

HW 2

Wednesday, September 4, 2019 4:41 PM

Problem 1.

		$b + d$				
		a	b	c	d	e
P	q	$p \vee q$	$p \wedge q$	$p \rightarrow (p \vee q)$	$p \rightarrow (p \wedge q)$	$(p \wedge q) \rightarrow q$
F	F	F	F	T	T	T
F	T	T	F	T	T	F
T	F	T	F	T	F	F
T	T	T	T	T	T	T

Problem 2.

a. If it's raining, then it's cold

$$R \rightarrow C$$

N = $R \wedge \bar{C} \Rightarrow$ It's raining, and it's not cold.

C = $\bar{C} \rightarrow R \Rightarrow$ If it's not cold, then it's not raining.

b. If it's cold, then it's raining

$$C \rightarrow R$$

N = $C \wedge \bar{R} \Rightarrow$ It's cold, and it's not raining

C = $\bar{R} \rightarrow \bar{C} \Rightarrow$ If it's not raining, then it's not cold.

c. If it's raining or it's windy, then it's cold.

$$(R \vee W) \rightarrow C$$

N = $(R \vee W) \wedge \bar{C} \Rightarrow$ It's raining or it's windy, but it's not cold.

C = $\bar{C} \rightarrow (\bar{R} \wedge \bar{W}) \Rightarrow$ If it's not cold, then it's not raining and it's not windy.

d. If it's raining, then it's cold & windy.

$$R \rightarrow (C \wedge W)$$

N = $R \wedge (\bar{C} \vee \bar{W}) \Rightarrow$ It's raining and it's either not cold or it's not windy.

C = $(\bar{C} \vee \bar{W}) \rightarrow \bar{R} \Rightarrow$ If it's either not cold or not windy, then it's not raining.

e. If $n \geq 2$, then $n^2 \geq 4$

$$P \rightarrow Q$$

N = $P \wedge \bar{Q} \Rightarrow$ $n \geq 2$, but $n^2 < 4$.

C = $\bar{Q} \rightarrow \bar{P} \Rightarrow$ If $n^2 < 4$, then $n < 2$.

f. If $n^2 \geq 4$, then $n \geq 2$

$$Q \rightarrow P$$

$$N = Q \wedge \overline{P} \Rightarrow n^2 \geq 4, \text{ but } n < 2$$

$$C = \overline{P} \rightarrow \overline{Q} \Rightarrow \text{if } n < 2, \text{ then } n^2 < 4$$

Problem 3.

a. 1. $(F \vee M) \rightarrow C$

2. \overline{F}

conc: \overline{C}

not valid bc you can still pass both midterms, and pass the class.

F	M	C	FVM	$(F \vee M) \rightarrow C$	\overline{F}	\overline{C}
0	0	0	0	1	1	1
0	0	1	0	1	0	0
0	1	0	1	1	1	1
0	1	1	1	1	0	0
1	0	0	1	0	0	0
1	0	1	1	1	0	0
1	1	0	1	0	0	0
1	1	1	1	1	0	0

b. 1. $(F \vee M) \rightarrow C$

2. $\overline{M} \rightarrow F$

conc: C

valid bc statement 2 ensures tautology.

F	M	C	FVM	$(F \vee M) \rightarrow C$	\overline{F}	$\overline{M} \rightarrow F$
F	F	F	F	T	T	F
F	F	T	F	T	T	F
F	T	F	T	F	F	T
F	T	T	T	F	T	T
T	F	F	F	T	T	T
T	F	T	F	T	T	T
T	T	F	T	T	T	F
T	T	T	T	T	F	T

c. 1. $(F \vee M) \rightarrow C$

2. \overline{C}

conc: \overline{M}

valid

d. 1. $(F \vee M) \rightarrow C$

2. \overline{C}

conc: $\overline{F} \wedge \overline{M}$

valid

F	M	C	FVM	$(F \vee M) \rightarrow C$	\overline{F}	\overline{C}
F	F	F	F	T	T	T
F	F	T	F	T	T	F
F	T	F	T	F	F	T
F	T	T	T	T	F	F
T	F	F	F	T	T	T
T	F	T	F	T	T	F
T	T	F	T	T	F	F
T	T	T	T	T	F	T

e. 1. $S \rightarrow F$

2. $F \rightarrow C$

conc: $S \rightarrow C$

Valid bc modus ponens.

f. 1. $s \rightarrow F$

2. $F \rightarrow C$

concl: $C \rightarrow S$

valid bc contrapositive of (e).

Problem 4.

a) All of you will pass

$$\forall s, P(s) \rightarrow \exists s, \overline{P(s)}$$

b) Everyone will pass.

$$\forall s, P(s) \rightarrow \exists s, \overline{P(s)}$$

c) Some of you will pass.

$$\exists s, P(s) \rightarrow \forall s, \overline{P(s)}$$

d) At least one of you will pass.

$$\exists s, P(s) \rightarrow \forall s, \overline{P(s)}$$

e) Some of you will not pass.

$$\exists s, \overline{P(s)} \rightarrow \forall s, P(s)$$

f) Not everyone will pass.

$$\exists s, \overline{P(s)} \rightarrow \forall s, P(s)$$

g) Everyone won't pass

$$\forall s, \overline{P(s)} \rightarrow \exists s, P(s)$$

Answer:

a+b are equivalent
c+d are equivalent
e+f are equivalent

a+b negate e+f
c+d negate g