7 - Non Lineari

Sunday, 26 June 2022 17:38

SISTEMA 1

$\dot{x} = 2(x-y)$

$\dot{y} = x^2 - 4x - y$ $\longrightarrow M = \begin{vmatrix} 2x & -2y \\ x^2-4x & -y \end{vmatrix}$

EQUILIBRIO

$\dot{x} = \dot{y} = 0 \qquad x \to \bar{x} \qquad y \to \bar{y}$

$\begin{cases} 0 = x - y \\ 0 = x^2 - 4x - y \end{cases} \quad \begin{cases} y = x \\ x^2 - 4x - x = 0 \end{cases} \quad \begin{cases} y = x \\ x(x-5) = 0 \end{cases}$

$E_1(0,0) \qquad E_2(5,5)$

LINEARIZZAZIONE

$$J = \begin{vmatrix} \frac{d}{dx}(2x) & \frac{d}{dy}(-2y) \\ \frac{d}{dx}(x^2-4x) & \frac{d}{dy}(-y) \end{vmatrix} = \begin{vmatrix} 2 & -2 \\ 2x-4 & -1 \end{vmatrix}$$

STABILITA' EQUILIBRIO E_1

$J(E_1) = \begin{vmatrix} 2 & -2 \\ -4 & -1 \end{vmatrix} \qquad \begin{matrix} TR(J) = +1 > 0 \\ DET(J) = -10 < 0 \end{matrix} \longrightarrow$ INSTABILE SELLA

AUTOVALORI

$\Delta_J(\lambda) = (\lambda-2)(\lambda+1) - (4)(2) = 0$

$\longrightarrow \lambda^2 - \lambda - 10 = 0$

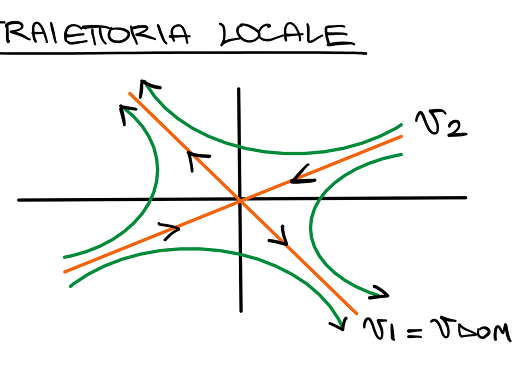
$\lambda_{1,2} = \frac{+1 \pm \sqrt{1-4(-10)}}{2} = \frac{1 \pm \sqrt{41}}{2}$ — 3.70 — -2.70

AUTOVETTORI

$Av = \lambda v$

$\begin{vmatrix} 2 & -2 \\ -4 & -1 \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \end{vmatrix} = \lambda \begin{vmatrix} v_1 \\ v_2 \end{vmatrix}$

TRAIETTORIA LOCALE



$2v_1 - 2v_2 = \lambda v_1$

$v_1(2-\lambda) = 2v_2 \qquad v_1 = \frac{2}{2-\lambda} v_2$ — $-1,176\ (\lambda^+)$ DOM — $+0,425\ (\lambda^-)$

STABILITA' EQUILIBRIO E_2

$J(E_2) = \begin{vmatrix} 2 & -2 \\ 6 & -1 \end{vmatrix} \qquad \begin{matrix} TR(J) = +1 > 0 \\ DET(J) = +10 > 0 \end{matrix} \longrightarrow$ INSTABILE FUOCO

INSTABILE

$\Delta_J(\lambda) = (\lambda-2)(\lambda+1) - (-6)(+2) = 0$

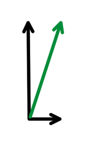
$\longrightarrow \lambda^2 - \lambda + 10 = 0$

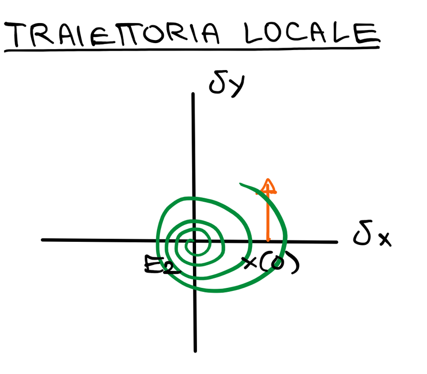
$\lambda_{1,2} = \frac{+1 \pm \sqrt{+1-4(10)}}{2} = \frac{+1 \pm i\sqrt{39}}{2} = \frac{1}{2} \pm i\frac{\sqrt{39}}{2}$

TRAIETTORIA LOCALE

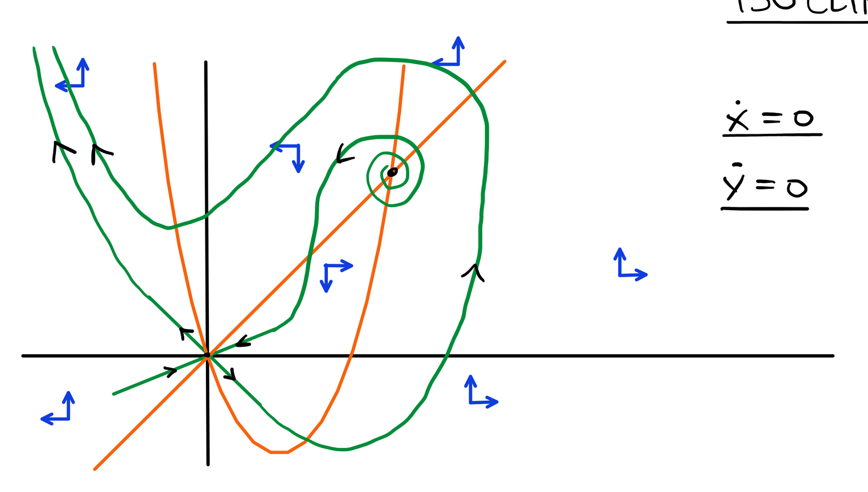
POSTO $x = \begin{vmatrix} x(0) \\ 0 \end{vmatrix}$

ALLORA $\begin{vmatrix} 2 & -2 \\ 6 & -1 \end{vmatrix} \begin{vmatrix} x(0) \\ 0 \end{vmatrix} = \begin{vmatrix} 2x(0) \\ 6x_0(0) \end{vmatrix}$





TRAIETTORIA GLOBALE



ISOCLINEE

$\dot{x} = 0 \qquad x = y$

$\dot{y} = 0 \qquad x^2 - 4x = 0$

SISTEMA 2

$\dot{x} = 2x(1-x) + y = f_1$

$\dot{y} = -y = f_2$

EQUILIBRI

$\dot{x} = \dot{y} = 0$

$\begin{cases} 0 = 2x(1-x) + y \\ y = 0 \end{cases} \quad \begin{cases} 2x(1-x) = 0 \\ y = 0 \end{cases}$

$E_1(0,0) \qquad E_2 = (1,0)$

LINEARIZZAZIONE

$M = \begin{vmatrix} 2x - 2x^2 & y \\ 0 & -y \end{vmatrix}$

$J = \begin{vmatrix} 2-4x & 1 \\ 0 & -1 \end{vmatrix}$

STABILITA' E_1

$J(E_1) = \begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix}$ MATRICE TRIANGOLARE $\lambda_1 = +2 \quad \lambda_2 = -1$

$\exists \lambda_i$ con $Re(\lambda_i) > 0 \longrightarrow$ INSTABILE SELLA

AUTOVETTORI

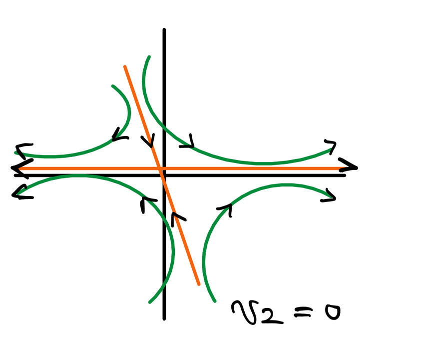
$Av = \lambda v$

$\begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \end{vmatrix} = \lambda \begin{vmatrix} v_1 \\ v_2 \end{vmatrix}$

$2v_1 + v_2 = \lambda v_1$

$v_2 = (\lambda - 2) v_1$

TRAIETTORIA LOCALE



$v_2 = 0 \qquad (\lambda = +2)$

$v_2 = -3v_1 \qquad (\lambda = -1)$

STABILITA' E 2

$J(E_2) = \begin{vmatrix} -2 & -1 \\ 0 & -1 \end{vmatrix}$ MATRICE TRIANGOLARE $\lambda_1 = -2 \quad \lambda_2 = -1$

$Re(\lambda_i) < 0 \ \forall i \longrightarrow$ NODO STABILE

AUTOVETTORI

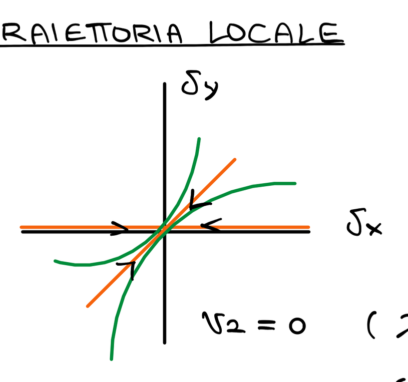
$Av = \lambda v$

$\begin{vmatrix} -2 & -1 \\ 0 & -1 \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \end{vmatrix} = \lambda \begin{vmatrix} v_1 \\ v_2 \end{vmatrix}$

$-2v_1 - v_2 = \lambda v_1$

$v_2 = (-2-\lambda) v_1$

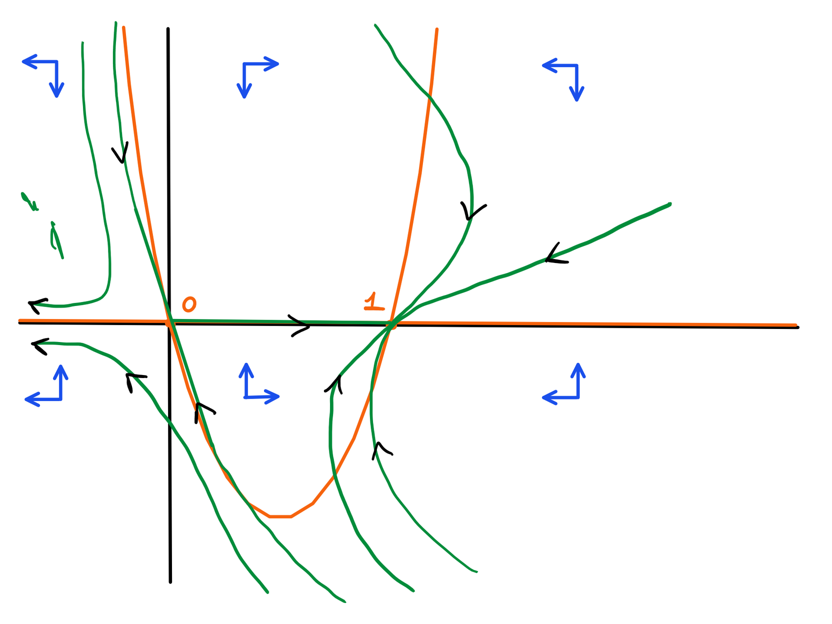
TRAIETTORIA LOCALE



$v_2 = 0 \qquad (\lambda = -2)$

$v_2 = +v_1 \qquad (\lambda = -1)$

TRAIETTORIA GLOBALE



ISOCLINE

$\dot{x} = 2x(1-x) + y \geq 0$

$\dot{x} \geq 0 \qquad \underline{y \geq 2x^2 - 2x}$

$\dot{y} \geq 0 \qquad -y \geq 0 \qquad \underline{y \leq 0}$

SISTEMA 3

$\dot{x} = -y - 2x + x^3$

$\dot{y} = y - Px$

EQUILIBRI

$\dot{x} = \dot{y} = 0$

$\begin{cases} y = x^3 - 2x \\ y = Px \end{cases} \qquad \begin{matrix} x^3 - 2x - Px = 0 \\ x(x^2 - 2 - P) = 0 \end{matrix} \begin{matrix} \longrightarrow x = 0 \\ \longrightarrow x = \pm\sqrt{2+P} \end{matrix}$

$E_1(0,0) \qquad E_2(\sqrt{2+P}, P\sqrt{2+P}) \qquad E_3(-\sqrt{2+P}, -P\sqrt{2+P})$

LINEARIZZAZIONE

Px

$\begin{vmatrix} 3x^2 - 2 & -1 \\ -P & +1 \end{vmatrix}$

STABILITA' E_1

$J(E_1) = \begin{vmatrix} -2 & -1 \\ -P & +1 \end{vmatrix} \qquad \begin{matrix} TR(J) = -1 \\ DET(J) = -2-P \end{matrix}$

DET < 0	P > -2	IN	
DET = 0	P = -2	?	(solo se $\exists \lambda = 0$)
DET > 0	P < -2	AS	

STABILITA' E_2 E E_3

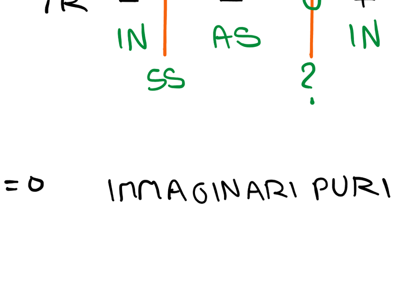
$J(E_2) = J(E_3) = \begin{vmatrix} 3(2+P)-2 & -1 \\ -P & +1 \end{vmatrix} = \begin{vmatrix} 3P-4 & -1 \\ -P & +1 \end{vmatrix}$

$TR(J) = 3P + 4 + 1 = 3P + 5$

$DET(J) = 3P + 4 - P = 2P + 4$

$DET(J) > 0 \longrightarrow \underline{P > -2}$

$TR(J) < 0 \longrightarrow \underline{P < -\frac{5}{3}}$



$\underline{P = -5/3}$ SS se $Re(\lambda_i) = 0$ IMMAGINARI PURI

STABILITA' DEL SISTEMA

	E_1	E_2	E_3
$P < -2$	AS	IN	IN
$-2 < P < -5/3$	IN	AS	AS
$P > -5/3$	IN	IN	IN