Assignment 6: for discussion and submission during **Week 7**

Assignment numbers are the ones as provided in the lecture notes. Please submit a written solution to Exercises 4.3.10 and 4.4.4 to receive the week 7 participation mark.

Exercise 4.2.13 Let $f(x, y, z) = x^2 - y^2 + z^3$. Find the equation of the level surface through $(1, 2, -1)$. Also find the equation of the straight line in $\mathbb{R}^3$ which is normal to this surface at $(1, 2, -1)$.

Exercise 4.2.14 Find the equation of the tangent plane to the surface $yz^2 + 2x^2 = 12$ at the point $(2, 1, 2)$.

Exercise 4.2.15 Show that the curve $\mathbf{r}(t) = 3t^{-1}\mathbf{i} - 2t^2\mathbf{j} + 2t\mathbf{k}$ meets the ellipsoid $x^2 + 3y^2 + z^2 = 25$ at the point $(3, -2, 2)$. Find the angle between the curve and the surface at this point.

Exercise 4.3.8 Find the maximum and minimum values of $f(x, y) = x^2 - 4y^2$ subject to the constraint $x^2 + 4xy + 6y^2 = 20$. Show that f has local minima at $(2\sqrt{5/3}, -2\sqrt{5/3})$ $(-2\sqrt{5/3}, 2\sqrt{5/3})$ and local maximma at $(4\sqrt{10/3}, -\sqrt{10/3})$ and $(-4\sqrt{10/3}, \sqrt{10/3})$.

Exercise 4.3.9 Show that the function $f(x, y) = e^{x^2-2y^2}$ has a critical point at $(0, 0$ and classify it.

Exercise 4.3.10 Please hand in this exercise Find all the critical points of $f(x, y) = x^2 + 2xy - y^2 + 3y$ and classify them.

Exercise 4.4.4 Please hand in this exercise Use the Lagrange multiplier method to maximize $f(x, y) = xy$ given that $x + y = 6$. Of course one can do this by substituting $y = 6 - x$, but it is good to practice the Lagrange multiplier method in this simple case.

Exercise 4.4.5 Maximise the function $f(x, y, z) = 2x + 4y + 4z$ on the sphere $x^2 + y^2 + z^2 = 36$.

Exercise 4.4.6 A factory can produce three products in quantities q_1, q_2, q_3 making a profit $P(q_1, q_2, q_3) = 2q_1 + 8q_2 + 24q_3$. Find the values of q_1, q_2, q_3 which maximize profit given that production is constrained by $q_1^2 + 2q_2^2 + 4q_3^2 = 9000$.

Exercise 4.4.7 Use the method of Lagrange multipliers to find the minimum distance from the origin to the plane $2x - 2y + z = 5$.

Exercise 5.1.6 Find the length of each of the following arcs:

(1) $\mathbf{r}(t) = (\sin 2\pi t, \cos 2\pi t, t)$ where $0 \leq t \leq 1$;
(2) $\mathbf{r}(t) = (3t, 4t)$ where $0 \leq t \leq 10$;
(3) $\mathbf{r}(t) = (e^t, e^t)$ where $0 \leq t \leq 1$.