

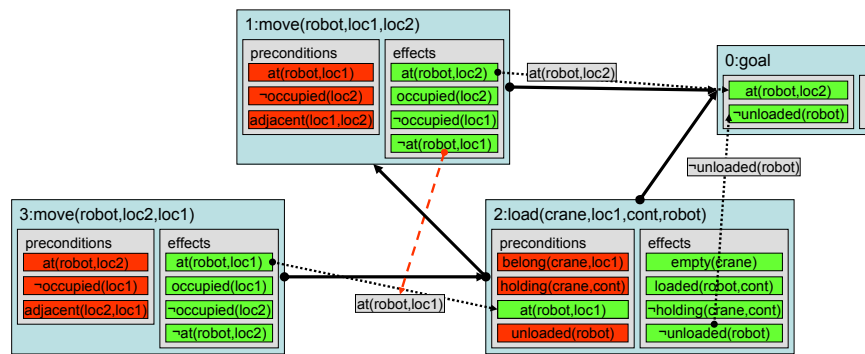
# **Artificial Intelligence Planning**

## **Plan-Space Search**

**Artificial Intelligence Planning**

- **Informed Search**

# Example: Partial Plan



## Overview

- Search States: Partial Plans
- Plan Refinement Operations
- The Plan-Space Search Problem
- Flawless Partial Plans
- The PSP Algorithm
- PSP Implementation Details
- Partial-Order Planning

### Overview

#### ➤ Search States: Partial Plans

- now: introducing a completely different search space with partial plans as search states

#### • Plan Refinement Operations

#### • The Plan-Space Search Problem

#### • Flawless Partial Plans

#### • The PSP Algorithm

#### • PSP Implementation Details

#### • Partial-Order Planning

## State-Space vs. Plan-Space Search

- state-space search:  
search through graph of nodes representing world states
- plan-space search:  
search through graph of partial plans
  - nodes: partially specified plans
  - arcs: plan refinement operations
  - solutions: partial-order plans

### State-Space vs. Plan-Space Search

#### •state-space search: search through graph of nodes representing world states

- search space directly corresponds to graph representation of state-transition system

#### •plan-space search: search through graph of partial plans

##### •nodes: partially specified plans

##### •arcs: plan refinement operations

- least commitment principle: do not add constraints to the plan that are not strictly needed

##### •solutions: partial-order plans

- partial-order plan: set of actions + set of orderings; not necessarily total order
- state-space algorithms also maintain partial plan – but always in total order

## Partial Plans

- plan: set of actions organized into some structure
- partial plan:
  - subset of the actions
  - subset of the organizational structure
    - temporal ordering of actions
    - rationale: what the action achieves in the plan
  - subset of variable bindings

### Partial Plans

#### •plan: set of actions organized into some structure

- organization e.g. sequence

#### •partial plan:

##### •subset of the actions

##### •subset of the organizational structure

##### •temporal ordering of actions

##### •rationale: what the action achieves in the plan

- refers only to subset of actions

##### •subset of variable bindings

- plan refinement operators accordingly: add actions, add ordering constraints, add causal links, add variable bindings

## Definition of Partial Plans

- A partial plan is a tuple  $\pi = (A, <, B, L)$ , where:
  - $A = \{a_1, \dots, a_k\}$  is a set of partially instantiated planning operators;
  - $<$  is a set of ordering constraints on  $A$  of the form  $(a_i < a_j)$ ;
  - $B$  is a set of binding constraints on the variables of actions in  $A$  of the form  $x=y$ ,  $x \neq y$ , or  $x \in D_x$ ;
  - $L$  is a set of causal links of the form  $\langle a_i - [p] \rightarrow a_j \rangle$  such that:
    - $a_i$  and  $a_j$  are actions in  $A$ ;
    - the constraint  $(a_i < a_j)$  is in  $<$ ;
    - proposition  $p$  is an effect of  $a_i$  and a precondition of  $a_j$ ; and
    - the binding constraints for variables in  $a_i$  and  $a_j$  appearing in  $p$  are in  $B$ .

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• sub-goals in a partial plan: preconditions without causal links

• different view: partial plan as set of (sequential) plans

- those that meet the specified constraints and can be refined to a total order plan by adding constraints

• note: partial plans with two types of additional flexibility:

- actions only partially ordered and
- not all variables need to be instantiated

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#### •Search States: Partial Plans

- just done: introducing a completely different search space with partial plans as search states

#### ➤Plan Refinement Operations

- now: state transitions in the new search space – refining partial plans

#### •The Plan-Space Search Problem

#### •Flawless Partial Plans

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## Adding Actions

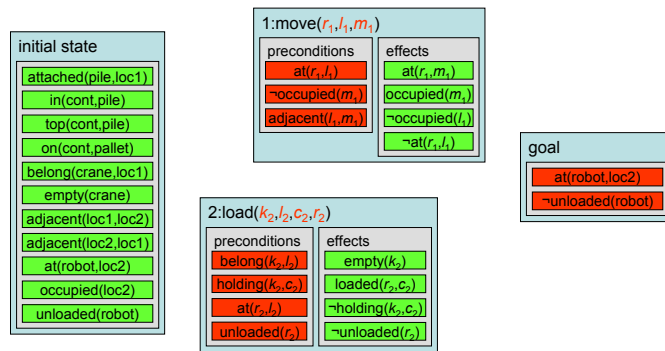
- partial plan contains actions
  - initial state
  - goal conditions
  - set of operators with different variables
- reason for adding new actions
  - to achieve unsatisfied preconditions
  - to achieve unsatisfied goal conditions

### Adding Actions

- **partial plan contains actions**
  - **initial state**
  - **goal conditions**
    - can be represented as two actions with only effects or preconditions
  - **set of operators with different variables**
- **least commitment principle: introduce actions only for a reason**
- **reason for adding new actions**
  - **to achieve unsatisfied preconditions**
  - **to achieve unsatisfied goal conditions**
- **note: new actions can be added anywhere in the current partial plan**



## Adding Actions: Example



### Adding Actions: Example

- empty plan:
  - initial state: all initially satisfied conditions (green)
  - goal: conditions that need to be satisfied (red)
- add operator:  $1:\text{move}(r_1, l_1, m_1)$ 
  - number (1) to provide unique reference to this operator instance
  - also used as variable index for unique variables
  - least commitment principle: choose values for variables only when necessary
- add operator:  $2:\text{load}(k_2, l_2, c_2, r_2)$

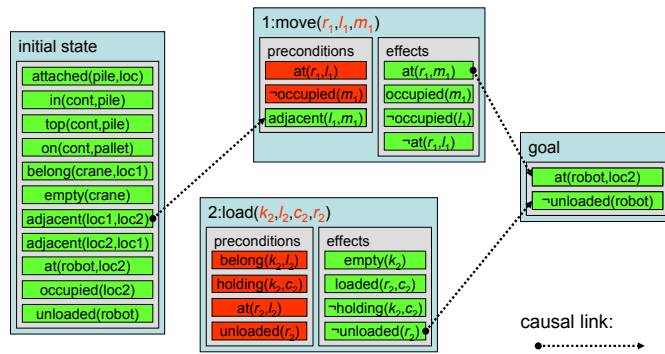
## Adding Causal Links

- partial plan contains causal links
  - links from the provider
    - an effect of an action or
    - an atom that holds in the initial state
  - to the consumer
    - a precondition of an action or
    - a goal condition
- reasons for adding causal links
  - prevent interference with other actions

### Adding Causal Links

- partial plan contains causal links
  - links from the provider
    - an effect of an action or
    - an atom that holds in the initial state
  - to the consumer
    - a precondition of an action or
    - a goal condition
  - causal link implies ordering constraint
    - but: provider need not come directly before consumer
- reasons for adding causal links
  - prevent interference with other actions
  - keeping track of rationale: any action inserted between provider and consumer must not clobber conditions in causal link
  - preconditions without a causal link pointing to them are open sub-goals

## Adding Causal Links: Example



### Adding Causal Links: Example

- add link from 1:move to goal
  - changes colour of goal to green – now satisfied
- add link from 2:load to goal
- add link from initial state to 1:move

## Adding Variable Bindings

- partial plan contains variable bindings
  - new operators introduce new (copies of) variables into the plan
  - solution plan must contain actions
  - variable binding constraints keep track of possible values for variables and co-designation
- reasons for adding variable bindings
  - to turn operators into actions
  - to unify and effect with the precondition it supports

### Adding Variable Bindings

#### •partial plan contains variable bindings

##### •new operators introduce new (copies of) variables into the plan

- each copy of an operator has its own set of variables that are different from variables in other operators instances

##### •solution plan must contain actions

##### •variable binding constraints keep track of possible values for variables and co-designation

- convention (here): give number to operator instances to distinguish them; let variables have index of operator they belong to least commitment principle:

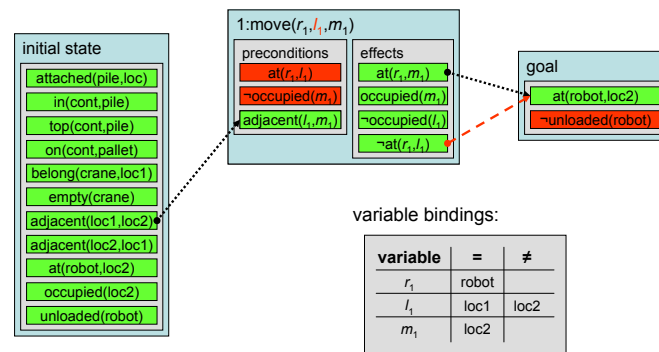
- least commitment principle: add only necessary variable binding constraints

#### •reasons for adding variable bindings

##### •to turn operators into actions

##### •to unify and effect with the precondition it supports

## Adding Variable Bindings: Example



### Adding Variable Bindings: Example

- bind variables due to causal link:
  - bind  $r_1$  to robot
  - bind  $m_1$  to loc2
  - note: variables in operator no longer red to indicate they are bound
- clobbering: move may also destroy goal condition
- introduce variable inequality:  $l_1 \neq loc2$
- clobbering now impossible
- introduce causal link from initial state
- bind  $l_1$  to loc1
  - note consistency with inequality

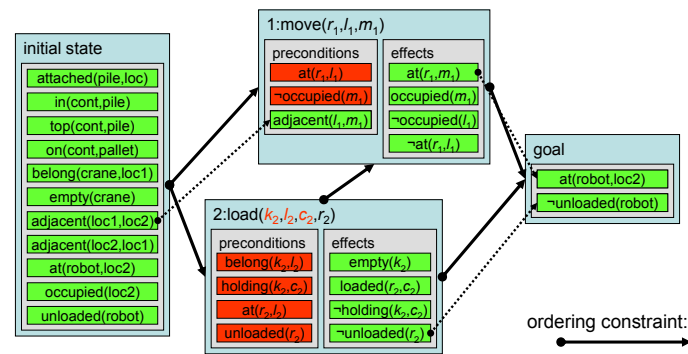
## Adding Ordering Constraints

- partial plan contains ordering constraints
  - binary relation specifying the temporal order between actions in the plan
- reasons for adding ordering constraints
  - all actions after initial state
  - all actions before goal
  - causal link implies ordering constraint
  - to avoid possible interference

### Adding Ordering Constraints

- **partial plan contains ordering constraints**
  - **binary relation specifying the temporal order between actions in the plan**
  - temporal relation: qualitative, not quantitative (at this stage)
- **reasons for adding ordering constraints**
  - **all actions after initial state**
  - **all actions before goal**
  - **causal link implies ordering constraint**
  - **to avoid possible interference**
    - interference can be avoided by ordering the potentially interfering action before the provider or after the consumer of a causal link
    - least commitment principle: introduce ordering constraints only if necessary
- **result: solution plan not necessarily totally ordered**

## Adding Ordering Constraints: Example



### Adding Ordering Constraints: Example

- ordering constraints
  - due to causal links
  - also: all actions before goal
- ordering: all actions after initial state
- orderings may occur between actions

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#### •Search States: Partial Plans

#### •Plan Refinement Operations

- just done: state transitions in the new search space – refining partial plans

#### ➤The Plan-Space Search Problem

- now: definition of the plan-space search problem and solutions

#### •Flawless Partial Plans

#### •The PSP Algorithm

#### •PSP Implementation Details

#### •Partial-Order Planning



## Plan-Space Search: Initial Search State

- represent initial state and goal as dummy actions
  - init: no preconditions, initial state as effects
  - goal: goal conditions as preconditions, no effects
- empty plan  $\pi_0 = (\{\text{init}, \text{goal}\}, \{(\text{init} < \text{goal})\}, \{\}, \{\})$ :
  - two dummy actions init and goal;
  - one ordering constraint: init before goal;
  - no variable bindings; and
  - no causal links.

### Plan-Space Search: Initial Search State

•problem: plan space representation does not maintain states, but need to give initial state and goal description

•**represent initial state and goal as dummy actions**

•**init: no preconditions, initial state as effects**

•**goal: goal conditions as preconditions, no effects**

•**empty plan  $\pi_0 = (\{\text{init}, \text{goal}\}, \{(\text{init} < \text{goal})\}, \{\}, \{\})$ :**

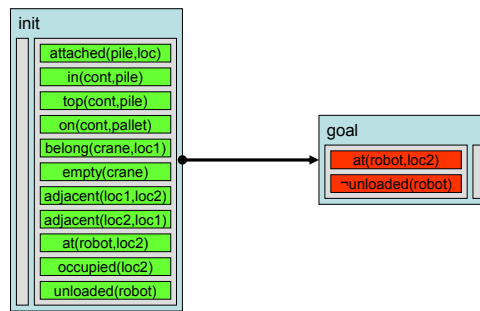
•**two dummy actions init and goal;**

•**one ordering constraint: init before goal;**

•**no variable bindings; and**

•**no causal links.**

## Plan-Space Search: Initial Search State Example



### Plan-Space Search: Initial Search State Example

- note empty box for preconditions in init and empty box for effects in goal

## Plan-Space Search: Successor Function

- states are partial plans
- generate successor through plan refinement operators (one or more):
  - adding an action to  $A$
  - adding an ordering constraint to  $<$
  - adding a binding constraint to  $B$
  - adding a causal link to  $L$

### Plan-Space Search: Successor Function

- **states are partial plans**
- **generate successor through plan refinement operators (one or more):**
  - more required to keep partial plans consistent, e.g. adding a causal link implies adding an ordering constraint
  - **adding an action to  $A$**
  - **adding an ordering constraint to  $<$**
  - **adding a binding constraint to  $B$**
  - **adding a causal link to  $L$**
- successors must be consistent: constraints in a partial plan must be satisfiable
- plan-space planning decouple two sub-problems:
  - which actions need to be performed
  - how to organize these actions
- partial plan as set of plans: refinement operation reduces the set to smaller subset
- next: to define planning as plan-space search problem: need to define goal state

## Total vs. Partial Order

- Let  $\mathcal{P}=(\Sigma, s_i, g)$  be a planning problem. A plan  $\pi$  is a solution for  $\mathcal{P}$  if  $\chi(s_i, \pi)$  satisfies  $g$ .
- problem:  $\chi(s_i, \pi)$  only defined for sequence of ground actions
  - partial order corresponds to total order in which all partial order constraints are respected
  - partial instantiation corresponds to grounding in which variables are assigned values consistent with binding constraints

### Total vs. Partial Order

• Let  $\mathcal{P}=(\Sigma, s_i, g)$  be a planning problem. A plan  $\pi$  is a solution for  $\mathcal{P}$  if  $\chi(s_i, \pi)$  satisfies  $g$ .

• solution defined for state transition system

• problem:  $\chi(s_i, \pi)$  only defined for sequence of ground actions

• partial order corresponds to total order in which all partial order constraints are respected

• partial ordering is consistent iff it is free of loops

• note: there may be an exponential number of total ordering consistent with a given partial ordering

• partial instantiation corresponds to grounding in which variables are assigned values consistent with binding constraints

• note: exponential combinatorics of assigning values to variables

## Partial Order Solutions

- Let  $\mathcal{P}=(\Sigma, s_i, g)$  be a planning problem. A plan  $\pi = (A, <, B, L)$  is a (partial order) solution for  $\mathcal{P}$  if:
  - its ordering constraints  $<$  and binding constraints  $B$  are consistent; and
  - for every sequence  $\langle a_1, \dots, a_k \rangle$  of all the actions in  $A - \{\text{init}, \text{goal}\}$  that is
    - totally ordered and grounded and respects  $<$  and  $B$
    - $\gamma(s_i, \langle a_1, \dots, a_k \rangle)$  must satisfy  $g$ .

### Partial Order Solutions

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  - $\gamma(s_i, \langle a_1, \dots, a_k \rangle)$  must satisfy  $g$ .

• note: causal links do not play a role in the definition of a solution

• with exponential number of sequences to check, definition is not very useful (as computational procedure for goal test)

• idea: use causal links to verify that every precondition of every action is supported by some other action

• problem: condition not strong enough

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### Overview

#### •Search States: Partial Plans

#### •Plan Refinement Operations

#### •The Plan-Space Search Problem

- just done: definition of the plan-space search problem and solutions (without goal test)

#### ➤Flawless Partial Plans

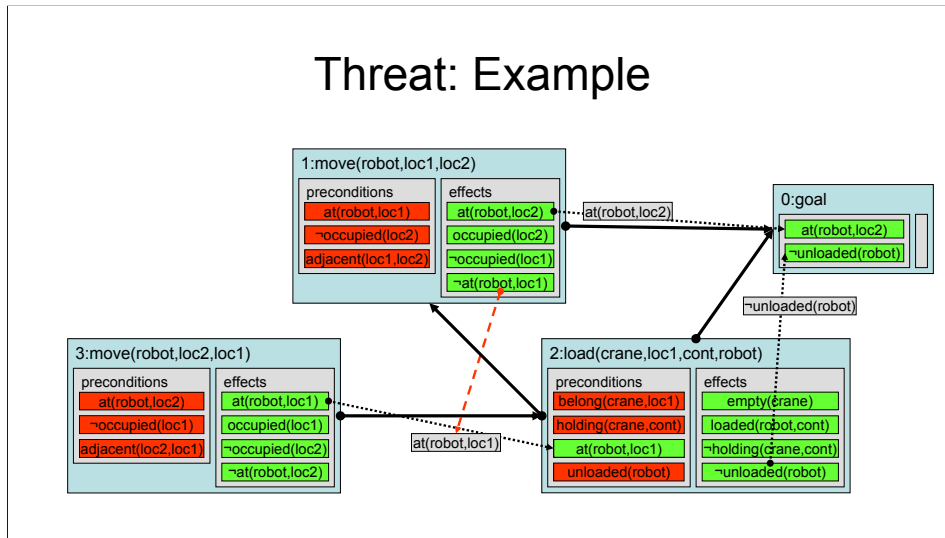
- now: the goal test that completes the search problem

#### •The PSP Algorithm

#### •PSP Implementation Details

#### •Partial-Order Planning

## Threat: Example



### Threat: Example

- start with partial plan from previous example (grounded; initial state not shown due to limited space on slide)
- introduce new 3:move action to achieve  $at(robot, loc1)$  precondition of 2:load action
  - note: still many unachieved preconditions – not a solution yet
- add causal link to maintain rationale
- add ordering to be consistent with causal link
- new: label causal link with condition it protects
- threat: effect of 1:move is negation of condition protected by causal link
  - if 1:move is executed between 3:move and 2:load the plan is no longer valid
- possible solution: additional ordering constraint

## Threats

- An action  $a_k$  in a partial plan  $\pi = (A, \prec, B, L)$  is a threat to a causal link  $\langle a_i - [p] \rightarrow a_j \rangle$  iff:
  - $a_k$  has an effect  $\neg q$  that is possibly inconsistent with  $p$ , i.e.  $q$  and  $p$  are unifiable;
  - the ordering constraints  $(a_i \prec a_k)$  and  $(a_k \prec a_j)$  are consistent with  $\prec$ ; and
  - the binding constraints for the unification of  $q$  and  $p$  are consistent with  $B$ .

### Threats

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- the binding constraints for the unification of  $q$  and  $p$  are consistent with  $B$ .



## Flaws

- A flaw in a plan  $\pi = (A, \prec, B, L)$  is either:
  - an unsatisfied sub-goal, i.e. a precondition of an action in  $A$  without a causal link that supports it; or
  - a threat, i.e. an action that may interfere with a causal link.

### Flaws

- A flaw in a plan  $\pi = (A, \prec, B, L)$  is either:
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  - a threat, i.e. an action that may interfere with a causal link.

## Flawless Plans and Solutions

- **Proposition:** A partial plan  $\pi = (A, <, B, L)$  is a solution to the planning problem  $\mathcal{P} = (\Sigma, s_i, g)$  if:
  - $\pi$  has no flaw;
  - the ordering constraints  $<$  are not circular; and
  - the variable bindings  $B$  are consistent.
- **Proof:** by induction on number of actions in  $A$ 
  - base case: empty plan
  - induction step: totally ordered plan minus first step is solution implies plan including first step is a solution:
 
$$\mathcal{V}(s_i, \langle a_1, \dots, a_k \rangle) = \mathcal{V}(\mathcal{V}(s_i, a_1), \langle a_2, \dots, a_k \rangle)$$

### Flawless Plans and Solutions

• **Proposition:** A partial plan  $\pi = (A, <, B, L)$  is a solution to the planning problem  $\mathcal{P} = (\Sigma, s_i, g)$  if:

- $\pi$  has no flaw;
- the ordering constraints  $<$  are not circular; and
- the variable bindings  $B$  are consistent.

• computation:

- let partial plans in the search space only violate the first condition (have flaws)
- partial plans that violate either of the last two conditions cannot be refined into a solution and need not be generated

• **Proof:** by induction on number of actions in  $A$

• **base case: empty plan**

- no flaws – every goal condition is supported by causal link from initial state

• **induction step: totally ordered plan minus first step is solution implies plan including first step is a solution:**

$$\mathcal{V}(s_i, \langle a_1, \dots, a_k \rangle) = \mathcal{V}(\mathcal{V}(s_i, a_1), \langle a_2, \dots, a_k \rangle)$$

- truncated plan is solution to different problem

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#### •Search States: Partial Plans

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#### •Flawless Partial Plans

- just done: the goal test that completes the search problem

#### ➤The PSP Algorithm

- now: a generic plan-space search planning algorithm

#### •PSP Implementation Details

#### •Partial-Order Planning

## Plan-Space Planning as a Search Problem

- given: statement of a planning problem  $P=(O,s_i,g)$
- define the search problem as follows:
  - initial state:  $\pi_0 = (\{\text{init}, \text{goal}\}, \{(\text{init} < \text{goal})\}, \{\}, \{\})$
  - goal test for plan state  $p$ :  $p$  has no flaws
  - path cost function for plan  $\pi$ :  $|\pi|$
  - successor function for plan state  $p$ : refinements of  $p$  that maintain  $<$  and  $B$

### Plan-Space Planning as a Search Problem

- given: statement of a planning problem  $P=(O,s_i,g)$
- define the search problem as follows:
  - initial state:  $\pi_0 = (\{\text{init}, \text{goal}\}, \{(\text{init} < \text{goal})\}, \{\}, \{\})$
  - goal test for plan state  $p$ :  $p$  has no flaws
  - path cost function for plan  $\pi$ :  $|\pi|$
  - successor function for plan state  $p$ : refinements of  $p$  that maintain  $<$  and  $B$
- note: plan space may be infinite even when state space is finite

## PSP Procedure: Basic Operations

- PSP: Plan-Space Planner
- main principle: refine partial  $\pi$  plan while maintaining  $\prec$  and  $B$  consistent until  $\pi$  has no more flaws
- basic operations:
  - find the flaws of  $\pi$ , i.e. its sub-goals and its threats
  - select one of the flaws
  - find ways to resolve the chosen flaw
  - choose one of the resolvers for the flaw
  - refine  $\pi$  according to the chosen resolver

### PSP Procedure: Basic Operations

#### •PSP: Plan-Space Planner

•main principle: refine partial  $\pi$  plan while maintaining  $\prec$  and  $B$  consistent until  $\pi$  has no more flaws

#### •basic operations:

##### •find the flaws of $\pi$ , i.e. its sub-goals and its threats

- simple for empty plan – all goal conditions are unachieved sub-goals and no threats

##### •select one of the flaws

##### •find ways to resolve the chosen flaw

##### •choose one of the resolvers for the flaw

##### •refine $\pi$ according to the chosen resolver

- modify the plan in such a way that  $\prec$  and  $B$  are in a consistent state for the generated successor
- aim: no need to verify consistency of  $\prec$  and  $B$  for goal test

## PSP: Pseudo Code

```
function PSP(plan)  
  allFlaws  $\leftarrow$  plan.openGoals() + plan.threats()  
  if allFlaws.empty() then return plan  
  flaw  $\leftarrow$  allFlaws.selectOne()  
  allResolvers  $\leftarrow$  flaw.getResolvers(plan)  
  if allResolvers.empty() then return failure  
  resolver  $\leftarrow$  allResolvers.chooseOne()  
  newPlan  $\leftarrow$  plan.refine(resolver)  
  return PSP(newPlan)
```

### •PSP: Pseudo Code

#### •function PSP(*plan*)

- refines the given partial plan into a solution plan; start with initial plan  $\pi_0$

#### •*allFlaws* $\leftarrow$ *plan*.openGoals() + *plan*.threats()

#### •**if** *allFlaws*.empty() **then return** *plan*

- see proposition in previous section: no flaws implies solution

#### •*flaw* $\leftarrow$ *allFlaws*.selectOne()

#### •*allResolvers* $\leftarrow$ *flaw*.getResolvers(*plan*)

- represents all possible ways of removing the selected flaw from the partial plan

#### •**if** *allResolvers*.empty() **then return** failure

- no resolvers means plan cannot be made flawless

#### •*resolver* $\leftarrow$ *allResolvers*.chooseOne()

#### •*newPlan* $\leftarrow$ *plan*.refine(*resolver*)

- must maintain consistency of  $\prec$  and  $B$ ; new plan may contain new flaws

#### •**return** PSP(*newPlan*)

## PSP: Choice Points

- *resolver*  $\leftarrow$  *allResolvers.chooseOne()*
  - non-deterministic choice
- *flaw*  $\leftarrow$  *allFlaws.selectOne()*
  - deterministic selection
  - all flaws need to be resolved before a plan becomes a solution
  - order not important for completeness
  - order is important for efficiency

### PSP: Choice Points

- *resolver*  $\leftarrow$  *allResolvers.chooseOne()*
  - non-deterministic choice
- *flaw*  $\leftarrow$  *allFlaws.selectOne()*
  - deterministic selection
  - all flaws need to be resolved before a plan becomes a solution
  - order not important for completeness
  - order is important for efficiency
    - for finding first plan, not so for finding all plans
    - deterministic implementation: using IDA\*, for example

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- **The Plan-Space Search Problem**
- **Flawless Partial Plans**
- **The PSP Algorithm**
  - just done: a generic plan-space search planning algorithm (overview)
- **PSP Implementation Details**
  - now: functions for identifying flaws and resolving them (used in PSP)
- **Partial-Order Planning**



## Implementing *plan.openGoals()*

- finding unachieved sub-goals (incrementally):
  - in  $\pi_0$ : goal conditions
  - when adding an action: all preconditions are unachieved sub-goals
  - when adding a causal link: protected proposition is no longer unachieved

### Implementing *plan.openGoals()*

- finding unachieved sub-goals (incrementally):
  - in  $\pi_0$ : goal conditions
  - when adding an action: all preconditions are unachieved sub-goals
  - when adding a causal link: protected proposition is no longer unachieved

## Implementing *plan.threats()*

- finding threats (incrementally):
  - in  $\pi_0$ : no threats
  - when adding an action  $a_{new}$  to  $\pi = (A, \prec, B, L)$ :
    - for every causal link  $\langle a_i - [p] \rightarrow a_j \rangle \in L$ 
      - if  $(a_{new} \prec a_i)$  or  $(a_i \prec a_{new})$  then next link
      - else for every effect  $q$  of  $a_{new}$ 
        - if  $(\exists \sigma: \sigma(p) = \sigma(\neg q))$  then  $q$  of  $a_{new}$  threatens  $\langle a_i - [p] \rightarrow a_j \rangle$
  - when adding a causal link  $\langle a_i - [p] \rightarrow a_j \rangle$  to  $\pi = (A, \prec, B, L)$ :
    - for every action  $a_{old} \in A$ 
      - if  $(a_{old} \prec a_i)$  or  $(a_j = a_{old})$  or  $(a_j \prec a_{old})$  then next action
      - else for every effect  $q$  of  $a_{old}$ 
        - if  $(\exists \sigma: \sigma(p) = \sigma(\neg q))$  then  $q$  of  $a_{old}$  threatens  $\langle a_i - [p] \rightarrow a_j \rangle$

## Implementing *plan.threats()*

### •finding threats (incrementally):

- in  $\pi_0$ : no threats
- when adding an action  $a_{new}$  to  $\pi = (A, \prec, B, L)$ :
  - for every causal link  $\langle a_i - [p] \rightarrow a_j \rangle \in L$
  - if  $(a_{new} \prec a_i)$  or  $(a_i \prec a_{new})$  then next link
    - if the new action must occur before the provider or after the consumer of the link
  - else for every effect  $q$  of  $a_{new}$
  - if  $(\exists \sigma: \sigma(p) = \sigma(\neg q))$  then  $q$  of  $a_{new}$  threatens  $\langle a_i - [p] \rightarrow a_j \rangle$ 
    - $\exists \sigma$ : test whether there is a substitution consistent with  $B$ !
- when adding a causal link  $\langle a_i - [p] \rightarrow a_j \rangle$  to  $\pi = (A, \prec, B, L)$ :
  - for every action  $a_{old} \in A$
  - if  $(a_{old} \prec a_i)$  or  $(a_j = a_{old})$  or  $(a_j \prec a_{old})$  then next action
  - else for every effect  $q$  of  $a_{old}$
  - if  $(\exists \sigma: \sigma(p) = \sigma(\neg q))$  then  $q$  of  $a_{old}$  threatens  $\langle a_i - [p] \rightarrow a_j \rangle$

## Implementing *flaw.getResolvers(plan)*

- for unachieved precondition  $p$  of  $a_g$ :
  - add causal links to an existing action:
    - for every action  $a_{old} \in A$ 
      - if  $(a_g = a_{old})$  or  $(a_g < a_{old})$  then next action
      - else for every effect  $q$  of  $a_{old}$ 
        - if  $(\exists \sigma: \sigma(p) = \sigma(q))$  then adding  $\langle a_{old} - [\sigma(p)] \rightarrow a_g \rangle$  is a resolver
  - add a new action and a causal link:
    - for every effect  $q$  of every operator  $o$ 
      - if  $(\exists \sigma: \sigma(p) = \sigma(q))$  then adding  $a_{new} = o.newInstance()$  and  $\langle a_{new} - [\sigma(p)] \rightarrow a_g \rangle$  is a resolver

## Implementing *flaw.getResolvers(plan)*

- for unachieved precondition  $p$  of  $a_g$ :
  - add causal links to an existing action:
    - for every action  $a_{old} \in A$ 
      - if  $(a_g = a_{old})$  or  $(a_g < a_{old})$  then next action
      - else for every effect  $q$  of  $a_{old}$ 
        - if  $(\exists \sigma: \sigma(p) = \sigma(q))$  then adding  $\langle a_{old} - [\sigma(p)] \rightarrow a_g \rangle$  is a resolver
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## Implementing *flaw.getResolvers(plan)*

- for effect  $q$  of action  $a_i$  threatening  $\langle a_i - [p] \rightarrow a_j \rangle$ :
  - order action before threatened link:
    - if  $(a_i = a_j)$  or  $(a_j < a_i)$  then not a resolver
    - else adding  $(a_i < a_j)$  is a resolver
  - order threatened link before action:
    - if  $(a_i = a_j)$  or  $(a_i < a_j)$  then not a resolver
    - else adding  $(a_j < a_i)$  is a resolver
  - extend variable bindings such that unification fails:
    - for every variable  $v$  in  $p$  or  $q$
    - if  $v \neq \sigma(v)$  is consistent with  $B$  then
    - adding  $v \neq \sigma(v)$  is a resolver

## Implementing *flaw.getResolvers(plan)*

- for effect  $q$  of action  $a_i$  threatening  $\langle a_i - [p] \rightarrow a_j \rangle$ :
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    - if  $(a_i = a_j)$  or  $(a_j < a_i)$  then not a resolver
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  - extend variable bindings such that unification fails:
    - for every variable  $v$  in  $p$  or  $q$
    - if  $v \neq \sigma(v)$  is consistent with  $B$  then
    - adding  $v \neq \sigma(v)$  is a resolver

## Implementing *plan.refine(resolver)*

- refines partial plan with elements in resolver by adding:
  - an ordering constraint;
  - one or more binding constraints;
  - a causal link; and/or
  - a new action.
- no testing required
- must update flaws:
  - unachieved preconditions (see: *plan.openGoals()*)
  - threats (see: *plan.threats()*)

### Implementing *plan.refine(resolver)*

#### •refines partial plan with elements in resolver by adding:

- an ordering constraint;
- one or more binding constraints;
- a causal link; and/or
- a new action.

#### •no testing required

- all testing already done in *flaw.getResolvers(plan)*

#### •must update flaws:

- unachieved preconditions (see: *plan.openGoals()*)
- threats (see: *plan.threats()*)

## Maintaining Ordering Constraints

- required operations:
  - query whether  $(a_i < a_j)$
  - adding  $(a_i < a_j)$
- possible internal representations:
  - maintain set of predecessors/successors for each action as given
  - maintain only direct predecessors/successors for each action
  - maintain transitive closure of  $<$  relation

### Maintaining Ordering Constraints

#### •required operations:

- query whether  $(a_i < a_j)$

- adding  $(a_i < a_j)$

- without consistency testing

#### •possible internal representations:

- maintain set of predecessors/successors for each action as given

- maintain only direct predecessors/successors for each action

- maintain transitive closure of  $<$  relation

- operations have different time and space complexity

- note: query performed more often than addition

## Maintaining Variable Binding Constraints

- types of constraints:
  - unary constraints:  $x \in D_x$
  - equality constraints:  $x = y$
  - inequalities:  $x \neq y$
- note: general CSP problem is NP-complete

### Maintaining Variable Binding Constraints

#### •types of constraints:

•**unary constraints:**  $x \in D_x$

•**equality constraints:**  $x = y$

•unary and equality constraints can be solved in linear time

•**inequalities:**  $x \neq y$

•inequalities give rise to general CSP problem

•**note: general CSP problem is NP-complete**

## PSP: Sound and Complete

- **Proposition:** The PSP procedure is sound and complete: whenever  $\pi_0$  can be refined into a solution plan,  $\text{PSP}(\pi_0)$  returns such a plan.
- **Proof:**
  - soundness:  $\prec$  and  $B$  are consistent at every stage of the refinement
  - completeness: induction on the number of actions in the solution plan

### PSP: Sound and Complete

• **Proposition:** The PSP procedure is sound and complete: whenever  $\pi_0$  can be refined into a solution plan,  $\text{PSP}(\pi_0)$  returns such a plan.

• **Proof:**

• **soundness:**  $\prec$  and  $B$  are consistent at every stage of the refinement

• **completeness:** induction on the number of actions in the solution plan

• **note:** non-deterministic version is complete, deterministic implementation must avoid infinite branches



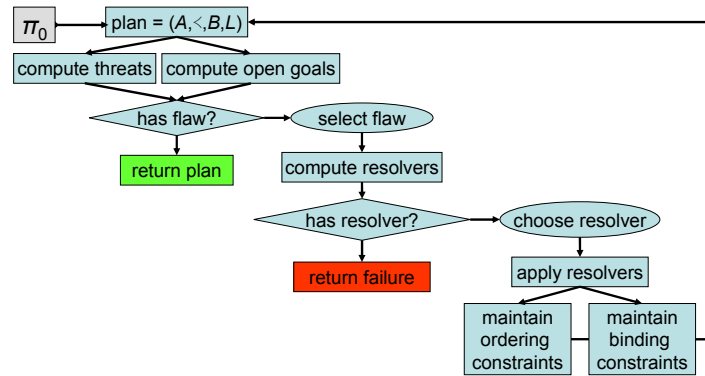
## Overview

- Search States: Partial Plans
- Plan Refinement Operations
- The Plan-Space Search Problem
- Flawless Partial Plans
- The PSP Algorithm
- PSP Implementation Details
- Partial-Order Planning

### Overview

- **Search States: Partial Plans**
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- **The Plan-Space Search Problem**
- **Flawless Partial Plans**
- **The PSP Algorithm**
- **PSP Implementation Details**
  - just done: functions for identifying flaws and resolving them (used in PSP)
- **Partial-Order Planning**
  - now: the algorithm implemented by the UCPOP planner

## PSP: Data Flow



### PSP: Data Flow

- deterministic step: selecting a a flaw
  - no backtracking required
  - selection important for efficiency
  - heuristic guidance required
- non-deterministic step: choosing a resolver for a flaw
  - implemented as backtracking
    - order in which resolvers are tried important for efficiency
    - heuristic guidance required
- note: admissible heuristics ( $A^*$ ) must have step cost greater than zero

## PSP Implementation: PoP

- extended input:
  - partial plan (as before)
  - agenda: set of pairs  $(a,p)$  where  $a$  is an action and  $p$  is one of its preconditions
- search control by flaw type
  - unachieved sub-goal (on agenda): as before
  - threats: resolved as part of the successor generation process

### PSP Implementation: PoP

•based on UCPOP

•**extended input:**

•**partial plan (as before)**

•**agenda: set of pairs  $(a,p)$  where  $a$  is an action and  $p$  is one of its preconditions**

•initial agenda: one pair for each precondition of the goal step

•**search control by flaw type**

•**unachieved sub-goal (on agenda): as before**

•**threats: resolved as part of the successor generation process**

## PoP: Pseudo Code (1)

```
function PoP(plan, agenda)  
  if agenda.empty() then return plan  
  (ag, pg)  $\leftarrow$  agenda.selectOne()  
  agenda  $\leftarrow$  agenda - (ag, pg)  
  relevant  $\leftarrow$  plan.getProviders(pg)  
  if relevant.empty() then return failure  
  (ap, pp,  $\sigma$ )  $\leftarrow$  relevant.chooseOne()  
  plan.L  $\leftarrow$  plan.L  $\cup$   $\langle a_p - [\sigma(p_g)] \rightarrow a_g \rangle$   
  plan.B  $\leftarrow$  plan.B  $\cup$   $\sigma$ 
```

### PoP: Pseudo Code (1)

- **function PoP(*plan*, *agenda*)**
- **if *agenda.empty()* then return *plan***
- **(*a<sub>g</sub>*, *p<sub>g</sub>*)  $\leftarrow$  *agenda.selectOne()***
  - deterministic choice point
- ***agenda*  $\leftarrow$  *agenda* - (*a<sub>g</sub>*, *p<sub>g</sub>*)**
- ***relevant*  $\leftarrow$  *plan.getProviders(p<sub>g</sub>)***
  - finds all actions
    - either from within the *plan* or
    - from new instances of an operator
  - that have an effect that unifies with *condition*
- **if *relevant.empty()* then return failure**
- **(*a<sub>p</sub>*, *p<sub>p</sub>*,  $\sigma$ )  $\leftarrow$  *relevant.chooseOne()***
  - non-deterministic choice point
- ***plan.L*  $\leftarrow$  *plan.L*  $\cup$   $\langle a_p - [p] \rightarrow a_g \rangle$**
- ***plan.B*  $\leftarrow$  *plan.B*  $\cup$   $\sigma$** 
  - must succeed for elements of relevant

## PoP: Pseudo Code (2)

```
if  $a_p \notin plan.A$  then
   $plan.add(a_p)$ 
   $agenda \leftarrow agenda + a_p.preconditions$ 
   $newPlan \leftarrow plan$ 
  for each threat on  $\langle a_p - [p] \rightarrow a_g \rangle$  or due to  $a_p$  do
     $allResolvers \leftarrow threat.getResolvers(newPlan)$ 
    if  $allResolvers.empty()$  then return failure
     $resolver \leftarrow allResolvers.chooseOne()$ 
     $newPlan \leftarrow newPlan.refine(resolver)$ 
  return PoP( $newPlan, agenda$ )
```

### PoP: Pseudo Code (2)

- **if  $a_p \notin plan.A$  then**
  - if the action is new and needs to be added to the plan
- **$plan.add(a_p)$** 
  - involves updating set of actions and ordering constraints
- **$agenda \leftarrow agenda + a_p.preconditions$** 
  - all preconditions of the new action are new sub-goals
- **$newPlan \leftarrow plan$**
- **for each *threat* on  $\langle a_p - [p] \rightarrow a_g \rangle$  or due to  $a_p$  do**
  - note: two sources of threats are treated identically
- **$allResolvers \leftarrow threat.getResolvers(newPlan)$**
- **if  $allResolvers.empty()$  then return failure**
- **$resolver \leftarrow allResolvers.chooseOne()$** 
  - second non-deterministic choice point
- **$newPlan \leftarrow newPlan.refine(resolver)$** 
  - note: loop does not add to agenda
- **return PSP( $newPlan, agenda$ )**

## State-Space vs. Plan-Space Planning

- |   |  |
|---|--|
| <ul style="list-style-type: none"><li>• state-space planning<ul style="list-style-type: none"><li>– finite search space</li><li>– explicit representation of intermediate states</li><li>– action ordering reflects control strategy</li><li>– causal structure only implicit</li><li>– search nodes relatively simple and successors easy to compute</li></ul></li></ul> | <ul style="list-style-type: none"><li>• plan-space planning<ul style="list-style-type: none"><li>– infinite search space</li><li>– no intermediate states</li><li>– choice of actions and organization independent</li><li>– explicit representation of rationale</li><li>– search nodes are complex and successors expensive to compute</li></ul></li></ul> |
|---|--|

### State-Space vs. Plan-Space Planning

#### •state-space planning vs. plan-space planning

##### •finite search space vs. infinite search space

- important: portion of search space explored/generated; both search trees potentially infinite

##### •explicit representation of intermediate states vs. no intermediate states

- explicit representation allows for efficient domain specific heuristics and control knowledge

##### •action ordering reflects control strategy vs. choice of actions and organization independent

##### •causal structure only implicit vs. explicit representation of rationale

- important for plan execution

##### •search nodes relatively simple and successors easy to compute vs. search nodes are complex and successors expensive to compute

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