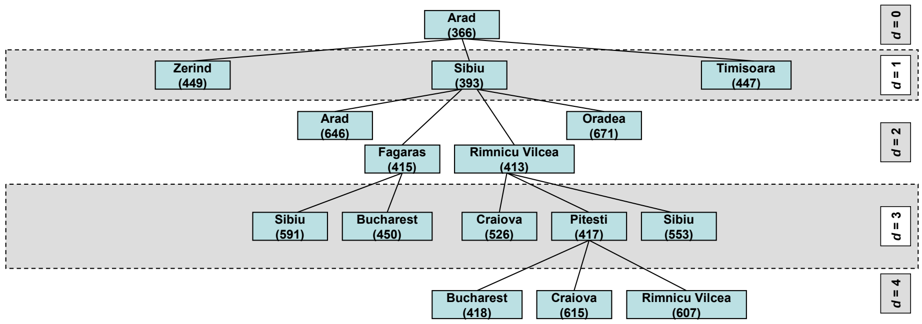
Artificial Intelligence Planning

•**Informed Search**

A* (Best-First) Search: Touring Romania



A* (Best-First) Search: Touring Romania

Overview

- Heuristic Search Strategies
- The A* Algorithm (for Tree Search)
- Properties of A*
- Graph Search with A*
- (Good) Heuristics

Overview

➢Heuristic Search Strategies

•The A* Algorithm (for Tree Search)

•Properties of A*

•Graph Search with A*

•(Good) Heuristics

Best-First Search

- an instance of the general tree search (or graph) search algorithm
  - strategy: select next node based on an <u>evaluation function</u> *f*: state space → ℝ
  - select node with lowest value *f*(*n*)
- implementation:
  selectFrom(*fringe*, *strategy*)
  - <u>priority queue</u>: maintains fringe in ascending order of *f*-values

Best-First Search

- **an instance of the general tree search (or graph) search algorithm**
  - tree or graph search: both possible; difference only lies in test for repeated states
  - **strategy: select next node based on an <u>evaluation function</u> *f*: state space → ℝ**
    - evaluation function: determines the search strategy
    - intuition: choose function that estimates the distance to the goal
  - **select node with lowest value *f*(*n*)**
    - lowest *f*-value means best node: hence best-first search
- **implementation: selectFrom(*fringe*, *strategy*)**
  - **<u>priority queue</u>: maintains fringe in ascending order of *f*-values**
    - implementation as binary tree: nodes can be added/retrieved in log-time (still expensive)

Heuristic Functions

- <u>heuristic function</u> *h*: state space → ℝ
- *h*(*n*) = estimated cost of the cheapest path from node *n* to a goal node
- if *n* is a goal node then *h*(*n*) must be 0
- heuristic function encodes problem-specific knowledge in a problem-independent way

Heuristic Functions

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•if *n* is a goal node then *h*(*n*) must be 0

•heuristic function encodes problem-specific knowledge in a problem-independent way

•difference between evaluation function and heuristic function:

  •good evaluation function makes sure nodes are expanded in an order that leads straight to the optimal solution

  •good heuristic function always gives the correct distance to the nearest goal node

  •evaluation function is not problem-specific, but uses heuristic function which is problem-specific

Greedy Best-First Search

- use heuristic function as evaluation function:

$$f(n) = h(n)$$

  - always expands the node that is closest to the goal node
  - eats the largest chunk out of the remaining distance, hence, “greedy”

Greedy Best-First Search

•use heuristic function as evaluation function: $f(n) = h(n)$

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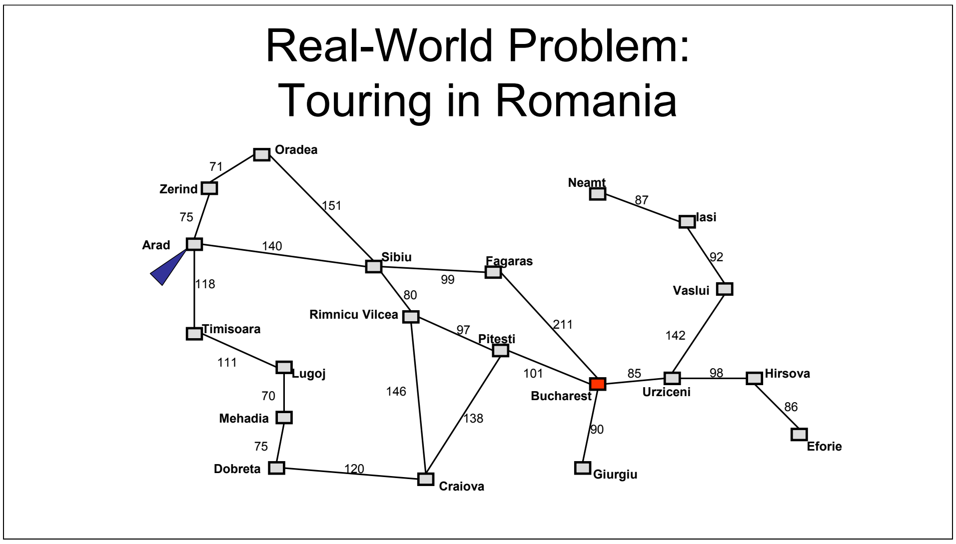


Real-World Problem: Touring in Romania

•shown: rough map of Romania

•initial state: on vacation in Arad, Romania

•goal? actions? -- “Touring Romania” cannot readily be described in terms of possible actions, goals, and path cost

Touring in Romania: Heuristic

- $h_{SLD}(n)$ = straight-line distance to Bucharest

Arad	366	Hirsova	151	Rimnicu Vilcea	193
Bucharest	0	Iasi	226		
Craiova	160	Lugoj	244	Sibiu	253
Dobreta	242	Mehadia	241	Timisoara	329
Eforie	161	Neamt	234	Urziceni	80
Fagaras	176	Oradea	380	Vaslui	199
Giurgiu	77	Pitesti	100	Zerind	374

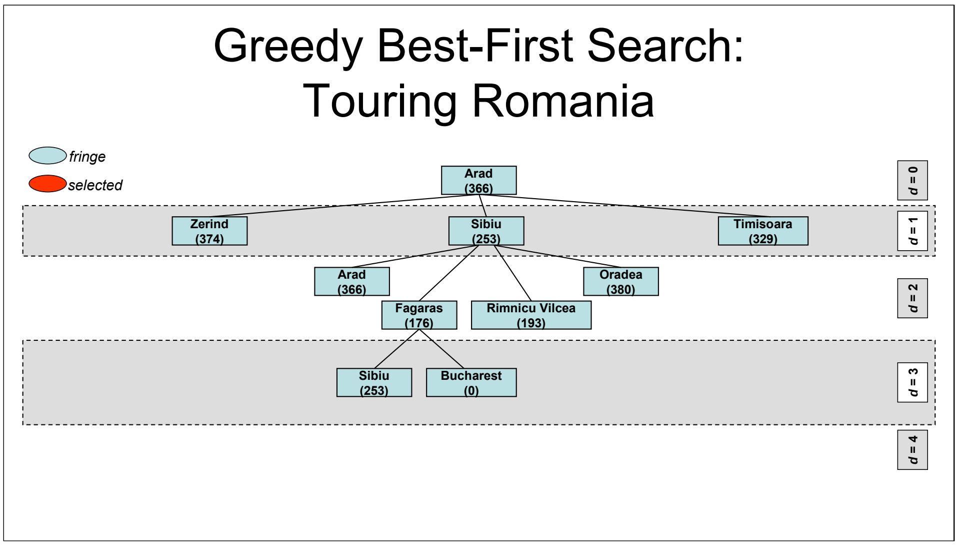
Touring in Romania: Heuristic

•$\boldsymbol{h_{SLD}(n)}$ **= straight-line distance to Bucharest**

- •straight-line distance: Euclidean distance
- •distance to Bucharest because our goal is to be in Bucharest

•**[table]**

- •h_{SLD}(Bucharest) = 0
- •h_{SLD}(Fagaras) = 176 < 211 driving distance
- •$h_{SLD}(n)$ cannot be computed from the problem description, it represents additional information



Greedy Best-First Search: Touring Romania

- •values are values of evaluation function = heuristic function

•select Arad; expand Arad

•select Sibiu; expand Sibiu

- •Fagharas has lowest *f*-value of all fringe nodes

•select Fagharas; expand Fagharas

•select Bucharest – goal node

•for this problem: search proceeds straight to the goal node:

- •minimal search cost
- •but not the optimal path

•uniform-cost search vs. greedy best-first search: both expand node with lowest number:

- •UCS: numbers start from 0 and increase – tendency to expand earlier nodes – breadth-first tendency
- •GBFS: number start from high and decreases – tendency to expand later nodes – depth-first tendency

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A* Search

- best-first search where
  $f(n) = h(n) + g(n)$
  - $h(n)$ the heuristic function (as before)
  - $g(n)$ the cost to reach the node n
- evaluation function:
  $f(n)$ = estimated cost of the cheapest solution through n
- A* search is optimal if $h(n)$ is <u>admissible</u>

A* Search

•best-first search where $f(n) = h(n) + g(n)$

•$h(n)$ the heuristic function (as before)

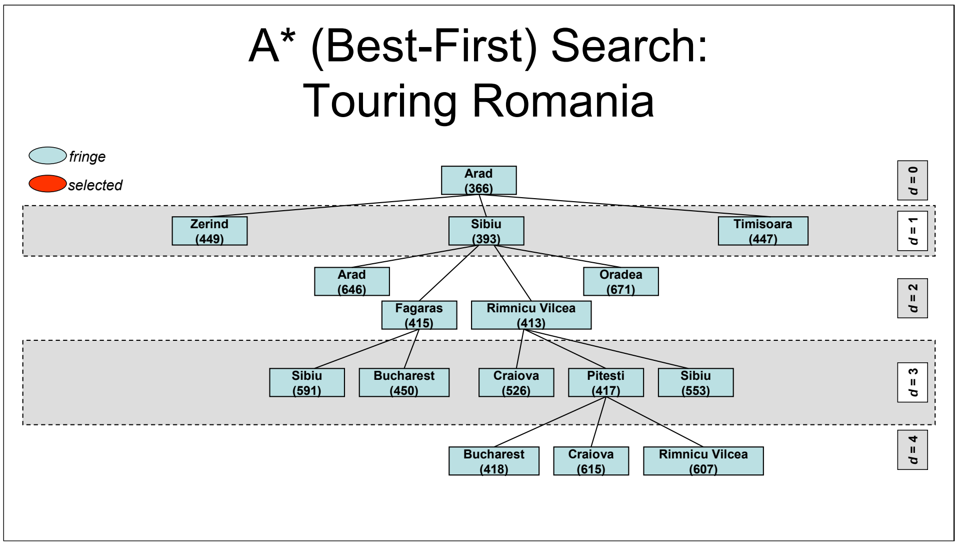
•$g(n)$ the cost to reach the node n

•adds a breadth-first component to GBFS

•evaluation function: $f(n)$ = estimated cost of the cheapest solution through n

•expand that node next which is on the cheapest path to a goal node

•A* search is optimal if $h(n)$ is <u>admissible</u>



A* (Best-First) Search: Touring Romania

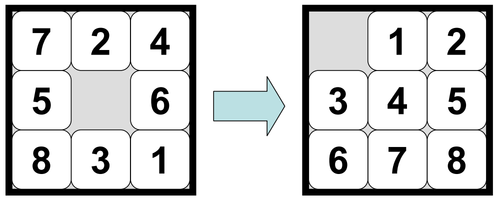
- initial state: in Arad; values shown are evaluation function $f(n) = h(n) + g(n)$

- select Arad; expand Arad
  - lowest f-value: Sibiu (393); means: possible path through Sibiu with cost 393
- select Sibiu; expand Sibiu
  - lowest f-value: Rimnicu Vilcea (413); means: possible path through Rimnicu Vilcea with cost 413
- select Rimnicu Vilcea; expand Rimnicu Vilcea
  - lowest f-value: Fagaras (415); expanding Rimnicu Vilcea showed f-value too optimistic
- select Fagaras; expand Fagaras
  - lowest f-value: Pitesti (417); expanding Fagaras showed f-value too optimistic
- select Pitesti; expand Pitesti
  - lowest f-value: Bucharest (418)
- select Bucharest
  - goal node test succeeds

- note: search cost not minimal as for GBFS but solution is optimal

The Eight-Puzzle

initial state goal state

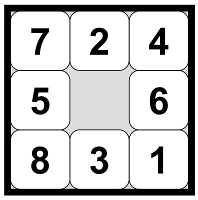


The Eight-Puzzle

Heuristics for the Eight-Puzzle

- h_1: number of misplaced tiles
- h_2: Manhattan block distance

  – example:
    - $h_1 = 8$
      all 8 tiles are misplaced
    - $h_2 = 3+1+2+2+2+3+3+2 = 18$



Heuristics for the Eight-Puzzle

•Both heuristics are admissible;

•Cost of the optimal solution: 26; both heuristics underestimate;

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Admissible Heuristics

A heuristic $h(n)$ is admissible if it *never overestimates* the distance from n to the nearest goal node.

- example: h_{SLD}
- A* search: If $h(n)$ is admissible then $f(n)$ never overestimates the true cost of a solution through n.

Admissible Heuristics

•A heuristic $h(n)$ is admissible if it *never overestimates* the distance from n to the nearest goal node.

- •admissible heuristics usually think the nearest goal node is closer than it actually is
- **•example: h_{SLD}**
  - •h_{SLD}: shortest distance between two point is straight line, hence h_{SLD} is admissible
- **•A* search: If $h(n)$ is admissible then $f(n)$ never overestimates the true cost of a solution through n.**
  - •since $f(n) = h(n) + g(n)$ and $g(n)$ is the exact cost of reaching n, $f(n)$ cannot overestimate the true cost of a solution through n

Optimality of A* (Tree Search)

Theorem:

A* using tree search is optimal if the heuristic $h(n)$ is admissible.

Optimality of A* (Tree Search)

- **Theorem: A* using tree search is optimal if the heuristic $h(n)$ is admissible.**
  - reminder: optimal means finds a minimal-path cost solution

Completeness of A*: Contours

- <u>contours</u>: sets of states that can be reached within a certain cost
  - prerequisite for drawing contours: *f*-values along a path are non-decreasing
- A* fans out from the start node, adding nodes in concentric bands (contours) of increasing *f*-values
- A* is complete: it must reach a contour that includes a goal node

Completeness of A*: Contours

•**<u>contours</u>: sets of states that can be reached within a certain cost**

•imagine like contours in a topographic map (with *f*-value instead of altitude)

•**prerequisite for drawing contours: *f*-values along a path are non-decreasing**

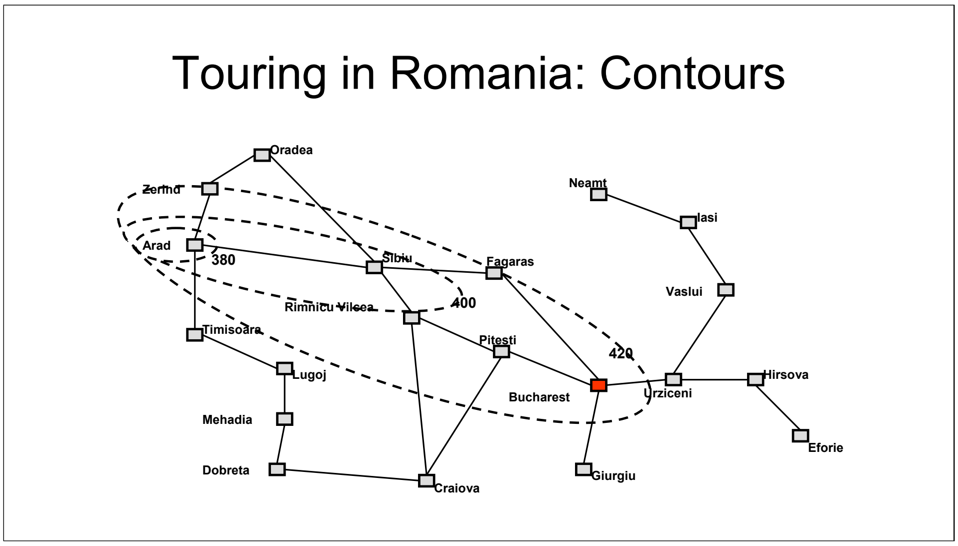
•**A* fans out from the start node, adding nodes in concentric bands (contours) of increasing *f*-values**

•uniform-cost search: draws circles

•A* search: ellipsis stretch towards the goal nodes around the optimal path; the more accurate the heuristic, the more they stretch

•**A* is complete: it must reach a contour that includes a goal node**

•each contour contains only a finite number of nodes because number of actions is finite and action cost must be greater than some positive value



Touring in Romania: Contours

- **[figure]**
    - contour for *f*=380 only contains initial state
    - contour for *f*=400 stretches towards the goal node
    - contour for *f*=420 stretches towards and includes the goal node

A*: Optimally Efficient

- A* is optimally efficient for a given heuristic function: no other optimal algorithm is guaranteed to expand fewer nodes than A*.
- any algorithm that does not expand all nodes with $f(n) < C^*$ runs the risk of missing the optimal solution

A*: Optimally Efficient

•A* is optimally efficient for a given heuristic function: no other optimal algorithm is guaranteed to expand fewer nodes than A*.

  •efficiency can still be increased with a different, more accurate heuristic for a given problem

  •but: efficiency does not only depend on number of nodes expanded

•any algorithm that does not expand all nodes with $f(n) < C^*$ runs the risk of missing the optimal solution

  •suppose there is a node with $f(n) < C^*$ that is not expanded before a goal node

  •then there could be a path of cost with $f(n) < C^*$ through that node which would be better than the goal node found

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A* Tree/Graph Search Algorithm

```
function aStarTreeSearch(problem, h)
  fringe ← priorityQueue(new searchNode(problem.initialState))
  allNodes ← hashTable(fringe)
  loop
    if empty(fringe) then return failure
    node ← selectFrom(fringe)
    if problem.goalTest(node.state) then
      return pathTo(node)
    for successor in expand(problem, node)
      if not allNodes.contains(successor) then
        fringe ← fringe + successor @ f(successor)
        allNodes.add(successor)
```

A* Tree/Graph Search Algorithm

•steps in grey for graph search

•also needed: detecting short-cuts if heuristic is not admissible

A* and Exponential Space

- A* has worst case time and space complexity of $O(b^l)$
- exponential growth of the fringe is normal
  - exponential time complexity may be acceptable
  - exponential space complexity will exhaust any computer's resources all too quickly

A* and Exponential Space

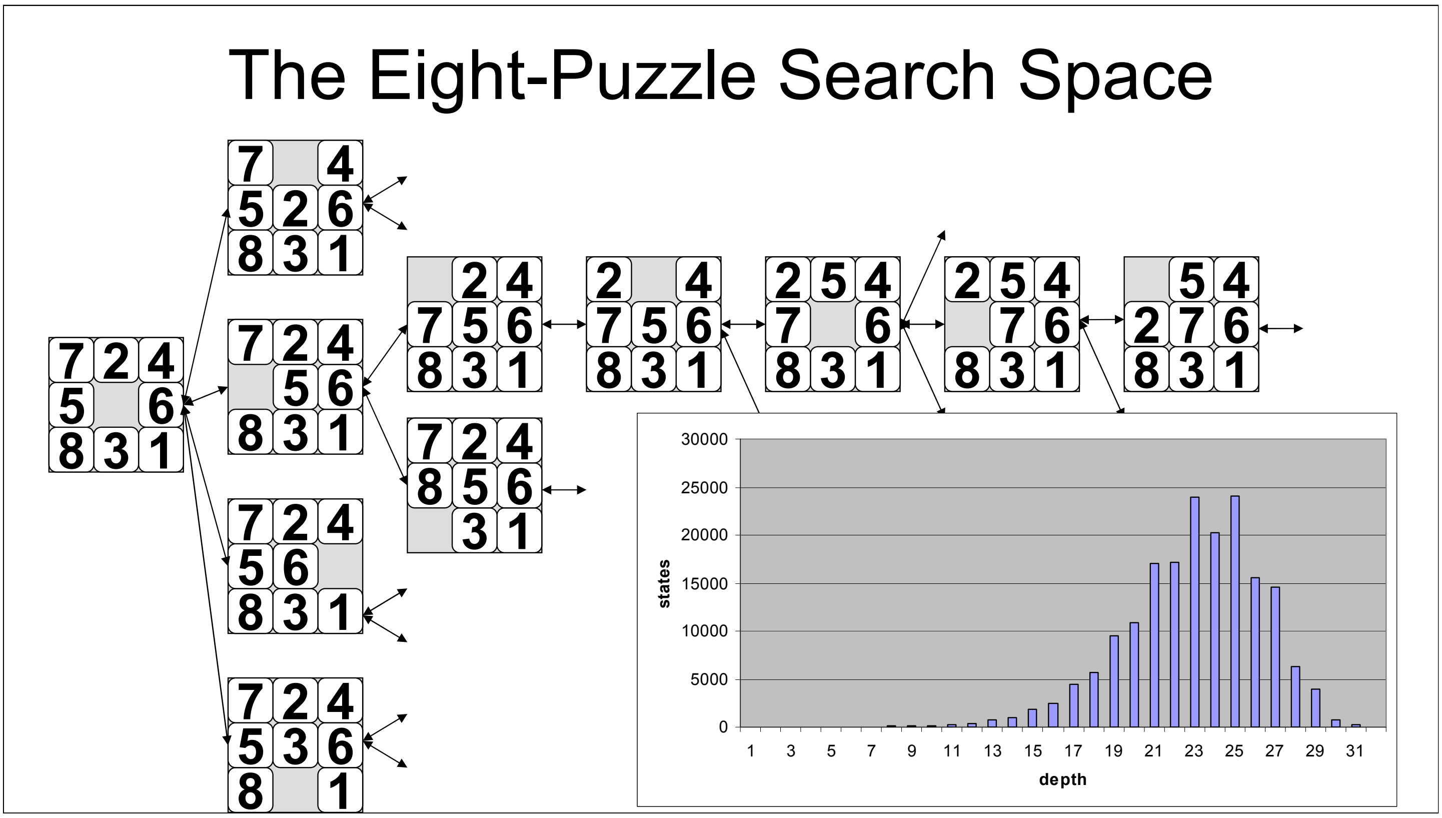
•A* has worst case time and space complexity of $O(b^l)$

•exponential growth of the fringe is normal

•exponential time complexity may be acceptable

•exponential space complexity will exhaust any computer's resources all too quickly

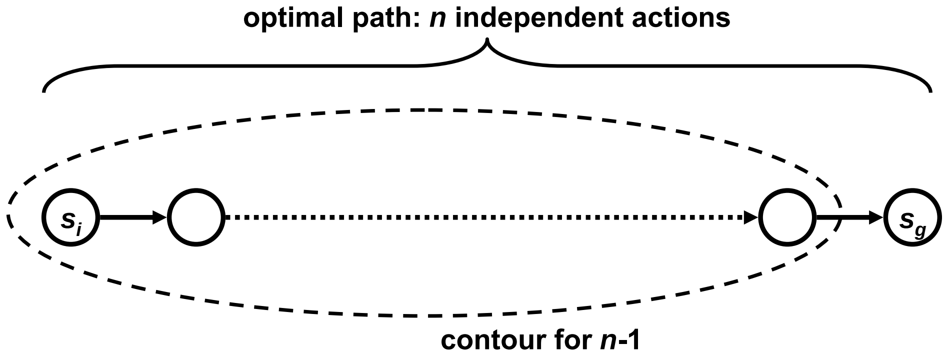
•and with the memory exhausted A* cannot continue and fails – no solution will be found



The Eight-Puzzle Search Space

Permutations of Solutions

- independent actions: for all states s:
  $\gamma(\gamma(s,a_1),a_2) = \gamma(\gamma(s,a_2),a_1)$
- worst case:



Permutations of Solutions

•problem: (n-1)! different paths in contour

•A* may explores all these nodes before exploring s_g (all optimal, so heuristic does not help)

•note: many puzzles do not have independent actions; real-world problems often do

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➢(Good) Heuristics

Heuristics

- heuristics are criteria, methods, or principles for deciding which among several alternative courses of action promises to be the most effective in order to achieve some goal
- example: ripe pineapple
  - inner leafs rip out easily
  - fruit smells like pineapple

Heuristics

•Heuristic (colloquial): **a rule of thumb**;

•Heuristic (general term) vs. heuristic function (technical term) $h(n)$ = estimated cost of the cheapest path from node n to a goal node;

•Another example: what made you (the student) choose this course over others?

Good Heuristics

- good heuristics
  - indicate a way to reduce the number of states that need to be evaluated
  - help obtain solutions within reasonable time constraints
- trade-off:
  - simplicity
    - must provide a simple means of discriminating between choices
  - accuracy
    - no guarantee that they identify the best course of actions
    - but they should do so sufficiently often

Good Heuristics

•Simplicity: easy to compute;

•Accuracy: should result in the best course of actions;

Finding Good Heuristics

- How can we find good heuristics for a given problem?
- Can this process be automated?

Finding Good Heuristics

•Automation would require problem description in a formal language, of course.

Heuristics from Simplified Problems

- relaxed problem: a problem with fewer restrictions on the actions than the original problem
- *The cost of an optimal solution for a relaxed problem is an admissible and consistent heuristic for the original problem.*

Heuristics from Simplified Problems

•Admissibility:

  •Optimal solution to original problem is also solution to relaxed problem;

  •Therefore: optimal solution to original problem at least as expensive as solution to relaxed problem;

•Consistency:

  •Because derived heuristic is exact cost for relaxed problem (finds “short cuts”) the triangle inequality must hold;

•ABSOLVER: program that generates heuristics based on the relaxed problem method;

  •Found best heuristic for 8-puzzle and first useful heuristic for Rubik’s cube;

8-Puzzle Actions

- a tile can move from square A to square B if A is horizontally or vertically adjacent to B and B is blank
- Relaxed conditions:
  - a tile can move from square A to square B if A is adjacent to B (⇒ Manhattan distance)
  - a tile can move from square A to square B if B is blank
  - a tile can move from square A to square B (⇒ misplaced tiles)

8-Puzzle Actions

•Heuristics are estimates for costs of actions still expected, hence modify constraints on actions

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