MA440 Worksheet

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1 Functions

1.1 Basic Concepts

Definition 1.1 A **relation** is a set of ordered pairs. The set of the first components of each ordered pair is called the **domain** and the set of the second components of each ordered pair is called the **range**.
A **function** is a relation that assigns each element in the domain a unique element in the range.
An arbitrary value in the domain is often represented by the lowercase letter x which is called an **independent variable**. An arbitrary output is often represented by the lowercase letter y which is called a **dependent variable**.
Each value in the domain is also known as an input value. Each value in the range is also known as an output value.

Example 1.1 The relation

$$\{(1,2),(2,4),(3,6),(4,8),(5,10)\}$$

is a function.
The domain is $\{1,2,3,4,5\}$. The range is $\{2,4,6,8,10\}$.

If a function has x as the independent variable and y as the dependent variable, then we often say that y is a function of x.

Example 1.2 In a grocery store, if we take items as the domain, and prices as the range, then the relation is a function. Because each item must have a unique price.
However, if we take prices as the domain and items as the range, then the relation is not a function in general. Because there are often multiple items with the same price.
Those two relations may be described as the follow. Price is a function of item. Item is not a function of price.

In mathematics, a function is often named by letters, such as f, F, p, or q. To describe a function named f, we often use the equation notation $y = f(x)$ which means that f assigns to the input x the output value y. Here $f(x)$ is read as f of x or f at x. The notation $f(x)$ is known as the function notation which represents the output of the function f when the input is f.

1.2 Domains and Ranges

Exercises

✍ **Exercise 1.1** Find the vertex, focus, and directrix of the parabola. Sketch the graph.
(1) $x^2 = -8y$. (2) $y^2 = 12x$. (3) $x^2 + 6y = 0$. (4) $2x - y^2 = 0$.

✍ **Exercise 1.2** An equation of an ellipse is given. Find the center, vertices, and foci of the ellipse, and the lengths of the major and minor axes. Sketch the graph.
(1) $\dfrac{x^2}{9}+\dfrac{y^2}{25} = 1$. (2) $\dfrac{y^2}{9}+\dfrac{x^2}{25} = 1$. (3) $9x^2+25y^2 = 1$. (4) $25x^2+9y^2-16 = 0$.

✍ **Exercise 1.3** An equation of an ellipse is given. Find the center, vertices, foci, and asymptotes of the hyperbola. Sketch the graph.

(1) $\frac{x^2}{9} - \frac{y^2}{25} = 1$. (2) $\frac{y^2}{9} - \frac{x^2}{25} = 1$. (3) $9x^2 - 25y^2 = 1$. (4) $25x^2 - 9y^2 - 4 = 0$.

✍ **Exercise 1.4** Find an equation for the conic section with the given properties.

(1) The parabola with vertex at the origin and focus $(0, 5)$.

(2) The parabola with vertex at the origin and the directrix $x = -2$.

(3) The ellipse with vertices $(\pm 2, 0)$ and foci $(\pm 1, 0)$.

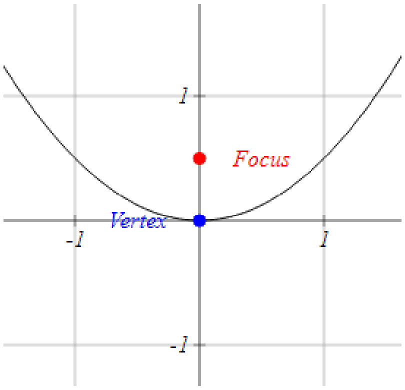
(4) the ellipse with foci $(0, \pm 3)$ and the eccentricity $e = \frac{3}{4}$.

(5) The hyperbola with foci $(0, \pm 3)$ and vertices $(\pm 2, 0)$.

(6) The hyperbola with foci $(\pm 5, 0)$ and asymptotes $y = \pm \frac{3}{4}$.

✍ **Exercise 1.5** Find an question for the conic section with the given graph.

(1)



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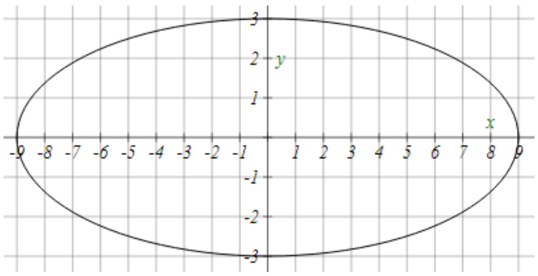

(3)