1. Consider using Lagrange's method to solve the following optimization problem.

$$\begin{aligned} \text{minimize:} \quad & 3x_1 - 4x_2 + x_3 - 2x_4 \\ \text{subject to:} \quad & -x_2^2 + x_3^2 + x_4^2 = 1 \\ & 3x_1^2 + x_3^2 + 2x_4^2 = 6 \end{aligned}$$

   Write down the Lagrangian and its gradient and explain why it is difficult to find a critical point.

2. Compute the gradient of the Lagrangian for the maximum expected return portfolio subject to risk = $(20\%)^2$.

3. Consider the Black-Scholes formula for the price of a European call option as a function of a single variable S (i.e., treat all the other inputs as constant). Write down a second order Taylor polynomial around S_0 for $C(S)$ in terms of Δ and Γ.

4. Compute the Taylor series expansion $T(x)$ of the function

$$f(x) = \frac{1}{1+x}$$

   around the point $a = 0$ and find its radius of convergence. Does $T(x) = f(x)$ on the domain of convergence?

5. Let $P_0, P_1, \ldots$ be a sequence of market prices for the same asset. The *arithmetic* return on the asset during period t is defined to be

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

   and the *log* return is defined to be

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right).$$

   Show that $r_t \approx R_t$ for small values of R_t (say on the order of 1%).

6. Use the integral form of the Taylor approximation error to find the order of the Taylor polynomial needed to compute $e^{0.25}$ to six digits of accuracy.