HW 8

Saturday, October 19, 2019 6:08 PM

Problem 1

consider $\leq$ + $=$ as relations from $\mathbb{Z} \to \mathbb{Z}$.

a) inverse of "$\leq$"

$\geq$

$\{1, 2, 3, 4\}$

$\{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

$L \leq R$

↳ flip values in ordered pairs, then $L \geq R$

b) inverse of "$=$"

$=$

$\{\ldots (1,1), (2,2), (3,3), \ldots\}$

↳ flip left + right values doesn't change anything

c) composition "$\leq \circ =$"

$x (\leq \circ =) y \leftrightarrow \exists b \in \mathbb{Z} \quad x = b \wedge b \leq y$

$\downarrow$

$x = b \leq y$

$x \leq y$

$\leq$

d) composition "$\leq \circ \leq$"

$x (\leq \circ \leq) y \leftrightarrow \exists b \in \mathbb{Z} \quad x \leq b \wedge b \leq y$

$x \leq b \leq y$

$x \leq y$

$\leq$

e) properties of $\leq$

reflexive, transitive, anti-symmetric

check: reflexive

✓ $x \leq x$ is true

check: transitive

✓ $x \leq b \wedge b \leq y \xrightarrow{?} x \leq y$

is true

check: symmetric

$x \leq y \xrightarrow{?} y \leq x$

is not true

check: anti-symmetric

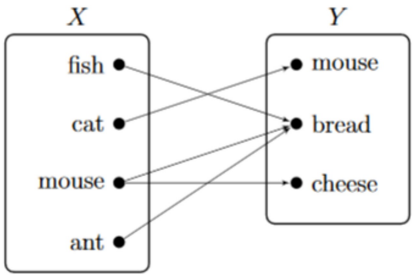
✓ if $x \leq y$, then $y \neq x$ unless $x = y$.

true

f) "$\leq$" equivalent relation?

No; because not symmetric

Problem 2



a) is mouse R cat?

no; not symmetric.

b) ordered pair form.

R = {(fish, bread), (cat, mouse),

$R = \{(\text{fish}, \text{bread}), (\text{cat}, \text{mouse}), (\text{mouse}, \text{bread}), (\text{mouse}, \text{cheese}), (\text{ant}, \text{bread})\}$

c) $\{x \in X \mid x \, R \, \text{bread}\}$

$\{\text{Fish}, \text{mouse}, \text{ant}\}$

Problem 3

Let G be an arbitrary directed graph, and define the relation R on $V(G)$ by

$u \, R \, v \Leftrightarrow$ there is a walk from u to v.

a) is R reflexive?

Yes; a "walk" of length $\emptyset$ from u to u is a walk.

b) is R transitive?

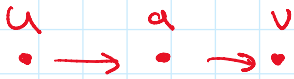
Yes; if there is a point p in a walk where $u \, R \, p \wedge p \, R \, v$, the there is a walk from u to v. You can go from u to p. And from p you can walk to v. That means p is just another vertex on walk u to v.

c) is R symmetric?

No; just because there is a walk from u to v, there

from u to v, there doesn't have to be a walk from v. This is because a return walk is not guaranteed.

ex:



There is no walk from v to u.