Problem 1.

(1) Show that the Zariski topology on $\mathbb{R}^n$ does not coincide with the usual Euclidean topology.

(2) Show that the Zariski topology on $\mathbb{R}^{n+m}$ does not coincide with the product of the Zariski topologies on $\mathbb{R}^n$ and $\mathbb{R}^m$.

Problem 2. Let $X \subset \mathbb{K}^n$ be an algebraic set, and let $f \in \mathbb{K}[x_1, \ldots, x_n]$.

(1) Show that in general $X \cap \{f \neq 0\}$ is not an algebraic set.

(2) Show that $X \cap \{f \neq 0\}$ can be naturally identified with an algebraic set in $\mathbb{K}^{n+1}$. (Hint: consider the equation $yf - 1 = 0$.)

Problem 3. Consider the map $\mathbb{P}^1 \to \mathbb{P}^n$ given by $[S : T] \mapsto [S^n : S^{n-1}T : \cdots : ST^{n-1} : T^n]$. Show that the image of this map is a projective algebraic set.

Problem 4. Consider the map $b \colon \mathbb{K}^2 \to \mathbb{K}^2$ given by $(x, y) \mapsto (x, xy)$. What is the image of b? Is it Zariski open or closed (or both or neither)?

Problem 5. Let $f \colon \mathbb{K}^n \to \mathbb{K}^m$ be a map defined by polynomials. Show that the graph of f inside $\mathbb{K}^n \times \mathbb{K}^m$ is an algebraic set.

Problem 6. We say that a subset $X \subset \mathbb{C}^n$ is an analytic set if $X = \{f_1 = \cdots = f_r = 0\}$ where $f_i \colon \mathbb{C}^n \to \mathbb{C}$ are holomorphic. Show that there exists an analytic set which is not algebraic.