

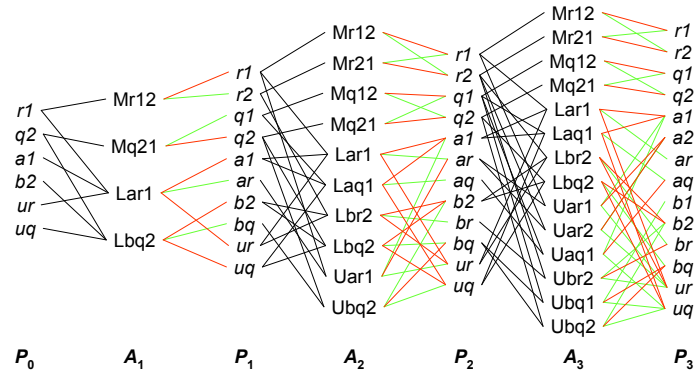
Artificial Intelligence Planning

Graphplan

Artificial Intelligence Planning

•Graphplan

Graphplan: Overview



Graphplan: Overview

- given a propositional planning domain and problem
- step 1: extend the graph with 2 layers (forward, left to right)
 - edges shown are preconditions and effects
 - other edges (not shown) express mutual exclusivity
 - worst-case time complexity is polynomial
- step 2: search for a plan in the graph
 - search backwards (right to left)
 - worst-case time complexity is exponential
- repeat steps 1 and 2

Overview

- A Propositional DWR Example
- The Basic Planning Graph (No Mutex)
- Layered Plans
- Mutex Propositions and Actions
- Forward Planning Graph Expansion
- Backwards Search in the Planning Graph
- The Graphplan Algorithm

Overview

➤ A Propositional DWR Example

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Classical Representations

- propositional representation
 - world state is set of propositions
 - action consists of precondition propositions, propositions to be added and removed
- STRIPS representation
 - like propositional representation, but first-order literals instead of propositions
- state-variable representation
 - state is tuple of state variables $\{x_1, \dots, x_n\}$
 - action is partial function over states

Classical Representations

• propositional representation

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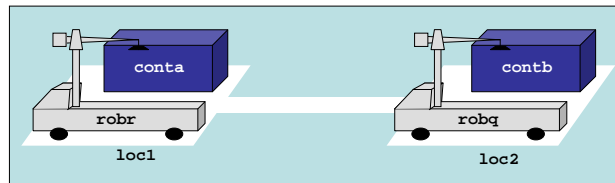
• STRIPS representation

- named after STRIPS planner
- like propositional representation, but first-order literals instead of propositions
- most popular for restricted state-transitions systems

• state-variable representation

- state is tuple of state variables $\{x_1, \dots, x_n\}$
 - action is partial function over states
 - useful where state is characterized by attributes over finite domains
- equally expressive: planning domain in one representation can also be represented in the others

Example: Simplified DWR Problem



- robots can load and unload autonomously
- locations may contain unlimited number of robots and containers
- problem: swap locations of containers

Example: Simplified DWR Problem

•[figure]

•initial state:

- 2 locations: loc1 and loc2, connected by path
- 2 robots: robr and robq, both unloaded initially at loc1 and loc2 respectively
- 2 containers: conta and contb, initially at loc1 and loc2 respectively

•robots can load and unload autonomously

•locations may contain unlimited number of robots and containers

•problem: swap locations of containers

Simplified DWR Problem: STRIPS Operators

- **move(r, l, l')**
 - precondition: $\text{at}(r, l), \text{adjacent}(l, l')$
 - effects: $\text{at}(r, l'), \neg \text{at}(r, l)$
- **load(c, r, l)**
 - precondition: $\text{at}(r, l), \text{in}(c, l), \text{unloaded}(r)$
 - effects: $\text{loaded}(r, c), \neg \text{in}(c, l), \neg \text{unloaded}(r)$
- **unload(c, r, l)**
 - precondition: $\text{at}(r, l), \text{loaded}(r, c)$
 - effects: $\text{unloaded}(r), \text{in}(c, l), \neg \text{loaded}(r, c)$

Simplified DWR Problem: STRIPS Actions

•**move(r, l, l')**

•move robot r from location l to adjacent location l' (4 possible actions; with rigid adjacent relation evaluated)

•**precond:** $\text{at}(r, l), \text{adjacent}(l, l')$

•**effects:** $\text{at}(r, l'), \neg \text{at}(r, l)$

•**load(c, r, l)**

•load container c onto robot r at location l (8 possible actions)

•**precond:** $\text{at}(r, l), \text{in}(c, l), \text{unloaded}(r)$

•**effects:** $\text{loaded}(r, c), \neg \text{in}(c, l), \neg \text{unloaded}(r)$

•**unload(c, r, l)**

•unload container c from robot r at location l (8 possible actions)

•**precond:** $\text{at}(r, l), \text{loaded}(r, c)$

•**effects:** $\text{unloaded}(r), \text{in}(c, l), \neg \text{loaded}(r, c)$

Simplified DWR Problem: State Proposition Symbols

- robots:
 - $r1$ and $r2$: $at(rob_r, loc1)$ and $at(rob_r, loc2)$
 - $q1$ and $q2$: $at(rob_q, loc1)$ and $at(rob_q, loc2)$
 - ur and uq : $unloaded(rob_r)$ and $unloaded(rob_q)$
- containers:
 - $a1$, $a2$, ar , and aq : $in(conta, loc1)$, $in(conta, loc2)$, $loaded(conta, rob_r)$, and $loaded(conta, rob_q)$
 - $b1$, $b2$, br , and bq : $in(contb, loc1)$, $in(contb, loc2)$, $loaded(contb, rob_r)$, and $loaded(contb, rob_q)$
- initial state: $\{r1, q2, a1, b2, ur, uq\}$

Simplified DWR Problem: State Proposition Symbols

•idea: represent each atom that may occur in a state by a single (short) proposition symbol

•robots:

- $r1$ and $r2$: $at(rob_r, loc1)$ and $at(rob_r, loc2)$
- $q1$ and $q2$: $at(rob_q, loc1)$ and $at(rob_q, loc2)$
- ur and uq : $unloaded(rob_r)$ and $unloaded(rob_q)$

•containers:

- $a1$, $a2$, ar , and aq : $in(conta, loc1)$, $in(conta, loc2)$, $loaded(conta, rob_r)$, and $loaded(conta, rob_q)$
- $b1$, $b2$, br , and bq : $in(contb, loc1)$, $in(contb, loc2)$, $loaded(contb, rob_r)$, and $loaded(contb, rob_q)$

•14 state propositions

•initial state: $\{r1, q2, a1, b2, ur, uq\}$

Simplified DWR Problem: Action Symbols

- move actions:
 - Mr12: move(robr,loc1,loc2), Mr21: move(robr,loc2,loc1), Mq12: move(robq,loc1,loc2), Mq21: move(robq,loc2,loc1)
- load actions:
 - Lar1: load(conta,robr,loc1); Lar2, Laq1, Laq2, Lbr1, Lbr2, Lbq1, and Lbq2 correspondingly
- unload actions:
 - Uar1: unload(conta,robr,loc1); Uar2, Uaq1, Uaq2, Ubr1, Ubr2, Ubq1, and Ubq2 correspondingly

Simplified DWR Problem: Action Symbols

•move actions:

•Mr12: move(robr,loc1,loc2), Mr21: move(robr,loc2,loc1), Mq12: move(robq,loc1,loc2), Mq21: move(robq,loc2,loc1)

•load actions:

•Lar1: load(conta,robr,loc1); Lar2, Laq1, Laq2, Lar1, Lbr2, Lbq1, and Lbq2 correspondingly

•unload actions:

•Uar1: unload(conta,robr,loc1); Uar2, Uaq1, Uaq2, Uar1, Ubr2, Ubq1, and Ubq2 correspondingly

•14 state symbols: lower case, italic

•20 action symbols: uppercase, not italic

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•A Propositional DWR Example

➤The Basic Planning Graph (No Mutex)

•Layered Plans

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Solution Existence

- **Proposition:** A propositional planning problem $\mathcal{P}=(\Sigma, s_i, g)$ has a solution iff $S_g \cap \Gamma^>(\{s_i\}) \neq \{\}$.
- **Proposition:** A propositional planning problem $\mathcal{P}=(\Sigma, s_i, g)$ has a solution iff $\exists s \in \Gamma^<(\{g\}) : s \subseteq s_i$.

Solution Existence

• **Proposition:** A propositional planning problem $\mathcal{P}=(\Sigma, s_i, g)$ has a solution iff $S_g \cap \Gamma^>(\{s_i\}) \neq \{\}$.

- ... iff there is a goal state that is also a reachable state

• **Proposition:** A propositional planning problem $\mathcal{P}=(\Sigma, s_i, g)$ has a solution iff $\exists s \in \Gamma^<(\{g\}) : s \subseteq s_i$.

- ... iff there is a minimal set of propositions amongst all regression sets that is a subset of the initial state

Reachability Tree

- tree structure, where:
 - root is initial state s_i
 - children of node s are $\Gamma(\{s\})$
 - arcs are labelled with actions
- all nodes in reachability tree are $\Gamma^>(\{s_i\})$
 - all nodes to depth d are $\Gamma^d(\{s_i\})$
 - solves problems with up to d actions in solution
- problem: $O(k^d)$ nodes;
 k = applicable actions per state

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Planning Graph: Nodes

- layered directed graph $G=(N,E)$:
 - $N = P_0 \cup A_1 \cup P_1 \cup A_2 \cup P_2 \cup \dots$
 - state proposition layers: P_0, P_1, \dots
 - action layers: A_1, A_2, \dots
- first proposition layer P_0 :
 - propositions in initial state s_i : $P_0=s_i$
- action layer A_j :
 - all actions a where: $\text{precond}(a) \subseteq P_{j-1}$
- proposition layer P_j :
 - all propositions p where: $p \in P_{j-1}$ or $\exists a \in A_j: p \in \text{effects}^+(a)$

Planning Graph: Nodes

•layered directed graph $G=(N,E)$:

- layered = each node belongs to exactly one layer

• $N = P_0 \cup A_1 \cup P_1 \cup A_2 \cup P_2 \cup \dots$

- proposition and action layers alternate

•state proposition layers: P_0, P_1, \dots

•action layers: A_1, A_2, \dots

•first proposition layer P_0 :

- propositions in initial state s_i : $P_0=s_i$

•action layer A_j :

- all actions a where: $\text{precond}(a) \subseteq P_{j-1}$

•proposition layer P_j :

- all propositions p where: $p \in P_{j-1}$ or $\exists a \in A_j: p \in \text{effects}^+(a)$

- propositions at layer P_j are all propositions in the union of all nodes in the reachability tree at depth j

- note: negative effects are not deleted from next layer

- note: $P_{j-1} \subseteq P_j$; propositions in the graph monotonically increase from one proposition layer to the next

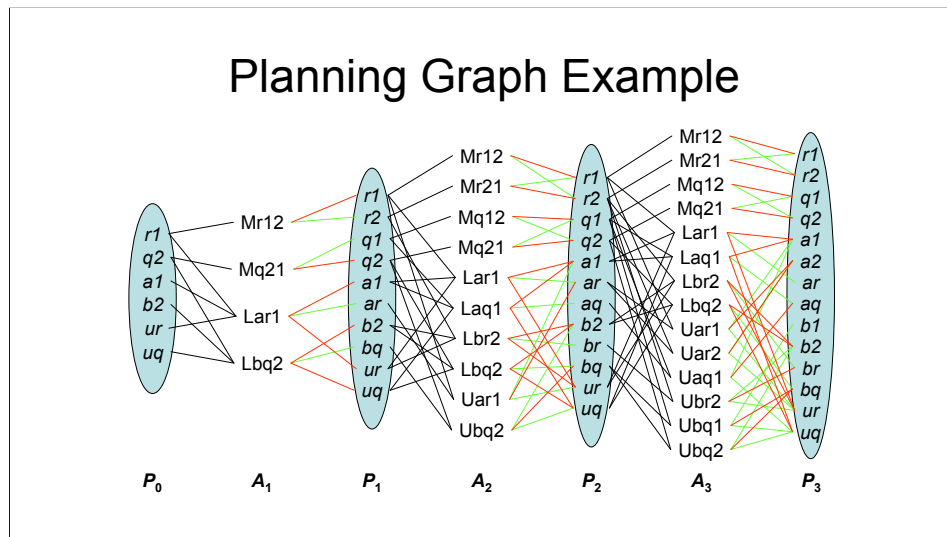
Planning Graph: Edges

- from proposition $p \in P_{j-1}$ to action $a \in A_j$:
 - if: $p \in \text{precond}(a)$
- from action $a \in A_j$ to layer $p \in P_j$:
 - positive arc if: $p \in \text{effects}^+(a)$
 - negative arc if: $p \in \text{effects}^-(a)$
- no arcs between other layers

Planning Graph: Arcs

- directed and layered = arcs only from one layer to the next
- from proposition $p \in P_{j-1}$ to action $a \in A_j$:
 - if: $p \in \text{precond}(a)$
- from action $a \in A_j$ to layer $p \in P_j$:
 - positive arc if: $p \in \text{effects}^+(a)$
 - negative arc if: $p \in \text{effects}^-(a)$
- no arcs between other layers
- note: $A_{j-1} \subseteq A_j$; actions in the graph monotonically increase from one action layer to the next

Planning Graph Example



Planning Graph Example

•[figure]

- start with initial proposition layer
- next action layer: applicable action; links from preconditions (black)
- next proposition layer: previous proposition plus positive effects; links to positive effects (green); links to negative effects (red)
- next action layer (A_2); precondition links; next proposition layer (P_2); effect links
- next action layer (A_3); precondition links; next proposition layer (P_3); effect links
- action layers contain “inclusive disjunctions” of actions

Reachability in the Planning Graph

- reachability analysis:
 - if a goal g is reachable from initial state s_i
 - then there will be a proposition layer P_g in the planning graph such that $g \subseteq P_g$
- necessary condition, but not sufficient
- low complexity:
 - planning graph is of polynomial size and
 - can be computed in polynomial time

Reachability in the Planning Graph

•reachability analysis:

- if a goal g is reachable from initial state s_i
- then there will be a proposition layer P_g in the planning graph such that $g \subseteq P_g$
- or: if no proposition layer contains g then g is not reachable

•necessary condition, but not sufficient

•necessary vs. sufficient:

•reachability tree:

- nodes contain propositions that must necessarily hold
- propositions in one node are consistent

•planning graph:

- proposition layers contains propositions that may possibly hold
- propositions in one layer usually inconsistent (e.g. robots/containers in two places at once)
- similarly, incompatible actions in one layer may interfere with each other

•low complexity:

- planning graph is of polynomial size and
- can be computed in polynomial time

•need more conditions (for sufficient criterion)

Overview

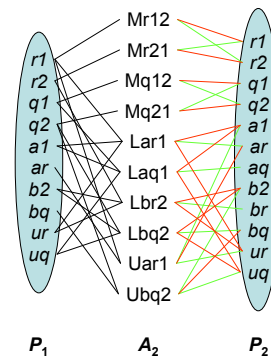
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Independent Actions: Examples

- Mr12 and Lar1:
 - cannot occur together
 - Mr12 deletes precondition *r1* of Lar1
- Mr12 and Mr21:
 - cannot occur together
 - Mr12 deletes positive effect *r1* of Mr21
- Mr12 and Mq21:
 - may occur in same action layer



Independent Actions: Examples

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- Mr12 and Mq21:
 - may occur in same action layer

Independent Actions

- Two actions a_1 and a_2 are independent iff:
 - $\text{effects}^-(a_1) \cap (\text{precond}(a_2) \cup \text{effects}^+(a_2)) = \{\}$ and
 - $\text{effects}^-(a_2) \cap (\text{precond}(a_1) \cup \text{effects}^+(a_1)) = \{\}$.
- A set of actions π is independent iff every pair of actions $a_1, a_2 \in \pi$ is independent.

Independent Actions

•idea: independent actions can be executed in any order (in same layer)

•Two actions a_1 and a_2 are independent iff:

• $\text{effects}^-(a_1) \cap (\text{precond}(a_2) \cup \text{effects}^+(a_2)) = \{\}$ and

• $\text{effects}^-(a_2) \cap (\text{precond}(a_1) \cup \text{effects}^+(a_1)) = \{\}$.

•two actions are dependent iff:

•one deletes a precondition of the other or

•one deletes a positive effect of the other

•A set of actions π is independent iff every pair of actions $a_1, a_2 \in \pi$ is independent.

•note: independence does not depend on planning problem; can be pre-computed

•note: independence relation is symmetrical (follows from definition)

Pseudo Code: independent

```
function independent( $a_1, a_2$ )  
  for all  $p \in \text{effects}^-(a_1)$   
    if  $p \in \text{precond}(a_2)$  or  $p \in \text{effects}^+(a_2)$  then  
      return false  
  for all  $p \in \text{effects}^-(a_2)$   
    if  $p \in \text{precond}(a_1)$  or  $p \in \text{effects}^+(a_1)$  then  
      return false  
  return true
```

Pseudo Code: independent

- **function** independent(a_1, a_2)

- returns true iff the two given actions are independent

- **for all** $p \in \text{effects}^-(a_1)$

- **if** $p \in \text{precond}(a_2)$ **or** $p \in \text{effects}^+(a_2)$ **then**

- **return** false

- **for all** $p \in \text{effects}^-(a_2)$

- **if** $p \in \text{precond}(a_1)$ **or** $p \in \text{effects}^+(a_1)$ **then**

- **return** false

- **return** true

- complexity:

- let b be max. number of preconditions, positive, and negative effects of any action

- element test in hash-set takes constant time

- complexity: $O(b)$

Applying Independent Actions

- A set π of independent actions is applicable to a state s iff $\bigcup_{a \in \pi} \text{precond}(a) \subseteq s$.
- The result of applying the set π in s is defined as:
 $\gamma(s, \pi) = (s - \text{effects}^-(\pi)) \cup \text{effects}^+(\pi)$, where:
 - $\text{precond}(\pi) = \bigcup_{a \in \pi} \text{precond}(a)$,
 - $\text{effects}^+(\pi) = \bigcup_{a \in \pi} \text{effects}^+(a)$, and
 - $\text{effects}^-(\pi) = \bigcup_{a \in \pi} \text{effects}^-(a)$.

Applying Independent Actions

• A set π of independent actions is applicable to a state s iff $\bigcup_{a \in \pi} \text{precond}(a) \subseteq s$.

• note: applying a set of independent actions can be done in any order

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- $\text{effects}^-(\pi) = \bigcup_{a \in \pi} \text{effects}^-(a)$.

Execution Order of Independent Actions

- **Proposition:** If a set π of independent actions is applicable in state s then, for any permutation $\langle a_1, \dots, a_k \rangle$ of the elements of π :
 - the sequence $\langle a_1, \dots, a_k \rangle$ is applicable to s , and
 - the state resulting from the application of π to s is the same as from the application of $\langle a_1, \dots, a_k \rangle$, i.e.:
$$\gamma(s, \pi) = \gamma(s, \langle a_1, \dots, a_k \rangle).$$

Execution Order of Independent Actions

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$$\gamma(s, \pi) = \gamma(s, \langle a_1, \dots, a_k \rangle).$$

Layered Plans

- Let $P = (A, s_i, g)$ be a statement of a propositional planning problem and $G = (N, E)$, $N = P_0 \cup A_1 \cup P_1 \cup A_2 \cup P_2 \cup \dots$, the corresponding planning graph.
- A layered plan over G is a sequence of sets of actions: $\Pi = \langle \pi_1, \dots, \pi_k \rangle$ where:
 - $\pi_i \subseteq A_i \subseteq A$,
 - π_i is applicable in state P_{i-1} , and
 - the actions in π_i are independent.

Layered Plans

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 - $\pi_i \subseteq A_i \subseteq A$,
 - π_i is applicable in state P_{i-1} , and
 - the actions in π_i are independent.

Layered Solution Plan

- A layered plan $\Pi = \langle \pi_1, \dots, \pi_k \rangle$ is a solution to a to a planning problem $P=(A, s_i, g)$ iff:
 - π_1 is applicable in s_i ,
 - for $j \in \{2 \dots k\}$, π_j is applicable in state $\gamma(\dots \gamma(\gamma(s_i, \pi_1), \pi_2), \dots \pi_{j-1})$, and
 - $g \subseteq \gamma(\dots \gamma(\gamma(s_i, \pi_1), \pi_2), \dots, \pi_k)$.

Layered Solution Plan

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- π_1 is applicable in s_i ,
- for $j \in \{2 \dots k\}$, π_j is applicable in state $\gamma(\dots \gamma(\gamma(s_i, \pi_1), \pi_2), \dots \pi_{j-1})$, and
- $g \subseteq \gamma(\dots \gamma(\gamma(s_i, \pi_1), \pi_2), \dots, \pi_k)$.

• note: independence of actions still not sufficient criterion for solution

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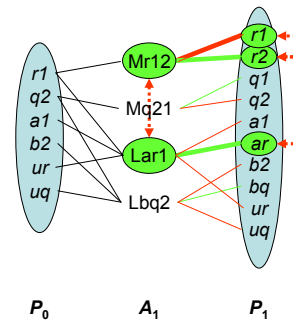
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Problem: Dependent Propositions: Example

- $r2$ and ar :
 - $r2$: positive effect of Mr12
 - ar : positive effect of Lar1
 - but: Mr12 and Lar1 not independent
 - hence: $r2$ and ar incompatible in P_1
- $r1$ and $r2$:
 - positive and negative effects of same action: Mr12
 - hence: $r1$ and $r2$ incompatible in P_1



Problem: Dependent Propositions: Example

• $r2$ and ar :

- $r2$: positive effect of Mr12
- ar : positive effect of Lar1
- but: Mr12 and Lar1 not independent

• dependent actions cannot occur together same set of actions in a layered plan, e.g. in π_1

- hence: $r2$ and ar incompatible in P_1

• $r1$ and $r2$:

- positive and negative effects of same action: Mr12
- hence: $r1$ and $r2$ incompatible in P_1

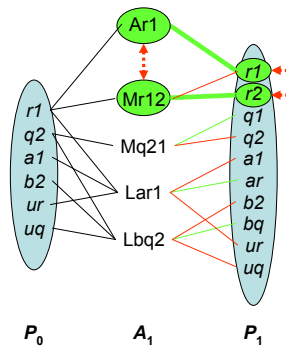
• both cases: compatible if they are also

- two positive effects of one action
- the positive effects of two independent actions

• incompatible propositions: cannot be reached through preceding action layer (A_1)

No-Operation Actions

- No-Op for proposition p :
 - name: Ap
 - precondition: p
 - effect: p
- $r1$ and $r2$:
 - $r1$: positive effect of $Ar1$
 - $r2$: positive effect of $Mr12$
 - but: $Ar1$ and $Mr12$ not independent
 - hence: $r1$ and $r2$ incompatible in P_1
- only one incompatibility test



No-Operation Actions

•No-Op for proposition p :

- for every action layer and every proposition that may persist

•name: Ap

•precondition: p

•effect: p

• $r1$ and $r2$:

• $r1$: positive effect of $Ar1$

• $r2$: positive effect of $Mr12$

•but: $Ar1$ and $Mr12$ not independent

•hence: $r1$ and $r2$ incompatible in P_1

•only one incompatibility test

•previous slide: two types of incompatibility (positive effects of dependent actions + positive and negative effects of same action)

•with no-ops: only first type needed (simplification)

Mutex Propositions

- Two propositions p and q in proposition layer P_j are mutex (mutually exclusive) if:
 - every action in the preceding action layer A_j that has p as a positive effect (incl. no-op actions) is mutex with every action in A_j that has q as a positive effect, and
 - there is no single action in A_j that has both, p and q , as positive effects.
- notation: $\mu P_j = \{ (p,q) \mid p,q \in P_j \text{ are mutex} \}$

Mutex Propositions

- Two propositions p and q in proposition layer P_j are mutex (mutually exclusive) if:
 - every action in the preceding action layer A_j that has p as a positive effect (incl. no-op actions) is mutex with every action in A_j that has q as a positive effect, and
 - need to define when two actions are mutex
 - obvious case: if they are dependent
 - there is no single action in A_j that has both, p and q , as positive effects.
- notation: $\mu P_j = \{ (p,q) \mid p,q \in P_j \text{ are mutex} \}$
- note: mutex relation for propositions is symmetrical (follows from definition)
- proposition layer P_1 contains 8 mutex pairs

Pseudo Code: mutex for Propositions

```
function mutex( $p_1, p_2, \mu A_j$ )  
  for all  $a_1 \in p_1.\text{producers}()$   
    for all  $a_2 \in p_2.\text{producers}()$   
      if  $(a_1, a_2) \notin \mu A_j$  then  
        return false  
  return true
```

Pseudo Code: mutex for Propositions

•function mutex($p_1, p_2, \mu A_j$)

- input: two propositions (from same layer), mutex relation between the actions in the preceding layer

•for all $a_1 \in p_1.\text{producers}()$

- producers: actions in the preceding layer that have p_1 as a positive effect; should be stored with proposition node

•for all $a_2 \in p_2.\text{producers}()$

- producers: see above

•if $(a_1, a_2) \notin \mu A_j$ then

- test whether the action are in the given set of mutually exclusive actions

•return false

- if not: consistent producers found; propositions are not mutex

•return true

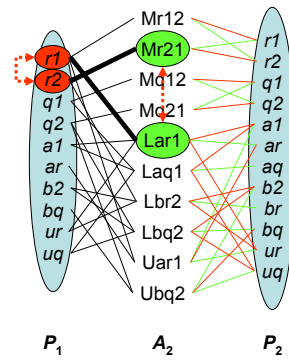
- no consistent producers found; propositions are mutex

- note: single action producing both is covered: action cannot be mutex with itself

- complexity: let m be number of actions in domain (incl. no-ops); $O(m^2)$

Mutex Actions: Example

- $r1$ and $r2$ are mutex in P_1
- $r1$ is precondition for Lar1 in A_2
- $r2$ is precondition for Mr21 in A_2
- hence: Lar1 and Mr21 are mutex in A_2



Mutex Actions: Example

- $r1$ and $r2$ are mutex in P_1
- $r1$ is precondition for Lar1 in A_2
- $r2$ is precondition for Mr21 in A_2
- hence: Lar1 and Mr21 are mutex in A_2
- dependency between actions in action layer A_j leads to mutex between propositions in P_j
- mutex between propositions in P_j leads to mutex between actions in action layer A_{j+1}

Mutex Actions

- Two actions a_1 and a_2 in action layer A_j are mutex if:
 - a_1 and a_2 are dependent, or
 - a precondition of a_1 is mutex with a precondition of a_2 .
- notation: $\mu A_j = \{ (a_1, a_2) \mid a_1, a_2 \in A_j \text{ are mutex} \}$

Mutex Actions

- Two actions a_1 and a_2 in action layer A_j are mutex if:
 - a_1 and a_2 are dependent, or
 - dependent actions are necessarily mutex
 - a precondition of a_1 is mutex with a precondition of a_2 .
 - dependency is domain-specific, i.e. not problem-specific
 - mutex-relation is problem specific
 - pair of actions/propositions may be mutex in one layer but not so in another
- notation:
 $\mu A_j = \{ (a_1, a_2) \mid a_1, a_2 \in A_j \text{ are mutex} \}$
 - action layer A_1 contains 2 mutex (dependent) pairs
 - action layer A_2 contains 24 mutex pairs (not all dependent)
 - note: mutex relation (for actions and propositions) is symmetrical (follows from definition)

Pseudo Code: mutex for Actions

```
function mutex( $a_1, a_2, \mu P$ )  
  if  $\neg$ independent( $a_1, a_2$ ) then  
    return true  
  for all  $p_1 \in \text{precond}(a_1)$   
    for all  $p_2 \in \text{precond}(a_2)$   
      if  $(p_1, p_2) \in \mu P$  then return true  
  return false
```

Pseudo Code: mutex for Actions

- **function** mutex($a_1, a_2, \mu P$)
 - μP – mutex relations from the preceding proposition layer
- **if** \neg independent(a_1, a_2) **then**
- **return** true
- **for all** $p_1 \in \text{precond}(a_1)$
- **for all** $p_2 \in \text{precond}(a_2)$
- **if** $(p_1, p_2) \in \mu P$ **then return** true
- **return** false
- complexity: let b = max number preconditions/pos. effects/neg effects: $O(b^2)$

Decreasing Mutex Relations

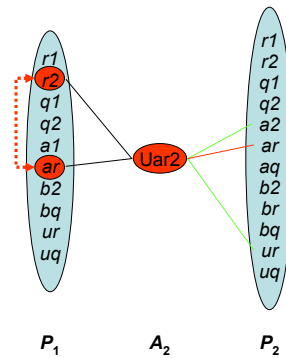
- **Proposition:** If $p, q \in P_{j-1}$ and $(p, q) \notin \mu P_{j-1}$ then $(p, q) \notin \mu P_j$.
 - Proof:
 - if $p, q \in P_{j-1}$ then $Ap, Aq \in A_j$
 - if $(p, q) \notin \mu P_{j-1}$ then $(Ap, Aq) \notin \mu A_j$
 - since $Ap, Aq \in A_j$ and $(Ap, Aq) \notin \mu A_j$, $(p, q) \notin \mu P_j$ must hold
- **Proposition:** If $a_1, a_2 \in A_{j-1}$ and $(a_1, a_2) \notin \mu A_{j-1}$ then $(a_1, a_2) \notin \mu A_j$.
 - Proof:
 - if $a_1, a_2 \in A_{j-1}$ and $(a_1, a_2) \notin \mu A_{j-1}$ then
 - a_1 and a_2 are independent and
 - their preconditions in P_{j-1} are not mutex
 - both properties remain true for P_j
 - hence: $a_1, a_2 \in A_j$ and $(a_1, a_2) \notin \mu A_j$

Decreasing Mutex Relations

- **Proposition:** If $p, q \in P_{j-1}$ and $(p, q) \notin \mu P_{j-1}$ then $(p, q) \notin \mu P_j$.
 - **Proof:**
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 - a_1 and a_2 are independent and
 - their preconditions in P_{j-1} are not mutex
 - both properties remain true for P_j
 - hence: $a_1, a_2 \in A_j$ and $(a_1, a_2) \notin \mu A_j$
- mutex relations are monotonically decreasing (between layers with the same propositions)

Removing Impossible Actions

- actions with mutex preconditions p and q are impossible
 - example: preconditions $r2$ and ar of $Uar2$ in A_2 are mutex
- can be removed from the graph
 - example: remove $Uar2$ from A_2



Removing Impossible Actions

- actions with mutex preconditions p and q are impossible
 - example: preconditions $r2$ and ar of $Uar2$ in A_2 are mutex
- action with mutex preconditions can never be part of any layered plan (will violate applicability condition in definition)
- can be removed from the graph
 - example: remove $Uar2$ from A_2
- mutex pair of actions must remain in graph because one of the actions may be used in final plan
- note: still consistent with monotonically increasing actions

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Reachability in Planning Graphs

- **Proposition:** Let $P = (A, s_i, g)$ be a propositional planning problem and $G = (N, E)$, $N = P_0 \cup A_1 \cup P_1 \cup A_2 \cup P_2 \cup \dots$, the corresponding planning graph. If
 - g is reachable from s_i
 then
 - there is a proposition layer P_g such that
 - $g \subseteq P_g$ and
 - $\neg \exists g_1, g_2 \in g: (g_1, g_2) \in \mu P_g$.

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 then
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 - $g \subseteq P_g$ and
 - $\neg \exists g_1, g_2 \in g: (g_1, g_2) \in \mu P_g$.
- still only necessary condition, but relatively efficient to compute

The Graphplan Algorithm: Basic Idea

- expand the planning graph, one action layer and one proposition layer at a time
- from the first graph for which P_g is the last proposition layer such that
 - $g \subseteq P_g$ and
 - $\neg \exists g_1, g_2 \in g: (g_1, g_2) \in \mu P_g$
- search backwards from the last (proposition) layer for a solution

The Graphplan Algorithm: Basic Idea

• **expand the planning graph, one action layer and one proposition layer at a time**

• similar to iterative deepening: discover new part of the search space with each iteration

• **from the first graph for which P_g is the last proposition layer such that**

• $g \subseteq P_g$ and

• $\neg \exists g_1, g_2 \in g: (g_1, g_2) \in \mu P_g$

• no need to search for solutions in graph with fewer layers; see last proposition

• **search backwards from the last (proposition) layer for a solution**

• two major steps:

• expansion of planning graph to next proposition layer

• searching a given planning graph for a solution

Planning Graph Data Structure

- k -th planning graph G_k :
 - nodes N :
 - array of proposition layers $P_0 \dots P_k$
 - proposition layer j : set of proposition symbols
 - array of action layers $A_1 \dots A_k$
 - proposition layer j : set of action symbols
 - edges E :
 - precondition links: $pre_j \subseteq P_{j-1} \times A_j, j \in \{1 \dots k\}$
 - positive effect links: $e_j^+ \subseteq A_j \times P_j, j \in \{1 \dots k\}$
 - negative effect links: $e_j^- \subseteq A_j \times P_j, j \in \{1 \dots k\}$
 - proposition mutex links: $\mu P_j \subseteq P_j \times P_j, j \in \{1 \dots k\}$
 - action mutex links: $\mu A_j \subseteq A_j \times A_j, j \in \{1 \dots k\}$

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- positive effect links: $e_j^+ \subseteq A_j \times P_j, j \in \{1 \dots k\}$
- negative effect links: $e_j^- \subseteq A_j \times P_j, j \in \{1 \dots k\}$
- proposition mutex links: $\mu P_j \subseteq P_j \times P_j, j \in \{1 \dots k\}$
- action mutex links: $\mu A_j \subseteq A_j \times A_j, j \in \{1 \dots k\}$

• note: instance of this data structure does not depend on problem

• initial planning graph: $P_0 = s_i$; rest is empty sets

Pseudo Code: expand

```

function expand( $G_{k-1}$ )
   $A_k \leftarrow \{a \in A \mid \text{precond}(a) \subseteq P_{k-1} \text{ and } \{(p_1, p_2) \mid p_1, p_2 \in \text{precond}(a)\} \cap \mu P_{k-1} = \{\}\}$ 
   $\mu A_k \leftarrow \{(a_1, a_2) \mid a_1, a_2 \in A_k, a_1 \neq a_2, \text{ and mutex}(a_1, a_2, \mu P_{k-1})\}$ 
   $P_k \leftarrow \{p \mid \exists a \in A_k : p \in \text{effects}^+(a)\}$ 
   $\mu P_k \leftarrow \{(p_1, p_2) \mid p_1, p_2 \in P_k, p_1 \neq p_2, \text{ and mutex}(p_1, p_2, \mu A_k)\}$ 
  for all  $a \in A_k$ 
     $\text{pre}_k \leftarrow \text{pre}_k \cup (\{p \mid p \in P_{k-1} \text{ and } p \in \text{precond}(a)\} \times a)$ 
     $e_k^+ \leftarrow e_k^+ \cup (a \times \{p \mid p \in P_k \text{ and } p \in \text{effects}^+(a)\})$ 
     $e_k^- \leftarrow e_k^- \cup (a \times \{p \mid p \in P_k \text{ and } p \in \text{effects}^-(a)\})$ 

```

Pseudo Code: expand

- **function** expand(G_{k-1})
- $A_k \leftarrow \{a \in A \mid \text{precond}(a) \subseteq P_{k-1} \text{ and } \{(p_1, p_2) \mid p_1, p_2 \in \text{precond}(a)\} \cap \mu P_{k-1} = \{\}\}$
 - actions with satisfied, non-mutex preconditions (incl. no-ops)
- $\mu A_k \leftarrow \{(a_1, a_2) \mid a_1, a_2 \in A_k, a_1 \neq a_2, \text{ and mutex}(a_1, a_2, \mu P_{k-1})\}$
- $P_k \leftarrow \{p \mid \exists a \in A_k : p \in \text{effects}^+(a)\}$
 - union of all positive effects
- $\mu P_k \leftarrow \{(p_1, p_2) \mid p_1, p_2 \in P_k, p_1 \neq p_2, \text{ and mutex}(p_1, p_2, \mu A_k)\}$
- **for all** $a \in A_k$
- $\text{pre}_k \leftarrow \text{pre}_k \cup (\{p \mid p \in P_{k-1} \text{ and } p \in \text{precond}(a)\} \times a)$
- $e_k^+ \leftarrow e_k^+ \cup (a \times \{p \mid p \in P_k \text{ and } p \in \text{effects}^+(a)\})$
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Planning Graph Complexity

- **Proposition:** The size of a planning graph up to level k and the time required to expand it to that level are polynomial in the size of the planning problem.
- **Proof:**
 - problem size: n propositions and m actions
 - $|P_j| \leq n$ and $|A_j| \leq n+m$ (incl. no-op actions)
 - algorithms for generating each layer and all link types are polynomial in size of layer

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- algorithms for generating each layer and all link types are polynomial in size of layer

Fixed-Point Levels

- A fixed-point level in a planning graph G is a level κ such that for all i , $i > \kappa$, level i of G is identical to level κ , i.e. $P_i = P_\kappa$, $\mu P_i = \mu P_\kappa$, $A_i = A_\kappa$ and $\mu A_i = \mu A_\kappa$
- **Proposition:** Every planning graph G has a fixed-point level κ , which is the smallest k such that $|P_k| = |P_{k+1}|$ and $|\mu P_k| = |\mu P_{k+1}|$.
- **Proof:**
 - P_i grows monotonically and μP_i shrinks monotonically
 - A_i and P_i only depend on P_{i-1} and μP_{i-1}

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• $|P_k| = |P_{k+1}|$ implies $P_k = P_{k+1}$

• **Proof:**

• P_i grows monotonically and μP_i shrinks monotonically

• μP_i shrinks monotonically: for equal P_i

• A_i and P_i only depend on P_{i-1} and μP_{i-1}

• time complexity: $O(n+m)$ from fixed point level; only copying required

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Searching the Planning Graph

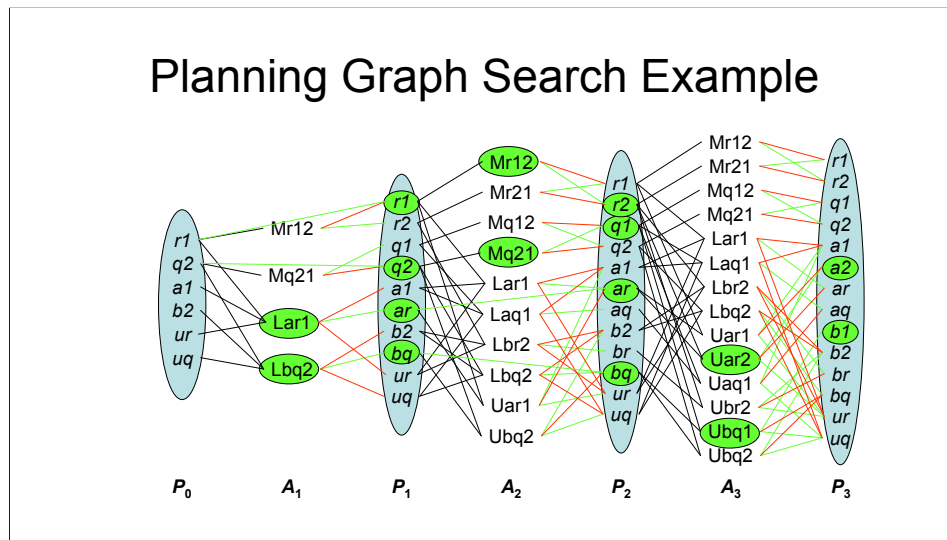
- general idea:
 - search backwards from the last proposition layer P_k in the current graph
 - let g be the set of goal propositions that need to be achieved at a given proposition layer P_j (initially the last layer)
 - find a set of actions $\pi_j \subseteq A_j$ such that these actions are not mutex and together achieve g
 - take the union of the preconditions of π_j as the new goal set to be achieved in proposition layer P_{j-1}

Searching the Planning Graph

•general idea:

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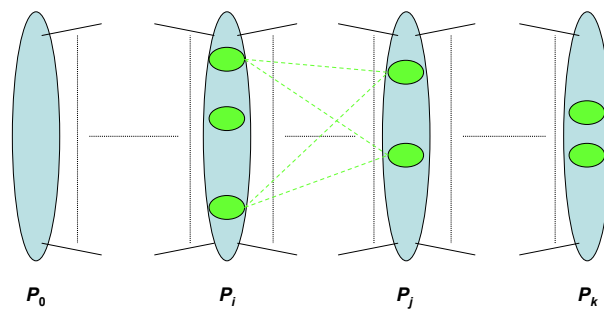
Planning Graph Search Example



Planning Graph Search Example

- initial goal: a_2 and b_1
- only one incoming positive effect link per goal (but no-ops not shown)
- achievable with Uar_2 and Ubq_1 (which are not mutex; mutex relations not shown)
- precondition links indicate sub-goal at next layer
- new sub-goal at P_2 : r_2 , q_1 , ar , bq
- only one incoming positive effect link per goal condition (but no-ops not shown)
 - achieve ar and bq with no-ops
 - achieve r_2 with Mr_{12} and q_1 with Mq_{21}
- precondition links (for Mr_{12} and Mq_{21}) indicate some sub-goal at next layer
- complete sub-goal (incl. preconditions of no-ops) at P_1 : r_1 , q_2 , ar , bq
- only one incoming positive effect link per goal condition (but no-ops not shown)
 - achieve r_1 and q_2 with no-ops
 - achieve ar with Lar_1 and bq with Lbq_2
- precondition links (for Lar_1 and Lbq_2) indicate some sub-goal at next layer
- complete sub-goal (incl. preconditions of no-ops) at P_0 : complete initial state

Repeated Sub-Goals



Repeated Sub-Goals

- ultimate goal leads to possible sub-goals at P_j
- possible sub-goals at P_j lead to possible sub-goals at P_i
 - search to initial proposition layer to see whether sub-goals can be achieved
 - suppose: sub-goals at P_i cannot be achieved
- backtrack to later layer, say P_j
- possible sub-goals at P_j may lead to same possible sub-goals at P_i , but in a different way
 - no need to repeat search: same sub-goals at same layer still cannot be achieved
 - generalization: same some sub-goals at same or earlier layer still cannot be achieved
 - otherwise no-op would achieve sub-goal at later layer

The *nogood* Table

- *nogood* table (denoted ∇) for planning graph up to layer k :
 - array of k sets of sets of goal propositions
 - inner set: one combination of propositions that cannot be achieved
 - outer set: all combinations that cannot be achieved (at that layer)
- before searching for set g in P_j :
 - check whether $g \in \nabla(j)$
- when search for set g in P_j has failed:
 - add g to $\nabla(j)$

The *nogood* Table

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 - array of k sets of sets of goal propositions
 - inner set: one combination of propositions that cannot be achieved
 - outer set: all combinations that cannot be achieved (at that layer)
 - mutex only gives pairs of propositions that cannot be achieved together, *nogood* table gives impossible tuples
- before searching for set g in P_j :
 - check whether $g \in \nabla(j)$
 - actually: in j or later layer
- when search for set g in P_j has failed:
 - add g to $\nabla(j)$
 - or move?

Pseudo Code: extract

```
function extract( $G, g, i$ )  
  if  $i=0$  then return  $\langle \rangle$   
  if  $g \in \nabla(i)$  then return failure  
   $\Pi \leftarrow \text{gpSearch}(G, g, \{\}, i)$   
  if  $\Pi \neq \text{failure}$  then return  $\Pi$   
   $\nabla(i) \leftarrow \nabla(i) + g$   
  return failure
```

Pseudo Code: extract

•function extract(G, g, i)

- inputs: planning graph G , set of propositions (sub-goals) g , and layer at which sub-goals need to be achieved i

- output: a layered plan $\langle \pi_1, \dots, \pi_i \rangle$ that achieves g at i in G or failure if there is no such plan

•if $i=0$ then return $\langle \rangle$

- trivial success with empty plan

•if $g \in \nabla(i)$ then return failure

- sub-goals have resulted in failure before

• $\pi_i \leftarrow \text{gpSearch}(G, g, \{\}, i)$

- perform the search

•if $\pi_i \neq \text{failure}$ then return π_i

- the search was successful

• $\nabla(i) \leftarrow \nabla(i) + g$

- unsuccessful search: remember unachievable sub-goals

•return failure

Pseudo Code: gpSearch

```

function gpSearch( $G, g, \pi, i$ )
  if  $g = \{\}$  then
     $\Pi \leftarrow \text{extract}(G, U_{a \in \pi} \text{precond}(a), i-1)$ 
    if  $\Pi = \text{failure}$  then return failure
    return  $\Pi \bullet \langle \pi \rangle$ 
   $p \leftarrow g.\text{selectOne}()$ 
   $\text{providers} \leftarrow \{a \in A_i \mid p \in \text{effects}^+(a) \text{ and } \neg \exists a' \in \pi: (a, a') \in \mu A_i\}$ 
  if  $\text{providers} = \{\}$  then return failure
   $a \leftarrow \text{providers.chooseOne}()$ 
  return gpSearch( $G, g - \text{effects}^+(a), \pi + a, i$ )

```

Pseudo Code: gpSearch

•function gpSearch(G, g, π, i)

•inputs: planning graph G , remaining sub-goals g , and set of actions already committed to π , both at level i

•outputs: layered plan

•if $g = \{\}$ then

•all actions chosen

• $\Pi \leftarrow \text{extract}(G, U_{a \in \pi} \text{precond}(a), i-1)$

•if $\Pi = \text{failure}$ then return failure

•return $\Pi \bullet \langle \pi \rangle$

• $p \leftarrow g.\text{selectOne}()$

•no need to backtrack here; order only important for efficiency

• $\text{resolvers} \leftarrow \{a \in A_i \mid p \in \text{effects}^+(a) \text{ and } \neg \exists a' \in \pi: (a, a') \in \mu A_i\}$

•if $\text{resolvers} = \{\}$ then return failure

• $a \leftarrow \text{resolvers.chooseOne}()$

•non-deterministic choice point; backtrack to here

•return GPSearch($G, g - \text{effects}^+(a), \pi + a, i$)

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Pseudo Code: Graphplan

```

function graphplan( $A, s_i, g$ )
   $i \leftarrow 0; \nabla \leftarrow []; P_0 \leftarrow s_i; G \leftarrow (P_0, \{\})$ 
  while ( $g \notin P_i$  or  $g^2 \cap \mu P_i \neq \{\}$ ) and  $\neg \text{fixedPoint}(G)$  do
     $i \leftarrow i+1; \text{expand}(G)$ 
    if  $g \notin P_i$  or  $g^2 \cap \mu P_i \neq \{\}$  then return failure
     $\eta \leftarrow \text{fixedPoint}(G) ? |\nabla(\kappa)| : 0$ 
     $\Pi \leftarrow \text{extract}(G, g, i)$ 
    while  $\Pi = \text{failure}$  do
       $i \leftarrow i+1; \text{expand}(G)$ 
       $\Pi \leftarrow \text{extract}(G, g, i)$ 
      if  $\Pi = \text{failure}$  and  $\text{fixedPoint}(G)$  then
        if  $\eta \neq |\nabla(\kappa)|$  then return failure
         $\eta \leftarrow |\nabla(\kappa)|$ 
    return  $\Pi$ 

```

Pseudo Code: graphplan

• **function graphplan(A, s_i, g)**

• given planning problem, return layered solution plan

• $i \leftarrow 0; \nabla \leftarrow []; P_0 \leftarrow s_i; G \leftarrow (P_0, \{\})$

• **while** ($g \notin P_i$ or $g^2 \cap \mu P_i \neq \{\}$) **and** $\neg \text{fixedPoint}(G)$ **do**

• $i \leftarrow i+1; \text{expand}(G)$

• planning graph expanded until solution possible or fixed point reached

• **if** $g \notin P_i$ or $g^2 \cap \mu P_i \neq \{\}$ **then return failure**

• test necessary criterion

• $\eta \leftarrow \text{fixedPoint}(G) ? |\nabla(\kappa)| : 0$

• used to test when expansion will not work

• $\Pi \leftarrow \text{extract}(G, g, i)$

• **while** $\Pi = \text{failure}$ **do**

• $i \leftarrow i+1; \text{expand}(G)$

• $\Pi \leftarrow \text{extract}(G, g, i)$

• **if** $\Pi = \text{failure}$ **and** $\text{fixedPoint}(G)$ **then**

• **if** $\eta \neq |\nabla(\kappa)|$ **then return failure**

• $\eta \leftarrow |\nabla(\kappa)|$

• **return** Π

Graphplan Properties

- **Proposition:** The Graphplan algorithm is sound, complete, and always terminates.
 - It returns failure iff the given planning problem has no solution;
 - otherwise, it returns a layered plan Π that is a solution to the given planning problem.
- Graphplan is orders of magnitude faster than previous techniques!

Graphplan Properties

• **Proposition:** The Graphplan algorithm is sound, complete, and always terminates.

• It returns failure iff the given planning problem has no solution;

• otherwise, it returns a layered plan Π that is a solution to the given planning problem.

• Graphplan is orders of magnitude faster than previous techniques!

• caveat: restriction to propositional STRIPS

Overview

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- The Basic Planning Graph (No Mutex)
- Layered Plans
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