MA440 Worksheet

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1 Functions

1.1 Basic Concepts

Definition 1.1 A **relation** is a set of ordered pairs. The set of the first components of each ordered pair is called the **domain** and the set of the second components of each ordered pair is called the **range**.

A **function** is a relation that assigns each element in the domain a unique element in the range.

An arbitrary value in the domain is often represented by the lowercase letter x which is called an **independent variable**. An arbitrary output is often represented by the lowercase letter y which is called a **dependent variable**.

Example 1.1 Determine if the relation

$$\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)\}$$

is a function. Find the domain and the range.

If a function has x as the independent variable and y as the dependent variable, then we often say that y is a function of x.

Example 1.2 Consider items and prices in a grocery store. Is price a function of item? Is item a function of price?

A function is often named by letters, such as f, F, p, or q. If f is a function of x, then we denote it as $y = f(x)$ which is called the function notation. Here $f(x)$ is read as f of x or f at x. The notation $f(x)$ represents the output of the function f for a given input x.

Example 1.3 Use function notation to represent a function whose input is the name of a month and output is the number of days in that month.

Example 1.4 A function $N = f(y)$ gives the number of police officers, N, in a town in year y. What does $f(2005) = 300$ represent?

Example 1.5 Using a table to represent the days in the month as the function of month.

Example 1.6 Consider the function $f(x) = x^2+3x-4$. Find the values of the following expressions.

(1) $f(2)$ (2) $f(a)$ (3) $f(a+h)$ (4) $\dfrac{f(a+h)-f(a)}{h}$

Example 1.7 Consider the function $f(x) = x^2 - 2x$. Find all x values such that $f(x) = 3$.

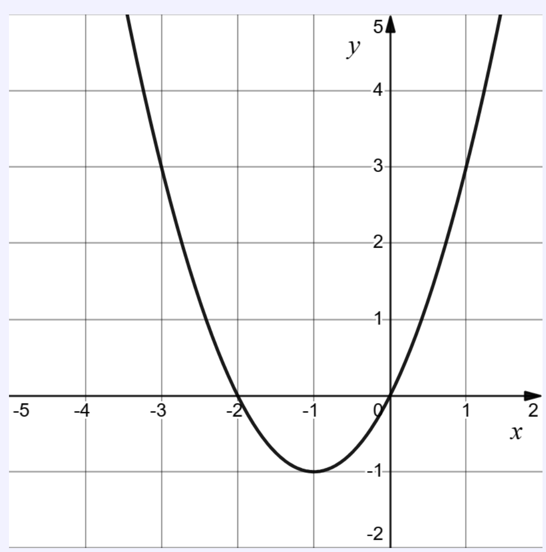
Example 1.8 Express the relationship defined by the function $2x - y - 3 = 0$ as a function $y = l(x)$.

Example 1.9 Does the equation $x^2 + y^2 = 1$ defines y as a function x. If so, express the relationship as a function $y = f(x)$. If not, under what extra condition does the function $y = f(x)$ exist?

Example 1.10 Consider the function $f(x)$ defined by a graph below.

(1) Find $f(-1)$.

(2) Find all x such that $f(x) = 3$.



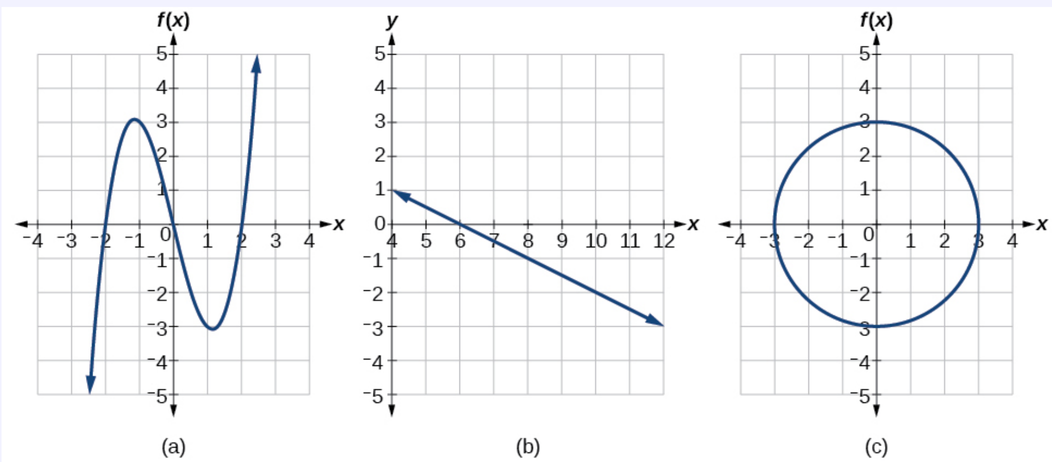
Definition 1.2 A function is a **one-to-one function** if each output value corresponds to exactly one input value.

Example 1.11 Is the area of a circle a function of its radius? If yes, is the function one-to-one?

A graph is a function if very vertical line crosses the graph at most once. This method is known as the **vertical line test**.

A function is an one-to-one if very horizontal line crosses the graph at most once. This method is known as the **horizontal line test**.

Example 1.12 Determine if the graph defines a function. If so, is it a one-to-one function?



Exercises

✍ **Exercise 1.1** Consider the function $f(x) = 2x^2 + x - 3$. Find the values of the following expressions.

(1) $f(-1)$ (2) $f(a)$ (3) $f(a+h)$ (4) $\dfrac{f(a+h) - f(a)}{h}$

✍ **Exercise 1.2** For the function $f(x) = -4x + 5$, evaluate and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

✍ **Exercise 1.3** Consider the function $f(x) = -x^2 - 4x$. Find all x values such that $f(x) = 3$.

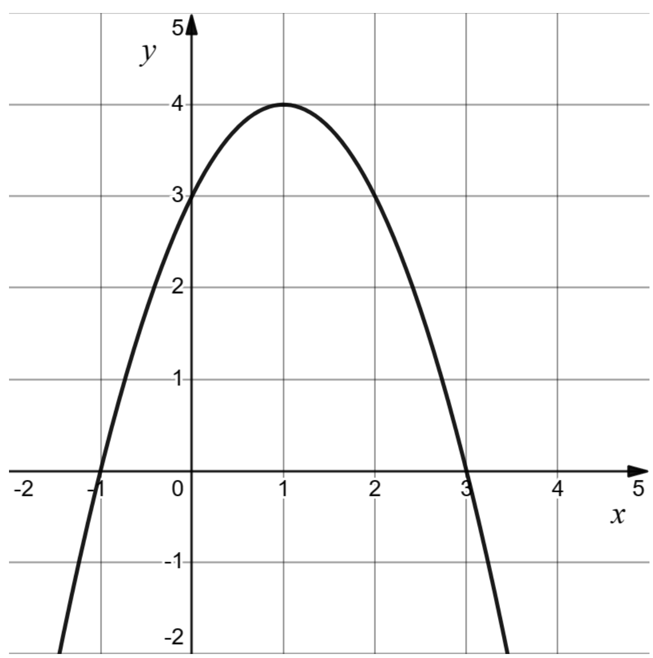
✍ **Exercise 1.4** Express the relationship defined by the function $3x - 2y - 6 = 0$ as a function $y = l(x)$.

✍ **Exercise 1.5** If $8x - y^3 = 0$, express y as a function of x.
Is y a one-to-one function of x?

✍ **Exercise 1.6** Consider the function $f(x)$ defined by a graph below.

(1) Find $f(1)$.

(2) Find all x such that $f(x) = 3$.



1.2 Domains and Ranges

The domain of a function f consists of possible input values x. Or equivalently, the domain consists of all x values except those that will make the function is undefined.

The range of a function f consists of all possible output values y. Equivalently, the range consists of y value such that equation $y = f(x)$ has a solution x.

Example 1.13 Find the domain of the function

$$f(x) = \frac{x+1}{2-x}.$$

Example 1.14 Find the domain of the function

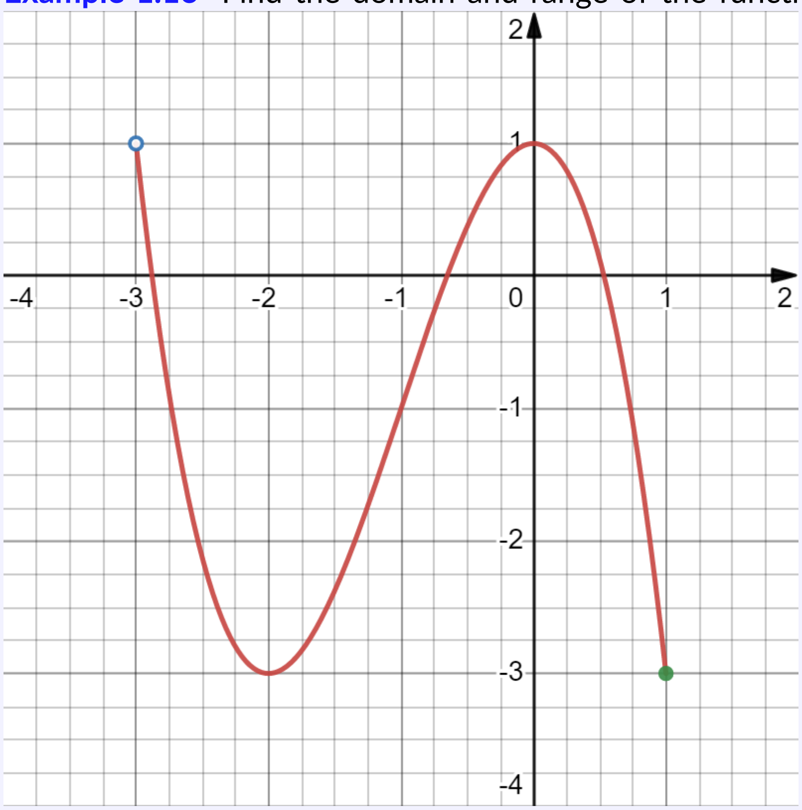
$$f(x) = \sqrt{7-x}.$$

Set-builder notation is a method of specifying a set of elements that satisfy a certain condition. It takes the form $\{x|$ statement about x$\}$ which is read as, “the set of all x such that the statement about x is true."

Interval notation is a way of describing sets that include all real numbers between a lower limit that may or may not be included and an upper limit that may or may not be included. The endpoint values are listed between brackets or parentheses. A square bracket indicates inclusion in the set, and a parenthesis indicates exclusion from the set.

Example 1.15 Find the domain of the function $f(x) = \frac{\sqrt{x+2}}{x-1}$. Write your answer in set-builder notation and interval notation.

Example 1.16 Find the domain and range of the function f whose graph is shown in Figure.



Example 1.17 Find the domain and range of the function
$$f(x) = \frac{2}{x+3}.$$

Example 1.18 Find the domain and range of the function
$$f(x) = 3\sqrt{x+2}.$$

Example 1.19 Consider the piecewise function
$$f(x) = \begin{cases} 2x-3 & \text{if} \quad x \le -1 \\ -x^2 & \text{if} \quad -1 < x < 1 \\ -2x+4 & \text{if} \quad 1 \le x. \end{cases}$$

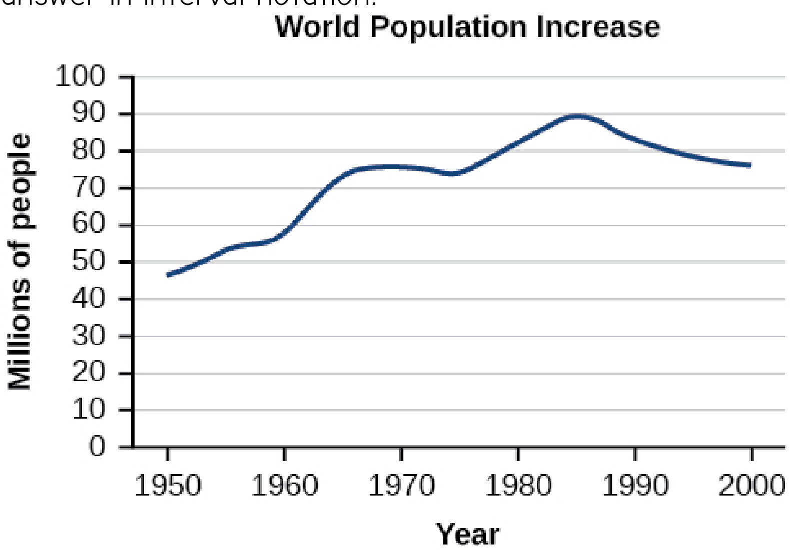
(1) Sketch the graph (2) Find $f(-4)$ (3) Find $f(2)$

Exercises

✍ **Exercise 1.7** Find the domain of the function

(1) $f(x) = \dfrac{1+4x}{2x-1}$ (2) $f(x) = \sqrt{5+2x}$ (3) $f(x) = \dfrac{\sqrt{x+1}}{x-1}$ (4) $f(x) = \dfrac{x-2}{x^2+7x-44}$

✍ **Exercise 1.8** Estimate the domain and range for the function defined by the graph. Write your answer in interval notation.



✍ **Exercise 1.9** Find the domain and range of each of the following functions. Write your answer in set-builder notation and interval notation.

(1) $f(x) = \frac{3}{x-2}$

(2) $f(x) = -2\sqrt{x+4}$

✍ **Exercise 1.10** Consider the piecewise function

$$f(x) = \begin{cases} -2x+5 & \text{if} \quad x < -2 \\ x^2 - 1 & \text{if} \quad -2 \leq x \leq 2 \\ 2x - 3 & \text{if} \quad 2 < x. \end{cases}$$

(1) Sketch the graph

(2) Find $f(-4)$

(3) Find $f(2)$

1.3 Rates of Change and Behavior of Graphs