

# Introduction to Dynamical Systems

King's College London

2023-2024

4CCM131A

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## Homework 2

### Instructions

This homework has been set to be similar to a potential exam question.

As it is a participation homework, so long as you attempt the homework, the actual mark will not matter for your module mark - a serious attempt at the homework will earn the participation mark.

You do not need to complete all parts of the question to earn the participation mark, but I hope you will try so long as you do not spend too long on it.

This is a typical question: it starts with an easy part and then gets progressively harder. The marks are also not distributed evenly - the last few marks are much harder to get than the first few marks.

1. (10 marks) Find the general solution to the matrix differential equation

$$\dot{\mathbf{x}} = M \mathbf{x}, \quad M = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}, \quad (1)$$

and find a real matrix  $P$  such that  $P^{-1}MP$  is in normal form.

2. Consider the first order autonomous dynamical system

$$\frac{dx}{dt} = (x - 2)^2(x - 3). \quad (2)$$

- a) (5 marks) Sketch the phase portrait including a sketch of the velocity function.
- b) (4 marks) Find the fixed points and state their nature (stable or unstable).
- c) (6 marks) Find the exact time it takes the system to go
  - (i) From  $x = 4$  to  $x \rightarrow \infty$ .
  - (ii) From  $x = 5/2$  to  $x = 2$ .