10 - Risposta in Frequenza

Monday, 27 June 2022 18:22

ES 1

$A = \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} \quad b = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad c = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \quad d = 0$

TRASFERIMENTO

$$\begin{cases} \dot{x}_1 = -x_1 + u \\ \dot{x}_2 = x_1 - x_2 \\ y = x_2 \end{cases} \quad \begin{cases} s x_1 = -x_1 + u \\ s x_2 = x_1 - x_2 \\ y = x_2 \end{cases} \rightarrow G(s) = \frac{1}{(s+1)^2}$$

STABILITA'

$\lambda_1 = \lambda_2 = -1 \qquad Re(\lambda_{1,2}) < 0 \rightarrow AS$

GUADAGNO $G(0) = +1$

CASO 1 $u(t) = -2$

$$y(\infty) = \lim_{s \to 0} s \cdot L(y) = \lim_{s \to 0} s \cdot L(-2) \cdot G(s)$$
$$= \lim_{s \to 0} \cancel{s} \frac{-2}{\cancel{s}} G(s) = -2\,G(0) = -2$$

CASO 2 $u(t) = 10 \sin(2t)$

TEOREMA $y(\infty) = |G(i\omega)|\, A \sin(\omega t + ARG(G(i\omega)))$

NEL NOSTRO CASO $\omega = +2$

$$G(i\omega) = \frac{1}{(1+2i)^2} = \frac{1}{4i-3} \cdot \frac{4i+3}{4i+3} = -\frac{4i+3}{25} = -\frac{3}{25} - \frac{4}{25} i$$

$$|G(i\omega)| = \left| -\frac{4i+3}{25} \right| = \frac{1}{25}\sqrt{4^2+3^2} = \frac{5}{25} = \frac{1}{5}$$

$$ARG(G(i\omega)) = ARCTAN\left(-\frac{4}{25} : -\frac{3}{25}\right) = ARCTAN(4/3) = 53° \underline{+180°}$$

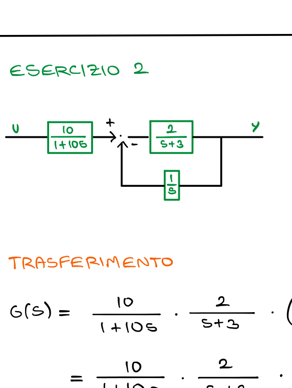
RISPOSTA $y(\infty) = 10 \cdot \frac{1}{5} \cdot \sin(2t + 53°) = 2 \sin(2t + 233°)$

CASO 3 $u(t) = -2 + 2\sin(2t)$

PER SOVRAPPOSIZIONE EFFETTI

$y(\infty) = -2 + 2\sin(2t + 233°)$

DIAGRAMMA DI BODE



2 POLI A $\omega = +1$

$\rightarrow -2 \cdot 20dB$

2 POLI A $\omega = +1$

$\rightarrow -2 \cdot 90°$

ESERCIZIO 2

TRASFERIMENTO

$$G(s) = \frac{10}{1+10s} \cdot \frac{2}{s+3} \cdot \left(1 + \frac{2}{s(s+3)}\right)^{-1} =$$
$$= \frac{10}{1+10s} \cdot \frac{2}{\cancel{s+3}} \cdot \frac{s\,\cancel{(s+3)}}{s^2+3s+2} =$$
$$= \frac{20\,s}{(1+10s)(s+1)(s+2)}$$

RISPOSTA A $u(t) = -5 + 20\cos(t)$

TEOREMA RISPOSTA FREQUENZA

$y(\infty) = \cancel{-5\,G(0)} + 20\,|G(i)| \cos(t + ARG(G(i)))$

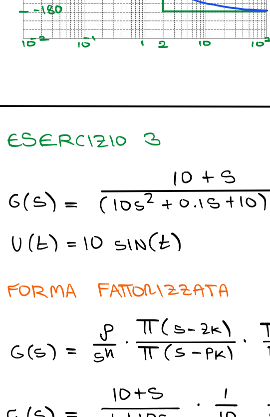
GUADAGNO $G(0) = 0$ POICHE' $s = 0$ E' UNO ZERO

$$|G(i)| = \left| \frac{20i}{(1+10i)(i+1)(i+2)} \right| = \frac{20}{\sqrt{1+10^2} \cdot \sqrt{1^2+1^2} \cdot \sqrt{1^2+2^2}} \cong 0.63$$

$$ARG(G(i)) = \frac{\pi}{2} - ARCTAN\,10 - ARCTAN\,1 - ARCTAN\,2 = -65°$$

RISPOSTA $y(\infty) = 12.3 \cos(t - 65°)$

DIAGRAMMI DI BODE



ZERO $\omega = 0$

POLI $\omega = \frac{1}{10}, 1, 2$

ESERCIZIO 3

$$G(s) = \frac{10+s}{(10s^2 + 0.1s + 10)(1+10s)}$$

$u(t) = 10 \sin(t)$

FORMA FATTORIZZATA

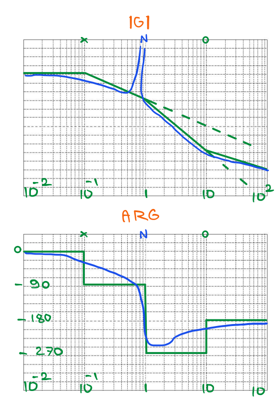
$$G(s) = \frac{\rho}{s^h} \cdot \frac{\prod(s - z_k)}{\prod(s - p_k)} \cdot \frac{\prod(s^2 + 2\xi\alpha s + \alpha^2)}{\prod(s^2 + 2\delta\omega s + \omega^2)}$$

$$G(s) = \frac{10+s}{1+10s} \cdot \frac{1}{10} \quad \frac{1}{s^2 + 0.01s + 1}$$

$$G(s) = \frac{1}{10} \quad \frac{10+s}{1+10s} \cdot \frac{1}{s^2 + 2 \cdot 0.005\,s + 1^2}$$

LA PULSAZIONE NATURALE E' $\omega = +1$

DIAGRAMMI DI BODE



RISPOSTA A SCALINO

ZERI $s = -10$

POLI $s_1 = -\frac{1}{10} \qquad s_{2,3} = -0.005 \pm \sqrt{0.005^2 - 1}$

STABILITA' $Re(s_i) < 0\ \forall i \rightarrow AS$

OSCILLAZIONI $\exists\, s_{2,3} \in \mathbb{C} \rightarrow$ PERMANENTI

PERIODO $\omega_N = 1 \qquad T = \frac{2\pi}{\omega_N} \cong 6$ SECONDI

TEMPO $\lambda_D = -0.005 \qquad T_R = 5\,\frac{-1}{\lambda_D} = 1000$

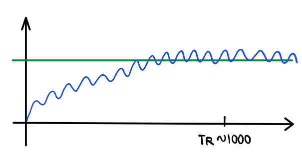
GUADAGNO $G(0) = 1 = y(\infty)$

GRADO $R = 2 > 0$ PROPRIO

$y(0) = \lim_{s \to \infty} s L(y) = \lim_{s \to \infty} s \frac{1}{s} G(s) = 0$

$y(\infty) = \lim_{s \to 0} s L(y) = \lim_{s \to 0} s \frac{1}{s} G(s) = G(0) = 1$

$\dot{y}(0) = \lim_{s \to \infty} s(sL(y) - \cancel{y(0)}) = \lim_{s \to \infty} s^{\cancel{2}} \frac{1}{\cancel{s}} G(s) = 0$



RISPOSTA IN FREQUENZA

$u(t) = 10 \sin(t)$

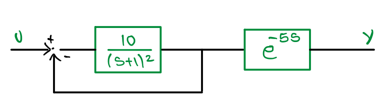
$y(\infty) = 10 \cdot |G(i)| \cdot \sin(t + ARG(G(i)))$

$$|G(i)| = \frac{|10+i|}{|1+10i|\,|\cancel{-10} + 0.1i\,\cancel{+10}|} = \frac{\sqrt{10^2+1^2}}{\sqrt{1^2+10^2}\ \sqrt{0.1^2}} = 10$$

$$ARG(i) = ARG\left(\frac{1}{10}\right) - ARG(10) - ARG(\infty) = -169°$$

$\rightarrow y(\infty) = 100 \sin(t - 169°)$

ESERCIZIO 4



TRASFERIMENTO

$$G(s) = \frac{10}{(s+1)^2}\left(1 + \frac{10}{(s+1)^2}\right)^{-1} e^{-5s} =$$
$$= \frac{10}{\cancel{(s+1)^2}} \frac{\cancel{(s+1)^2}}{s^2+2s+11} e^{-5s}$$
$$= \frac{10\,e^{-5s}}{s^2+2s+11}$$

FORMA FATTORIZZATA

$$G(s) = \frac{\rho}{s^h} \frac{\prod(s-z)}{\prod(s-p)} \frac{\prod(s^2 + 2\xi\alpha + \alpha^2)}{\prod(s^2 + 2\delta\omega + \omega^2)}$$

$$G(s) = 10\,e^{-5s} \frac{1}{s^2 + 2 \cdot \frac{1}{\sqrt{11}} \sqrt{11}\,s + \sqrt{11}^2}$$

FREQUENZA NATURALE $\omega = \sqrt{11} \sim 3$

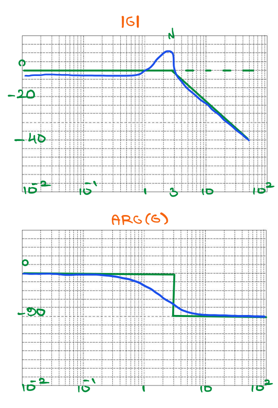
POLI CC $P_{1,2} = -1 \pm \sqrt{10}\, i$

STABILITA' $Re(P_{1,2}) < 0 \rightarrow \underline{AS}$

DIAGRAMMA DI BODE

$$\omega = 3 \rightarrow |G(3i)| = \frac{|10\,e^{-5i}|}{|2+6i|} = \frac{10}{\sqrt{2^2+6^2}} \cong 1.6$$

$|G(3i)|_{dB} = 20 \log(1.6) = 4dB$



-40 dB / DECADE

RITARDATO DI 5s

RISPOSTA IN FREQUENZA

POSSO APPLICARLA POICHE' AS

$u(t) = 1 + 2\sin(0.1t)$

GUADAGNO $G(0) = \frac{10}{11} \sim 1$

$u(t) = 1 \rightarrow y(\infty) = G(0) \cdot 1 = 1$

SINUSOIDE

$u(t) = 2\sin(0.1t)$

$$|G(0.1i)| = \frac{|10\,e^{-0.1i}|}{|11 - 0.01 + 0.2i|} = \frac{10}{\sqrt{(11-0.01)^2 + 0.2^2}} = 0.91 \sim 1$$

$$ARG(G(0.1i)) = \cancel{ACTAN\left(\frac{0.2}{11-0.01}\right)} + ARG(e^{-5i\,0.1}) = -0.5\,RAD = -30°$$

RISPOSTA $y(\infty) = |G| \cdot 2 \cdot \sin(0.1t + ARG)$

OVVERO $y(\infty) = 2\sin(0.1t - 30°)$

RISPOSTA COMPLESSIVA

PER LINEARITA' OTTENIAMO

$y(\infty) = 1 + 2\sin(0.1t - 30°)$

ESERCIZIO 5

DISEGNA BODE DI

$$G(s) = -\frac{1}{10} \frac{(10s+1)s}{(s+1)^3}$$