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Practical No. 1

Topic : Limit & Continuity

(1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right.$

$\lim_{x \to a} \frac{(a+2x-3x)\,(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x\,(\sqrt{a+2x} + \sqrt{3x})}$

$\lim_{x \to a} \frac{(a-x)\,(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)\,(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\frac{1}{3} \lim_{x \to a} \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}$

$= \frac{2}{3\sqrt{3}}$

EE :

③ $\lim_{Y\to 0}\left[\frac{\sqrt{a+Y}-\sqrt{a}}{Y\sqrt{a+Y}}\right]$

$\lim_{Y\to 0}\left(\frac{\sqrt{a+Y}-\sqrt{a}}{Y\sqrt{a+Y}}\times\frac{\sqrt{a+Y}+\sqrt{a}}{\sqrt{a+Y}+\sqrt{a}}\right)$

$\lim_{Y\to 0}\frac{a+Y-a}{Y\sqrt{a+Y}(\sqrt{a+Y}+\sqrt{a})}$

$=\frac{1}{\sqrt{a+0}(\sqrt{a+0}+\sqrt{a})}$

$=\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$=\frac{1}{\sqrt{a}(2\sqrt{a})}=\frac{1}{2a}$

④ $\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right]$

$\lim_{x\to\infty}\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$\lim_{x\to\infty}\frac{8}{2}\left(\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right)$

$\lim_{x\to\infty}\frac{4\left(\sqrt{x^2(1+\frac{3}{x^2})}+\sqrt{x^2(1+\frac{1}{x^2})}\right)}{\sqrt{x^2(1+\frac{5}{x^2})}+\sqrt{x^2(1-\frac{3}{x^2})}}$

$4\lim_{x\to\infty}\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}$

$=4$

⑤ Examine the Continuity

i) $f(x)=\frac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $a<x<\frac{\pi}{2}$

Sol^n: $f\left(\frac{\pi}{2}\right)=\frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2\left(\frac{\pi}{2}\right)}}$

$f\left(\frac{\pi}{2}\right)=0$

f at $\frac{\pi}{2}$ defined

ii) $\lim_{x\to\frac{\pi}{2}^+} f(x)=\lim_{x\to\frac{\pi}{2}^+}\frac{\cos x}{\pi-2x}$

$x-\frac{\pi}{2}=h$

$x=h+\frac{\pi}{2}$

where $h\to 0$

a) $\lim\limits_{h\to 0} \dfrac{\cos(h+\frac{\pi}{2})}{\pi-2(h+\frac{\pi}{2})}$

$\lim\limits_{h\to 0} \dfrac{\cos(h+\frac{\pi}{2})}{\pi-2h-\pi}$

$\lim\limits_{h\to 0} \dfrac{\cos(h+\frac{\pi}{2})}{-2h}$

$\lim\limits_{h\to 0} \dfrac{\cos h \cos\frac{\pi}{2} - \sin h \sin\frac{\pi}{2}}{-2h}$

$\lim\limits_{h\to 0} \cos h \; 0 - \dfrac{\sin h}{-2h}$

$\lim\limits_{h\to 0} \dfrac{-\sin h}{-2h}$

$=\frac{1}{2}$

(b) $\lim\limits_{x\to \pi/2} f(x) = \lim\limits_{x\to\pi/2} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim\limits_{x\to \pi/2} \dfrac{2\sin x\cos x}{\sqrt{2\sin^2 x}}$

$\lim\limits_{x\to \pi/2} \dfrac{2\sin x}{\sqrt{2}\sin x}$

$\lim\limits_{x\to \pi/2} \dfrac{2\cos x}{\sqrt{2}}$

$\dfrac{2}{\sqrt{2}} \lim\limits_{x\to\pi/2} \cos x$

L.H.S $\neq$ R.H.S

f is not Continuous at $x=\frac{\pi}{2}$

ii) $f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$ $\qquad$ at $x=3$ & $x=0$

$= x+3$ $\quad 3 \leq x < 6$

$= \dfrac{x^2-9}{x+3}$ $\quad 6 \leq x < 9$

$f(3) = \frac{x^2-9}{x-3} = 0$ at $x=3$

f at $x=3$ define

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}$

1. $f(3) = x+3 = 3+3 = 6$
f is define at $x=3$

2. $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} \cdot \frac{(x-3)(x+3)}{(x-3)}$

L.H.S = R.H.S
f is Continuous at $x=3$
for $x=6$

b) $f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

2. $\lim_{x\to 6^+} = \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

L.H.S ≠ R.H.S
function is not Continuous

6)
i) $\left.\begin{aligned} f(x) &= \frac{1-\cos 4x}{x^2} & x<0 \\ &= K & x=0 \end{aligned}\right\}$ at $x=0$

Soln: f is cts at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$

$2\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = K$

$2\lim_{x\to 0} \left(\frac{\sin^2 2x}{x}\right)^2 = K$

$2(2)^2 = K$

$K = 8$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$= k$ $\quad x = 0$ $\Big\}$ at $x = 0$

$= (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$
$= e = k$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \dfrac{\pi}{3}$
$= k$ $\quad x = \dfrac{\pi}{3}$ $\Big\}$ at $x = \dfrac{\pi}{3}$

$x - \dfrac{\pi}{3} = h$
$x = h + \dfrac{\pi}{3}$
$h \to 0$

$f(\frac{\pi}{3} + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$$\frac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{\pi - \pi - 3h}$$

$$= \frac{\dfrac{-4\tan h}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$= \dfrac{4}{3} \lim_{h\to 0} \dfrac{\tan h}{h} \lim_{h\to 0} \dfrac{1}{1-\sqrt{3}\tan h}$

$= \dfrac{4}{3} \dfrac{1}{1-3(0)}$

$= \dfrac{4}{3} \dfrac{1}{1} = \dfrac{4}{3}$

7)
i) $f(x) = \dfrac{1-\cos 3x}{x \tan x}$ $\quad x \neq 0$
$= 9$ $\quad x = 0$ $\Big\}$ at $x = 0$

$f(x) = \dfrac{1-\cos 3x}{x \tan x}$

$= \dfrac{2\sin^2 3/2\, x}{x \tan x}$

$$\frac{\dfrac{2\sin^2 \frac{3x}{2} \times x^2}{x^2}}{\dfrac{x \tan x}{x^2} \times x^2}$$

$$= 2\lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$2 \times \frac{9}{4} = \frac{9}{2}$$

$\lim_{x\to 0} f(x) = \frac{9}{2}$ $9 = f(0)$

$\therefore$ f is not at $x=0$

Redefine funct?

$$f(x) = \begin{cases} \frac{1-\cos 3x}{x \cdot \tan x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable at $x=0$

ii) $f(x) = \frac{[e^{3x}-1]\sin x^\circ}{x^2}$ $x \neq 0$ $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ at $x=0$

$= \frac{\pi}{6}$ $x=0$

at $x=0$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3\log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

f is cts at $x=0$

(8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x=0$

is Continuous at $x=0$

$\therefore$ f is Continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$$= \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$= \log e + 2\left(\frac{\sin x/2}{x}\right)^2$$

multiply with 2 on Num & Deno

$$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

2) $f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \frac{\pi}{2}$

$f(x)$ is cont at $x = \pi/2$

$$\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$\frac{2-1-\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$\frac{1-\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1-\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2\times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}}$$

3 $\lim_{x\to\pi/6}\left[\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}\right]$

Put $x \longrightarrow \pi/6 + h \qquad h \to 0$

$$\lim_{h\to 0} \frac{\cos(\pi/6+h)-\sqrt{3}\sin(\pi/6+h)}{-6h}$$

$$\lim_{h\to 0} \frac{(\cos \pi/6 \cos h - \sin \pi/6 \sin h) - \sqrt{3}(\sin \pi/6 \cos h + \cos \pi/6 \sin h)}{-6h}$$

$$\lim_{h\to 0} \frac{(\sqrt{3}/2 \cos h - 1/2 \sin h) - \sqrt{3}(1/2 \cos h + \sqrt{3}/2 \sin h)}{-6h}$$

$$\lim_{h\to 0} \left(\frac{\sqrt{3}/2 \cos h - 1/2 \sin h - \sqrt{3}/2 \cos h - 3/2 \sin h}{-6h}\right)$$

$$\lim_{h\to 0} \left(\frac{-4/2 \sin h}{-6h}\right)$$

$$\lim_{h\to 0} \left(\frac{-2\sin h}{-6h}\right)$$

$$\frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h}$$

$$\frac{1}{3}$$

P.8

Practical-02

Topic : Derivative

Q.1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x-a}$$

$$= \lim_{x\to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

Put $x-a=h$

$x = a+h$

as $x\to a$, $h\to a$

$$Df(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(\cancel{a}+h-\cancel{a})\tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

formula : $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

$\tan A - \tan B = \tan(A-B)(1+\tan A \tan B)$

$$= \lim_{h\to 0} \frac{(\cancel{a}-\cancel{a}-h)-(1+\tan a \tan(a+h))}{h\times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in \mathbb{R}$

ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Put $x-a=h$

$x = a+h$

as $x\to a$, $h\to 0$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

formula: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{\cancel{a}-\cancel{a}-h}{2}\right)}{h\times \sin a \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-\frac{1}{2}\times 2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)} = -\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

11 B.

iii) Sec x

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a, h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{\cancel{a}-\cancel{a}-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\frac{-h}{2}}{\cos a \cos(a+h) \times \frac{-h}{2}} \times \frac{-1}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= \cancel{+}\frac{1}{\cancel{2}} \times \cancel{2} \frac{\sin a}{\cos a \times \cos a}$$

$= \tan a \sec a$

Q2 If $f(x) = 4x + 1, \quad x \leq 2$

$= x^2 + 5 \quad x > 0$, at $x = 2$, then find function is differentiable or not:

Solution:

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - (4 \times 2 + 1)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - 9}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x - 8}{x + 2}$$

$$= \lim_{x \to 2} \frac{4(x-2)}{(x-2)} = 4$$

$Df(2^-) = 4$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$= 2 + 2 = 4$

$Df(2^+) = 4$

RHD = LHD

f is differentiable at $x = 2$

Q3) If $f(x) = 5x+7,\ x<3$
$= x^2+3x+1;\ x \geq 3$ at $x=3$, then find f is differentiable or not?

Solution:

RHD:

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3+3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6 = 9$$

$Df(3^+) = 9$

LHD: $Df(3^-)$

$$= \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{(5x+7)-19}{x-3}$$

$$= \lim_{x\to 3^-} \frac{5x-12}{x-3}$$

$$\lim_{x\to 3} \frac{5(x-3)}{(x-3)}$$

$Df(3^-) = 5$

RHD $\neq$ LHD

f is not differentiable at $x=3$

Q4) If $f(x) = 8x-5,\ x \leq 2$
$= 3x^2-4x+7,\ x>2$ at $x=2$, then find f is differentiable or not.

Solution:

$f(2) = 8\times 2 - 5 = 16-5 = 11$

RHD:

$$Df(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 3\times 2+2 = 8$

$Df(2^+) = 8$

LHD:

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 8$

$Df(2^-) = 8$

LHD = RHD

f is differentiable at $x=3$.

11/12/15

Practical-3

Topic: Application of Derivatives

1) Find the intervals in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards

i) $Y = 3x^2 - 2x^3$

ii) $Y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $Y = x^3 - 27x + 5$

iv) $Y = 69 - 24x - 9x^2 + 2x^3$

v) $Y = 2x^3 + x^2 - 20x + 4$

Solution:

Q1

i) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\because f$ is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x - 2) > 0$

$\therefore x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x + 2) - 5(x + 2) > 0$

$\therefore (x + 2)(3x - 5) > 0$

$+ \quad - \quad +$

$-2 \qquad 5/3$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2+2x-20<0$

$\therefore 2(3x^2+x-10)<0$

$\therefore 3x^2+x-10<0$

$\therefore 3x^2+6x-5x-10<0$

$\therefore (x+2)(3x-5)<0$

$+ \quad -2 \quad - \quad 5/3 \quad +$ $\qquad x \in (-2, 5/3)$

4) $f(x)=x^3-27x+5$

$f'(x)=3x^2-27$

$\because$ f is increasing iff $f'(x)>0$

$\therefore 3(x^2-9)>0$

$\therefore (x-3)(x+3)>0$

$+ \quad -3 \quad - \quad 3 \quad +$

$\therefore x \in (-\infty,3) \cup (3,\infty)$

and f is decreasing iff $f'(x)<0$

$\therefore 3x^2-27<0$

$\therefore 3(x^2-9)<0$

$\therefore (x-3)(x+3)<0$

$+ \quad -3 \quad - \quad 3 \quad +$ $\qquad \therefore x \in (-3,3)$

5) $f(x)=2x^3-9x^2-24x+69$

$f'(x)=6x^2-18x-24$

$\therefore$ f is increasing iff $f'(x)>0$

$\therefore 6x^2-18x-24>0$

$\therefore 6(x^2-3x-4)>0$

$\therefore x^2-4x+x-4>0$

$\therefore x(x-4)+1(x-4)>0$

$\therefore (x-4)(x+1)>0$

$+ \quad -1 \quad - \quad 4 \quad +$

$\therefore x \in (-\infty,-1) \cup (4,\infty)$

and f is decreasing iff $f'(x)<0$

$\therefore 6x^2-18x-24<0$

$\therefore 6(x^2-3x-4)<0$

$\therefore x^2-4x+x-4<0$

$\therefore x(x-4)+1(x-4)<0$

$\therefore (x-4)(x+1)<0$

$+ \quad -1 \quad 4 \quad +$

$\therefore x \in (-1,4)$

Q2)

1) $y=3x^2-2x^3$

$\therefore f(x)=3x^2-2x^3$

$\therefore f'(x)=6x-6x^2$

$\therefore f''(x)=6-12x$

f is concave upward iff $f''(x)>0$

$\therefore (6-12x)>0$

$\therefore 12(6/12-x)>0$

$\cancel{x-1/2>0}$

$x>1/2$

$\because f''(x)>0$

$\therefore x \in (1/2,\infty)$

[illegible]

6) [illegible]

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Practical :- 4

Q1

(i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now Consider

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum Value at $x = 2$

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum Value at $x = -2$

$\therefore$ Function reaches minimum

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider,

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$\therefore f(1) = -30 + 60$

$= 30 > 0$

$\therefore$ f has minimum Value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore$ f has maximum Value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore$ f has the maximum Value 5 at $x = -1$ and has the minimum Value at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum Value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum Value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum Value 1 at $x = 0$ and

f has minimum Value -3 at $x = 2$.

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum Value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore$ f has maximum Value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12\{-1\} + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum Value 8 at $x = -1$ and

f has minimum Value -19 at $x = 2$

TA

Q.2

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$\therefore x_1 = 0 + \frac{9.5}{55}$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \frac{0.0829}{55.9467}$

$= 0.1712$

$x_0 = 0 \rightarrow$ Given

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.1712 + \frac{0.0011}{55.9393}$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$ $[2,3]$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approxiation

$\therefore$ By Newton's Method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{6}{23}$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5095 - 4$

$= 18.5095$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.7392 - \frac{0.596}{18.5095}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 19.8386 - 10.8284 - 9$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9$

$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$

$= 21.8943 - 4$

$= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7015 + 0.0050$

$= 2.7065$

$f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17$

$= -2.2$

Let $x_0 = 2$ be initial approximation

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{2.2}{5.2}$$

$$= 2 - 0.4230$$

$$= 1.577$$

$$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$$

$$= 3.9219 - 4.4764 - 15.77 + 17$$

$$= 0.6765$$

$$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$$

$$= 7.4608 - 6.6772 - 10$$

$$= -8.2164$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.577 + \frac{0.6765}{8.2164}$$

$$= 1.577 + 0.0822$$

$$= 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$$

$$= 4.5677 - 4.9653 - 16.592 + 17$$

$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$

$$= 8.2588 - 5.99312 - 10$$

$$= -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.6592 + \frac{0.0204}{f'(x_2)}$$

$$= 1.6592 + \frac{0.0204}{7.7143}$$

$$= 1.6592 + 0.0026$$

$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$

$$= 4.5892 - 4.9708 - 16.618 + 17$$

$$= 0.0004$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$

$$= 8.2847 - 5.9824 - 10$$

$$= -7.6977$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.8618 + \frac{0.0004}{7697}$$

$$= 1.6618$$

$\therefore$ The root of equation is 1.6618

Practical. 4

Topic : Application of derivative & Newton's Method

Q1) Find maximum & minimum Value of following

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1 \quad [-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$

Q2) Find the root of the following equation by Newton's (Take 4 st iteration only) Correct upto 4 decimal

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

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Q1)

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now Consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$\therefore 2x = 32/x^3$

$\therefore x^4 = 32/2$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum Value at $x = 2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum Value at $x = -2$

$\therefore$ Function reaches minimum Value at $x = 2$, and $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore 15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore$ f has minimum Value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore$ f has maximum Value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has maximum Value 5 at $x = -1$ and has the minimum Value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$ $\therefore$ f has maximum Value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum Value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore f$ has maximum Value 1 at $x = 0$ and f has minimum Value -3 at $x = 2$.

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore f$ has minimum Value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore f$ has maximum Value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has maximum Value 8 at $x = -1$ and f has minimum Value -19 at $x = 2$

Q2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$\therefore x_1 = x_0 - f(x_0) / f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's Method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 6.596/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7671)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9 = -0.0901$

$f(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0056$

$= 2.7065$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = \cancel{8-7} \cdot 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation. By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 2 - 2.2/5.2$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 6.6755$

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$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.577 + 0.0822$
$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2)/f'(x_2)$
$= 1.6592 + 0.0204/7.7143$
$= 1.6592 + 0.0026$
$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$
$= 1.618 + \frac{0.0004}{7.6977}$
$= 1.6618$

Practical - 5

Topic : Integration

Q1) Solve the following integration

i) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

iv) $\int \dfrac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int t^7 \sin(2t^4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right)dx$

viii) $\int \dfrac{\cos x}{3\sqrt{\sin^2 x}}\,dx$

ix) $\int e^{\cos^2 x} \sin 2x\,dx$

x) $\int \left(\dfrac{x^2-2x}{x^3-3x^2+1}\right)dx$

1) $\int \frac{1}{x^2+2x-3}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

※ $a^2+2ab+b^2=(a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$

Substitute

Put $x+1=t$

$dx = \frac{1}{t} \times dt$ where $t=1$ $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}}\,dt$

using

※ $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+C$

2) $\int(4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$ ※ $\int e^{\lambda x}\,dx = \frac{1}{\lambda} \times e^{\lambda x}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$ ※ $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3+10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4\,dx}{\sqrt{x}}$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

※ Split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}dx + \int 3x^{1/2}dx + \int \frac{4}{x^{1/2}}\,dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5 $\int t^7 \times \sin(2t^4)\,dt$

Put $4 = 2t^4$

$d4 = 2\times4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times4t^3}\,d4$

$= \int t^4 \sin(2t^4) \frac{x\,1}{2\times4}\,d4$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,d4 = \frac{t^4 \times \sin(2t^4)}{8}\,d4$

Substitute t^4 with $\frac{4}{2}$

$= \int \frac{4/2 \times \sin(1+4)}{8}\,d4$

$= \int \frac{4\times\sin(4)}{2} / 8\,d4$

$= \int \frac{4\times\sin(4)}{16}\,d4$

$= \frac{1}{16} \int 4\times\sin(4)\,d4$

※ $\int 4\,dv = 4v - \int v\,d4$

where $4 = 4$

$dv = \sin(4) \times d4$

$d4 = 1d4 \quad V = -\cos(4)$

$= \frac{1}{16} \quad (4\times(-\cos(4)) - \int -\cos(4)\,d4$

$= \frac{1}{16} \quad \times (4\times(-\cos 4)) + \int \cos(4)\,d4$

※ $\int \cos x\,dx = \sin(x)$

$= \frac{1}{16} \quad \times (4\times(-\cos(4)) + \sin(4))$

Return the Satisfied $4 = 2+4$

$= 1/16 \quad \times (2t^4 \times (-\cos(2+4)) + \sin(2+4))$

$= -\frac{t^4 \times \cos(2+4)}{8} + \frac{\sin(2+4)}{16} + C$

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vi) $\int \sqrt{x}(x^2-1)dx$

$= \int \sqrt{x}x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \times x^2 - x^{1/2} dx$

$= \int x^{5/2} - x^{1/2} dx$

$= \int x^{5/2} dx - \int x^{1/2} dx$

$= I_1 \cdot \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$

iii) $\int \frac{\cos x}{3\sqrt{\sin(x)^2}} dx$

$= \int \frac{\cos x}{\sin x^{2/3}} dx$

Put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{2/3}} \times \frac{1}{\cos(x) dt}$

$= \frac{1}{\sin x^{2/3}} dt$

$= \frac{1}{t^{2/3}} dt$

$I = \int \frac{1}{t^{2/3}} dt = \frac{-1}{(2/3-1)t^{2/3-1}} =$

$= \frac{-1}{-1/3 t^{2/3-1}} = \frac{1}{1/3 t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

Return Substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + C$

(x) $\int \frac{x^2-2x}{x^3-3x^2+1} dx$

Put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x} dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x} dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)} dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3} dt$

$= \int \frac{1}{3(x^3-3x^2+1)} dt = \int \frac{1}{3t} dt$

$= 1/3 \int 1/t \, dt$ $\quad \int 1/x \, dx = \ln|x|$

$= 1/3 \times \ln|t| + C$

$= 1/3 \times \ln(|x^3 - 3x^2 + 11|) + C$

7) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

let $\frac{1}{x^2} = t$

$x^{-2} = t$

$\frac{-2}{x^3} dx = dt$

$I = \frac{-1}{2} \int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= \frac{-1}{2} \int \sin t$

$= \frac{-1}{2}(-\cos t) + C$

$= \frac{1}{2} \cos t + C$

Resubstitution $t = 1/x^2$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + C$

8) $\int \frac{\cos x}{3\sqrt{\sin^2 x}}$

$I = \int \frac{\cos x}{3\sqrt{\sin^2 x}}$

let $\sin x = t$

$\cos x \, dx = dt$

$I = \int \frac{dt}{3\sqrt{t^2}}$

$= \int \frac{dt}{t^{2/3}}$

$= \int t^{-2/3} dt$

$= 3t^{1/3} + C$

$= 3(\sin x)^{1/3} + C$

$= 3 \cdot \sqrt[3]{\sin x} + C$

9) $\int \cos^2 x \cdot \sin^2 x \, dx$

$I \int \cos^2 x \sin^2 x \, dx$

$10 + \cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$-2\sin 2x \, dx = dt$

$I = \int -\sin 2x \cos^2 x \, dx$

$= -\int e^{t} dt$

$= e^t + C$

Resubstituting $t = \cos^2 x$

$I = e^{\cos^2 x} + C$

AK 01/01/2022

Practical: 6

Topic:- Application of integration & Numerical integration

Q1) Find the length of the following curve

i) $x = t - \sin t$ $\quad y = 1 - \cos t$ $\quad [0, 2\pi]$
for t belong to $[0, 2\pi]$

Sol^n:- $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t - \sin t$

$\frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = 0 - (-\sin t)$

$\frac{dy}{dt} = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2}\, dt$

$\int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$\int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$\int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt \quad \therefore \sin^2 \frac{t}{2} = \frac{1 - \cos t}{2}$

$\int_0^{2\pi} 2 \sin \frac{t}{2}\, dt$

$\left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$

$= 4 + 4$

$= 8.$

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Q.1) $Y = \sqrt{4-x^2}$ $\quad x \in [-2,2]$

Sol^n $2 = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$\frac{dy}{dx} = 2\int_0^2 \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$$

$$= 2\pi$$

3) $Y = x^{3/2}$ $\ln [0,4]$

Sol^n $f'(x) - \frac{3}{2}x^{1/2}$

$$[f'(x)]^2 = \frac{9}{4}x$$

$$l = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$$

$$= \int_0^x \sqrt{1+\frac{9}{4}x}\, dx$$

Put $u = 1+\frac{9}{4}x$, $du = \frac{9}{4}dx$

$$l = \int_1^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u}\, du = \left[\frac{4}{9}\cdot\frac{2}{3}(u^{3/2})\right]^{1+\frac{9}{4}x}$$

$$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right) - 1\right]$$

4) $x = 3\sin t$ & $Y = 3\cos t$

Sol^n $\frac{dx}{dt} = 3\cos t$

$$\frac{dY}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dY}{dt}\right)^2 + \left(\frac{dx}{dt}\right)}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\, dt$$

$$= 3\int_0^{2\pi} x\, dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3[2\pi - 0]$$

$$L = 6\pi \text{ units}$$

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5) $x = \frac{1}{6} y^3 + \frac{1}{2y}$ on $y: [1,2]$

Sol^n $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$L = \int \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2 dy}{(2y)^2}}$

$= \int_1^2 \frac{y^4+1}{2y^2} dy$

$= \frac{1}{2} \int_1^2 y^2 dy + \frac{1}{2} \int_1^2 y^{-2} dy$

$= \frac{1}{2} \left[\frac{y^3}{3} - \frac{y^{-1}}{1} \right]_1^2$

$= \frac{1}{2} \left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1 \right]$

$= \cancel{\frac{1}{2} \left[\frac{7}{3} - \frac{1}{2} \right]}$

$= \frac{17}{12}$ units

1) $\int_0^2 e^{x^2} dx$ with $n = 4$

Sol^n $\int_0^2 e^{x^2} dx = 16.4526$

In each case the width of the subinterval be $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

and so the subintervals will be $[0, 0.5]$ $[0.5, 1]$ $[1, 1.5]$ $[1.5, 2]$

By Simpson rule

$\int_0^2 e^{x^2} dx = \frac{1/2}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$

$= \frac{1/2}{3} (e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}$

$= 7.3536$

2) $\int_0^4 x^2 dx \; n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$\int_0^4 f(x) dx = \frac{\Delta x}{3} [y_0 + 4y_1 + 2y_2 + 4y_3 + y_4]$

$= \frac{1}{3} [y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2]$

$= \frac{1}{3} [0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2]$

$= \frac{64}{3}$

3) $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ $n = 6$

Sol^n $\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/12$	$2\pi/12$	$3\pi/12$	$4\pi/12$	$5\pi/12$
y	0	0.4167	0.524	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$\int_0^{\pi/3} \sqrt{\sin x}\,dx = \frac{\Delta x}{3}\left[(y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

$= \frac{\pi/12}{3}\left(0 + 4(0.4167 + 0.707 + 0.875) + 0.930\right)$

$= 2(0.624 + 0.801) + 0.930$

$= 0.621$

Q1 Solve the following differential equations

1) $x\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $Q(x) = \frac{e^x}{x}$

$IF = e^{\int 1/x\,dx}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{e^x}{x}\,x\,dx + c$

$= \int e^x\,dx + c$

$\therefore xy = e^x + c$

2) $e^x\frac{dy}{dx} + 2e^x y = 1$

Sol^n $\frac{dy}{dx} + \frac{2e^x}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $Q(x) = e^{-x}$

$\int P(x)\,dx$

$I = e^{\int 2\,dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int e^{x}\,dx + C$

$= y \cdot e^{2x}$

$= e^{x} + C$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

Solⁿ:- $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \quad Q(x) = \frac{\cos x}{x^2}$

$IF = e^{\int P(x)dx}$

$= e^{\int 2x\,dx}$

$y(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int \frac{\cos x}{x^2} \cdot x^2\,dx + C$

$= \int \cos x + C$

$\therefore x^2 y = \sin x + C$

4) $x \cdot \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Solⁿ:

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$P(x) = \int 3/x\,dx$

$= x^3$

$IF = e^{\int P(x)dx}$

$= x^3$

$y(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + C$

$= \int \sin x + C$

$x^3 y = -\cos x + C$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

Solⁿ: $\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2$

$Q(x) = \frac{2x}{e} = 2xe^{-2x}$

$(IF) = e^{\int P(x)dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int 2x\, e^{-2x}\, e^{2x} + C$

$ye^{2x} = \int 2x + C = x^2 + C$

6) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \tan x \, dy$

$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 x}{\tan y} dy$

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x + \tan y| = c$

$\tan x + \tan y = e^c$

7) $\frac{dy}{dx} = \sin^2(x - y + 1)$

Put $x - y + 1 = v$

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan(x - y + 1) = x + c$

8) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

Put $2x + 3y = v$

$2 + \frac{3dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$= \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{(v+2)}$

$\int \frac{v+2}{v+1} dv = 3dx$

$= \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = \int 3dx$

$v \log|v| = 3x + c$

$2x + 3y + \log|2x + 3y + 1| = 3x + c$

$3y = x - \log|2x + 3y + 1| + c$

Practical-8

TOPIC: Euler's Method

Q1. $\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

Soln:
$f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$
0	0	2	1
1	0.5	2.5	2.1487
2	1	3.5743	4.2925
3	1.5	5.7205	8.2021
4	2	9.8215	

$\therefore y(2) = 9.8215$

Q2. $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$y_0 = 0$, $y_0 = 0$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

Q3 $\frac{dy}{dx} = \frac{\sqrt{x}}{y}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1.0894	0.4472	1.0894
2	0.4	1.2105	6.7050	1.2105
3	0.6	1.2105	8.7040	1.3513
4	0.8	1.3513	6.7696	1.5051
5	1	1.5051		

$y(1) = 1.6051$

Q4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$ $h = 0.5$
$h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	7.754	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

Q5) $Y_0 = 2$ $X_0 = 1$ $h = 0.25$

n	X_n	Y_n	$f(x_n, Y_n)$	Y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$Y(2) = 299.9960$

Q6) $\frac{dy}{dx} = \sqrt{x}Y + 2$ $y(1) = 1$ $h = 0.2$

$x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	X_n	Y_n	$f(x_n Y_n)$	Y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$Y(1.2) = 3.6$

AV

Practical- 9

i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{-4(-1) + 5}$

$= \frac{-64 + 3 + 1 - 1}{4 + 5}$

$= \frac{-61}{9}$

ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(0+1)((2)^2 + 0 - 4(2))}{2+0}$

$= \frac{1(4 + 0 - 8)}{2}$

$= \frac{-4}{2}$

$= -2$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2}{x^3-x^2y}$

At (1,1,1), Denominator = 0

$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

$\lim_{(x,y,z)\to(1,1,1)} \frac{\cancel{(x-yz)}(x+yz)}{x^2(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

On Applying limit

$= \frac{1+1(1)}{(1)^2}$

$= 2$

Q.2

(i) $f(x,y) = xye^{x^2+y^2}$

$\therefore f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= \cancel{ye^{x^2+y^2}}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

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ii) $f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$= f_y = -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_y = 2x^3y - 3x^2 + 3y^2$

Q3)

(i) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\partial}{\partial x}(2x - 2x\frac{\partial}{\partial x}(1+y^2))}{(1+y^2)^2}$

$= \frac{2+2y^2-0}{(1+y^2)^2}$

$$= \frac{2(1+y^2)}{(1+y^2)(1+y^2)}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$.

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

(4)

(1) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (x^2 y - 2xy^2 + 2x^2 y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy^2 + 2x^2 y)}{x^8} \quad (1)$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \frac{\partial}{\partial x}(2y - x) - (2y - x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

from (3) & (4);

$f_{xy} = f_{yx}$

(ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 0 + 6x^2y - 0$

$= 6x^2y$

$f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1\,\frac{\partial(2x)}{\partial x} - 2x\,\frac{\partial(x^2+1)}{\partial x}}{(x^2+1)^2}\right)$

$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$ —— (1)

$f_{yy} = \frac{\partial}{\partial y}(6x^2y)$

$= 6x^2$ —— (2)

$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy$ —— (3)

$f_{yx} = \frac{\partial}{\partial x}(6x^2y)$

$= 12xy$

from (3) & (5),

$\therefore f_{xy} = f_{yx}$ —— (5)

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$\therefore f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$ —— (1)

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) - e^{x+y}$ —— (2)

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ —— (3)

$f_{yx} = \frac{\partial}{\partial x}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ —— (5)

$\therefore$ from (3) & (5)

$f_{xy} \neq f_{yx}$

Q5

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\quad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad = \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\quad f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{2}y - \cancel{\frac{2}{\sqrt{2}}}$

$= \frac{x+y}{\sqrt{2}}$

$f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$ $\quad f_y = 0 - 0 + \sin x$

f_x at $(\frac{\pi}{2}, 0) = -1 + 0 = -1$ $\quad f_y$ at $(\frac{\pi}{2}, 0) = \sin\frac{\pi}{2} = 1$

$(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)(x - \frac{\pi}{2}) + 1(y-0)$

$= 1 - \cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$

$= 1 - x + y$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\quad f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\quad f_y$ at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

Practical 10

Q1)

(1). $f(x,y) = x+2y-3 \quad a = (1,-1) \quad u = 3i-j$

Here $u = 3i-j$ is not unit Vector

$|u| = \sqrt{3^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit Vector along is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3i-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1)+h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1)+2(-1)-3 = 1-2-3 = -4$

$f(a+hu) = f\left((1,-1)+h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$

$f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right)+2\left(-1-\frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right)+2\left(-1\ \frac{h}{\sqrt{10}}\right)-3$

$= 1+\frac{3}{\sqrt{10}} - 2 \ \frac{-2h}{\sqrt{10}} - 3$

$f(a+hu) = -4+\frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$

$= \lim_{h\to 0} : \frac{-4+h/\sqrt{10}+4}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$

ii) $f(x) = y^2 - 4x+1 \quad a = (3,4) \quad u = i+5j$

Here $u = i+5j$ is not a unit Vector

$|u| = \sqrt{(1)^2+(5)^2} = \sqrt{26}$

Unit Vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2-4(3)+1 = 5$

$f(a+hu) = f(3,4)+h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f: \left(3+\frac{h}{\sqrt{26}}, 4+\frac{5h}{\sqrt{26}}\right)$

$f_{x,y}(a+hu) = \left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right)+1$

$= 16+\frac{25h^2}{26}+\frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}}+1$

$= \frac{25h^2}{26}+\frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}}+5$

$= \frac{25h^2}{26}+\frac{40h-4h}{\sqrt{26}}+5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$\therefore D_u f(a) = \lim \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + \cancel{5} - \cancel{5}}{h}$

$\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)$

$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

iii) $2x + 3y \quad a = (1, 2).\ u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit Vector

$|\vec{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

Unit Vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$f(a+hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + \cancel{8} - \cancel{8}}{\cancel{h}}$

$= \frac{18}{5}$

2) Find gradient Vector for the following funct^n at given Point

i) $f(x,y) = x^y + y^x \quad a = (1,1)$

$f_x = yx^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$\therefore f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y,\ x^y \log x + xy^{x-1})$

$f(1,1) = (1+0,\ 1+0)$

$= (1, 1)$

ii) $f(x,y) = (\tan^{-1} x)\, y^2 \qquad a = (1, -1)$

$f_x = \frac{1}{1+x^2}\, y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2},\ 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2},\ \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2},\ \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = \left((-1)(0) - e^{(1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0}\right)$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1,-1,-2)$

Q3) Find the equation of tangent & normal to each of the following using curves at given points

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} x$

$(x_0, y_0) = (1,0)$ ∴ $x_0 = 1$ $y_0 = 0$

eqn tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0,y_0) = \cos 0 \, 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0,y_0) = (1)^2(-\sin 0) + e^0 1$

$= 0 + 1.1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ It is the required eqn of tangent

eqn of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(x) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$1 + 2(0) + d = 0$

$d + 1 = 0$

∴ $d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2)$ ∴ $x_0 = 2, y_0 = -2$

$f_x(x_0,y_0) = 2(2) - 2 = 2$

$f_y(x_0,y_0) = 2(-2) + 3 = -1$

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2)+(-1(y+2))=0$

$2x-2-y-2=0$

$2x-y-4=0 \longrightarrow$ It is required eqn of tangent

eqn of Normal

$= ax+by+c=0$

$bx+ay+d=0$

$=-1(x)+2(y)+d=0$

$-x+2y+d=0$ at $(2,-2)$

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$\therefore d=6$

Q4) Find the eqn of tangent & normal line to each of the following Surface:

i) $x^2-2yz+3y+xz=7$ at $(2,1,0)$

$f_x=2x-0+0+z$

$f_x=2x+z$

$f_y=0-2z+3+0$

$=2z+3$

$f_z=0-2y+0+x$

$=-2y+x$

$(x_0,y_0,z_0)=(2,1,0)$ $\therefore x_0=2, y_0=1, z_0=0$

$f_x(x_0,y_0,z_0)=2(2)+0=4$

$f_y(x_0,y_0,z_0)=2(0)+3=3$

$f_z(x_0,y_0,z_0)=-2(1)+2=0$

eqn of tangent

$f_x(x_0-x_0)+f_y(y_0-y_0)+f_z(z-z_0)=0$

$=4(x-2)+3(y-1)+0(z-0)=0$

$=4x-8+3y-3=0$

$4x+3y-11=0 \longrightarrow$ This required eqn of tangent

Eqn of normal at $(4,3,-11)$

$$\frac{x-x_0}{f_x}=\frac{y-y_0}{f_y}=\frac{z-z_0}{f_z}$$

$$=\frac{x-2}{4}=\frac{y-1}{3}=\frac{z+11}{0}$$

ii) $3xyz-x-y+z=-4$ at $(1,-1,2)$

$3xyz-x-y+z+4=0$ at $(1,-1,2)$

$f_x=3yz-1-0+0+0$

$=3yz-1$

$f_y=3xz-0-1+0+0$

$=3xz-1$

$f_z=3xy-0-0+1+0$

$=3xy+1$

$(x_0,y_0,z_0)=(1,-1,2)$ $\therefore x_0=1, y_0=-1, z_0=2$

$f_x(x_0,y_0,z_0)=3(-1)(2)-1=-7$

$f_y(x_0,y_0,z_0)=3(+1)(2)-1=5$

$f_z(x_0,y_0,z_0)=3(1)(-1)+1=-2$

Eqn of normal at (-7, 5, -2)

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q5. Find the local maxima & minima for the following

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2 \longrightarrow$ 1

$f_y = 0$

$2y - 3x - 4 = 0$
$2y - 3x = 4 \longrightarrow$ 2

Multiply eqn 1 with 2

$4x = 2\cancel{y} = -4$
$\cancel{2y} - 3x = 4$

$x = 0$

Substitute value of x in eqn 1

$2(0) - y = -2$
$-y = +2$
$\therefore y = 2$

$\therefore$ Critical points are (0, 2)

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

$\therefore$ f has maximum at (0, 2)

$3x^2 + y^2 - 3xy + 6x - 4y$ at (0, 2)
$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$0 + 4 - 0 + 0 - 8$
$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$
$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$
$2x(4x^2 + 3y) = 0$
$4x^2 + 3y = 0 \longrightarrow$ 1

$f_y = 0$

$3x^2 - 2y = 0 \longrightarrow$ 2

Multiply eq^n ① with 3
② with 4

ii)

$12x^2 + 9y = 0$
$-12x^2 - 8y = 0$
$11y = 0$
$\therefore y = 0$

Substitute Value of y in eq^n ①
$4x^2 + 3(0) = 0$
$4x^2 = 0$
$x = 0$

Critical Point is $(0,0)$
$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 - 2 = -2$
$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$
$= 24(0) + 6(0) = 0$
$\therefore r = 0$
$rt - s^2 = 0(-2) - (s)^2$
$= 0 - 0 = 0$
$r = 0$ & $rt - s^2 = 0$
(nothing to say)

$f(x,y)$ at $(0,0)$
$2(0)^4 + 3(0)^3(0) - (0)$
$= 0 + 0 - 0$
$= 0$

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$
$f_x = 2x + 2$
$f_y = -2y - 8$
$f_x = 0 \quad \therefore 2x + 2 = 0$
$x = \frac{-2}{2} \quad \therefore x = -1$
$f_y = 0 \quad -2y + 8 = 0$
$y = \frac{-8}{-2}$
...

Critical Point is $(-1, 4)$

$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$
$= -4 < 0$
$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$
$1 + 16 - 2 + 32 - 70$
$= 17 + 30 - 70$
$= 37 - 70$
$= \underline{33}$

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