HW 10

Friday, November 1, 2019 1:49 PM

Problem 1

Let $B = \{\text{pancakes, bacon, coffee}\}$, $L = \{\text{sandwich, salad, coffee}\}$, and $D = \{\text{pasta, soup, coffee}\}$.

a) Compute $|B|$, $|L|$, and $|D|$.

b) Compute $|B \times L|$ and $|B \times L \times D|$.

c) Compute $|B \cap L|$ and $|B \cup L|$.

a) $|B| = \boxed{3}$

$|L| = \boxed{3}$

$|D| = \boxed{3}$

b) $|B \times L| = |B| \cdot |L| = 3 \cdot 3 = \boxed{9}$

$|B \times L \times D| = |B| \cdot |L| \cdot |D| = 3 \cdot 3 \cdot 3 = \boxed{27}$

c) $B \cap L = \{\text{coffee}\}$

$\therefore |B \cap L| = \boxed{1}$

$|B \cup L| = |B| + |L| - |B \cap L| = 3 + 3 - 1 = \boxed{5}$

Suppose that your favorite restaurant has different options for breakfast, lunch, and dinner. B is their breakfast menu, L is their lunch menu, and D is their dinner menu.

d) How many options are there for breakfast?

e) You go to the restaurant for all three meals, and order one item for each meal. How many possible daily diets are there?

f) If you go to the restaurant for either breakfast or lunch (but not both), how many possible items could you order?

d) 3 options for breakfast

e) $|B \times L \times D| = |B| \cdot |L| \cdot |D| = 3^3 = \boxed{27}$

f) $|B \cup L| = |B| + |L| - |B \cap L| = \boxed{5}$

Problem 2

a) How many functions are there from $\{1\}$ to $\{a, b, c, d, e\}$?

$\forall x \in \{1\}, \exists y \in \{a, b, c, d, e\}$ s.t. $x f y$

There are $\boxed{5}$ functions.

namely, $1 \to a$, $1 \to b$, $1 \to c$, $1 \to d$, $1 \to e$

b) How many functions are there from $\{1, 2\}$ to $\{a, b, c, d, e\}$?

$\forall x \in \{1, 2\}, \exists y \in \{a, b, c, d, e\}$ s.t. $x f y$

There $5 \times 5 = \boxed{25}$ functions.

1 can map to one of 5 letters in $\{a, b, c, d, e\}$

2 can map to one of the 5 letters in $\{a, b, c, d, e\}$

c) How many injective functions are there from $\{1, 2\}$ to $\{a, b, c, d, e\}$?

one-to-one

There are $5 \times 4 = \boxed{20}$ functions

1 can map to one of 5 letters in $\{a, b, c, d, e\}$
2 can map to one of the 4 remaining letters in $\{a, b, c, d, e\}$.

d) How many surjective functions are there from $\{1, 2\}$ to $\{a, b, c, d, e\}$?

There are $\boxed{0}$ such surjections.
values in $\{1, 2\}$ can each only map to one value in $\{a, b, c, d, e\}$. The range then is only, at max, of size 2. But $|\{a, b, c, d, e\}| = 5$. $2 \neq 5$. So the range is not the codomain.

e) How many functions are there from $\{1, \ldots, m\}$ to $\{1, \ldots, n\}$?

There are $\boxed{n^m}$ such functions
Each value in $\{1, \ldots, m\}$ can map to one of n values, so $n \times n \times n \times \cdots$ m times.

Problem 3

There are 10 math majors, 10 ECE majors, and 3 CS majors in a room. Two of the people are math- ECE double majors and 1 is a math-CS double major (no triple majors). How many people are there?

a) How many people are either math majors or ECE majors?

b) How many people are there in total?

a) $|M| = 10$

$|E| = 10$

$|C| = 3$

$|M \cap E| = 2$

$|M \cap C| = 1$

$|M \cup E| = |M| + |E| - |M \cap E|$

$= 10 + 10 - 2$

$= \boxed{18}$

b) $|M \cup E \cup C| = |M| + |E| + |C| - |M \cap E| - |M \cap C| + |M \cap E \cap C|$

$= 10 + 10 + 3 - 2 - 1 + 0$

$= \boxed{20}$

Problem 4

How many ways are there to permute the letters 'a' through 'z' so that at least one of the strings "fish," "cat," or "rat" appears as a substring?

Total possible permutations

$26!$

A_1 - strings w/ "fish"

A_2 - strings w/ "cat"

A_3 - strings w/ "rat"

$|A_1| =$ (1, 4 letter word option + 22 remaining letters)!

$= 23!$

$|A_2| =$ (1, 3 letter word option + 23 remaining letters)!

$= 24!$

$|A_3| =$ (1, 3 letter word option + 23 remaining letters)!

$= 24!$

$|A_1 \cap A_2| =$ (1, 4 letter word + 1, 3 letter word + 19 remaining letters)

$= 21!$

$|A_1 \cap A_3| =$ (1, 4 letter word +

1, 3 letter words

19 remaining letters)

$= 21!$

$|A_2 \cap A_3| = 0$ bc both contain 'a' 't'

$|A_1 \cap A_2 \cap A_3| = 0$ $\Downarrow$

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$$
$$- |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3|$$
$$+ |A_1 \cap A_2 \cap A_3|$$
$$= 23! + 24! + 24! - 21! - 21! - 0 + 0$$

$$\boxed{23! + 24! + 24! - 21! - 21!}$$