

10 - Risposta in Frequenza

Monday, 27 June 2022 18:22

ES 1

$$A = \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} \quad b = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad c = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \quad d = 0$$

TRASFERIMENTO

$$\begin{aligned} x_1 &= -x_1 + u & s x_1 &= -x_1 + u \\ x_2 &= x_1 - x_2 & s x_2 &= x_1 - x_2 \rightarrow G(s) = \frac{1}{(s+1)^2} \\ y &= x_2 & y &= x_2 \end{aligned}$$

STABILITÀ

$$\lambda_1 = \lambda_2 = -1 \quad \operatorname{Re}(\lambda_{1,2}) < 0 \rightarrow \text{AS}$$

GUADAGNO $G(0) = +1$

CASO 1 $U(t) = -2$

$$\begin{aligned} y(\infty) &= \lim_{s \rightarrow 0} s \cdot L(y) = \lim_{s \rightarrow 0} s \cdot L(-2) \cdot G(s) \\ &= \lim_{s \rightarrow 0} s \cdot \frac{-2}{s+1} \cdot G(s) = -2 G(0) = -2 \end{aligned}$$

CASO 2 $U(t) = 10 \sin(2t)$

$$\text{TEOREMA } y(\infty) = |G(i\omega)| A \sin(\omega t + \operatorname{ARG}(G(i\omega)))$$

NEL NOSTRO CASO $\omega = +2$

$$G(i\omega) = \frac{1}{(1+2i)^2} = \frac{1}{4i-3} \cdot \frac{4i+3}{4i+3} = -\frac{4i+3}{25} = -\frac{3}{25} - \frac{4}{25}i$$

$$|G(i\omega)| = \left| -\frac{4i+3}{25} \right| = \frac{1}{25} \sqrt{4^2+3^2} = \frac{5}{25} = \frac{1}{5}$$

$$\operatorname{ARG}(G(i\omega)) = \arctan\left(\frac{4}{3}\right) = 53^\circ + 180^\circ$$

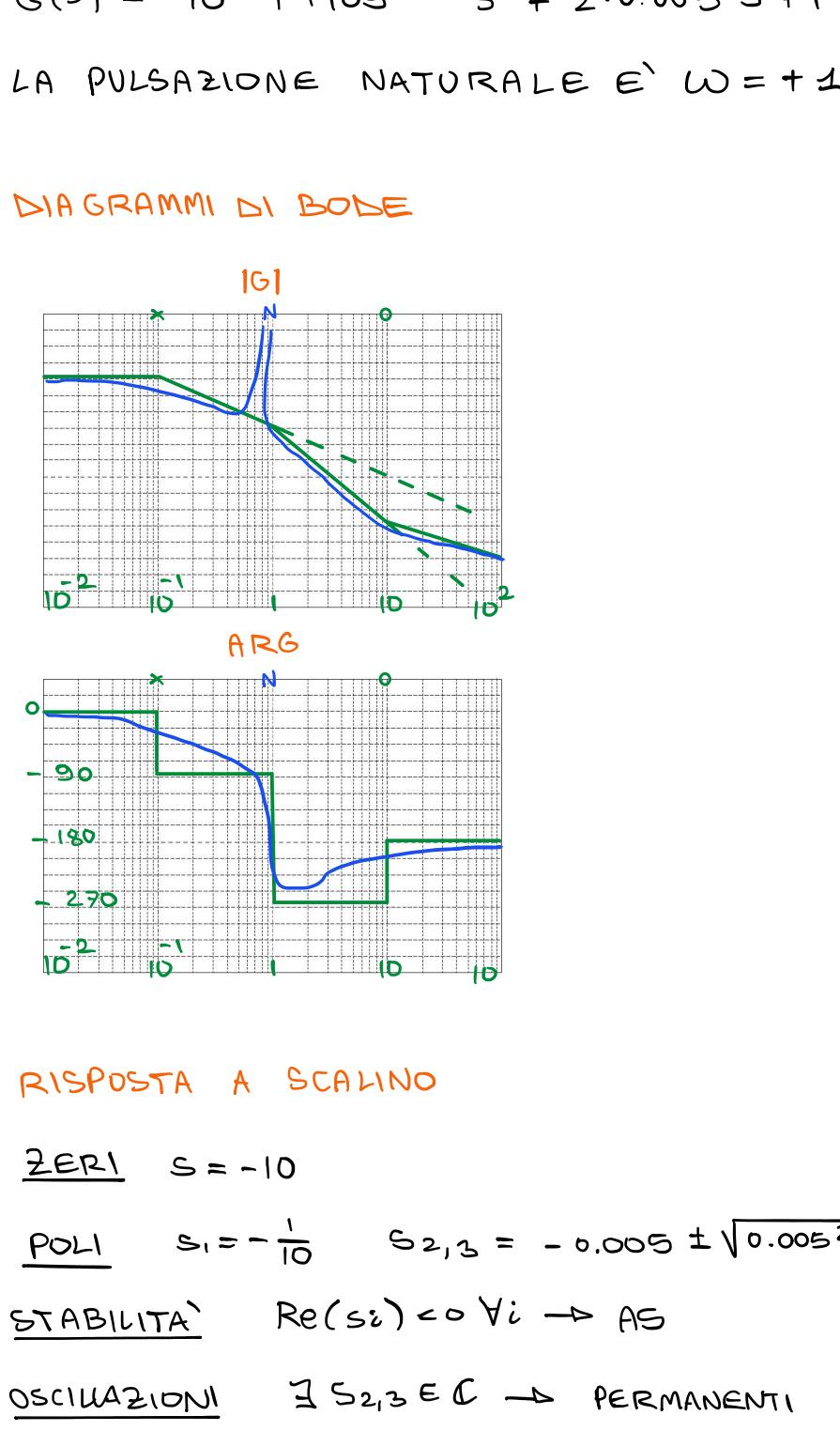
$$\text{RISPOSTA } y(\infty) = 10 \cdot \frac{1}{5} \cdot \sin(2t + 53^\circ) = 2 \sin(2t + 233^\circ)$$

CASO 3 $U(t) = -2 + 2 \sin(2t)$

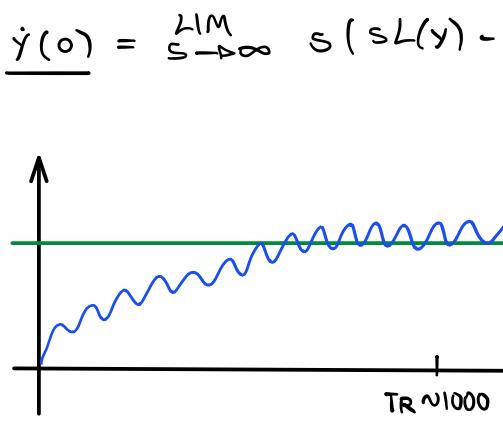
PER SOVRAPPOSIZIONE EFFETTI

$$y(\infty) = -2 + 2 \sin(2t + 233^\circ)$$

DIAGRAMMI DI BODE



ESERCIZIO 2



TRASFERIMENTO

$$\begin{aligned} G(s) &= \frac{10}{1+i0s} \cdot \frac{2}{s+3} \cdot \left(1 + \frac{2}{s(s+3)} \right)^{-1} = \\ &= \frac{10}{1+i0s} \cdot \frac{2}{s+3} \cdot \frac{s(s+3)}{s^2+3s+2} = \\ &= \frac{20s}{(1+i0s)(s+1)(s+2)} \end{aligned}$$

$$\text{RISPOSTA A } V(t) = -5 + 20 \cos(t)$$

TEOREMA RISPOSTA FREQUENZA

$$y(\infty) = -5 G(0) + 20 |G(i\omega)| \cos(t + \operatorname{ARG}(G(i\omega)))$$

GUADAGNO $G(0) = 0$ POICHÉ $s = 0$ È UNO ZERO

$$|G(i\omega)| = \left| \frac{20i}{(1+i0s)(i+1)(i+2)} \right| = \frac{20}{\sqrt{1+i^2} \cdot \sqrt{1^2+1^2} \cdot \sqrt{1^2+2^2}} \approx 0.63$$

$$\operatorname{ARG}(G(i\omega)) = \frac{\pi}{2} - \arctan 10 - \arctan 1 - \arctan 2 = -65^\circ$$

$$\text{RISPOSTA } y(\infty) = 12.3 \cos(t - 65^\circ)$$

DIAGRAMMI DI BODE

ESERCIZIO 3

$$G(s) = \frac{10+s}{(10s^2+0.1s+10)(1+i0s)}$$

$$U(t) = 10 \sin(t)$$

FORMA FATTOORIZZATA

$$G(s) = \frac{P}{s^n} \cdot \frac{\pi(s-z_k)}{\pi(s-p_k)} \cdot \frac{\pi(s^2+2\zeta\omega s+\omega^2)}{\pi(s^2+2\delta\omega s+\omega^2)}$$

$$G(s) = \frac{10}{1+i0s} \cdot \frac{1}{s^2+2 \cdot 0.005 s + 1^2}$$

LA PULSAZIONE NATURALE È $\omega = +1$

DIAGRAMMI DI BODE

RISPOSTA A SCALINO

ZERI $s = -10$

$$\text{POLI } s_1 = -\frac{1}{10}, s_2,3 = -0.005 \pm \sqrt{0.005^2 - 1}$$

STABILITÀ $\operatorname{Re}(s_{1,2}) < 0 \forall i \rightarrow \text{AS}$

OSCILLAZIONI $\exists s_{2,3} \in \mathbb{C} \rightarrow \text{PERMANENTI}$

$$\text{PERIODO } \omega_N = 1 \quad T = \frac{2\pi}{\omega_N} \approx 6 \text{ SECONDI}$$

$$\text{TEMPO } 2\tau_b = -0.005 \quad T_R = 5 \frac{1}{2\tau_b} = 1000$$

$$\text{GUADAGNO} \quad G(0) = 1 = y(\infty)$$

GRADO $R = 2 > 0$ PROPRIO

$$y(0) = \lim_{s \rightarrow \infty} s L(y) = \lim_{s \rightarrow \infty} s \frac{1}{s} G(s) = 0$$

$$y(\infty) = \lim_{s \rightarrow 0} s L(y) = \lim_{s \rightarrow 0} s \frac{1}{s} G(s) = G(0) = 1$$

$$\dot{y}(0) = \lim_{s \rightarrow \infty} s^2 L(y) - y(0) = \lim_{s \rightarrow \infty} s^2 \frac{1}{s} G(s) = 0$$

RISPOSTA IN FREQUENZA

POSso APPLICARLA POICHÉ AS

$$U(t) = 1 + 2 \sin(0.1t)$$

$$\text{GUADAGNO} \quad G(0) = \frac{10}{11} \approx 1$$

$$U(t) = 1 \rightarrow y(\infty) = G(0) \cdot 1 = 1$$

SINUSOIDA

$$U(t) = 2 \sin(0.1t)$$

$$|G(0.1i)| = \frac{|10e^{0.1i}|}{|1+i0.1|} = \frac{\sqrt{10^2+1^2}}{\sqrt{1^2+0.1^2}} = 0.91 \approx 1$$

$$\operatorname{ARG}(G(0.1i)) = \arctan\left(\frac{0.1}{10}\right) + \operatorname{ARG}(e^{0.1i}) = -0.6 \text{ RAD} = -35^\circ$$

$$\text{RISPOSTA } Y(\infty) = 16.1 \cdot 2 \cdot \sin(0.1t + 35^\circ)$$

$$\text{OPERO } Y(\infty) = 2 \sin(0.1t - 35^\circ)$$

RISPOSTA COMPLESSIVA

PER LINEARITÀ OTENIAMO

$$Y(\infty) = 1 + 2 \sin(0.1t - 35^\circ)$$

ESERCIZIO 5

DISEGNA BODE DI

$$G(s) = -\frac{10}{10(s+1)^2}$$

