# Notes on Maple for Calculus II

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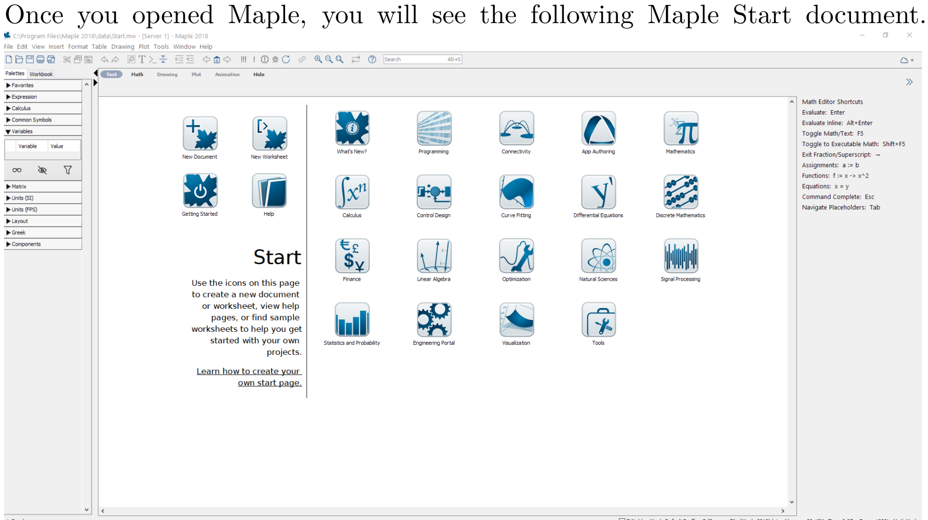
# Contents

# Introduction

This is a book written for labs for Calculus II.

## What should I do after I opened Maple

Once you opened Maple, you will see the following Maple Start document.




- If you already know what you want to do, then you may open a new document by clicking `New Document` icon in the start document. The following shows what an new (empty document) looks like.

In this new document you may type in text under `Text` mode or evaluat a Maple syntax in the `Math` mode. (See the following picture).

- If you want to explore some featured sample documents, you may go to `Start.mw` document and click on different icons to open a new document.
- You may alway reopen the start page by click the home icon to reopen the start page.

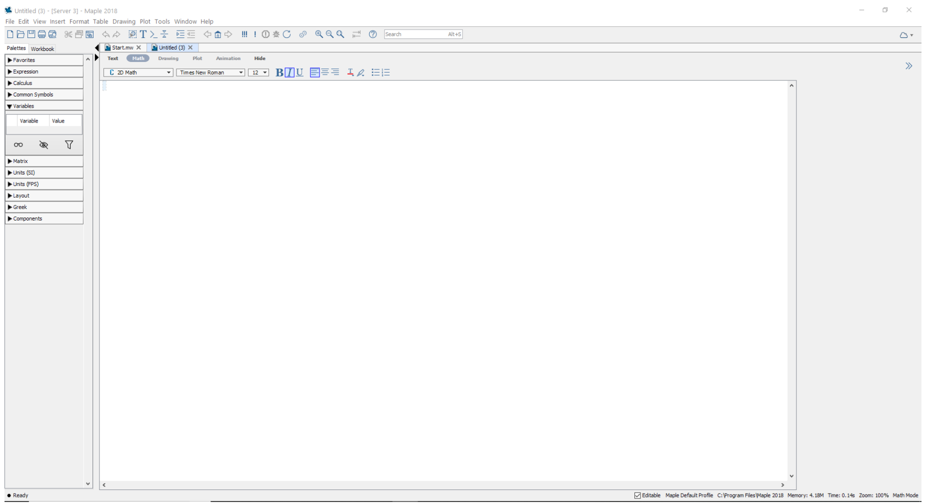


Figure 1:

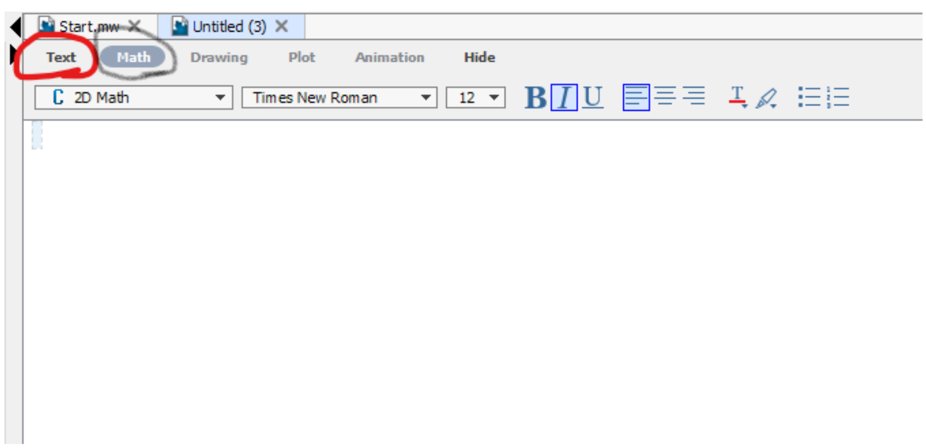



Figure 2:

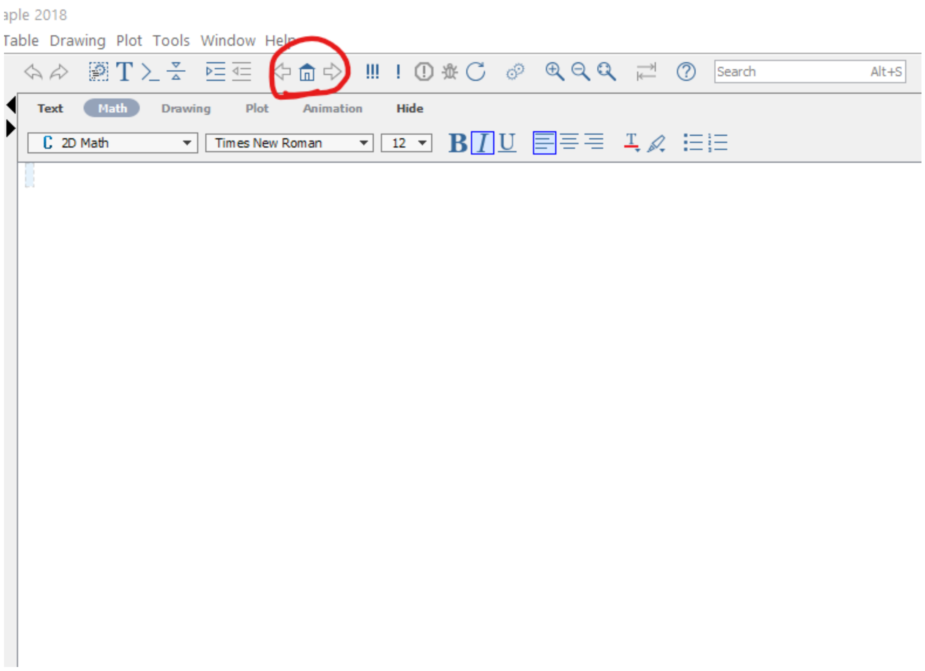



Figure 3:

- For Caculus, the most useful document is `Calculus`.

If you click the `Calculus` icon on the Start page and click `OK`, you will see the following document.

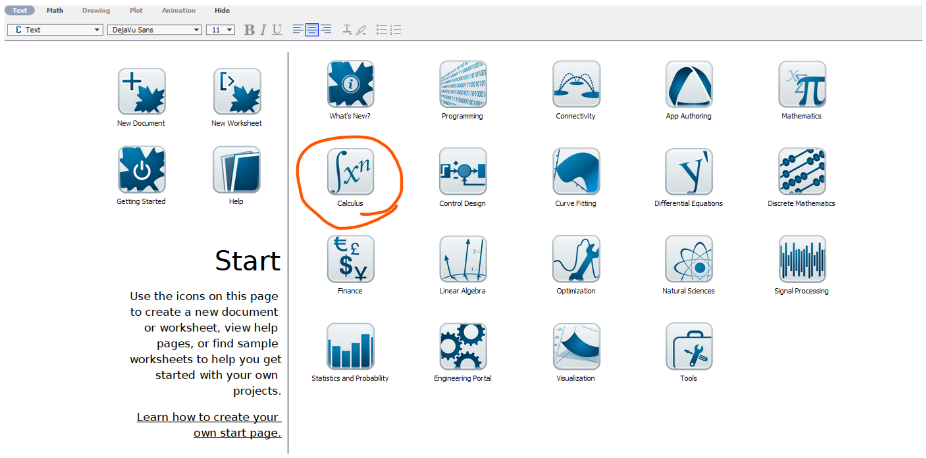



Figure 4:

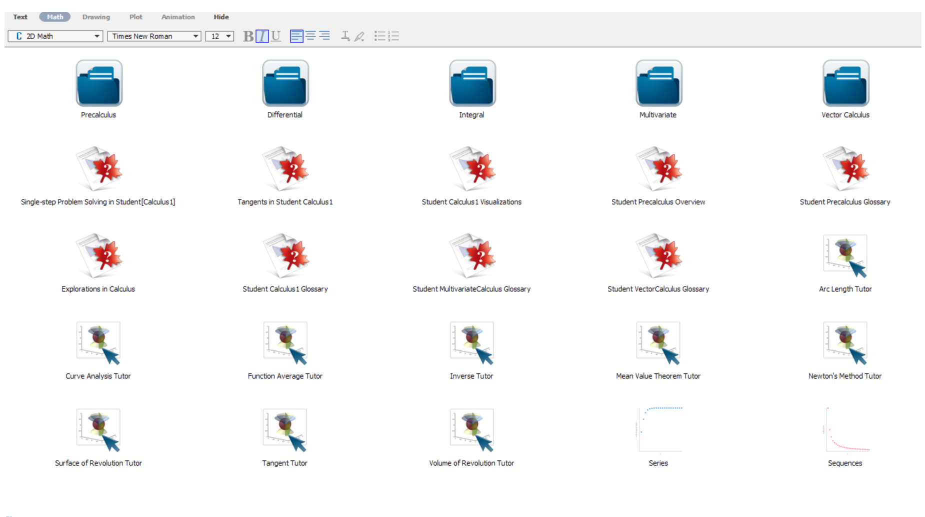



Figure 5:

# Chapter 1

# Volume of Revolution

In terms of definite integrals, the volume of a solid obtained by rotating a region about the $x$-axis can be calculated by

$$\int_a^b \pi(r_1(x)^2 - r_2(x)^2)\mathrm{d}x \qquad \text{disk/washer method,}$$

or

$$\int_a^b 2\pi r(y)h(y)\mathrm{d}y \qquad \text{shell method method,}$$

where $r_1(x)$, $r_2(x)$ and $r(x)$ represents the radius and $h(x)$ represents the height of a cylindrical shell.

In practice, it's better to recognize the shape of a cross section, find the volume of a slice of the solid and then set up the integral.

In the following, you will see some tools/commands from Maple which are very helpful to calculate the volume of a solid.

In Maple, the following command, supported by the package `Student[Calculus1]`, can be used to get the graph, the integral and the volume of the solid obtained by rotation the region bounded by $f(x)$, $g(x)$, $x = a$ and $x = b$.

```
VolumeOfRevolution(f(x), g(x), x = a..b, opts)
```

To learn what options does the command `VolumeOfRevolution` have, you may type

```
?VolumeOfRevolution
```

in the Math mode and hit enter. You will see the help page.
**Example 1.1.** Show the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ about $y$-axis. Set up an integral for the volume. Find the volume.

**Student[Calculus1]**

VolumeOfRevolution

find the volume of revolution of a curve

| | |
|---|---|
| Calling Sequence | Notes |
| Parameters | Examples |
| Description | Compatibility |

### Calling Sequence

```
VolumeOfRevolution(f(x), x = a..b, opts)
VolumeOfRevolution(f(x), g(x), x = a..b, opts)
VolumeOfRevolution(f(x), a..b, opts)
VolumeOfRevolution(f(x), g(x), a..b, opts)
```

### Parameters

| | | |
|---|---|---|
| f(x), g(x) | - | algebraic expressions in variable **x** |
| x | - | name; specify the independent variable |
| a,b | - | algebraic expressions; specify the endpoints of the curve |
| opts | - | equation(s) of the form **option**=**value** where **option** is one of **axis**, **distancefromaxis**, **functionoptions**, **function2options**, **lineoptions**, **method**, **numpoints**, **output**, **partition**, **regionoptions**, **revolutionpoints**, **showfunction**, **showrotationline**, **showsum**, **showvolume**, **sumvolumeoptions**, **volumeoptions**, **volume2options**, **border**, or Student plot options; specify output options |

Figure 1.1:

*Solution.*

Chapter 2

# Load the package

```
with(Student[Calculus1])
```

# Chapter 3

# Show the solid

```
VolumeOfRevolution(x^2, x, x = 0 .. 1, axis = vertical, output = plot)
```

## Chapter 4

# Set up an integral

```
VolumeOfRevolution(x^2, x, x = 0 .. 1, axis = vertical, output = integral)
```

# Chapter 5

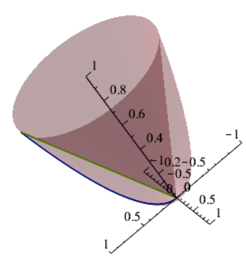
# Find the volume

`VolumeOfRevolution(x^2, x, x = 0 .. 1, axis = vertical, output = value)`

The outputs in Maple can be seen in the following picture

Visualize the solid

*VolumeOfRevolution*$(x\text{^}2, x, x = 0\,..\,1, axis = vertical, output = plot)$



The solid of revolution created on $0 \le x \le 1$ by rotation of $f(x) = x^2$ and $g(x) = x$ about the axis $x = 0$.

Setup the integral

*VolumeOfRevolution*$(x\text{^}2, x, x = 0\,..\,1, axis = vertical, output = integral)$

$$\int_0^1 -2\,\pi x^2\,(x-1)\,dx$$

Find the volume

*VolumeOfRevolution*$(x\text{^}2, x, x = 0\,..\,1, axis = vertical, output = value)$

$$\frac{\pi}{6}$$

*Remark.* 1. If you change the function to `VolumeOfRevolutionTutor`, you will see an interactive popup windows which does exactly the same thing.

2. If the rotation axis is not an axis of the coordinate system, you need add the option `distancefromaxis = numeric` into the function. For example,

if in the above example, the rotation is about $y = -2$, then the Maple command should be the following

```
VolumeOfRevolution(x^2, x, x = 0 .. 1, axis = vertical, distancefromaxis = -2, out
```

**Exercise 5.1.** Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $x = 0$, $y = 1$ about $y$-axis

**Exercise 5.2.** Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 0$, $x = 1$ about (a) $y = 0$, and (b) $x = 2$.

# Chapter 6

# Inverse Functions

Maple package `Student[Calculus1]` provides the following command

```
InversePlot(f(x), x = a..b);
```

which graphs the original function $f(x)$ and the inverse function $f^{-1}(x)$ together over the interval $[a, b]$.

You will see clearly that the graphs of a function and its inverse are symmetric with respect to the line $y = x$.

**Example 6.1.** 1. Graph the function $f(x) = x^3 - 2$, its inverse function, and the line $y = x$ over the interval $[-2, 2]$.

2. Find the inverse function.

*Solution.* One way to plot the function and its inverse together is to use the following command which is supported by the package `Student[Calculus1]`.

```
InversePlot(x^3-3, x = -2 .. 2)
```

Here is the output in Maple

Another way to plot the function $f$ and its inverse $g$ together uses the `plot` function.

```
plot([f(x), g(x), x], x = -2 .. 4, y = -5 .. 5, color = [red, black, blue])
```

To find the inverse function, we replace $f(x)$ by $y$, then switch $x$ and $y$, and solve for $y$. In Maple, you may use the command `solve(equation/inequality, variable)` to solve an equation or an inequality (even system of equations/inequalities).

In this example, we may find the inverse function by type in the following command. Note I have switch $x$ and $y$.

```
solve(x=y^3, y)
```

*InversePlot*$(x^\wedge 3 - 3, x = -2..2)$

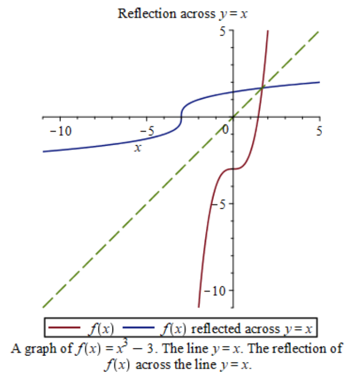



Figure 6.1: Graph of a pair of functions inverse to each other

To find the derivative of the inverse function of a function $f$ at a given point $x = a$, we may apply the formula

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}.$$

In Maple, we may use the following commands to calculate the value of the derivative function.

- Calculate the derivative of the function $f$.

```
diff(f(x), x)
```

- Find $f^{-1}(a)$ which is the solution of the equaiton $f(x) = a$.

```
solve(f(x)=a, x)
```

- Plug in the formula to evaluate.

```
eval(subs(x=f^{-1}(a), 1/f'(x)))
```

**Example 6.2.** Find $(f^{-1})'(0)$, where $f(x) = \cos(x)$ and $0 \le x \le \pi$.
*Solution.* Find the derivative of $f$

```
diff(cos(x), x)
```

Find the value of $f^{-1}(0)$

```
solve(cos(x)=0, x)
```

Apply the formula

```
eval(subs(x=Pi/2, -1/sin(x)))
```

Using Maple, we find $(f^{-1})'(0) = -1$.

**Exercise 6.1.** 1. Graph the function $f(x) = 3 + 2\sin x$, its inverse function, and the line $y = x$ over the interval $[-2, 2]$.

2. Find the value $(f^{-1})'(5)$.

# Chapter 7

# Logarithmic and Exponential Functions

## 7.1 Basic properties and graphs

The natural logarithmic function $y = \ln(x)$ is defined by $\ln(x) = \int_1^x \frac{1}{t} \mathrm{d}t$.

The natural exponential function $y = e^x$ is defined as the inverse function of $y = \ln(x)$.

From the definition, we have very important identities

$$\ln(e^x) = x \qquad \text{and} \qquad e^{\ln x} = x.$$

Using those two identities, we may define general exponential functions and general logarithmic function, and deduce the Law of Logarithms and Law of Exponents.

- For any positive number $b \neq 1$, we have $b^x = (e^{\ln b})^x = e^{x \ln b}$.
- For any positive number $b \neq 1$, we define $y = \log_b x$ to be the inverse function of $y = b^x$
- By solving $x = b^y$ for $y$, we find that $\log_b x = \dfrac{\ln x}{\ln b}$. This identity is called the change of base property.

How do graphs of logarithmic functions and exponential functions look like?
**Example 7.1.** Graph the following functions together.

$$y = \ln x, \qquad y = e^x, \qquad y = 2^x, \qquad y = \log_2 x, y = x.$$

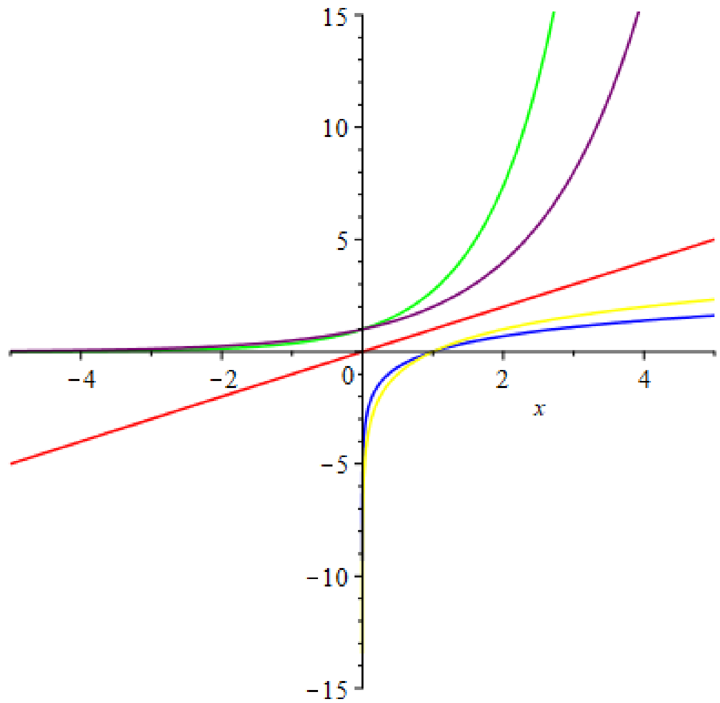



Figure 7.1: Graph of some logarithmic and exponential functions

*Solution.* In Maple, the logarithm $\log_b x$ is given by `log[b](x)`. When $b = e$, you simply use `ln(x)` for $\ln x$. When $b = 10$, you may also use `log(x)` or `log10(x)` for $\log_{10} x$.

The exponent $b^x$ is given by `b^x` in Maple. When $b = e$, you may also use `exp(x)` to represent $e^x$.

To graph the functions together with different colors, we use the following command

```
plot([ln(x), exp(x), 2^x, log[2](x), x], x=-5..5, color=[blue, green, purple, yellow, r
```

Here is the output

**Exercise 7.1.** Graph the following functions together.

$$y = \log_3 x, \qquad y = 3^x, \qquad y = (1/3)^x, \qquad y = \log_{1/3} x.$$

Find the pairs that are symmetric to each other with respect to a certain line.

**Exercise 7.2.** Graph the following functions together.

$$y = 0.5^x, \qquad y = 2^x, \qquad y = 5^x.$$

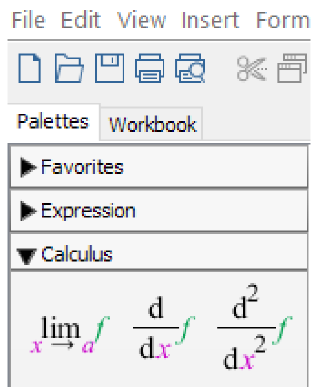



Figure 7.2: Calculus Palette in Maple

Describe the monotonicity (increasing/decreasing) of the functions?

Fix an input $x$. Describe how $y$-values change when bases changes from small number to bigger number?
**Exercise 7.3.** Graph the following functions together.

$$y = \log_{0.5} x, \qquad y = \log_2 x, \qquad y = \log_5 x.$$

Describe the monotonicity (increasing/decreasing) of the functions?

Fix an input $x$. Describe how $y$-values change when bases changes from small number to bigger number?

## 7.2 Differentiation and integration

In Maple, one way to do differentiation and integration is to use the `Calculus Palette` on the left side.

The other way is to use the commands `diff(f(x), x)` , `int(f(x), x)`, and `int(f(x), x=a..b)`.

Supported by the `Student[Calculus1]` package, Maple also provides the tutor commands `DiffTutor()` and `IntTutor()` which can show step-by-step solution of differentiation and integration.

Note you may also access tutor commands from the `Start` page (click the home button in the toolbar and look for Calculus).
**Example 7.2.** Find $y'$, where $y = \ln\left(x^3 + 5x + 1\right)$.
*Solution.* Using `diff`:

```
diff(ln(x^3+5*x+1), x)
```

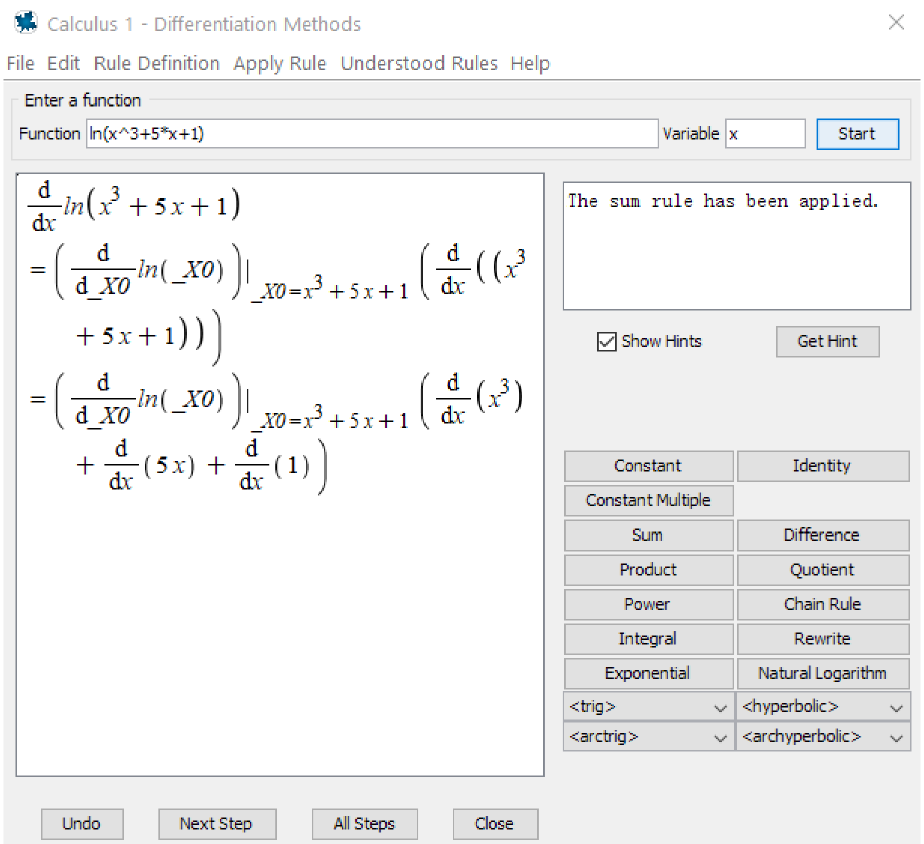



Figure 7.3:

We get

$$y' = \frac{3x^2+5}{x^3+5x+1}.$$

Type in (assume that `with(Student[Calculus1])` was run)

`DiffTutor(ln(x^3+5*x+1), x)`

and hit enter you will see

By click `Next Step` or `All Steps` you will see detailed solution with rules used.

**Example 7.3.** Evaluate the integral

$$\int \frac{e^x-1}{e^x+1}\mathrm{d}x.$$

*Solution.* Using `int`:

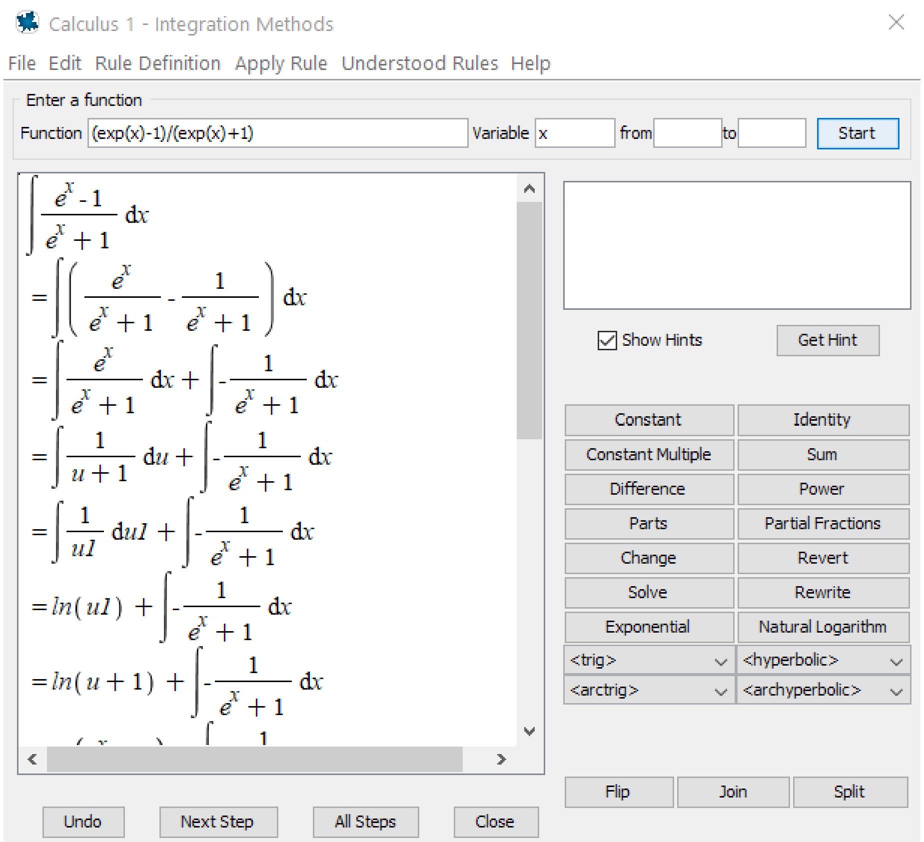



Figure 7.4:

```
int((exp(x)-1)/(exp(x)+1), x)
```

We get

$$\int \frac{e^x - 1}{e^x + 1} \mathrm{d}x = 2\ln\left(\mathrm{e}^x + 1\right) - x + C.$$

Type in (assume that `with(Student[Calculus1])` was run)

```
IntTutor((exp(x)-1)/(exp(x)+1), x)
```

and hit enter you will see

By click `Next Step` or `All Steps` you will see detailed solution with rules used.

**Exercise 7.4.** Find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$, where $y = \ln|\cos x|$

**Exercise 7.5.** Find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$, where $y = x^{\cos x}$

**Exercise 7.6.** Evaluate the integral

$$\int \frac{\left(e^{4x} + e^{2x}\right)}{e^{3x}} dx$$

**Exercise 7.7.** Evaluate the integral

$$\int 2^{3x} dx$$

# Chapter 8

# Solve differential equations

In Maple, you may solve the equation $y'(x) = ky(x)+c$ (which is called an ODE) using the command `dsolve({ics, eq})`, where `ics` stands for initial condition $y(0) = c$ and `eq` stands for the differential equation. Without the `ics`, `dsolve` will provide a general solution.

**Example 8.1.** Find the function $f(x)$ which satisfies the differential equation $f'(x) = kf(x)$ with $f(0) = 5$ and $f(2) = 3$.

*Solution.* Use the following command

```
dsolve({f(0)=5, f'(x)=k f(x)})
```

we get $f(x) = 5e^{kx}$.

To find $k$, we solve the equation $3 = 5e^{2k}$ by

```
solve(3=5*e^(2*k), k)
```

which shows that $k = \frac{\ln 3 - \ln 5}{2} \approx -0.255$. Here we use `evalf(%)` (% represents the previous result) to get the approximation.

So the function $f$ is given by

$$f(x) = 5e^{\frac{x(\ln 3 - \ln 5)}{2}} \approx 5e^{-0.255x}.$$

**Exercise 8.1.** Find the function $y$ which satisfies the differential equation $y'(x) = ky(x)$ with $y(0) = 2$ and $y(5) = 11$.