CCM115a Sequences and Series: Assignment 2

You should submit solutions to all of the problems on the list below that are marked with the symbol '†'. The deadline for submission of these solutions is

4pm (UK time), Monday 16th October.

Submission is online, via the Keats pages for this course (and full instructions are available there).

1. Give a more direct description of the following sets:

(a)[†]
$$(-\infty, 1) \cap (-1, 3) \cap (2, \infty)$$

(b)
$$([0,\infty)\cap[1,10))\setminus(1,2)$$

$$(c)^{\dagger} ([0,\infty) \setminus (1,3)) \cap [2,10]$$

(d)
$$\{x \in \mathbb{R} \mid x^2 \ge 10 \text{ and } 2x + 1 < 0\}$$

(e)[†]
$$\{x \in \mathbb{R} \mid x^2 \le 1 \text{ or } x > 0\}.$$

- $2.^{\dagger}$ Following the pattern of the proof of Theorem 4.2(i) from the lecture notes, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for any sets A, B, C.
- 3. Prove that $\bigcup_{n=1}^{\infty} (0,3^n) = (0,\infty)$.
- 4. Prove that $\bigcap_{n=1}^{\infty} (1 \frac{1}{n^2}, 1 + \frac{1}{n^2}) = \{1\}.$
- 5.† Prove that $\bigcup_{n=1}^{\infty} (-2n^3, 2n^3) = \mathbb{R}$.
- 6. Prove that $\bigcap_{n=0}^{\infty} [0, 2^{-n}] = \{0\}.$