Classical Dynamics – Problem Sheet 2

to be discussed in tutorial the week of 7 October

1. Lindsey kicks a 0.5 kg football (think of this as a particle) at the last minute of the match. The ball follows the trajectory

$$x = (35\,\mathrm{m\,s^{-1}})t \quad , \qquad y = (5\,\mathrm{m}) - (2\,\mathrm{m\,s^{-1}})t \quad , \qquad z = (1.5\,\mathrm{m}) + (10\,\mathrm{m\,s^{-1}})t - (9.8\,\mathrm{m\,s^{-2}})t^2$$

All distances are in meters. Assume Lindsey is at the origin of our co-ordinate system and the goal is situated at $x = 35\,\mathrm{m}$. Assume also that the goal spans $-3.66\,\mathrm{m} < y < 3.66\,\mathrm{m}$ and is 2.44 m high.

   (a) Compute the momentum just after the ball is kicked (at $t = 0$).

   (b) Does Lindsey score a goal? i.e. when $x = 35\,\mathrm{m}$, is the ball within the goal's parameters?

   (c) In either case, compute the momentum when the ball is at $x = 35\,\mathrm{m}$.

(*Turn the page for a hint if you get stuck on part (b).*)

2. A drop of rain of mass $m = 1$ g $= 10^{-3}$ kg falls from the sky from an altitude $r(0) = 1$ km $= 1000$ m. Its initial velocity is $v(0) = 0$. Under the effect of the gravitational attraction (without any friction), the drop is falling according to the following law

$$r(t) = r(0) - \frac{1}{2}gt^2 + v(0)t \; .$$

Use $g \sim 10$ m s^{-2}.

   (a) Evaluate at what time the drop of rain will hit the ground.

   (b) Calculate velocity and momentum of the falling drop of rain.

   (c) Evaluate its velocity and momentum when it hits the ground.

   (d) Compare it with the momentum of a brick of $m = 1$ kg $= 1000$ g falling from your hands (altitude $r(0) = 1$ m and initial velocity $v(0) = 0\,\mathrm{m\,s^{-1}}$) on your feet.

3. A puffin of mass m at time t has position: $\quad \underline{r} = t^2\underline{e}_x + \underline{e}_y - (\sin t)\underline{e}_z$.

   (a) Calculate the velocity $\dot{\underline{r}}(t)$, the acceleration $\ddot{\underline{r}}(t)$, and the momentum $\underline{p}(t) = m\dot{\underline{r}}$.

   (b) Use Newton's 2nd Law to determine the force $\underline{F}(t)$ required in order that the puffin move along this trajectory.

   (c) Calculate the angular momentum $\underline{L}(t) = \underline{r} \times \underline{p}$ and the torque $\underline{N}(t) = \underline{r} \times \underline{F}$ about the origin $\underline{r} = \underline{0}$.

4. In lecture, we considered circular motion ($\dot{r} = 0$, where $r \equiv |\underline{r}|$) at constant frequency ($\dot{\theta} = 2\pi f \Rightarrow \ddot{\theta} = 0$) and calculated the angular momentum $\underline{L}$ and torque $\underline{N}$. Calculate again the angular momentum $\underline{L}$ and the torque $\underline{N}$ and check whether $\underline{N} = \dot{\underline{L}}$ for a spiral motion (i.e. assume now that $\dot{r} \neq 0$, $\ddot{\theta} = 0$).

5. Which of the following forces are conservative (a is a constant scalar, $\underline{c}$ a constant vector, $r = |\underline{r}|$ as usual)

   (i) $\underline{F} = a\,x\underline{e}_z$,

   (ii) $\underline{F} = \frac{a\,\underline{r}}{|\underline{r}|^3}$,

   (iii) $\underline{F} = a\underline{e}_x + 2yzr^2\underline{e}_y + y^2|\underline{r}|^2\underline{e}_z$,

(iv) $\underline{F} = \underline{c}$.

Find the potentials for those that are.

A particle of mass m moves in a potential V. Calculate the force $\underline{F}$ when:

(i) $V(\underline{r}) = x^2 + 2y^2 - z$.

(ii) $V(\underline{r}) = \frac{k}{|\underline{r}|} = \frac{k}{(x^2+y^2+z^2)^{\frac{1}{2}}}$.

Are energy and angular momentum conserved in each case?

6. The potential energy is given by the expression

$$V(\underline{r}) = \sum_{b=1}^{3} \sum_{c=1}^{3} C_{bc} r^b r^c$$

where C_{bc} is symmetric: $C_{bc} = C_{cb}$. Keeping careful track of indices, find an expression for the a^{th}component of the force where,

$$F_a(\underline{r}) = -\frac{\partial V}{\partial r^a} \ .$$

(Suggestion for 2b: Compute t when the ball reaches $x = 35\,\mathrm{m}$ and then use t to compute the y- and z-positions.)