The Black-Scholes Price for a European put option is

$$P(S,t,K,T,r,q,\sigma) = Ke^{-r(T-t)}\Phi(-d_2) - Se^{-q(T-t)}\Phi(-d_1) \tag{1}$$

where

$$d_1 = \frac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T-t} = \frac{\log\left(\frac{S}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

Compute each of

(a) $\Delta(P) = \dfrac{\partial P}{\partial S}$

(b) $\Gamma(P) = \dfrac{\partial^2 P}{\partial S^2}$

(c) $\theta(P) = \dfrac{\partial P}{\partial t}$

(d) $\rho(P) = \dfrac{\partial P}{\partial r}$

by taking derivatives of (1). Verify that your answers are correct using put-call parity.

Example

$$\begin{aligned}
\text{vega}(P) = \frac{\partial P}{\partial \sigma} &= \frac{\partial}{\partial \sigma}\left[Ke^{-r(T-t)}\Phi(-d_2) - Se^{-q(T-t)}\Phi(-d_1)\right] \\
&= Ke^{-r(T-t)}\phi(-d_2)\frac{\partial}{\partial \sigma}(-d_2) - Se^{-q(T-t)}\phi(-d_1)\frac{\partial}{\partial \sigma}(-d_1)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial \sigma}(-d_1) &= \frac{\partial}{\partial \sigma}\left[\frac{-\log\left(\frac{S}{K}\right) - \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right] \\
&= \frac{\partial}{\partial \sigma}\left[\frac{-\frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}\right] \\
&= \frac{\partial}{\partial \sigma}\left[-\frac{\sigma}{2}\sqrt{T-t}\right] = -\frac{1}{2}\sqrt{T-t}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial \sigma}(-d_2) &= \frac{\partial}{\partial \sigma}(-d_1 + \sigma\sqrt{T-t}) \\
&= \frac{\partial}{\partial \sigma}(-d_1) + \sqrt{T-t} \\
&= \frac{1}{2}\sqrt{T-t}
\end{aligned}
$$

$$
\begin{aligned}
\text{vega}(P) = \frac{\partial P}{\partial \sigma} &= Ke^{-r(T-t)}\phi(-d_2)\frac{\partial}{\partial \sigma}(-d_2) - Se^{-q(T-t)}\phi(-d_1)\frac{\partial}{\partial \sigma}(-d_1) \\
&= \frac{1}{2}Ke^{-r(T-t)}\phi(-d_2)\sqrt{T-t} + \frac{1}{2}Se^{-q(T-t)}\phi(-d_1)\sqrt{T-t} \\
&= \frac{1}{2}\left[Ke^{-r(T-t)}\phi(-d_2) + Se^{-q(T-t)}\phi(-d_1)\right]\sqrt{T-t} \\
&= Se^{-q(T-t)}\phi(-d_1)\sqrt{T-t}
\end{aligned}
$$

Check result using the put-call parity formula:

$$
\begin{aligned}
P &= C - Se^{-q(T-t)} + Ke^{-r(T-t)} \\
\text{vega}(P) = \frac{\partial P}{\partial \sigma} &= \frac{\partial}{\partial \sigma}\left[C - Se^{-q(T-t)} + Ke^{-r(T-t)}\right] \\
&= \frac{\partial C}{\partial \sigma}
\end{aligned}
$$

The vega of a European put option is the same as the vega for a European call option (which is easy to find on the internet).