Linear Algebra and Geometry I   Skills Session 1

We often use 'sigma' notation to write a sum in a more concise form:

$$\sum_{k=1}^{n} f(k) = f(1) + f(2) + \cdots + f(n-1) + f(n).$$

Here k is 'dummy variable' (so you can replace it with your favourite letter instead).

Problem 1

Let $n \geq 2$ be an integer and define the sum

$$S = \sum_{j=1}^{n}(3j+1)^2.$$

Which of the following expressions are equal to S?

(A) $\displaystyle\sum_{j=0}^{n-1} 9j^2$ (B) $\displaystyle\sum_{j=0}^{n-1}(3j+4)^2$ (C) $\displaystyle\sum_{j=2}^{n+1}(3j-2)^2$ (D) $3n(3n+2)+\displaystyle\sum_{j=0}^{n-1}(3j+1)^2$

Problem 2

Show that
$$\sum_{j=0}^{n}\sum_{k=0}^{n}(j + 2k + 3) = 3(n+1)\sum_{k=0}^{n+1}k.$$

Hence find a formula in terms of n for the left hand side.

Which of the following expressions is equal to $\displaystyle\sum_{j=1}^{n}\sum_{k=1}^{n} jk$?

(A) $\displaystyle\sum_{k=1}^{n} jk$ (B) $\displaystyle\sum_{k=1}^{n} k^2$ (C) $\displaystyle\left(\sum_{k=1}^{n} k\right)^2$ (D) None of them

Problem 3

Find a formula, in terms of n, for the sum

$$\sum_{k=0}^{n} k\sin^2(k\pi/2).$$

(Hint: consider n even and n odd separately.)