HW 10 - proofs

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Problem 1

Theorem

Prove that if A and B are finite sets then there is an injection from A to B if and only if $|A| \leq |B|$.

Proof: Let A + B be finite sets

Part 1:

Assume there is an injection from A to B
Then, no two element from set A will map to the same element in set B by way of the injection.
This means there is a one-to-one relation between values in set A and values in set B.
In order to be a function, all elements in A have to map to one value in B
Therefore, the number of elements in B have to be at least the number of elements in A, to maintain the 1-1 relationship
so, $|B| \geq |A|$. It's greater than or equal to because not all elements of B have to be mapped to from an element in A.

Part 2:

Choose $m = |A|$ and $n = |B|$.
If $m \leq n$, then all the elements in A can map to unique elements of B (meaning no other element of A also maps to that element in B).

$\therefore$ The theorem is true.

$\square$

Problem 2

Theorem

Prove that if A and B are finite sets then there is an surjection from A to B if and only if $|A| \geq |B|$.

Proof: Let A + B be finite sets.

Part 1: Assume there is an surjection from A to B
Then, all the elements in B have an element in A that map to it

Then, all the elements in B have an element in A that map to it

In order to be a function, all elements in A have to map to one value in B

so, $|A|$ + $|B|$ have to be at least equal

A surjection does not require an element in B to only have only element in A to map to it.

Therefore, the number of elements in B can be smaller than $|A|$ since two values in A can map to the same B.

$|B| \not> |A|$ because an element in A can only map to one el. in B.

so, $|B| \leq |A|$.

Part 2: Choose $m = |A|$ and $n = |B|$.

If $m \geq n$, then all the elements in A maps to some one element in B, with overlap (meaning 2 elements in A can map to the same element in B). All the while, each in every element in B has an element in A that maps to it.

$\therefore$ Theorem proven

D.

Problem 3

Theorem

Prove that if A and B are finite sets then there is a bijection from A to B if and only if $|A| = |B|$.

Proof Let A and B be finite sets.

Part 1:

If there is a bijection from A to B, then there is also an injection and a surjection.

A surjection from A to B occurs only if $|A| \geq |B|$.

An injection from B to A occurs only if $|B| \geq |A|$.

so for both an injection and surjection to occur,

$|A| = |B|$.

Part 2:

If $|A| = |B|$, then all the elements in A can map to an element in B.

If all the elements in A map to an unique, independent element in B, then there will be a 1-1 relationship as well as a surjection.

$\therefore$ Theorem true

$\square$