HW 8 - proofs

Saturday, October 19, 2019 6:43 PM

Problem 1

Let $X = \{-1, 0, 1\}$, and define the relation R on $\mathscr{P}(X)$ by

$$A \; R \; B \Leftrightarrow \text{ the sum of elements of } A \text{ equals the sum of elements of } B.$$

a) Prove R is an equivalence relation

Proof: Let A, B, C be partitions of X.

Part 1: checking the reflexive property.

The sum of elements of A would always equal itself.

Part 2: checking the transitive property.

If the sum of elements of A equals the sum of elements of C. And if the sum of elemts of C equals the sum of elements in B. Then, the sum of elements of A equals the sum of elements of B.

Part 3: checking the symmetric property.

If the sum of elements of A equals the sum of elements of B, then sum of elements of B equals the sum of elements of A.

$\therefore$ by definition of "equivalence relation",

R is an equiv. relation.

$\square$

b) find equivalence classes.

$[\{0\}] = \{S \in \mathcal{P}(X) \mid \text{sum of el in } S = 0\}$

$= \{\phi, \{0\}, \{-1, 1\}, \{-1, 0, 1\}\}$

Three distinct classes

$[\{1\}] = \{S \in \mathcal{P}(X) \mid \text{sum of el in } S = 1\}$

$= \{\{1\}, \{0, 1\}\}$

$[\{-1\}] = \{S \in \mathcal{P}(X) \mid \text{sum of el in } S = -1\}$

$= \{\{-1\}, \{-1, 0\}\}$

Problem 2

Theorem: If R & S are equivalence relations on the same set, then $R \circ S$ is an equivalence relation.

Prove: Let R & S be equivalence relations on the same set A.

Then, $\exists\, x, y, z \in A$ st

(x,x), (y,y), (x,y), (y,x),
(x,z), (z,y) $\in$ R and S.

By def of "equivalence relations", which includes the reflexive, transitive, and symmetric properties.

Then, $\exists\, v \in A$ s.t.

$(x,v) \in R$ and

$(v,y) \in S$, namely

z.

$\therefore$ by definition of "composition", the theorem is true, assuming the extra assumption:

$$R \circ S = S \circ R$$

$\blacksquare$