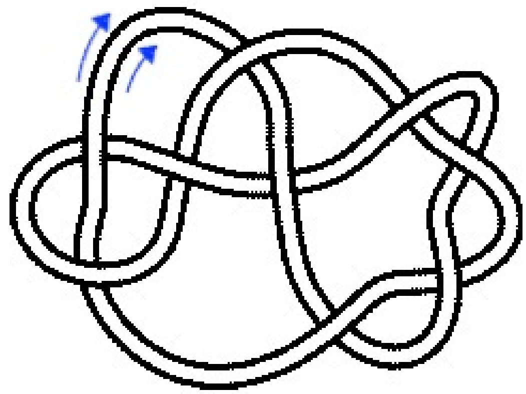
Geometric Topology 2024/25	Sheet 2

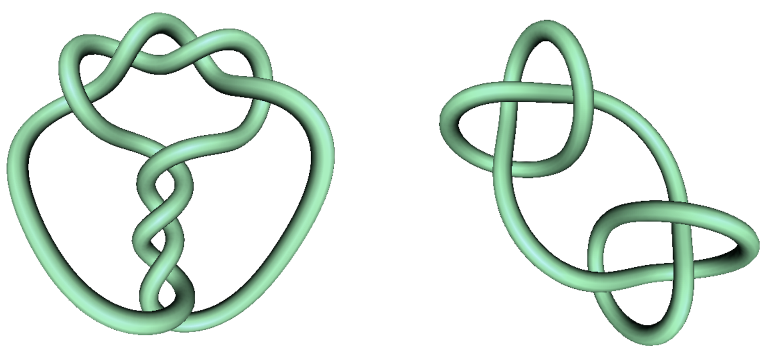
1. Below is a diagram D of the knot 9_{14} with 9 alternating crossings. Doubling the curve is a traditional means to make clear the overpasses and underpasses.

(i) Determine the sign of each of the 9 crossings, and hence find the writhe $w(D)$.

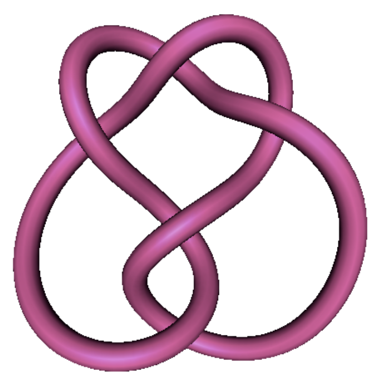
(ii) Now interpret D as the diagram of a link with 36 crossings and two components oriented in parallel. Write down their linking number.



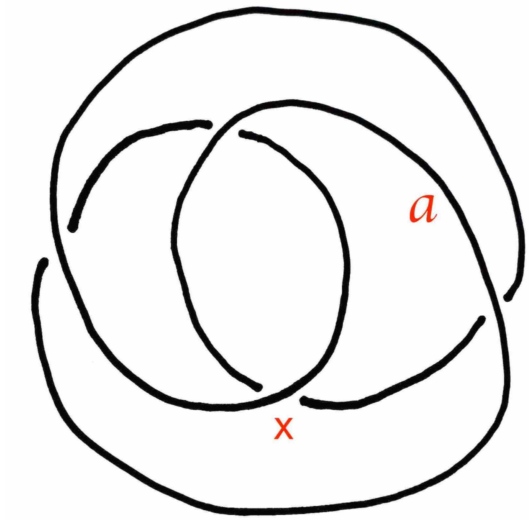
2. The diagrams below represent the prime knot 8_3 and the composite knot $R3_1\#L3_1$. Show that each be converted into the diagram of an unknot by reversing 2 crossings. This implies that both knots have unknotting number at most 2.



3. Find the value of n with $3 \leqslant n \leqslant 10$ such that the knot represented by the image shown on the right is n-colourable. To do this, label the arcs $a, b, c, d, 0$ with 0 bottom left, and write down five equations to find a non-zero solution.



4. (i) Copy the knot diagram D below. By modifying your sketch, or otherwise, describe two Reidemeister moves that (together with deformations R0) convert D into an alternating diagram D' of a trefoil knot. Is the latter $L3_1$ or $R3_1$? Hint: start by deforming arc a so that it passes much closer to crossing x.



(ii) Explain why D is not *regularly* isotopic to D'.

5. Let K be a knot having a projection to a diagram D with c crossings, and let G denote its shadow (the underlying planar graph with vertices in place of crossings).

(i) Explain how G can be made into a diagram D' of the unknot (as done on 23 Sep).

(ii) Deduce that the unknotting number $u(K)$ is at most $c/2$.