Introduction to Dynamical Systems
King's College London
2023-2024
4CCM131A

Homework 1

Instructions
This homework has been set to be similar to a potential exam question.

As it is a participation homework, so long as you attempt the homework, the actual mark will not matter for your module mark - a serious attempt at the homework will earn the participation mark.

You do not need to complete all parts of the question to earn the participation mark, but I hope you will try so long as you do not spend too long on it.

This is a typical question: it starts with an easy part and then gets progressively harder. The marks are also not distributed evenly - the last few marks are much harder to get than the first few marks.

1. (9 marks) A first order ODE is an equation of the form

$$\frac{dx}{dt} = f(x,t)\,. \tag{1}$$

   (a) Find $f(x,t)$ such that the ODE is homogeneous but not separable.

   (b) Find $f(x,t)$ such that the ODE is separable but not homogeneous.

   (c) Find $f(x,t)$ such that the ODE both separable and homogeneous.

2. (9 marks) Consider the first order ODE

$$t^2\frac{dx}{dt} = (t+x)^2 + tx\,. \tag{2}$$

   (a) Show that the ODE can be written in the form

$$\frac{dx}{dt} = G(v)\,, \tag{3}$$

   where $v = x/t$.

   (b) Find the general solution $x(t)$ of the ODE.

3. (7 marks) Consider the initial value problem that consists of the ODE of Eq. (2) and the initial condition

$$x(1) = 0\,. \tag{4}$$

(a) Find the solution of the initial value problem.

(b) Apply Picard's theorem and show that the solution to the initial value problem exists in $[t_0 - \delta_1, t_0 + \delta_1]$ and it is unique. What is the range of possible values of δ_1?