Probability and Statistics 1

Participation Homework 2

Submit by 16:00 on 4 March 2024

Participation homeworks function as follows:

- There are 3 participation homeworks for this module (set in weeks 4, 7 and 10).
- Each participation homework will contain questions related to the previous three weeks of lectures.
- 10% of the total marks available on this course are for participation homeworks.
- Each of the three participation homeworks is equally weighted, so accounts for $3\frac{1}{3}$% of the overall module mark.
- Students who make reasonable attempts at the questions in a participation homework will gain full participation marks for that homework. Students who do not will gain no participation marks for that homework.
- In other words, the participation mark for each participation homework is either a "0" or a "1" (Bernoulli style!).
- You are permitted to work in groups on the problems, but you must submit your own solutions in order to earn the participation marks.
- Participation homeworks are marked by your tutorial tutor. They will provide written feedback as well as assessing your participation mark. Participation homeworks provide the only opportunity in the course for you to receive individual, formative feedback on your work.

Problem 1. A light switch has two states *On* and *Off*. Initially the light is Off. Each minute $n = 1, 2, 3, \ldots$, someone walks past the switch and, independently for each instance,

- If the light is Off, the person will always turn it On.
- If the light is On, the person will turn it Off with probability 1/3.

Let p_n be the probability that the light is On after the person walks past at minute n.

(a) Show that $p_{n+1} = 1 - \frac{1}{3}p_n$ for all $n \geq 0$.

(b) You may assume that when n is large, p_n is approximately equal to a fixed constant C. Find C.

Problem 2. Ibrahim runs bird-watching tours. On a particular tour, the number of falcons they see is F, and the number of hawks is H.
We assume that $F \sim \text{Poisson}(3)$ and $H \sim \text{Poisson}(5)$, and that F, H are independent.

(a) Find the probability that the tour sees at least 3 birds in total.

(b) A randomly-chosen guest on the tour has falcons as their favourite bird with probability 2/3 and hawks as their favourite bird with probability 1/3. Find the expected number of a randomly-chosen guest's favourite bird that are seen on the tour, and the probability that this number is at least one.

(c) Ibrahim acknowledges that there is a 5% chance he will have to cancel the tour because of vehicle issues. With this additional assumption, find the expected total number of birds that will be seen by a guest who has just bought a ticket for the tour.

Problem 3. Joyce also goes bird-watching. On each day $n = 1, 2, 3, \ldots$ independently, Joyce goes out with probability 1/4. On each trip, independently, there is a 1/10 chance that she sees a golden eagle.

(a) Let N be the number of *trips* (not days) until Joyce sees a golden eagle. State the distribution of N and calculate the probability generating function of N.

[*You may find it helpful to revisit Sheet 6, Problem 4.*]

(b) Let D be the number of the day on which Joyce first sees a golden eagle. Explain how to write D as a random sum

$$D = X_1 + X_2 + \ldots + X_N,$$

where the X_is are IID.

(c) Find the generating function of D, and deduce the distribution of D.

(d) Explain briefly how you could have established the distribution of D without generating functions.