1. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

   (a) Use elimination to turn A into an upper triangular matrix. How many pivots does A have?

   (b) Let $b = (2, 8, 3)$. Does $Ax = b$ have a solution?

   (c) Let $b = (2, 8, 1)$. Does $Ax = b$ have a solution?

   (d) Can you find multiple solutions in either part (b) or part (c)? If so, find 2.

   (e) Does A have an inverse? Justify your answer using results from this exercise.

2. Let $AB = I$ and $CA = I$ where I is the $n \times n$ identity matrix.

   (a) What are the dimensions of the matrices A, B and C?

   (b) Show that $B = C$.

   (c) Is A invertible?

   [Hint: you can write $B = IB$]

3. Let A be a square matrix with the property that $A^2 = A$. Find expressions for $(I - A)^2$ and $(I - A)^5$.

4. (a) Can the vector $(9, 2, -5)$ be written as a linear combination of the vectors $(1, 2, 3)$ and $(6, 4, 2)$?

   (b) Does a system of equations with coefficient matrix

$$A = \begin{bmatrix} 1 & 6 & 9 \\ 2 & 4 & 2 \\ 3 & 2 & -5 \end{bmatrix}$$

   have a full set of pivots?