Choose one of the following problems to complete for feedback- but I encourage you to attempt solving all of them!

Problem 1. Let $f(x_1, \ldots, x_n)$ be a non-constant polynomial in n variables with coefficients in $\mathbb{C}$. Show that the set $\{(a_1, \ldots, a_n) : f(a_1, \ldots, a_n) = 0\} \subset \mathbb{C}^n$ is non-empty and is not finite if $n \geq 2$. Show by example that this need not be true if we replace $\mathbb{C}$ by $\mathbb{R}$.

Problem 2. Consider the following conic $C = \{x^2 + 2y^2 - 3 = 0\} \subset \mathbb{R}^2$.

(1) Show that the point $(1, 1)$ lies on C.
(2) Let L be a line passing through $(1, 1)$. Show that (with one exception) L intersects C at exactly one other point
(3) Show that the point of intersection of L and C has rational coordinates (i.e., is of the form (p, q) where $p, q \in \mathbb{Q}$) if and only if the slope of L is rational.
(4) Deduce that there is a bijection between the points of C with rational coordinates and $\mathbb{P}^1_{\mathbb{Q}}$. Use this to find all the rational solutions to $x^2 + 2y^2 = 3$.

Problem 3. Below is a list of polynomials in affine coordinates $x = X/Z$ and $y = Y/Z$ on the set $\{Z \neq 0\}$. Calculate what these polynomials are in homogeneous coordinates.

- $f(x, y) = x^2 + y^3$
- $g(x, y) = x + y^2 + 1$
- $h(x, y) = xy + x^5$

Problem 4. For f, g, h above, find f', g', h' such that there is no linear change of coordinates which sends f' to f (resp. g' to g, resp. h' to h) **but** there is a projective linear change of coordinates which sends f' to f (resp. g' to g, resp. h' to h)