5 - Analisi

Friday, 24 June 2022 01:57

1 - MEDUSE

FASE 1 LARVOIDALE $\alpha_1 = 10\%$

FASE 2 POLIPOIDALE $\alpha_2 = 8\%$

FASE 3 MEDUSOIDALE $\alpha_3 = 10\%$ 1000 UOVA

MODELLO DINAMICO

x_i # MEDUSE IN FASE i

$\dot{x}_1 = \alpha_3 K x_3$

$\dot{x}_2 = \alpha_1 x_1$

$\dot{x}_3 = \alpha_2 x_2$

$y = x_1 + x_2 + x_3$

$$A = \begin{vmatrix} 0 & 0 & 100 \\ 0.1 & 0 & 0 \\ 0 & 0.08 & 0 \end{vmatrix}$$

STABILITÀ DEL SISTEMA

$$\det(\lambda I - A) = \det \begin{vmatrix} \lambda & 0 & -100 \\ -0.1 & \lambda & 0 \\ 0 & -0.08 & \lambda \end{vmatrix} = \Delta_A(\lambda)$$

$$\Delta_A(\lambda) = \lambda(\lambda^2) + 0.1(-8) = \lambda^3 - 0.8$$

$$\Delta_A(\lambda) = 0 \rightarrow \lambda^3 = 0.08$$

$$\lambda^3 = 0.08\, e^{2k\pi i} \qquad \lambda_{1,2,3} = \sqrt[3]{0.08}\, e^{\frac{2k\pi i}{3}} \qquad k \in (0,1,2)$$

$\lambda_1 = 0.43$

$\lambda_2 = 0.43\, e^{\frac{2}{3}\pi i} = 0.43\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$

$\lambda_3 = 0.43\, e^{\frac{4}{3}\pi i} = 0.43\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$

$$\{\lambda_A\} = \left\{ \sqrt[3]{0.08},\ \sqrt[3]{0.08}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right),\ \sqrt[3]{0.08}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) \right\}$$

$Re(\lambda_i) < 1\ \forall i$ (DISCRETO) $\rightarrow$ AS

STABILITÀ ASINTOTICA $\rightarrow x(t) \rightarrow 0\ \forall x(0)$

LA SPECIE È DESTINATA ALL'ESTINZIONE

CALCOLA LA PROBABILITÀ DI SOPRAVVIVENZA AFFINCHÈ LA SPECIE VADA IN INVASIONE

$$A = \begin{vmatrix} 0 & 0 & 1000p \\ 0.1 & 0 & 0 \\ 0 & 0.08 & 0 \end{vmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda & 0 & -1000p \\ -0.1 & \lambda & 0 \\ 0 & -0.08 & \lambda \end{vmatrix} = \Delta_A(p, \lambda)$$

$$\Delta_A(p,\lambda) = \lambda^3 + 0.1(-80p) = \lambda^3 - 8p$$

$$\Delta_A(p,\lambda) = 0 \rightarrow \lambda = \sqrt[3]{8p} \qquad p \in [0,1]$$

INVASIONE = INSTABILITÀ $\rightarrow |\lambda| > 1$

$\sqrt[3]{8p} > 1 \qquad 8p > 1 \qquad p > \frac{1}{8}$

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2 - NOLEGGIO AUTO

NM MENSILE NB BIMESTRALE

$U(t)$ # AUTO IN AFFITTO AD INIZIO MESE $NM = \frac{2}{3}$ $BM = \frac{1}{3}$

α € A FINE NM

β € A FINE NB

SISTEMA DINAMICO

$x_1 =$ # AUTO IN NM

$x_2 =$ # AUTO IN NB (MESE 1)

$x_3 =$ # AUTO IN NB (MESE 2)

$x_1(t+1) = \frac{2}{3} U(t)$

$x_2(t+1) = \frac{1}{3} U(t)$

$x_3(t+1) = x_2(t)$

$y(t) = \alpha x_1(t) + \beta(x_2(t) + x_3(t))$

SPAZIO DEGLI STATI

$$A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \qquad b = \begin{vmatrix} 2/3 \\ 1/3 \\ 0 \end{vmatrix}$$

$c^T = |\alpha \quad \beta \quad \beta| \qquad d = 0$

STABILITÀ

$\{\lambda\}_A = \{0, 0, 0\} \qquad |\lambda_i| < 1\ \forall i \rightarrow$ AS

NON CI SONO AUTOVALORI NEGATIVI O COMPLESSI $\rightarrow$ NO OSCILLAZIONI

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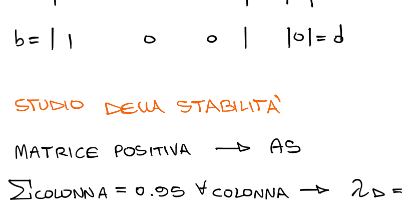
3 - CAR SHARING

3 DIVERSE CATEGORIE (SI INIZIA DA 3)

10% OGNI CATEGORIA SUPERA LA SOGLIA $(1 \rightarrow 2,\ 2 \rightarrow 3)$

5% OGNI CATEGORIA NON RINNOVA

MODELLO A GRAFO



MODELLO DINAMICO

$x_1(t+1) = 0.85 x_1 + 0.85 x_2$

$x_2(t+1) = 0.1 x_1 + 0.85 x_3$

$x_3(t+1) = U(t) + 0.1 x_2 + 0.1 x_3$

$y(t) = x_1(t)$

SPAZIO DEGLI STATI

$$A = \begin{vmatrix} 0.85 & 0.85 & 0 \\ 0.1 & 0 & 0.85 \\ 0 & 0.1 & 0.1 \end{vmatrix} \qquad \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} = b$$

$b = |1 \quad 0 \quad 0| \qquad |0| = d$

STUDIO DELLA STABILITÀ

MATRICE POSITIVA $\rightarrow$ AS

$\sum$ COLONNA $= 0.95\ \forall$ COLONNA $\rightarrow \lambda_D = 0.95$

$T_D = \frac{1}{\ln(\lambda d)} \qquad T_R = 5 T_D$

EQUILIBRIO

$x_i(t+1) = x_i(t) = \overline{x_i} \qquad U(t) = \overline{U} = 1000$

$\overline{U} = 1000,\ \overline{y} = \overline{x_1}$

$\overline{x_1} = 0.85(\overline{x_1} + \overline{x_2})$

$\overline{x_2} = 0.1\, \overline{x_1} + 0.85\, \overline{x_3}$

$\overline{x_3} = 0.1(\overline{x_2} + \overline{x_3}) + \overline{U}$

$\overline{x_1}(1 - 0.85) = \overline{x_2} \rightarrow \overline{x_1} = 5{,}67\, \overline{x_2}$

$5{,}67\, \overline{x_1} = 0.1\, \overline{x_1} + 0.85\, \overline{x_3} \rightarrow \overline{x_2} = 1.96\, \overline{x_3}$

$\overline{x_3} = 1.96\, \overline{x_3} = 5.67 \cdot 1.96\, \overline{x_3} + \overline{U} \rightarrow \overline{x_3} = 1.42\, \overline{U}$

$$\overline{x} = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \begin{vmatrix} 15781 \\ 2783 \\ 1420 \end{vmatrix}$$

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3 - PIANETA TERNO

MORTE DOPO 3 ANNI

α CONTRIBUTO (1 ANNO)

$K = \frac{1}{2}$ FIGLI MEDI (1 ANNO)

2α CONTRIBUTO (2 ANNO)

β PENSIONE (3 ANNO)

$U(t) = 200$ NUOVI NEONATI

MODELLO MATEMATICO

$x_i =$ # INDIVIDUI DI ANNO i

$y =$ € IN CASSE STATALI

$x_1(t+1) = U(t) + K x_1(t)$

$x_2(t+1) = x_1(t)$

$x_3(t+1) = x_2(t)$

SPAZIO DEGLI STATI

$$A = \begin{vmatrix} K & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \qquad \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = b$$

EQUILIBRIO

$x_i(t+1) = x_i(t) = \overline{x_i} \qquad K = \frac{1}{2}$

$\overline{x_1} = \overline{U} + K\overline{x_1} \qquad \overline{x_1}(1-K) = \overline{U} \rightarrow \overline{x_1} = 2\overline{U}$

$\overline{x_2} = \overline{x_1} \qquad \overline{x_2} = 2\overline{U}$

$\overline{x_3} = \overline{x_2} = \overline{x_1} \qquad \overline{x_3} = 2\overline{U}$

STABILITÀ DISCRETA

$$A = \begin{vmatrix} K & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

MATRICE TRIANGOLARE $\lambda_A = \{K, 0, 0\}$

$\forall i\ |\lambda_i| < 1 \rightarrow$ ASINTOTICAMENTE STABILE

TEMPO DI RISPOSTA

$\lambda_D = 0.5 \qquad T_D = -\frac{1}{\ln \lambda_D} \qquad T_R = 5T_R = -\frac{5}{\ln\frac{1}{2}} = +\frac{5}{\ln 2}$

MODELLIZZA CASSE STATALI ALL'EQUILIBRIO (INTERESSE γ ANNUO)

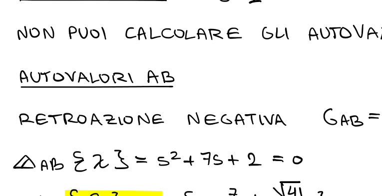
$y(t+1) = \alpha x_1(t) + 2\alpha x_2(t) - \beta x_3(t) + (1+\gamma)\, y(t)$

$\overline{y} = \alpha\overline{x_1} + 2\alpha\overline{x_2} - \beta\overline{x_3} + (1+\gamma)\overline{y}$

$-\gamma\overline{y} = \alpha\overline{x_1} + 2\alpha\overline{x_2} - \beta\overline{x_3}$

$\overline{y} = \frac{\overline{x}}{\gamma}(\beta - 3\alpha) = 4000(\beta - 3\alpha)$

SISTEMA



$G_A(s) = \frac{10}{s-1} \qquad G_B(s) = \frac{s}{s-2}$

C: $\ddot{y}_c + \dot{y}_c + 4y_c = -4\dot{u}_c + 2u_c$

$$E: \underset{A}{\begin{vmatrix} -1 & 0 & 0 & 0 \\ 2 & -2 & -1 & -2 \\ 3 & 0 & -1 & 1 \\ 4 & 0 & 0 & -2 \end{vmatrix}} \quad \underset{b}{\begin{vmatrix} 1 \\ -1 \\ 1 \\ -1 \end{vmatrix}} \quad \underset{c^T}{|0\ 1\ 0\ 1|}$$

PROPONI D AFFINCHÈ SISTEMA AS (ORDINE 2)

AGGREGATO STABILE SE I SINGOLI SONO STABILI

~~AUTOVALORI A~~ $\cancel{\frac{10}{s-1}} \quad \cancel{\lambda_A = +1}$

~~AUTOVALORI B~~ $\cancel{\frac{5}{s-2}} \quad \cancel{\lambda_B = +2}$

NON PUOI CALCOLARE GLI AUTOVALORI IN RETROAZIONE

AUTOVALORI AB

RETROAZIONE NEGATIVA $G_{AB} = \frac{G_A}{1 + G_A G_B} = \frac{10(s-2)}{s^2 + 7s + 2}$

$\Delta_{AB}\{\lambda\} = s^2 + 7s + 2 = 0$

$\rightarrow \{\lambda\}_{AB} = \left\{-\frac{7}{2} \pm \frac{\sqrt{41}}{2}\right\}$

AUTOVALORI C

$\ddot{y}_c + \dot{y}_c + 4y_c = -4\dot{u}_c + 2u_c \rightarrow G_c(s) = \frac{2 - 4s}{s^2 + s + 4}$

$\Delta_c(\lambda) = \lambda^2 + \lambda + 4 = 0$

$\rightarrow \{\lambda\}_c = \left\{-\frac{1}{2} \pm \frac{\sqrt{15}}{2} i\right\}$

AUTOVALORI E

MATRICE TRIANGOLARE A BLOCCHI

$\{\lambda\}_E = \{-1, -1, -2, -2\}$

AUTOVALORI AGGREGATO F

$\{\lambda\}_F = \left\{-1, -1, -2, -2, -\frac{1}{2} \pm \frac{\sqrt{15}}{2} i, -\frac{7}{2} \pm \frac{\sqrt{41}}{2}\right\}$

$Re(\lambda_i) < 0\ \forall i \rightarrow$ STABILITÀ ASINTOTICA SE ANCHE D HA AUTOVALORI CON $Re(\lambda_D) < 0$

ESEMPIO $G(s) = \frac{1}{(s+1)^2} \qquad \lambda_{D1,D2} = -1$

CALCOLA TEMPO DI RISPOSTA

$\lambda_D = \max(Re(\lambda_i)) = -\frac{1}{2}$

$T_D = \frac{1}{-\lambda_D} \qquad T_R = 5T_D = 16{,}7$

POICHE $\exists \lambda \in \mathbb{C} \rightarrow \exists \infty$ OSCILLAZIONI

SE $G_c(s) = \frac{-4s + 2}{s^3 + s^2 + s + 4}$ STUDIA LA STABILITÀ

AUTOVALORI C

$\Delta_c(\lambda) = \lambda^3 + \lambda^2 + \lambda + 4$

HURWITZ $\alpha_0 = 1 \quad \alpha_1 = 1 \quad \alpha_2 = 1 \quad \alpha_3 = 4$

$$H = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 4(1-4) = -12 < 0$$

FALLISCE IL CRITERIO DI H. NON AS

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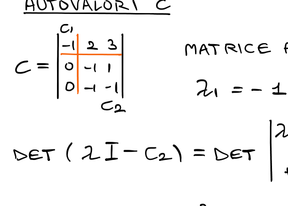
5 - SISTEMA



$G_A(s) = \frac{1}{s-1} \qquad D: \dot{y}_D + 2y_D = -\dot{u}_D$

$$C: \begin{vmatrix} -1 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & -1 \end{vmatrix} \quad \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} \quad |1\ 1\ 1|$$

PROPONI B AFFINCHÈ SISTEMA AS (ORDINE 1)

TRASFERIMENTO AB

RETROAZIONE NEGATIVA $G_{AB} = \frac{G_A}{1 + G_A G_B}$

AUTOVALORI D

$G_D(s) = \frac{-s}{s+2} \qquad \lambda_D = -2$

AUTOVALORI C

$$C = \begin{vmatrix} -1 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & -1 \end{vmatrix}$$

(C_1, C_2) MATRICE A BLOCCHI $\lambda_1 = -1$

$$\det(\lambda I - C_2) = \det \begin{vmatrix} \lambda+1 & -1 \\ +1 & \lambda+1 \end{vmatrix} = \Delta_{C2}(\lambda)$$

$\Delta_{C2}(\lambda) = (\lambda+1)^2 + 1 = \lambda^2 + 2\lambda + 2 = 0$

$\lambda_{2,3} = -1 \pm \sqrt{1-2} = -1 \pm i$

$\{\lambda\}_C = \{\lambda\}_{C_1 + C_2} = \{-1, -1 \pm i\}$

$Re(\lambda_i) < 0\ \forall i$ DI C e D

SE AB HA AUTOVALORI CON $Re(\lambda_i) < 0$ ALLORA L'AGGREGATO È AS

AUTOVALORI DI AB

GENERICO (GRADO 1) $G_B = \frac{\alpha}{\beta + s} \qquad \alpha, \beta \in \mathbb{R}$

$$G_{AB} = \frac{G_A}{1 + G_A G_B} = \frac{1}{s-1}\left(1 + \frac{1}{s-1} \cdot \frac{\alpha}{\beta + s}\right)^{-1} =$$

$$= \frac{1}{\cancel{s-1}}\left(\frac{s^2 + s\beta - s - \beta + \alpha}{\cancel{(s-1)}(s+\beta)}\right)^{-1} = \frac{s + \beta}{s^2 + s(\beta - 1) + (\alpha - \beta)}$$

CRITERIO ROUTH

$\beta - 1 > 0 \qquad \beta > 1$

$\alpha - \beta > 0 \qquad \alpha > \beta \quad \rightarrow \quad 1 < \beta < \alpha$

ESEMPIO $\alpha = 9 \quad \beta = 5 \qquad G_{AB} = \frac{s+5}{(s+2)^2}$

$\{\lambda\}_{AB} = \{-2, -2\}$

$\{\lambda\}_{AGGREGATO} = \{-2, -2, -2, -1, -1 \pm i\}$

AGGREGATO ASINTOTICAMENTE STABILE

DETERMINA TUTTE LE COSTANTI DI TEMPO E IL TEMPO DI RISPOSTA DEL SISTEMA

$\{\lambda\}_{AGGREGATO} = \{-2, -2, -2, -1, -1 \pm i\}$

$T = \left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, 1, 1\right\}$

$T_D = 1 \qquad T_R = 5T_D = 5$

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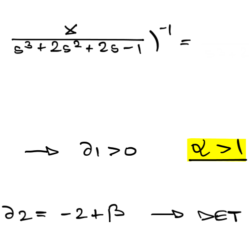
6 - AGGREGATO

$F(s) = \frac{1}{s-1}$

$G(s) = \frac{1}{s^2 + 2s - 2}$

$H(s) = \frac{1}{s^3 + 2s^2 + 2s - 1}$

$\alpha, \beta, \gamma \in \mathbb{R}$



PER QUALI α, β, γ IL SISTEMA È AS

R_F, R_G, R_H RETROAZIONE NEGATIVA

$G_{RF} = \frac{G_F}{1 + \alpha G_F} \qquad G_{RG} = \frac{G_G}{1 + \beta G_G} \qquad G_{RH} = \frac{G_H}{1 + \gamma G_H}$

CALCOLO I TRASFERIMENTI

$G_{RF}(s) = \frac{1}{s-1}\left(1 + \frac{\alpha}{s-1}\right)^{-1} = \frac{1}{\cancel{s-1}}\left(\frac{s - 1 + \alpha}{\cancel{s-1}}\right)^{-1} = \frac{1}{s + (\alpha - 1)}$

$G_{RG}(s) = \frac{1}{s^2 + 2s - 2}\left(1 + \frac{\beta}{s^2 + 2s - 2}\right)^{-1} = \frac{1}{s^2 + 2s - 2 + \beta}$

$G_{RH}(s) = \frac{1}{s^3 + 2s^2 + 2s - 1}\left(1 + \frac{\gamma}{s^3 + 2s^2 + 2s - 1}\right)^{-1} =$

HURWITZ $\forall a_i > 0$

$G_{RF} \qquad a_0 = 1 \quad a_1 = \alpha - 1 \rightarrow a_1 > 0 \qquad \alpha > 1$

$G_{RG} \qquad a_0 = 1 \quad a_1 = 2 \quad a_2 = -2 + \beta \rightarrow \det \begin{vmatrix} a_1 & 1 \\ a_3 & a_2 \end{vmatrix} > 0 \quad \forall a_i > 0$

$\det \begin{vmatrix} 2 & 1 \\ 0 & -2+\beta \end{vmatrix} = 2(-2+\beta) > 0 \qquad \beta > 2$

$G_{RH} \qquad a_0 = 1 \quad a_1 = 2 \quad a_2 = 2 \quad a_3 = -1 + \gamma \rightarrow \det \begin{vmatrix} a_1 & 1 \\ a_3 & a_2 \end{vmatrix} > 0 \quad \forall a_i > 0$

- $a_3 > 0 \qquad -1 + \gamma > 0 \qquad \gamma > 1$
- $\det \begin{vmatrix} 2 & 1 \\ -1+\gamma & 2 \end{vmatrix} = 4 + 1 - \gamma > 0 \qquad \gamma < 5$

$1 < \gamma < 5$

Σ È ASINTOTICAMENTE STABILE SE $\begin{cases} \alpha > 1 \\ \beta > 2 \\ 1 < \gamma < 5 \end{cases}$

FUNZIONE DI TRASFERIMENTO COMPLESSIVA

— RH —

— RG —

— RF —

PARALLELO

$$G_{TOT} = G_{RH} + G_{RG} + G_{RF} = \frac{F}{1 + \alpha F} + \frac{G}{1 + \beta G} + \frac{H}{1 + \gamma H}$$