HW 3a

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1. $\forall n \in \mathbb{Z}$, n even $\to n^2$ even

↳ For every integer, if n is even, so is n^2.

a. Every even integer has an even square. ⇒ if n is even, then n^2 is even.

$(\forall n \in \mathbb{Z}$ st. n is even$) \to n^2$ is even

can be rewritten as:

$\forall n \in \mathbb{Z}$, n even $\to n^2$ even

$\boxed{\text{Equivalent}}$

b. Some integers have even squares

$\exists\, n \in \mathbb{Z}$ st. n^2 even

$\boxed{\text{Not equivalent}}$

c. For every integer n, n^2 is even whenever n is even

$\forall n \in \mathbb{Z}$, n even $\to n^2$ even

$\boxed{\text{Equivalent}}$

d. Whenever n^2 even, n even

$\forall n$, n^2 even $\to$ n even

Since the def of even established in class says evens are always integers:

$\forall n \in \mathbb{Z}$, n^2 even $\to$ n even

$\boxed{\text{Not equivalent}}$

e. Whenever n is even, n^2 is even

$\forall n$, n even $\to$ n^2 even

since the def of even established in class says evens are always integers:

$\forall n \in \mathbb{Z}$, n even $\to$ n^2 even

$\boxed{\text{Equivalent}}$

f. n^2 even for every even n

($\forall n$ st. n even) $\to$ n^2 even

since the def of even established in class says evens are always integers:

$\forall n \in \mathbb{Z}$, n even $\to$ n^2 even

$\boxed{\text{Equivalent}}$

2. $\forall n \in \mathbb{Z}$, $\exists m \in \mathbb{Z}$ st $m > n$

a. For every integer, there is a strictly larger integer

$\forall n \in \mathbb{Z}$, $\exists m \in \mathbb{Z}$ st $m > n$

$\boxed{\text{Equivalent}}$

b. There is an integer that is smaller than all other integers

$\exists n \in \mathbb{Z}$ st $\forall m \in \mathbb{Z}$, $m > n$

$\boxed{\text{Not equivalent}}$

c. There is no integer $\geq$ all other integers

$\neg(\exists n \in \mathbb{Z}$ st $\forall m \in \mathbb{Z}$, $n \geq m)$

$\forall n \in \mathbb{Z}$, $\exists m \in \mathbb{Z}$ st $n < m$

$\boxed{\text{Equivalent}}$

d. Every integer is smaller than all other integers

$\forall n \in \mathbb{Z}$, $\forall m \in \mathbb{Z}$ st $m \neq n$, $n < m$

$\boxed{\text{Not equivalent}}$

e. There is no largest integer.

$\forall n \in \mathbb{Z}$, $\exists m \in \mathbb{Z}$ st $m > n$

$\boxed{\text{Equivalent}}$

f. There is no smallest integer

$\forall n \in \mathbb{Z}$, $\exists m \in \mathbb{Z}$ st $n > m$

$\boxed{\text{Not Equivalent}}$

3. Negations

a. Everybody trusts somebody

For all ppl there exists ppl they trust

$$\neg(\forall p \in \mathbb{P} \ \exists s \in \mathbb{P} \text{ st. } T(p,s))$$

$$\exists p \in \mathbb{P} \text{ st } \forall s \in \mathbb{P}, \overline{T(p,s)}$$

$\boxed{\text{Somebody trusts nobody.}}$

b. Somebody trusts everybody

$$\neg(\exists p \in \mathbb{P} \text{ st } \forall s \in \mathbb{P}, T(p,s))$$

$$\forall p \in \mathbb{P}, \exists s \in \mathbb{P} \text{ st } \overline{T(p,s)}$$

$\boxed{\text{Everybody distrusts somebody.}}$

c. There is a person who everyone trusts.

$$\neg(\exists p \in \mathbb{P} \text{ st } \forall s \in \mathbb{P}, T(s,p))$$

$$\forall p \in \mathbb{P}, \exists s \in \mathbb{P} \text{ st } \overline{T(s,p)}$$

$\boxed{\text{Everyone (e) has somebody who distrusts them (e).}}$

d. Every person has someone that they trust

$$\neg(\forall p \in \mathbb{P}, \exists s \in \mathbb{P} \text{ st } T(p,s))$$

$$\exists p \in \mathbb{P} \text{ st } \forall s \in \mathbb{P} \ \overline{T(p,s)}$$

$\boxed{\text{Somebody trusts nobody.}}$

e. Every person has someone they distrust

$\neg(\forall p \in \mathbb{P}, \exists s \in \mathbb{P} \text{ st } \overline{T(p,s)})$

$\exists p \in \mathbb{P} \text{ st } \forall s \in \mathbb{P}, T(p,s)$

Somebody trusts everyone.

f. $\exists x \in \mathbb{N} \text{ st } \forall y \in \mathbb{N}, y \geq x$

$\forall x \in \mathbb{N}, \exists y \in \mathbb{N} \text{ st } y < x$

For all natural #s (x), there exists another natural # (y) such that y is less than x.