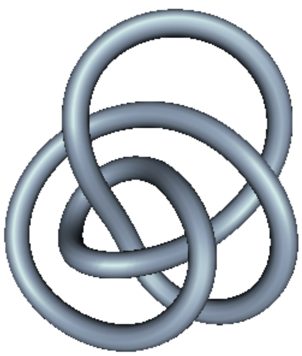
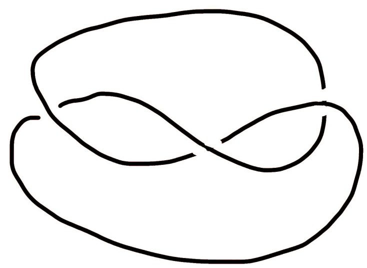
Geometric Topology 2024/25 — Sheet 1

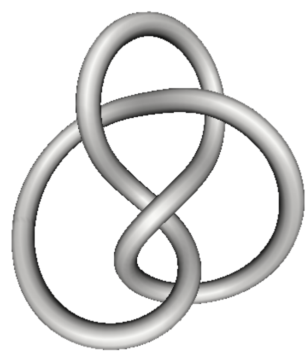
1. (i) The diagram below left represents a knot K. Find an alternating diagram D that also represents K, so that as one traverses D the crossings alternate under/over. How many crossings does D have?

(ii) Is the knot represented by the diagram below right ambient isotopic to the left trefoil knot $L3_1$ or the right one $R3_1$?





2. The diagram D shown right represents the figure-eight knot 4_1.

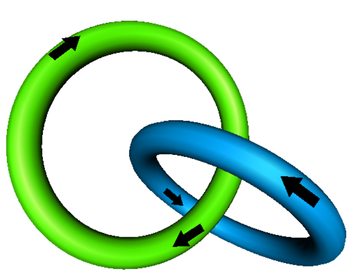


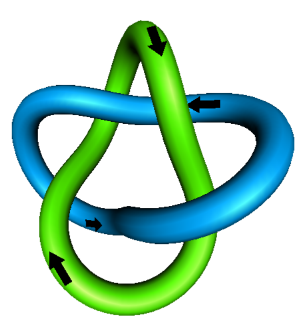
(i) Show that if *any one* crossing is reversed, then the resulting diagram D' is isotopic to one with two crossings, and so the unknot.

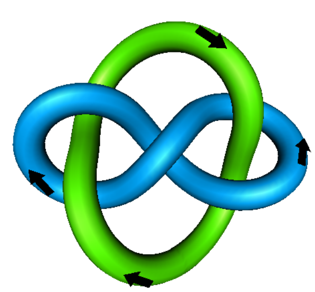
(ii) Show that it is possible to reverse two crossings so as to obtain the left-hand trefoil $L3_1$, and a different two so as to obtain $R3_1$.

3. (i) Treat the images below as link diagrams in the plane, with crossing data indicated. Compute the writhes of each of the three diagrams with respect to the indicated orientations.

(ii) Compute the linking numbers between the two components in each case.

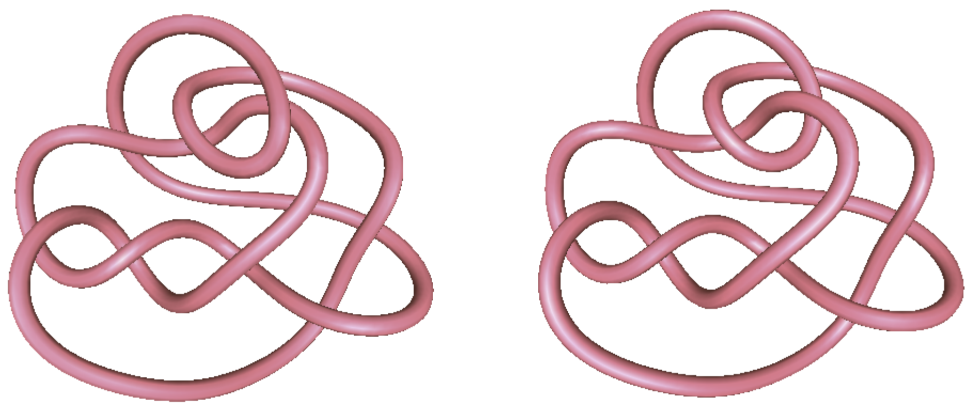






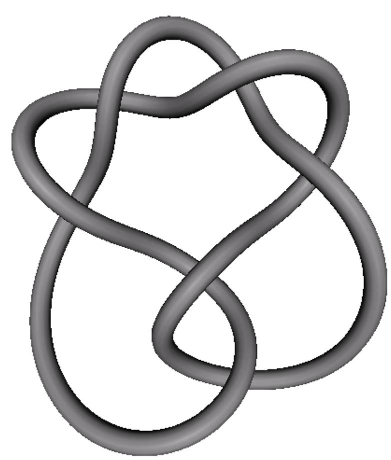
What conclusion can you draw about zero linking number?

4. The two diagrams below represent the projections of two different knots. Both diagrams have 10 crossings. Spot the difference between them. One corresponds to a loop of string that is actually unknotted, whereas the other is an over-complicated representation of the trefoil knot. Which is which? Identify the trefoil first, then try to produce some sketches to describe how the other can be unknotted.



5. Recall that an **arc** of a knot diagram is a strand that starts and ends at an underpass but passes under no other strand.

(i) Show that it is possible to colour each of the six arcs in this alternating diagram Red, Green or Blue such that at each crossing either all the colours are the same or they are all different.



(ii) Is it possible to find colourings such that at every crossing all three colours are different?