

Funzione di trasferimento

$$G(p) = \frac{n(p)}{d(p)}$$

zeri : radici di $n(p)$ ($G=0$)

poli : radici di $d(p)$ ($G=\infty$)

f.d.t. in forme tipiche

$$G(p) = \begin{cases} \frac{\beta_r p^{n-r} + \beta_{r+1} p^{n-r-1} + \dots + \beta_n}{p^n + \alpha_1 p^{n-1} + \dots + \alpha_n} \\ \rho \frac{(p-z_1)(p-z_2) \dots (p-z_{n-r})}{(p-p_1)(p-p_2) \dots (p-p_n)} \\ \text{t.c.} \left\{ \begin{array}{l} \mu \frac{(1+s\tau_1)(1+s\tau_2) \dots (1+s\tau_{n-r})}{s^g (1+sT_1)(1+sT_2) \dots (1+sT_{n-g})} \\ \mu \frac{s^{-g} (1+s\tau_1)(1+s\tau_2) \dots (1+s\tau_{n-r-g})}{(1+sT_1)(1+sT_2) \dots (1+sT_n)} \end{array} \right. \end{cases}$$

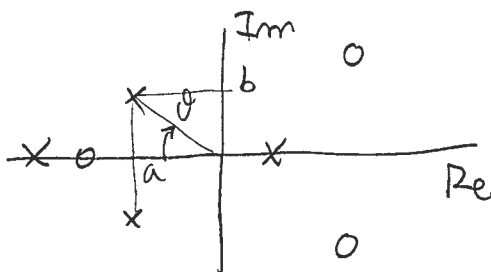
r : grado relativo o surplus di poli
 z_i : zeri, p_i : poli
 ρ : costante di trasferimento $= \beta_r$
 μ : guadagno (generalizzato)
 g : tipo
 $g \geq 0$
 τ_i, T_i : costanti di tempo
 $g \leq 0$

Il caso di poli (o zeri) complessi

$$(p-p_i)(p-\bar{p}_i) = (p-(a+ib))(p-(a-ib)) = p^2 - 2as + \overbrace{(a^2+b^2)}^{(b: \text{pulsazione})} = p^2 + 2\zeta\omega_n s + \omega_n^2$$

$\omega_n^2 = \text{pul. naturale}$

Il piano complesso



x : poli
 o : zeri

$$\zeta = -\frac{a}{\omega_n} = \cos \theta \quad \text{smorzamento}$$

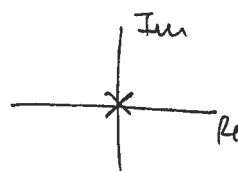
$\theta = \arctan(-\frac{b}{a})$

Esempi

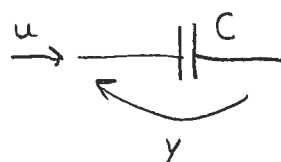
Integratore



$$G = \frac{1}{s}$$



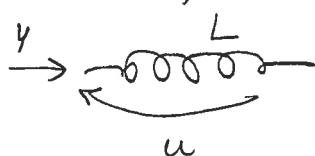
Condensatore



$$G = \frac{1}{Cs}$$

idem

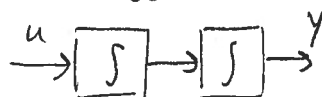
Induttore



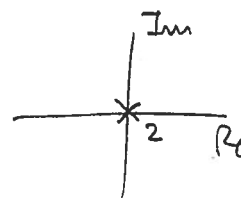
$$G = \frac{1}{Ls}$$

idem

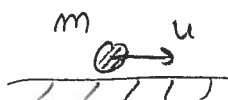
Doppio integratore



$$G = \frac{1}{s^2}$$



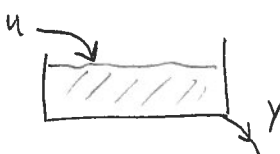
Massa senza attrito
(Legge di Newton)



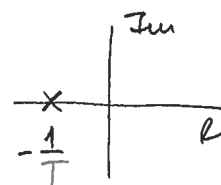
$$G = \frac{1}{ms^2}$$

idem

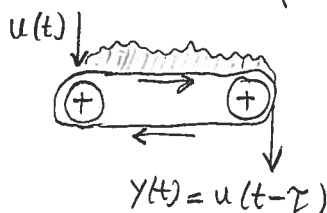
Serbatoio



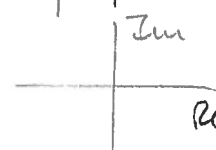
$$G = \frac{1}{1+sT}$$



Ritardatore puro



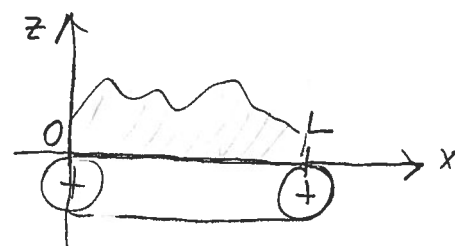
$$G = e^{-\tau s} \quad (?!) \quad \left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right)$$



Il ritardatore puro

$n = ?$

Lo stato del sistema è dato dall'altezza z della sabbia per $x \in [0, L]$, quindi dalle funzioni $z = z(x) \rightarrow n = \infty$!



$$y(t) = u(t - \tau) = u(t + \varepsilon) = \sum_{i=0}^{\infty} u^{(i)}(t) \frac{\varepsilon^i}{i!} = u(t) \sum_{i=0}^{\infty} \frac{(\varepsilon s)^i}{i!} =$$

$(\varepsilon = -\tau)$

$$= u(t) e^{\varepsilon s} = u(t) e^{-\tau s}$$

$$n(s) = e^{-\tau s}$$

$$d(s) = 1$$