1. ../Individual-Questions-by-Topics/Rational-Expressions/Simplify-Rational-Expression-I-001.tex Simplify the rational expression.

$$\frac{15x^2 - 37x + 20}{5x - 4}.$$

Solution:

$$\frac{15x^2 - 37x + 20}{5x - 4}$$

$$= \frac{(3x - 5)(5x - 4)}{5x - 4}$$

$$= 3x - 5.$$

2. ../Individual-Questions-by-Topics/Rational-Expressions/Simplify-Rational-Expression-I-002.tex Simplify the rational expression.

$$\frac{8x^2 - 26x + 15}{4x^2 - 4x - 15}.$$

Solution:

$$\frac{8x^2 - 26x + 15}{4x^2 - 4x - 15}$$

$$= \frac{(2x - 5)(4x - 3)}{(2x - 5)(2x + 3)}$$

$$= \frac{4x - 3}{2x + 3}.$$

3. ../Individual-Questions-by-Topics/Rational-Expressions/Simplify-Rational-Expression-I-003.tex Simplify the rational expression.

$$\frac{x^3 + 5x^2 + 5x + 25}{x + 5}.$$

$$\frac{x^3 + 5x^2 + 5x + 25}{x + 5}$$

$$= \frac{(x^3 + 5x^2) + (5x + 25)}{x + 5}$$

$$= \frac{x^2(x + 5) + 5(x + 5)}{x + 5}$$

$$= \frac{(x^2 + 5)(x + 5)}{x + 5}$$

$$= x^2 + 5$$

4. ../Individual-Questions-by-Topics/Rational-Expressions/Multiply-Simplify-I-001.tex Simplify the rational expression.

$$\frac{4x^2 - 11x - 3}{3x^2 - 14x + 15} \cdot \frac{9x^2 - 25}{16x^2 - 1}.$$

$$\frac{4x^2 - 11x - 3}{3x^2 - 14x + 15} \cdot \frac{9x^2 - 25}{16x^2 - 1}$$

$$= \frac{(x - 3)(4x + 1)}{(x - 3)(3x - 5)} \cdot \frac{(3x - 5)(3x + 5)}{(4x - 1)(4x + 1)}$$

$$= \frac{3x + 5}{4x - 1}.$$

5. ../Individual-Questions-by-Topics/Rational-Expressions/Multiply-Simplify-I-002.tex Simplify the rational expression.

$$\frac{8x+2}{9x^2-16} \cdot \frac{3x^2-13x+12}{4x^2+1x}.$$

Solution:

$$\frac{8x+2}{9x^2-16} \cdot \frac{3x^2-13x+12}{4x^2+1x}$$

$$= \frac{2(4x+1)}{(3x-4)(3x+4)} \cdot \frac{(x-3)(3x-4)}{x(4x+1)}$$

$$= \frac{2(x-3)}{x(3x+4)}.$$

6. ../Individual-Questions-by-Topics/Rational-Expressions/Divide-Simplify-I-001.tex Simplify the rational expression.

$$\frac{12x+9}{4x^2-25} \div \frac{4x^2+3x}{2x+5}.$$

Solution:

$$\frac{12x+9}{4x^2-25} \div \frac{4x^2+3x}{2x-5}$$

$$= \frac{12x+9}{4x^2-25} \cdot \frac{2x-5}{4x^2+3x}$$

$$= \frac{3(4x+3)}{(2x-5)(2x+5)} \cdot \frac{2x-5}{x(4x+3)}$$

$$= \frac{3}{x(2x+5)}.$$

7. ../Individual-Questions-by-Topics/Rational-Expressions/Divide-Simplify-I-002.tex Simplify the rational expression.

$$\frac{5x^2 - 7x - 6}{6xy - 2y} \div \frac{25x^2 - 9}{3x^2 - 7x + 2}.$$

$$\frac{5x^2 - 7x - 6}{6xy - 2y} \div \frac{25x^2 - 9}{3x^2 - 7x + 2}$$

$$= \frac{5x^2 - 7x - 6}{6xy - 2y} \cdot \frac{3x^2 - 7x + 2}{25x^2 - 9}$$

$$= \frac{(x - 2)(5x + 3)}{2y(3x - 1)} \cdot \frac{(x - 2)(3x - 1)}{(5x - 3)(5x + 3)}$$

$$= \frac{(x - 2)^2}{2y(5x - 3)}$$

8. ../Individual-Questions-by-Topics/Rational-Expressions/Divide-Simplify-I-003.tex Simplify the rational expression.

$$\frac{x^2 - 9}{x^2 - 2x - 15} \div \frac{x}{x - 5}.$$

$$\frac{x^2 - 9}{x^2 - 2x - 15} \div \frac{x}{x - 5}$$

$$= \frac{x^2 - 9}{x^2 - 2x - 15} \cdot \frac{x - 5}{x}$$

$$= \frac{(x + 3)(x - 3)}{(x + 3)(x - 5)} \cdot \frac{x - 5}{x}$$

$$= \frac{x - 3}{x}.$$

9. ../Individual-Questions-by-Topics/Rational-Expressions/Add-Subtract-Same-Denominator-I-001.tex Simplify the rational expression.

$$\frac{3x^2 - 11x}{5x^2 - 18x - 8} - \frac{2x - 4}{5x^2 - 18x - 8}.$$

Solution:

$$\frac{3x^2 - 11x}{5x^2 - 18x - 8} - \frac{2x - 4}{5x^2 - 18x - 8}.$$

$$= \frac{3x^2 - 13x + 4}{5x^2 - 18x - 8}$$

$$= \frac{(x - 4)(3x - 1)}{(x - 4)(5x + 2)}$$

$$= \frac{3x - 1}{5x + 2}$$

10. ../Individual-Questions-by-Topics/Rational-Expressions/Add-Subtract-Diff-Denominator-I-001.tex Simplify the rational expression.

$$\frac{7}{x^2+x-12}+\frac{2}{x^2-8x+15}.$$

Solution:

$$\frac{7}{x^2 + x - 12} + \frac{2}{x^2 - 8x + 15}.$$

$$= \frac{7}{(x - 3)(x + 4)} + \frac{2}{(x - 3)(x - 5)}$$

$$= \frac{7(x - 5) + 2(x + 4)}{(x - 3)(x + 4)(x - 5)}$$

$$= \frac{9x - 27}{(x - 3)(x + 4)(x - 5)}$$

$$= \frac{9}{(x + 4)(x - 5)}$$

11. ../Individual-Questions-by-Topics/Rational-Expressions/Add-Subtract-Diff-Denominator-I-002.tex Simplify the rational expression.

$$\frac{-2}{x^2 - 6x + 8} - \frac{-3}{x^2 - 7x + 10}.$$

$$\frac{-2}{x^2 - 6x + 8} - \frac{-3}{x^2 - 7x + 10}.$$

$$= \frac{-2}{(x - 2)(x - 4)} - \frac{-3}{(x - 2)(x - 5)}$$

$$= \frac{-2(x - 5) - -3(x - 4)}{(x - 2)(x - 4)(x - 5)}$$

$$= \frac{x - 2}{(x - 2)(x - 4)(x - 5)}$$

$$= \frac{1}{(x - 4)(x - 5)}$$

12. ../Individual-Questions-by-Topics/Rational-Expressions/Add-Subtract-Diff-Denominator-I-003.tex Simplify the rational expression.

$$\frac{x}{x^2-4x+3}-\frac{-3}{x^2-8x+15}.$$

Solution:

$$\frac{x}{x^2 - 4x + 3} - \frac{-3}{x^2 - 8x + 15}.$$

$$= \frac{x}{(x - 3)(x - 1)} - \frac{-3}{(x - 3)(x - 5)}$$

$$= \frac{x(x - 5) - (-3)(x - 1)}{(x - 3)(x - 1)(x - 5)}$$

$$= \frac{x^2 - 2x - 3}{(x - 3)(x - 1)(x - 5)}$$

$$= \frac{(x - 3)(x + 1)}{(x - 3)(x - 1)(x - 5)}$$

$$= \frac{(x + 1)}{(x - 1)(x - 5)}$$

13. ../Individual-Questions-by-Topics/Rational-Expressions/Add-Subtract-Diff-Denominator-I-004.tex Simplify the rational expression.

$$\frac{x+2}{x+5} - \frac{7}{x+3}$$
.

Practic Problems

Solution:

$$\frac{x+2}{x+5} - \frac{7}{x+3}$$

$$= \frac{(x+2)(x+3) - 7(x+5)}{(x+5)(x+3)}$$

$$= \frac{x^2 - 2x - 29}{(x+5)(x+3)}$$

14. ../Individual-Questions-by-Topics/Rational-Expressions/Add-Subtract-Diff-Denominator-I-005.tex Simplify the rational expression.

$$\frac{x^2 - 26}{x^2 + 10x + 24} - \frac{x+1}{x+6}.$$

$$\frac{x^2 - 26}{x^2 + 10x + 24} - \frac{x+1}{x+6}$$

$$= \frac{x^2 - 26}{(x+4)(x+6)} - \frac{x+1}{x+6}$$

$$= \frac{(x^2 - 26) - (x+4)(x+1)}{(x+4)(x+6)}$$

$$= \frac{-5x - 30}{(x+4)(x+6)}$$

$$= \frac{-5}{x+4}$$

15. ../Individual-Questions-by-Topics/Rational-Expressions/Complex-Rational-Expressions-I-001.tex Simplify the rational expression.

$$\frac{\frac{2}{x} - \frac{1}{x^2}}{\frac{3}{x} + \frac{1}{x^2}}.$$

Solution:

$$\frac{\frac{2}{x} - \frac{1}{x^2}}{\frac{3}{x} + \frac{1}{x^2}} = \frac{\frac{2x - 1}{x^2}}{\frac{3x + 1}{x^2}}$$
$$= \frac{2x - 1}{x^2} \cdot \frac{x^2}{3x + 1}$$
$$= \frac{2x - 1}{3x + 1}$$

16. ../Individual-Questions-by-Topics/Rational-Expressions/Complex-Rational-Expressions-I-002.tex Simplify the complex rational expression.

$$\frac{\frac{3}{x-4} - \frac{1}{x+3}}{\frac{3}{x-4} + \frac{1}{x+3}}.$$

Solution:

$$\frac{\frac{3}{x-4} - \frac{1}{x+3}}{\frac{3}{x-4} + \frac{1}{x+3}} = \frac{\frac{3(x+3) - (x-4)}{(x-4)(x+3)}}{\frac{3(x+3) + (x-4)}{(x-4)(x+3)}}$$
$$= \frac{3(x+3) - (x-4)}{3(x+3) + (x-4)}$$
$$= \frac{2x+13}{4x+5}$$
$$= \frac{2x+13}{4x+5}$$

17. ../Individual-Questions-by-Topics/Rational-Expressions/Complex-Rational-Expressions-I-003.tex Simplify the rational expression.

$$\frac{2 - \frac{5}{x+1}}{2 + \frac{5}{x+1}}.$$

Practic Problems

Solution:

$$\frac{\frac{2(x+1)-5}{x+1}}{\frac{2(x+1)+5}{x+1}} = \frac{2(x+1)-5}{2(x+1)+5}$$
$$= \frac{2x-3}{2x+7}$$

 $\textbf{18.} \quad ../Individual-Questions-by-Topics/Rational-Expressions/Rational-Equation-I-001.tex$

Solve the rational equation

$$\frac{5}{x-4} - \frac{7}{x-3} = 0.$$

$$\frac{5}{x-4} - \frac{7}{x-1} = 0$$

$$(x-3)(x-4)\frac{5}{x-4} - (x-3)(x-4)\frac{7}{x-3} = 0$$

$$5(x-3) - 7(x-4) = 0$$

$$-2x + 13 = 0$$

$$x = \frac{13}{2}$$

19. ../Individual-Questions-by-Topics/Rational-Expressions/Rational-Equation-I-002.tex

Solve the rational equation

$$\frac{1}{x-3} - \frac{1}{x-2} = \frac{1}{2}.$$

Solution:

$$\frac{1}{x-3} - \frac{1}{x-2} = \frac{1}{2}$$

$$2[(x-2) - (x-3)] = (x-3)(x-2)$$

$$2 = x^2 - 5x + 6$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x - 4 = 0$$
 or $x - 1 = 0$
 $x = 4$ $x = 1$

20. ../Individual-Questions-by-Topics/Rational-Expressions/Rational-Equation-I-003.tex

Solve the rational equation

$$\frac{2}{3x-1} - \frac{16}{12x^2 + 11x - 5} = \frac{x}{4x+5}.$$

$$\frac{2}{3x-1} - \frac{16}{12x^2 + 11x - 5} = \frac{x}{4x+5}$$
$$\frac{2}{(3x-1)} - \frac{16}{(3x-1)(4x+5)} = \frac{x}{4x+5}$$
$$2(4x+5) - 16 = x(3x-1)$$
$$3x^2 - 9x + 6 = 0$$
$$(3x+6)(x-1) = 0$$

$$3x + 6 = 0$$
 or $x - 1 = 0$
 $x = 2$ $x = 1$

21. ../Individual-Questions-by-Topics/Rational-Expressions/Rational-Equation-I-004.tex Solve the rational equation

$$\frac{3}{5x-1} + \frac{1}{3x+5} = \frac{6}{15x^2 + 22x - 5}.$$

Solution:

$$\frac{3}{5x-1} + \frac{1}{3x+5} = \frac{6}{15x^2 + 22x - 5}$$
$$3(3x+5) + (5x-1) = 6$$
$$14x+8 = 0$$
$$x = -\frac{4}{7}$$

22. ../Individual-Questions-by-Topics/Rational-Expressions/Rational-Equation-I-005.tex Solve for y from the rational equation

$$\frac{4}{p} + \frac{1}{y} = \frac{6}{b}.$$

Solution:

$$\frac{4}{p} + \frac{1}{y} = \frac{6}{b}$$

$$4by + pb = 6py$$

$$4by - 6py = -pb$$

$$(4b - 6p)y = -pb$$

$$y = \frac{pb}{6p - 4b}$$

23. ../Individual-Questions-by-Topics/Radicals/Simplify-Radicals-I-001.tex

Simplify the radical expression.

$$\sqrt[6]{(-2)^6(x-5)^6b^{24}}.$$

$$\sqrt[6]{(-2)^6(x-5)^6b^{24}} = \sqrt[6]{64(x-5)^6b^{24}} = \sqrt[6]{[2(x-5)b^4]^6} = 2|x-5|b^4.$$

 ${\bf 24.}\ ../Individual-Questions-by-Topics/Radicals/Simplify-Radicals-I-002.tex$

Simplify the radical expression.

$$\sqrt[3]{-125x^9y^9}$$
.

Solution:

$$\sqrt[3]{-125x^9y^9} = \sqrt[3]{(-5x^3y^3)^3} = -5x^3y^3.$$

25. ../Individual-Questions-by-Topics/Radicals/Simplify-Radicals-I-003.tex

Simplify the radical expression.

$$\sqrt[3]{-\frac{64a^3}{8y^{12}}}.$$

Solution:

$$\sqrt[3]{-\frac{64a^3}{8y^{12}}} = \sqrt[3]{\left(-\frac{3a}{2y^4}\right)^3} = -\frac{4a}{2y^4}.$$

26. ../Individual-Questions-by-Topics/Radicals/Simplify-Radicals-I-004.tex

Simplify the radical expression (assume all variables are positive).

$$\sqrt[3]{8a^6y^9c^9}$$
.

Solution:

$$\sqrt[3]{8a^6y^9c^9} = \sqrt[3]{(2a^2y^3c^3)^3} = 2a^2y^3c^3.$$

27. ../Individual-Questions-by-Topics/Radicals/Multiply-Simplify-Radicals-I-001.tex

Simplify the radical expression as much as possible (assume all variables are positive).

$$\sqrt{7x^4b^3}\sqrt{7b^3c^{10}}$$

Practic Problems

Solution:

$$\sqrt{7x^4b^3}\sqrt{7b^3c^{10}} = \sqrt{49x^4b^6c^{10}} = \sqrt{(7x^2b^3c^5)^2} = 7x^2b^3c^5.$$

28. ../Individual-Questions-by-Topics/Radicals/Divide-Simplify-Radicals-I-001.tex
Simplify the radical expression as much as possible (assume all variables are positive).

$$\frac{\sqrt{1a^3y^2}}{\sqrt{4a^{11}z^{10}}}.$$

Solution:

$$\frac{\sqrt{1a^3y^2}}{\sqrt{4a^{11}z^{10}}} = \sqrt{\frac{1a^3y^2}{4a^{11}z^{10}}} = \sqrt{\frac{1y^2}{4a^8z^{10}}}. = \sqrt{\left(\frac{1y}{2a^4z^5}\right)^2} = \frac{1y}{2a^4z^5}.$$

29. ../Individual-Questions-by-Topics/Radicals/Radical-to-Rational-Exponent-I-001.tex Write the radical expression as rational exponent (assume all variables are positive).

$$\sqrt[3]{a^2y^3}$$
.

Solution:

$$\sqrt[3]{a^2y^3} = a^{\frac{2}{3}}y^1.$$

30. ../Individual-Questions-by-Topics/Radicals/Rational-Exponent-to-Radical-I-001.tex Write the rational exponent as radical expression (assume all variables are positive).

$$x^1$$
.

Solution:

$$x^1 = \sqrt[3]{x^3}.$$

31. ../Individual-Questions-by-Topics/Radicals/Rational-Exponent-to-Radical-I-002.tex Write the rational exponent as radical expression (assume all variables are positive).

$$a^{\frac{7}{4}}y^{\frac{3}{4}}$$
.

$$a^{\frac{7}{4}}y^{\frac{3}{4}} = \sqrt[4]{a^7y^3}.$$

 ${\bf 32.} \quad ../ Individual - Questions - by - Topics/Radicals/Simplify - Radicals - II - 001. tex$

Simplify the radical expression (assume all variables are positive).

$$\sqrt[4]{32x^{15}y^{23}}$$
.

Solution:

$$\sqrt[4]{32x^{15}y^{23}} = \sqrt[4]{2^4x^{12}y^{20} \cdot 2x^3y^3} = 2x^3y^4\sqrt[4]{2x^3y^3}.$$

 ${\bf 33.} \ \ ../Individual-Questions-by-Topics/Radicals/Simplify-Radicals-II-002.tex$

Simplify the radical expression (assume all variables are positive).

$$\sqrt[3]{-625a^4y^4}$$
.

Solution:

$$\sqrt[3]{-625a^4y^4} = \sqrt[3]{(-5)^3a^3y^3 \cdot 5ay} = -5ay \cdot \sqrt[3]{5ay}.$$

34. ../Individual-Questions-by-Topics/Radicals/Divide-Simplify-Radicals-II-001.tex

Simplify the radical expression (assume all variables are positive).

$$\frac{\sqrt{216x^7b^7}}{\sqrt{x^7b^7}}$$

Solution:

$$\frac{\sqrt{216x^7b^7}}{\sqrt{x^7b^7}} = \sqrt{\frac{216x^7b^7}{x^7b^7}} = \sqrt{216b^0} = 6\sqrt{6}.$$

35. ../Individual-Questions-by-Topics/Radicals/Divide-Simplify-Radicals-II-002.tex Simplify the radical expression (assume all variables are positive).

$$\frac{\sqrt[5]{2x^9y^{19}}}{\sqrt[5]{-64x^6y^2}}$$

$$\frac{\sqrt[5]{2x^9y^{19}}}{\sqrt[5]{-64x^6y^2}} = \sqrt[5]{\frac{2x^9y^{19}}{-64x^6y^2}} = \sqrt[5]{\frac{x^3y^{17}}{(-2)^5}} = -\frac{y^3\sqrt[5]{x^3y^2}}{2}.$$

36. ../Individual-Questions-by-Topics/Radicals/Simplify-Rational-Exponent-I-001.tex

Assume all variables are positive. Write the following expression with rational exponents in radical expression and simplify.

$$(100a^2y^8)^{\frac{1}{2}}$$

Solution:

$$(100a^2y^8)^{\frac{1}{2}} = \sqrt{100a^2y^8} = \sqrt{(10ay^4)^2} = 10ay^4.$$

 ${\bf 37.} \ \ ../Individual-Questions-by-Topics/Radicals/Simplify-Rational-Exponent-I-002.tex$

Assume all variables are positive. Simplify and write your answer in radical notation.

$$(32x^{14}b^{27})^{\frac{1}{5}}$$

Solution:

$$(32x^{14}b^{27})^{\frac{1}{5}} = \sqrt[5]{32x^{14}b^{27}} = \sqrt[5]{2^5x^{10}b^{25} \cdot x^4b^2} = 2x^2b^5\sqrt[5]{x^4b^2}.$$

 $\textbf{38.} \ \, ../Individual-Questions-by-Topics/Radicals/Simplify-Rational-Exponent-I-003.tex$

Assume all variables are positive. Write the following expression with rational exponents and simplify to the form $\frac{x^p}{x^q}$.

$$\left(\frac{x^2y}{y^4}\right)^{\frac{2}{5}}$$

Solution:

$$\left(\frac{x^2y}{y^4}\right)^{\frac{2}{5}} = \left(\frac{x^2}{y^3}\right)^{\frac{2}{5}} = \frac{x^{2 \cdot \frac{2}{5}}}{y^{3 \cdot \frac{2}{5}}} = \frac{x^{\frac{4}{5}}}{y^{\frac{8}{5}}}.$$

39. ../Individual-Questions-by-Topics/Radicals/Combining-Like-Radicals-I-001.tex

Simplify the following expression and write in radical form.

$$7\sqrt{3} - 5\sqrt{27} - 3\sqrt{81}$$

$$7\sqrt{3} - 5\sqrt{27} - 3\sqrt{81} = 7\sqrt{3} - 5\sqrt{3^23} - 3\sqrt{3^4}$$
$$= 7 \cdot 3^0 \cdot \sqrt{3} - 5 \cdot 3 \cdot \sqrt{3} - 3 \cdot 3^2$$
$$= 7\sqrt{3} - 15\sqrt{3} - 27$$
$$= -8\sqrt{3} - 27.$$

40. ../Individual-Questions-by-Topics/Radicals/Combining-Like-Radicals-I-002.tex

Assume the variable *x* is positive. Simplify the following expression and write in radical form.

$$\sqrt[4]{16x^9} - 5\sqrt[4]{x^{13}}$$

Solution:

$$\sqrt[4]{16x^9} - 5\sqrt[4]{x^{13}} = \sqrt[4]{2^4 \cdot x^8 \cdot x} - 5\sqrt[4]{x^{12} \cdot x}$$
$$= 2x^2\sqrt[4]{x} - 5x^3\sqrt[4]{x}$$
$$= (2x^2 - 5x^3)\sqrt[4]{x}.$$

41. ../Individual-Questions-by-Topics/Radicals/Multiply-Simplify-Radicals-I-002.tex

Assume all variables are positive. Simplify the following expression as much as possible.

$$2\sqrt{2}\left(5\sqrt{12}+7\sqrt{6}\right)$$

Solution:

$$2\sqrt{2}\left(5\sqrt{12}+7\sqrt{6}\right) = 10\sqrt{4\cdot 6}+14\sqrt{4\cdot 3}=10\sqrt{4}\cdot \sqrt{6}+14\sqrt{4}\cdot \sqrt{6}=20\sqrt{6}+28\sqrt{3}.$$

42. ../Individual-Questions-by-Topics/Radicals/Multiply-Simplify-Radicals-II-001.tex

Assume all variables are positive. Simplify the following expression as much as possible.

$$(3\sqrt{a}+4\sqrt{b})(5\sqrt{a}-7\sqrt{b})$$

$$(3\sqrt{a} + 4\sqrt{b})(5\sqrt{a} - 7\sqrt{b}) = 15a + 20\sqrt{ab} - 21\sqrt{ab} - 28b = 15a - 28b - 1\sqrt{ab}.$$

43. ../Individual-Questions-by-Topics/Radicals/Rationalize-Denominator-I-001.tex

Rationalize the denonimator and simplify as much as possible.

$$\frac{5\sqrt{2}+6}{7\sqrt{2}-5}$$

Solution:

$$\frac{5\sqrt{2}+6}{7\sqrt{2}-5} = \frac{(5\sqrt{2}+6)(7\sqrt{2}+5)}{(7\sqrt{2}-5)(7\sqrt{2}+5)}$$

$$= \frac{(5\sqrt{2}+6)(7\sqrt{2}+5)}{7^2 \cdot 2 - 5^2}$$

$$= \frac{352+42\sqrt{2}+25\sqrt{2}+30}{73}$$

$$= \frac{100+67\sqrt{2}}{73}$$

$$= \frac{100}{73} + \frac{67}{73}\sqrt{2}.$$

44. ../Individual-Questions-by-Topics/Radicals/Rationalize-Denominator-I-002.tex

Rationalize the denonimator and simplify as much as possible.

$$\frac{3\sqrt{y}-6}{\sqrt{y^3}}$$

$$\frac{3\sqrt{y} - 6}{\sqrt{y^3}} = \frac{(3\sqrt{y} - 6)\sqrt{y}}{\sqrt{y^3}\sqrt{y}} = \frac{3y - 6\sqrt{y}}{\sqrt{y^4}} = \frac{3y - 6\sqrt{y}}{y^2}.$$

45. ../Individual-Questions-by-Topics/Radicals/Rationalize-Denominator-I-003.tex

Rationalize the denonimator and simplify as much as possible.

$$\frac{5\sqrt{r}-2}{7\sqrt{r}-3}$$

Solution:

$$\frac{5\sqrt{r}-2}{7\sqrt{r}-3} = \frac{(5\sqrt{r}-2)(7\sqrt{r}+3)}{(7\sqrt{r}-3)(7\sqrt{r}+3)}$$
$$= \frac{(5\sqrt{r}-2)(7\sqrt{r}+3)}{49r-9}$$
$$= \frac{35r-14\sqrt{r}+15\sqrt{r}-6}{49r-9}$$
$$= \frac{35r+\sqrt{r}-6}{49r-9}$$

46. ../Individual-Questions-by-Topics/Radicals/Divide-Complex-Numbers-I-001.tex

Write the quotient of complex numbers in the form $a + b\mathbf{i}$, where \mathbf{i} is the imaginary unit.

$$\frac{4+5\mathbf{i}}{-4+7\mathbf{i}}$$

$$\frac{4+5i}{-4+7i} = \frac{(4+5i)(-4-7i)}{(-4+7i)(-4-7i)}$$

$$= \frac{-16-28i-20i+35}{7^2+4^2}$$

$$= \frac{-16-28i-20i+35}{65}$$

$$= \frac{19-48i}{65}$$

$$= \frac{19}{65} - \frac{48}{65}i$$

47. ../Individual-Questions-by-Topics/Radicals/Divide-Complex-Numbers-I-002.tex

Write the quotient in the form a + bi, where i is the imaginary unit.

$$\frac{2\sqrt{-1}+9}{\sqrt{-1}}$$

Solution:

$$\frac{2\sqrt{-1}+9}{\sqrt{-1}} = \frac{2\mathbf{i}+9}{\mathbf{i}} = \frac{(2\mathbf{i}+9)\mathbf{i}}{(\mathbf{i})^2} = \frac{-2+9\mathbf{i}}{-1} = 2-9\mathbf{i}.$$

 $\textbf{48.} \ \, .../Individual-Questions-by-Topics/Radicals/Divide-Complex-Numbers-II-001.tex$

Write the quotient of complex numbers in the form $a + b\mathbf{i}$, where \mathbf{i} is the imaginary unit.

$$\frac{(3-3\mathbf{i})(-3\mathbf{i})}{1+\mathbf{i}}$$

$$\frac{(3-3i)(-3i)}{1-3i} = \frac{-9-9i}{1+i}$$

$$= \frac{(-9-9i)(1-i)}{(1+i)(1-i)}$$

$$= \frac{0+0i}{1-1\cdot(-1)}$$

$$= \frac{0+0i}{2}$$

$$= 0$$

49. ../Individual-Questions-by-Topics/Radicals/Solve-Radical-Equations-I-001.tex

Solve the following radical equation

$$\sqrt[3]{3x+18}-4=-1.$$

$$\sqrt[3]{3x + 18} - 4 = -1$$
$$\sqrt[3]{3x + 18} = 3$$

$$3x + 18 = 27$$

$$3x = 9$$

$$x = 3$$

50. ../Individual-Questions-by-Topics/Radicals/Solve-Radical-Equations-I-002.tex

Solve the following radical equation

$$\sqrt{3x - 21} + x = 7$$
.

Solution:

$$\sqrt{3x - 21} + x = 7$$

$$\sqrt{3x - 21} = 7 - x$$

$$3x - 21 = (x - 7)^{2}$$

$$3x - 21 = x^{2} - 2 \cdot 7 \cdot x + 7^{2}$$

$$x^{2} - 17x + 70 = 0$$

$$x - 7 = 0 \quad \text{or} \quad x - 10 = 0$$

$$x = 7$$
 or $x = 10$

Check the solutions.

When x = 7, the left hand side of the equation equals

$$\sqrt{3 \cdot 7 - 21} + 7 = \sqrt{0} + 7 = 7.$$

So x = 7 is a solution.

When x = 10, the left hand side of the equation equals

$$\sqrt{3 \cdot 10 - 21} + 7 = \sqrt{3^2} + 7 = 3 + 7 \neq 7.$$

So x = 10 is not a solution.

Therefore, the equation has only one solution x = 7.

51. ../Individual-Questions-by-Topics/Radicals/Solve-Radical-Equations-I-003.tex

Solve the following radical equation

$$\sqrt{x+3} - \sqrt{x-37} = 4.$$

$$\sqrt{x+3} - \sqrt{x-37} = 4$$

$$\sqrt{x+3} = \sqrt{x-37} + 4$$

$$x+3 = (\sqrt{x-37} + 4)^2$$

$$x+3 = x-37 + 8\sqrt{x-37} + 16$$

$$24 = 8\sqrt{x-37}$$

$$\sqrt{x-37} = 3$$

$$x - 37 = 9$$

$$x = 46$$

Page 21

Check the solutions. The left hand side of the equation equals

$$\sqrt{46+3} - \sqrt{46-37} = 7-3 = 4.$$

Therefore, the equation has only one solution x = 46.

52. ../Individual-Questions-by-Topics/Radicals/Solve-Radical-Equations-I-004.tex

Solve the following radical equation

$$\sqrt[3]{1x+6} = 2.$$

Solution:

$$\sqrt[3]{1x+6} = 2$$

$$1x + 6 = 8$$

$$1x = 2$$

$$x = 2$$
.

53. ../Individual-Questions-by-Topics/Radicals/Solve-Radical-Equations-I-005.tex

Solve the following radical equation

$$x - \sqrt{4x - 16} = 4.$$

Solution:

$$x - \sqrt{4x - 16} = 4$$

$$x - 4 = \sqrt{4x - 16}$$

$$(x-4)^2 = 4x - 16$$

$$x^2 - 8x + 4^2 = 4x - 16$$

$$x^2 - 12x + 32 = 0$$

$$x - 4 = 0$$
 or $x - 8 = 0$

$$x = 4$$
 or $x = 8$

Check the solutions.

When x = 4, the left hand side of the equation equals

$$4 - \sqrt{4 \cdot 4 - 16} = 4 - \sqrt{0} = 4$$
.

So x = 4 is a solution.

When x = 8, the left hand side of the equation equals

$$4 - \sqrt{4 \cdot 8 - 16} = 4 - \sqrt{4^2} = 4 - 4 = 0 \neq 4$$
.

So x = 8 is not a solution.

Therefore, the equation has only one solution x = 4.

54. ../Individual-Questions-by-Topics/Rat-Rad-Functions/Domain-Rational-Function-I-001.tex

Find the domain of the following rational function and write your answer in interval notation

$$f(x) = \frac{4x^2}{x+1}$$

Solution: The rational function is well-defined if $x + 1 \neq 0$, equiavlently, $x \neq -1$. In interval notation, the domain is

$$(-\infty, -1) \cup (-1, \infty).$$

55. ../Individual-Questions-by-Topics/Rat-Rad-Functions/Domain-Rational-Function-I-002.tex

Find the domain of the following rational function and write your answer in interval notation

$$f(x) = \frac{-2x^2}{x^2 + 11x}$$

Solution: The rational function is well-defined except the values of x such that $x^2 + 11x = 0$. Since the equation has two solutions x = 0 or x = -11. Removing the two numbers from the number line, we find the domain of the function f is

$$(-\infty, -11) \cup (-11, 0) \cup (0, \infty).$$

56. ../Individual-Questions-by-Topics/Rat-Rad-Functions/Domain-Radical-Function-I-001.tex

Find the domain of the following rational function and write your answer in interval notation

$$f(x) = \sqrt{-x - 7}$$

Solution: The rational function is well-defined if $-x - 7 \ge 0$, equiavlently, $x \le -7$. In interval notation, the domain is

$$(-\infty, -7]$$
.

57. ../Individual-Questions-by-Topics/Rat-Rad-Functions/Domain-Radical-Function-I-002.tex

Find the domain of the following rational function and write your answer in interval notation

$$f(x) = \sqrt{x^2 + 6}$$

Solution: The rational function is well-defined if $x^2 + 7 \ge 0$. Since the $x^2 \ge 0$ for all real numbers. Then the domain of this function is the set of real numbers. In interval notation, the domain is

$$(-\infty, \infty)$$
.

58. ../Individual-Questions-by-Topics/Rat-Rad-Functions/Domain-Rational-Radical-Function-I-001.tex

Find the domain of the following rational function and write your answer in interval notation

$$f(x) = \frac{-3x^3}{\sqrt{-x-3}}$$

Solution: The function is well-defined if the denominator $\sqrt{-x-3}$ is a nonzero real number. Since the denominator is a radical, and cannot be zero, we know x must satisfy the inequality -x-3>0. Solve the inequality, we get x<-3. So the domain of f is

$$(-\infty, -3)$$
.

59. ../Individual-Questions-by-Topics/Quadratic/Completing-Square-I-001.tex

Write the following quadratice polynomial in the form $a(y - h)^2 + k$ form

$$y^2 + 6y - 4$$
.

$$y^2 + 6y - 4 = (y^2 + 6y) - 4 = (y + 3)^2 - 3^2 - 4 = (y + 3)^2 - 13.$$

 $\textbf{60.} \hspace{0.1in} ../ Individual - Questions-by-Topics/Quadratic/Completing-Square-I-002. tex$

Write the following quadratice polynomial in the form $a(y-h)^2+k$ form

$$3y^2 + 6y - 2$$
.

Solution:

$$3y^2 + 6y - 2 = 3(y^2 + 2y) - 2 = 3(y+1)^2 - 3 \cdot 1^2 - 2 = 3(y+1)^2 - 5.$$

61. ../Individual-Questions-by-Topics/Quadratic/Solve-by-Completing-Square-I-001.tex

Solve the following equation by completing the square

$$3y^2 + 6y - 1 = 0.$$

Solution:

$$3y^{2} + 6y - 1 = 0$$

$$3(y+1)^{2} - 4 = 0$$

$$(y+1)^{2} = \frac{4}{3}$$

$$y+1 = \pm \sqrt{\frac{4}{3}}$$

$$y = -1 \pm \frac{\sqrt{12}}{3}$$

62. ../Individual-Questions-by-Topics/Quadratic/Solve-by-Completing-Square-I-002.tex

Solve the following equation by completing the square

$$y^2 - 6y + 11 = 0.$$

Note: No credit for using methods other than completing the square.

$$y^{2} - 6y + 11 = 0$$
$$(y - 3)^{2} + 2 = 0$$
$$(y - 3)^{2} = -2$$
$$y - 3 = \pm \sqrt{-2}$$
$$y = 3 \pm i\sqrt{2}.$$

63. ../Individual-Questions-by-Topics/Quadratic/Solve-by-Quadratic-Formula-I-001.tex

Solve the following equation

$$-3y^2 + 5y + 2 = 0.$$

Solution: Since a = -3 and b = 5 and c = 2, the equation has to two solution

$$y = \frac{-5 \pm \sqrt{5^2 - 4 \cdot (-3) \cdot 2}}{2 \cdot (-3)} = \frac{-5 \pm \sqrt{49}}{-6}.$$

64. ../Individual-Questions-by-Topics/Quadratic/Quadratic-Formula-Application-I-001.tex

A rectangle whose length is 3 meters longer than its width has an area 5 square meters. What is the dimension of the rectangle? Round your answer to the nearest tenth of a meter.

Solution: Suppose the width is x meters. Then the length is x + 3. By the rectangular area formula: $Area = Width \times Length$, we have the following equation

$$x(x+3)=5.$$

Simplify the equation, we get

$$x^2 + 3x - 5 = 0.$$

By the quadratic formula, we get

$$x = \frac{-3 + \sqrt{29}}{2}$$

So the width is approximately 1.2 meters and the length is approximately 4.2 meters.

65. ../Individual-Questions-by-Topics/Quadratic/Quadratic-Formula-Application-I-002.tex

The hypotenuse of a **right triangle** is 6 feet long. One leg is 2 feet longer than the other. Find the length of each leg to the nearest tenth of a foot.

Solution: Suppose the shorter leg is x feet. Then the longer leg is is x + 2. By Pythagorean Theorem, we have the following equation

$$x^2 + (x+2)^2 = 6^2.$$

Simplify the equation, we get

$$x^2 + 2x - 16 = 0.$$

By the quadratic formula, we get

$$x = -1 + \sqrt{17}.$$

So the legs are approximately 3.1 feet and 5.1 feet.

66. ../Individual-Questions-by-Topics/Quadratic/Quadratic-Formula-Application-I-003.tex

A rectangle whose length is 3 meters less than twice its width has an area 75 square meters. What is the length of the diagonal? Round your answer to the nearest thousandth of a meter.

Solution: Suppose the width is x meters. Then the length is 2x - 3 meters. By the rectangular area formula: Area = Width \times Length, we have the following equation

$$x(2x-3)=75.$$

Simplify the equation, we get

$$2x^2 - 3x - 75 = 0.$$

By the quadratic formula, we get

$$x = \frac{-3 + \sqrt{(-3)^2 - 4 \cdot 2 \cdot (-75)}}{2 \cdot 2} = \frac{-3 + \sqrt{609}}{4}.$$

So the width is approximately $\frac{-3+\sqrt{609}}{4}$ meters and the length is $\frac{\sqrt{609}-9}{2}$ meters. The diagonal is

$$\sqrt{\left(\frac{-3+\sqrt{609}}{4}\right)^2 + \left(\frac{\sqrt{609}-9}{2}\right)^2} \approx 12.859$$

meters.

67. ../Individual-Questions-by-Topics/Quadratic/Quadratic-Formula-Application-I-004.tex

The hypotenuse of an isosceles right triangle is 8cm longer than either of its legs. Note that an isosceles right triangle is a right triangle whose legs are the same length. Find the exact length of its legs and its hypotenuse.

Solution: Suppose the length of a leg is *x* meters. Then the length of the hypotenuse is x + 8. By Pythagorean identity: hypotenuse² = Leg² + Leg², we have the following equation

$$x^2 + x^2 = (x+8)^2$$
.

Simplify the equation, we get

$$x^2 - 16x - 64 = 0.$$

By the quadratic formula, we get

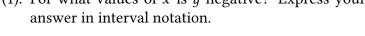
$$x = \frac{-(-16) + \sqrt{(-16)^2 - 4 \cdot (-64)}}{2} = 8 + 8\sqrt{2}.$$

So the leg is $8 + 8\sqrt{2}$ cm and the hypotenuse is $16 + 8\sqrt{2}$ cm.

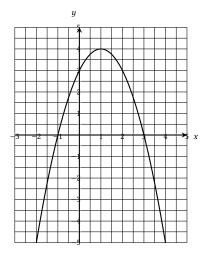
68. ../Individual-Questions-by-Topics/Quadratic/Graph-Quadratic-Function-I-001.tex

Consider the parabola of the function on the right.

(1). For what values of x is y negative? Express your



- (2). Find the domain of the function.
- (3). Find the range of the function.
- (4). Determine the value of f(0).
- (5). Determine the coordinates of the *x*-intercepts.
- (6). Determine the coordinate of the *y*-intercept.
- (7). Determine the coordinate of the vertex.
- (8). For what value of x is f(x) = 3



$$(1) (-1,3). \qquad (2) (-\infty,\infty). \qquad (3) (-\infty,4] \qquad (4) f(0) = 3.$$

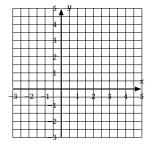
(5)
$$(-1,0)$$
 and $(3,0)$. (6) $(0,3)$. (7) $(1,4)$. (8) $x=0$ or $x=2$.

 $\textbf{69.} \quad ../ Individual-Questions-by-Topics/Quadratic/Graph-Quadratic-Function-I-002. tex$

Consider the quadratic function

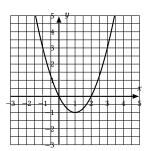
$$f(x) = (x - 1)^2 - 1.$$

- (1) Determine the coordinates of the *x*-intercepts, the coordinate of the *y*-intercept, the axis of symmetry and the coordinate of the vertex.
- (2) Sketch the graph of the quadratic function using the information in part A.



Solution:

- (1) The line of symmetry is x = 1. The x-intercepts are (0, 0) and (2, 0). The y-intercept is (0, 0). The vertex is (1, -1).
- (2) The graph of f is shown in the right.

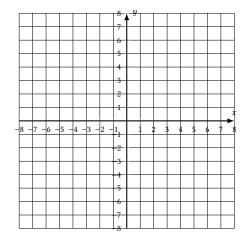


 $\textbf{70.} \quad .../Individual-Questions-by-Topics/Quadratic/Graph-Quadratic-Function-I-003.tex$

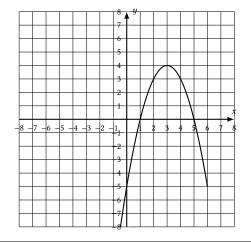
Consider the quadratic function

$$f(x) = -x^2 + 6x - 5.$$

- (1) Find the line of symmetry.
- (2) Find the coordinate of the vertex.
- (3) Find the coordinate of the *y*-intercept.
- (4) Find the coordinates of the *x*-intercepts.
- (5) Sketch the graph of the quadratic function.



- (1) The line of symmetry is x = 3.
- (2) The vertex is (3, 4).
- (3) The *y*-intercept is (0, -5).
- (4) The x-intercepts are (1, 0) and (5, 0).
- (5) The graph of f is shown in the right.



71. ../Individual-Questions-by-Topics/Quadratic/Quadratic-Function-I-001.tex

A quadratic function has its vertex at the point (-5, -1). The function passes through the point (4, 2). When written in vertex form, the function is $f(x) = a(x - h)^2 + k$. Find the a, h and k.

Solution: Since the vertex is (-5, -1), we know that h = -5 and k = -1. To find a, we plug the point (4, 2) in the equation and solve for a.

$$2 = a(4 - (-5))^{2} + (-1)$$
$$2 - (-1) = a(4 - (-5))^{2}$$
$$3 = 81a$$
$$a = \frac{1}{27}.$$

$\textbf{72.} \quad ../Individual-Questions-by-Topics/Exp-Log-Equations/Investment-Exponential-Function-I-001.tex \\$

A 5000 check was deposited to a bank account with the annual rate 5% compounded monthly. Using the formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt},\,$$

- (1) determine the account balance after 6 years. Round your answer to the nearest cent;
- (2) determine how many years, to the nearest tenth, it will take double the account.

(1)

$$A = 5000 \left(1 + \frac{0.05}{12} \right)^{12 \cdot 6} \approx 6745.09$$

(2) Solve t from the equation

$$10000 = 5000 \left(1 + \frac{0.05}{12} \right)^{12t}$$
$$2 = \left(1 + \frac{0.05}{12} \right)^{12t}$$
$$\ln 2 = 12t \ln \left(1 + \frac{0.05}{12} \right)$$
$$t = \frac{\ln 2}{12 \ln \left(1 + \frac{0.05}{12} \right)}$$
$$t \approx 13.9$$

73. ../Individual-Questions-by-Topics/Exp-Log-Equations/Investment-Exponential-Function-I-002.tex

Use the formula $A = P \cdot e^{rt}$ to find the balance of an investment \$3000 at 5% compounded continuously after 7 years. Round your answer to nearst dollars.

Solution:

$$A(7) = 3000e^{0.04999 \cdot 7} \approx 4256.$$

74. ../Individual-Questions-by-Topics/Exp-Log-Equations/Exponential-Equation-I-001.tex

Solve the exponential equation $2^{2x+1} = 18$. Round your result to the nearest thousandth.

$$2^{2x+1} = 18$$

$$2x + 1 = \log_2(18)$$

$$2x = \log_2(18) - 1$$

$$x = \frac{1}{2}(\log_2(18) - 1) \approx 1.585$$

75. ../Individual-Questions-by-Topics/Exp-Log-Equations/Exponential-Equation-I-002.tex Solve the exponential equation $2 \cdot 10^x - 4 = 2$. Round your result to the nearest hundredth.

Solution:

$$2 \cdot 10^{x} - 4 = 2$$
$$2 \cdot 10^{x} = 6$$
$$10^{x} = 3$$
$$x = \log 3 \approx 0.48$$

76. ../Individual-Questions-by-Topics/Exp-Log-Equations/Solve-log-equation-I-001.tex Solve the logarithmic equation

$$\log(2x+13) - \log x = 1.$$

Solution:

$$\log(2x + 13) - \log x = 1$$

$$\log(\frac{2x + 13}{x}) = 1$$

$$\frac{2x + 13}{x} = 10^{1}$$

$$2x + 13 = 10x$$

$$8x = 13$$

$$x = \frac{13}{8}.$$

77. ../Individual-Questions-by-Topics/Exp-Log-Equations/Solve-log-equation-I-002.tex Solve the following equations for x.

$$\log_7 x + \log_7 (x - 6) = 1.$$

$$\log_7 x + \log_7(x - 6) = 1$$
$$\log_7(x(x - 6)) = 1$$
$$x(x - 6) = 7$$
$$x^2 - 6x - 7 = 0$$
$$(x + 1)(x - 7) = 0$$

$$x + 1 = 0$$
 or $x - 7 = 0$
 $x = -1$ or $x = 7$

Since in $\log_7 x$, x must be positive. So x = -1 cannot be a solution. Plug in x = 5 and check:

$$\log_7 7 + \log_7 (7 - 6) = 1 + \log_7 1 = 1 + 0 = 1.$$

So the solution is x = 5.

78. ../Individual-Questions-by-Topics/Exp-Log-Equations/Solve-log-equation-I-003.tex

Solve the logarithmic equation

$$\log_3 \sqrt{x-6} = 1.$$

Solution:

$$\log_3 \sqrt{x - 6} = 1$$

$$\sqrt{x - 6} = 3$$

$$x - 6 = 3^2$$

$$x = 15.$$

79. ../Individual-Questions-by-Topics/Exp-Log-Equations/Solve-log-equation-I-004.tex

Solve the logarithmic equation

$$\log(9-7x) = 1 + \log(-x).$$

$$\log(9-7x) = 1 + \log(-x)$$

$$\log(9-7x) - \log(-x) = 1$$

$$\log\left(\frac{9-7x}{-x}\right) = 1$$

$$\frac{9-7x}{-x} = 10^{1}$$

$$\frac{9-7x}{-x} = 10$$

$$9-7x = -10x$$

$$9 = -3x$$

$$x = -3$$

Check: The left hand side is

$$\log(9 - 7 \cdot (-3)) = \log(3 \cdot 10) = \log 3 + 1$$

which is the same as right hand side. So the solution is x = -3.

80. ../Individual-Questions-by-Topics/Exp-Log-Equations/Solve-log-equation-I-005.tex Solve the logarithmic equation

$$\log_{\mathbf{y}} 9 = 2$$

Solution:

$$\log_x 9 = 2$$
$$x^2 = 9$$
$$x = \sqrt{9} = 3$$

81. ../Individual-Questions-by-Topics/Exp-Log-Equations/Domain-Log-Function-I-001.tex Find the domain of the function $f(x) = \log_{22}(x - 19)$. Write the domain in interval notation.

Solution: To get real numbers as output, the power x-19 must be positive. Solve the inequality x-19>0, we find that x>19 and domain of the function f is $(19, \infty)$.