x^2)}{(x^2)^2}$

$= \frac{-x^2 - 4xy + 2x^2}{x^4}$

From ③ and ④

$f_{xy} - f_{yx}$

(ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\partial}{\partial x}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 3x^2 + 6xy^2 - 2x/(x^2+1)$

$f_y = \frac{\partial}{\partial y}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 0 + 6x^2y - 0$

$= 6x^2y$

$f_{xx} = 6x + 6y^2 - \frac{\left(x^2+1\, d(2x)/dx - 2x\, \frac{d(x^2+1)}{dx}\right)}{(x^2+1)^2}$

$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$ — ①

$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$ — ②

$= 0 + 12xy + 0$

$= 12xy$

$f_{yx} = \frac{\partial}{\partial x}(6x^2y) = 12xy$

$\therefore f_{xy} = f_{yx}$.

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{\partial}{\partial x}\left(y\cos(xy) + e^{x+y}\right)$

$= -y\sin(xy)\cdot y + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$ — ①

$f_{yy} = \frac{\partial}{\partial y}\left(x\cos(xy) + e^{x+y}\right)$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ — ②

$f_{xy} = \frac{\partial}{\partial y}\left(y\cos(xy) + e^{x+y}\right)$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — ③

$f_{yx} = \frac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — ④

$\therefore$ from ③ and ④

$f_{xy} \neq f_{yx}$.

Q6. 83

① $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$f(x)$ at $(1,1) = \frac{1}{\sqrt{2}}$

$f(y)$ at $(1,1) = \frac{1}{\sqrt{2}}$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \cancel{\frac{x+y}{\sqrt{2}}}$

② $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$

f_x at $\left(\pi/2, 0\right) = -1 + 0$
$= -1$

$f_y = 0 - 0 + \sin x$

f_y at $(\pi/2, 0) = \sin\frac{\pi}{2} = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= 1 - x + y$

③ $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\qquad f_y = 0 + 1/y$

f_x at $(1,1) = 1$ $\qquad f_y$ at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

Practical no. 10.

Topic: Directional derivative, gradien vector, and maxima, minima, Tangent and Normal curves.

Q1. Find directional derivative of the following function at given points and in the direction of given vector.

① $f(x,y) = x+2y-3$, $a=(1,-1)$, $u = 3i-j$

Here, $u = 3i-j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1)+2(-1)-3 = -4$

$f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= 1+\frac{3}{\sqrt{10}} - 2 - \frac{24}{\sqrt{10}} - 3$

$f(a+hu) = \frac{-4+h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$

② Here $u = i+5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2+(5)^2} = \sqrt{26}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = 4^2 - 4(3)+1 = 5$

$f(a+hu) = f(3,4) + h\left(1/\sqrt{26}, 5/\sqrt{26}\right)$

$= f\left(3+h/\sqrt{26}, 4+5h/\sqrt{26}\right)$

$f(x,y)(a+hu) = \left(4+5h/\sqrt{26}\right)^2 - 4\left(3+h/\sqrt{26}\right)+1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{44}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$= \frac{\cancel{h}\left(\frac{25h}{26}+\frac{36}{\sqrt{26}}\right)}{\cancel{h}}$

$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

③ $2x+3y$ $a=(1,2)$ $u=(3i+4j)$

Here $u = 3i+4j$ is not unit vector.

$|\bar{u}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = 1/5\,(3,4)$

$= (3/5, 4/5)$

$f(a) = f(1,2) = 2(1)+3(2) = 8$

$f(a+hu) = f(1,2) + h(3/5, 4/5)$

$= f(1+3h/5, 2+4h/5)$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 2+\frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_uf(a) = \lim_{h\to 0} \frac{18h/5 + 8 - 8}{h}$

$= \frac{18}{5}$

Q2. Find gradient vector for the following function at given point.

(i) $fx = y \cdot x^{y-1} + y^x \log y$

$fy = x^4 \log x + xy^{x-1}$

$\nabla f(x,y) = (fx, fy)$

$= (y\,x^{y-1} + y^2 \log y,\ x^4 \log x + xy^{x-1})$

$f(1,1) = (1+0, 1+0) = (1,1)$

(ii) $f(x,y) = (\tan^{-1}x)\cdot y^2$ $a=(1,-1)$

$fx = \frac{1}{1+x^2}\cdot y^2$

$fy = 2y\tan^{-1}x$

$\nabla f(x,y) = (fx, fy)$

$= \left(y^2/_{1+x^2}, 2y\tan^{-1}x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

(iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a \equiv (1,-1,0)$

$f(x) = yz - e^{x+y+z}$

$f(y) = xz - e^{x+y+z}$

$f(z) = xy - e^{x+y+z}$

$\nabla f(x,y,z) = fx, fy, fz$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{1+(-1)+0}, (1)(0) - e^{1+(-1)+0}$

$= (0-e^0, 0-e^0, -1-e^0)$

$= (-1, -1, -2)$

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Q3. Find the equation of the tangent and normal to each of the following curves at given points:

① $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \cdot 2x + e^{xy} \cdot y$

$f_y = x^2 (-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0)$

$\therefore x_0 = 1, \; y_0 = 0$

Eq. of tangent.

$f_x (x - x_0) + f_y (y - y_0) = 0$

$f_x (x_0, y_0) = \cos (0)(2)(1) + e^0 \, 0$

$= 1(2) + 0$

$f_y(x_0, y_0) = (1)^2 = 2$

$= (-\sin 0) + e^{0} \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$, required Eq^n of tangent.

Eq^n of normal

$= ax + by + c = 0$

$= bx - ay + d = 0$

$(\cancel{1})(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$1 + 2y + d = 0$ at $(1, 0)$

$1 + 2(0) + d = 0$

$\therefore d + 1 = 0$

$d = -1$.

(ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$f_x = 2x + 0 - 2 + 0 + 0 = 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2) \therefore x_0 = 2, \; y_0 = -2$

$f_x (x_0, y_0) = 2(2) - 2$

$= 2$

$f_y (x_0, y_0) = 2(-2) + 3 = -1$

Eq^n of tangent :-

$f_x (x - x_0) + f_y (y - y_0) = 0$

$2(x-2) + (-1 (y+2)) = 0$

$2x - 2 - y - 2 = 0$

$\therefore 2x - y - 4 = 0$

$\therefore$ Required Eq^n of tangent.

Eq^n of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$= -1 (x) + 2(y) + d = 0$

$= -x + 2y + d = 0$ at $(2, -2)$

$= -2 + \cancel{2}(-2) + d = 0$

$-2 - 4 + d = 0$

$d = 6$.

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Q 4. Find the equation of tangent and normal to each of the following surfaces at given point.

(1) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$fx = 2x - 0 + 0 + z$

$fx = 2x + z$

$fy = 0 - 2z + 3 + 0$

$= 2z + 3$

$fz = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0)$ ∴ $x_0 = 2, y_0 = 1, z_0 = 0$

$fx(x_0, y_0, z_0) = 2(2) + 0 = 4$

$fy(x_0, y_0, z_0) = 2(0) + 3 = 3$

$fz(x_0, y_0, z_0) = -2(1) + 2 = 0$

∴ Eqn of Tangent:

$fx(x_0 - x_0) + fy(y_0 - y_0) + fz(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

∴ $4x + 3y - 11 = 0$

Required tangent

Eqn of normal at $(4, 3, -11)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

(2) $3xyz - x - y + z = -4$

$fx = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$fy = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$fz = 3yx = -0 - 0 + -1 - 1 + 0$

$= 3x + y + 1$

$(x_0 y_0, z_0) = (1, -1, 2)$.

∴ $x_0 = 1, y_0 = -1, z_0 = 2$

$fx(x_0, y_0, z_0) = (3)(-1)(2) - 1 = -7$

$fy(x_0, y_0, z_0) = (3)(1)(2)(-1) = 5$

$fz(x_0, y_0, z_0) = (3)(1)(-1) + 1 = -2$

Eqn of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7(x) + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

Required eq tangent

Eqn of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

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Q5. Find the local maxima and minima for the following function.

(i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$ $6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y = -2$ — ①

$f_y = 0$ $2y - 3x - 4 = 0$
$2y - 3x = 4$ — ②

Multiply eqn ① & ② $4x - 2y = -4$
$x = 0$

by substituting value of x in eqn ①
$2(0) - y = -2$
$-y = -2$ $\therefore y = 2$

$\therefore$ Critical Points are $(0,2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

$\therefore r > 0$

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$rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$ $= \boxed{3 > 0}$
f is maximum at $(0,2)$

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$\therefore 3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$3(0)^2 + 2(2)^2 - 3(0)(2) + 6(0) - 4(2)$
$= 0 + 4 - 0 + 0 - 8 = -4$

(ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$
$f_y = 3x^2 - 2y$

$f_x = 0$ $\therefore 8x^3 + 6xy = 0$
$\Rightarrow 2x(4x^2 + 3y) = 0$
$\therefore 4x^2 + 3y = 0$ — ①

$f_y = 0$ $3x^2 - 2y = 0$ — ②

Multiplying eq ① & ② $\therefore 12x^2 + 9y = 0$
$-12x^2 - 8y = 0$
$\therefore y = 0$

$\therefore$ By subst: $4x^2 + 3(0) = 0$
$\therefore x = 0$

$\therefore$ Critical Point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 - 2 = -2$
$s = f_{xy} = 6x - 0 = 6(0)$

$\therefore r$ at $(0,0)$
$= 24(0) + 6(0) = 0$

$rt - s^2 = 0(-2) - (s)^2 = 0 \therefore r = 0$

AB 27/01/2022

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