CCM115a Sequences and Series: Assignment 3

You are strongly encouraged to try as many of the problems on this sheet as possible (*if you are short of time*, then you may wish to focus on the problems marked with the symbol '†').

A selection of these problems will also be discussed in your tutorials.

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1. Solve the following inequalities for real $n > 0$:

   (a) $\frac{1}{10n^3} \leq 7$

   (b) $\frac{3}{n\sqrt{n}} < 5$

   (c) $2^{-n} \leq 20$

   (d) $2^n \leq 20$

   (e)† $(\frac{1}{2})^{5n} \geq 1/8$

   (f) $(\frac{1}{2})^{5n} \leq 1/8$

2. Solve the following equations and inequalities for $x \in \mathbb{R}$:

   (a) $|x - 3| = 1$

   (b) $|x + 1| \leq 3$

   (c)† $|x + 2| > 5$

   (d) $|x + 10| < 1$.

3. For each given set S, state whether it is bounded below and bounded above. If the set S is bounded above, find the set S_+ of all upper bounds of S; if S is bounded below, find the set S_- of all lower bounds for S. Find $\max S$ and $\min S$ if they exist. No proof is required.

   (a)† $S = [-10, -1) \cup (2, 5)$;

   (b) $S = \{1\} \cup (2, 3)$;

   (c) $S = \{\frac{1}{n^2} \mid n \in \mathbb{N}\}$;

   (d)† $S = \{\frac{(-1)^n}{n} \mid n \in \mathbb{N}\}$;

   (e) $S = \{(-1)^n - \frac{1}{n} \mid n \in \mathbb{N}\}$.

4. Prove that:

   (a) $S = [0, 1] \cup (3, \infty)$ is not bounded above;

   (b) The set $\mathbb{Q}$ of rational numbers is not bounded below;

(c)† $S = \{(-2)^n \mid n \in \mathbb{N}\}$ is not bounded above;

(d) $S = \cup_{n=1}^{\infty}[2n-1, 2n]$ is not bounded above.

5. Prove that:

   (a)† $\min S$ does not exist, where $S = (0, 1)$;

   (b) $\min S$ does not exist, where $S = \{2^n \mid n \in \mathbb{Z}\}$;

   (c)† $\min S$ does not exist, where $S = \{\frac{1}{\log n} \mid n \in \mathbb{N}, n \geq 2\}$.

6. Prove that S is bounded if and only if $\exists R \in \mathbb{R}$ such that $\forall x \in S$ one has $|x| \leq R$. Note that you have two statements to prove: "if" and "only if".