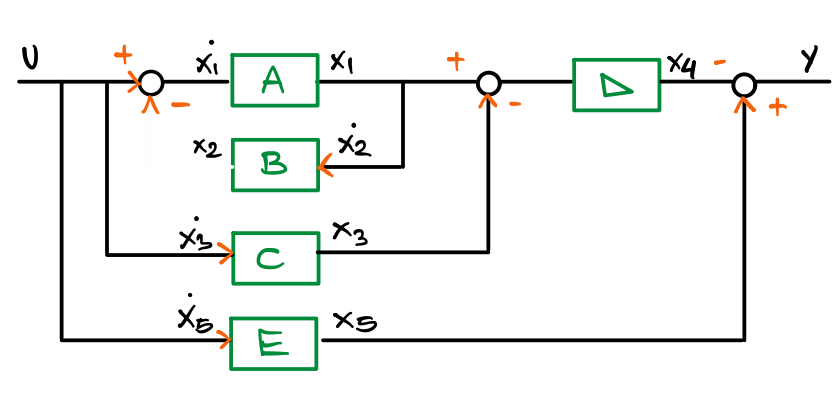
2 - Aggregati

Tuesday, 21 June 2022 12:57



CALCOLA G(P) AGGREGATO

OGNI TRASFERIMENTO $G_i(P) = 1/s$

$\dot{x}_1 = U - x_2$
$\dot{x}_2 = x_1$
$\dot{x}_3 = U$
$\dot{x}_4 = x_1 - x_3$
$\dot{x}_5 = U$
$y = x_5 - x_4$

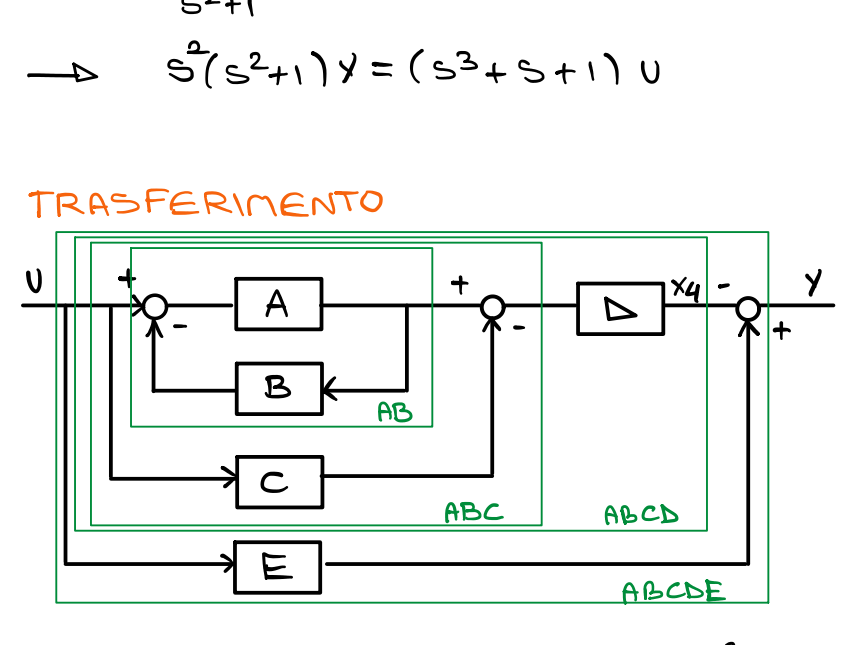
$$\begin{vmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{vmatrix}$$

ARMA I/O — REALIZZAZIONE

$Px_1 = U - x_2$	$x_1 = Px_2$	$x_1 = \frac{PU}{P^2+1}$
$Px_2 = x_1$	$P^2 x_2 = U - x_2$	$x_2 = \frac{U}{P^2+1}$
$Px_3 = U$	$x_3 = U/P$	$x_3 = U/P$
$Px_4 = x_1 - x_3$	——	$x_4 = \frac{-U}{(P^2+1)P^2}$
$Px_5 = U$	$x_5 = U/P$	$x_5 = U/P$
$y = x_5 - x_4$	——	$y = \frac{(P^3+P+1)U}{P^2(P^2+1)}$

$sy = sx_5 - sx_4$
$sy = U - (x_1 - x_3)$
$s^2y = sU - (sx_1 - sx_3)$
$s^2y = sU - (U - x_2 - U)$
$s^2y = sU + x_2 = sU + \frac{U}{s^2+1}$
$s^2y = \frac{s^3U + sU + U}{s^2+1}$
$\rightarrow s^2(s^2+1)y = (s^3+s+1)U$

TRASFERIMENTO



$G_{AB} = \frac{G_A}{1+G_AG_B} = \frac{1}{s} \cdot \frac{1}{1+1/s^2} = \frac{1}{s} \cdot \frac{s^2}{s^2+1} = \frac{s}{(s^2+1)}$

$G_{ABC} = G_{AB} - G_C = \frac{s}{s^2+1} - \frac{1}{s} = \frac{-1}{s(s^2+1)}$

$G_{ABCD} = G_{ABC} \cdot G_D = \frac{-1}{s^2(s^2+1)}$

$G_{ABCDE} = G_{AGGREGATO} = G_E - G_{ABCD} = \frac{s(s^2+1)+1}{s^2(s^2+1)}$

CANONICA DI RICOSTRUZIONE

$$G(z) = \frac{\beta_{N-1} z^{N-1} + \dots + \beta_1 z^1 + \beta_0}{z^N + \alpha_{N-1} z^{N-1} + \dots + \alpha_1 z + \alpha_0}$$

$$A = \begin{vmatrix} 0 & 0 & \dots & 0 & -\alpha_1 \\ 1 & 0 & \dots & 0 & -\alpha_2 \\ 0 & 1 & \dots & 0 & -\alpha_3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -\alpha_{N-1} \end{vmatrix} \quad \begin{vmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{N-1} \end{vmatrix} = B$$

$c = |0 \ 0 \ \dots \ 0 \ 1 \ 1| \qquad \beta_N = D$

RICOSTRUZIONE DEL SISTEMA

$G(s) = \frac{s(s^2+1)+1}{s^2(s^2+1)} = \frac{s^3+s+1}{s^4+s^2}$

$\alpha_2 = 1 \qquad \beta_1 = 1 \qquad \beta_3 = 1$

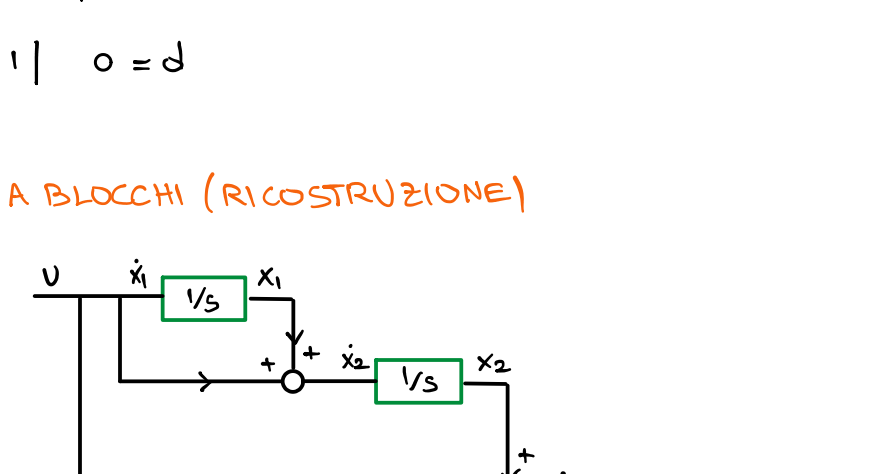
$$A = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{vmatrix} \quad \begin{vmatrix} 1 \\ 0 \\ 1 \\ 0 \end{vmatrix} = b$$

(righe x_1, x_2, x_3, x_4; colonne $x_1\ x_2\ x_3\ x_4$, U)

$c^T = |0\ 0\ 0\ 1| \qquad 0 = d$

DIAGRAMMA A BLOCCHI (RICOSTRUZIONE)

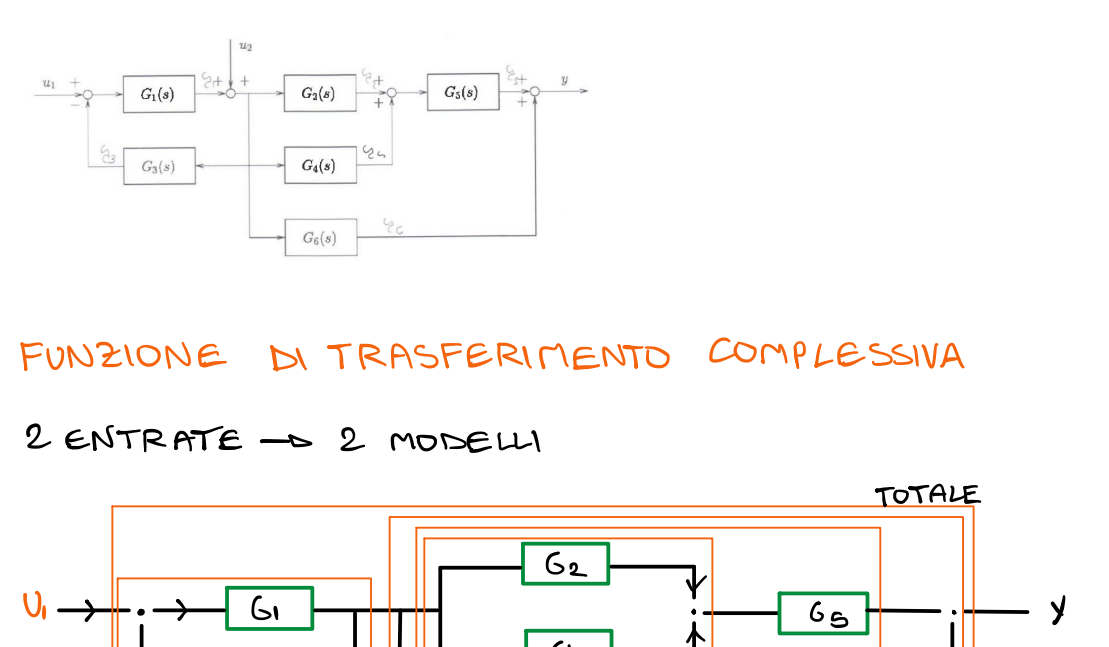
$\dot{x}_1 = U$
$\dot{x}_2 = x_1 + U$
$x_3 = x_2 - x_4$
$x_4 = x_3 + U$
$y = x_4$



2 - DOPPIA ENTRATA

FUNZIONE DI TRASFERIMENTO COMPLESSIVA

2 ENTRATE → 2 MODELLI



$G_{13} = \frac{G_1}{1+G_1G_3}$

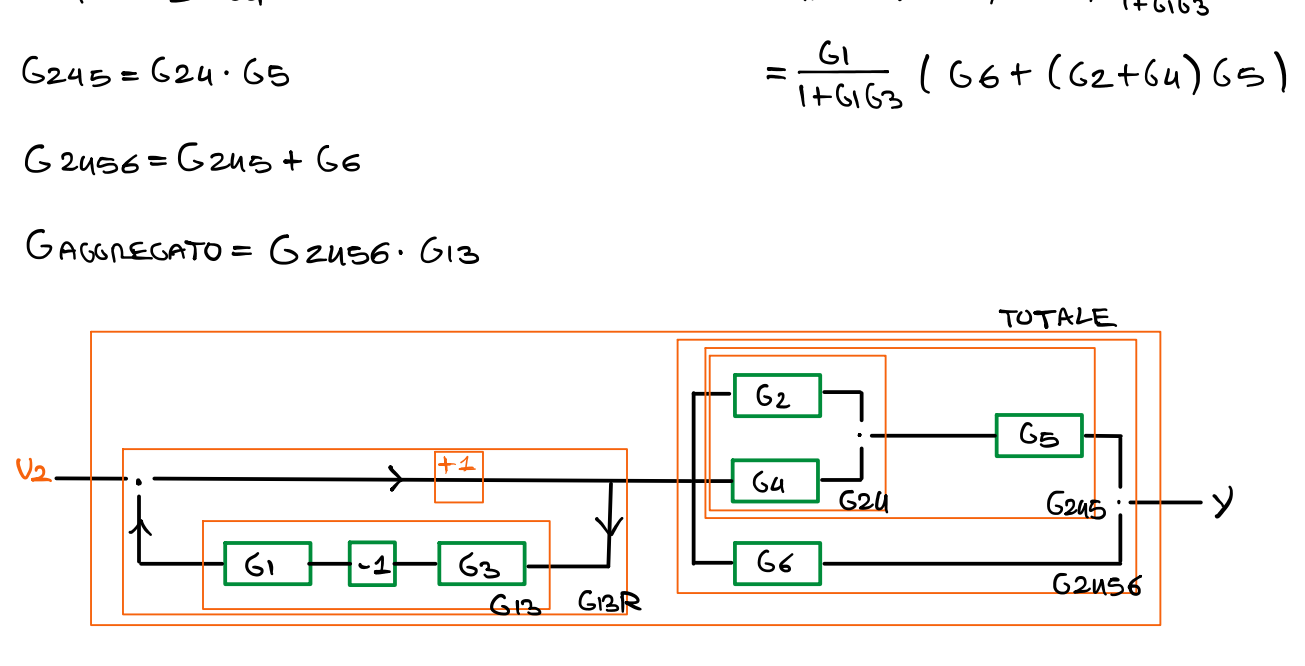
$G_{24} = G_2 + G_4$

$G_{245} = G_{24} \cdot G_5$

$G_{2456} = G_{245} + G_6$

$G_{AGGREGATO} = G_{2456} \cdot G_{13}$

$G_{AGGREGATO} = (G_{245} + G_6) \cdot G_{13}$
$= ((G_{24} \cdot G_5) + G_6) \frac{G_1}{1+G_1G_3}$
$= \frac{G_1}{1+G_1G_3} (G_6 + (G_2+G_4)G_5)$



$G_{13} = G_1(-G_3)$

$G_{13R} = \frac{1}{1+G_{13}}$

$G_{24} = G_2 + G_4$

$G_{245} = G_{24}G_5$

$G_{2456} = G_{245} + G_6$

$G_{AGGREGATO} = G_{13R} \cdot G_{2456}$

$G_{AGGREGATO} = G_{13R} \cdot G_{2456} =$
$= \frac{1}{1+G_1G_3}(G_{245} + G_6)$
$= \frac{1}{1+G_1G_3}(G_{24}G_5 + G_6)$
$= \frac{1}{1+G_1G_3}(G_6 + (G_2+G_4)G_5)$

STATI INTERNI

$\dot{y}_1 = -y_3 + U_1$	$y_1 = G_1(-y_3 + U_1)$
$\dot{y}_2 = y_1 + U_2$	$y_2 = G_2(y_1 + U_2)$
$\dot{y}_3 = y_1 + U_2$	$y_3 = G_3(y_1 + U_2)$
$\dot{y}_4 = y_1 + U_2$	$y_4 = G_4(y_1 + U_2)$
$\dot{y}_5 = y_2 + y_4$	$y_5 = G_5(y_2 + y_4)$
$\dot{y}_6 = y_1 + U_2$	$y_6 = G_6(y_1 + U_2)$
$y = y_5 + y_6$	

$y_1 = G_1(-y_3 + U_1)$
$y_3 = G_3(y_1 + U_2)$ $\qquad y_3 = G_3(G_1(-y_3+U_1)+U_2)$

$y_3 = G_3G_1U_1 - G_3G_1y_3 + G_3U_2$

$y_3(1+G_1G_3) = G_3G_1U_1 + G_3U_2$

$y_3 = \frac{G_3G_1U_1 + G_3U_2}{1+G_1G_3}$

$y_1 = G_1\left(-\frac{G_3G_1U_1 + G_3U_2}{1+G_1G_3} + U_1\right)$

$y_1 = \frac{G_1}{1+G_1G_3}(-G_3G_1U_1 - G_3U_2 + U_1 + G_1G_3U_1) = \frac{G_1}{1+G_1G_3}(1 \cdot U_1 - G_3U_2)$

$y_2 = G_2(y_1 + U_2) = \frac{G_1G_2}{1+G_1G_3}(1 \cdot U_1 - G_3U_2) + G_2U_2$

$y_2 = \frac{G_1G_2}{1+G_1G_3}(1 \cdot U_1 + (G_2 - G_3 + G_1G_2G_3)U_2)$

$y_4 = G_4(y_1 + U_2) = \frac{G_1G_4}{1+G_1G_3}(1 \cdot U_1 + (G_4 - G_3 + G_1G_3G_4)U_2)$

$y_6 = G_6(y_1 + U_2) = \frac{G_1G_6}{1+G_1G_3}(1 \cdot U_1 + (G_6 - G_3 + G_1G_3G_6)U_2)$

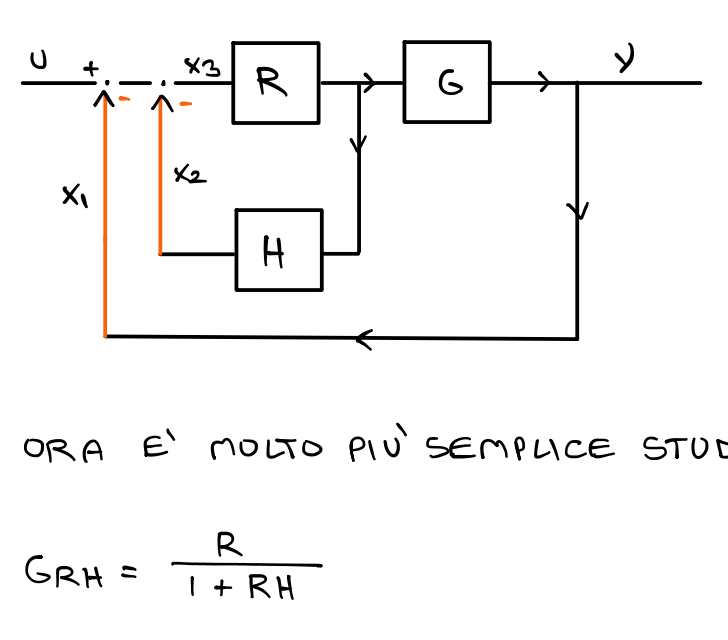
$y_5 = G_5(y_2 + y_4) = G_5\left(\frac{G_1G_4}{1+G_1G_3}(1 \cdot U_1 + (G_4 - G_3 + G_1G_3G_4)U_2) + \frac{G_1G_2}{1+G_1G_3}(1 \cdot U_1 + (G_2 - G_3 + G_1G_2G_3)U_2)\right)$

$= \frac{G_5}{1+G_1G_3}(G_1G_4U_1 + G_1G_4(G_4 - G_3 + G_1G_3G_4)U_2 + G_1G_2U_1 + G_1G_2(G_2 - G_3 + G_1G_2G_3)U_2)$

$= \frac{G_5}{1+G_1G_3}((G_1G_4 + G_1G_2)U_1 + G_1G_4^2U_2 - G_1G_3G_4U_2 + G_1^2G_3G_4^2U_2 + G_1G_2^2U_2 - G_1G_2G_3U_2 + G_1^2G_2^2G_3U_2)$

$= \frac{G_1G_5}{1+G_1G_3}((G_4+G_2)U_1 + (G_4^2 - G_3G_4 + G_1G_3G_4^2 + G_2^2 - G_2G_3 + G_1G_2^2G_3)U_2)$

3



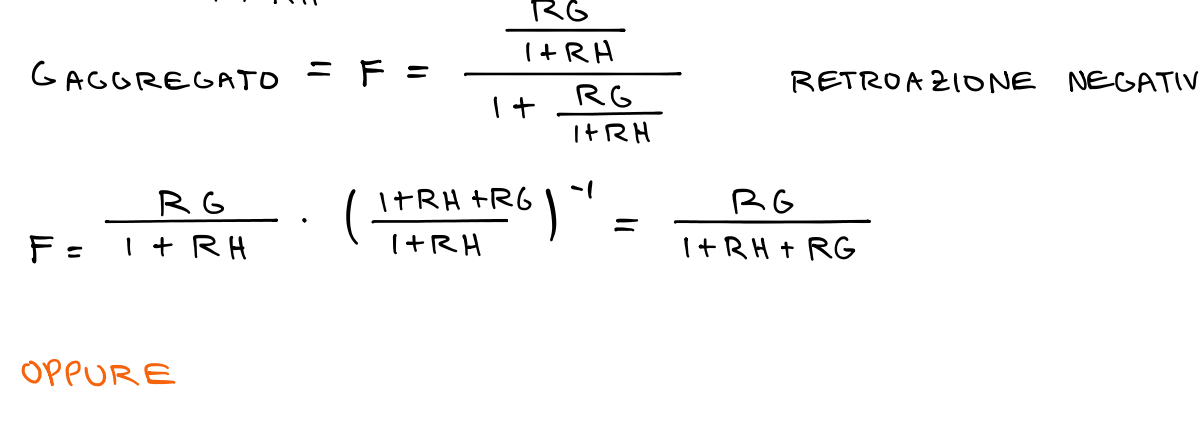
$R(s) = K$

$H(s) = \frac{1}{1+5s}$

$G(s) = \frac{1}{1+10s}$

FUNZIONE DI TRASFERIMENTO TOTALE

$\dot{x}_3 = +U - (x_1 + x_2) = U - x_2 - x_1$



ORA E' MOLTO PIU' SEMPLICE STUDIARLO

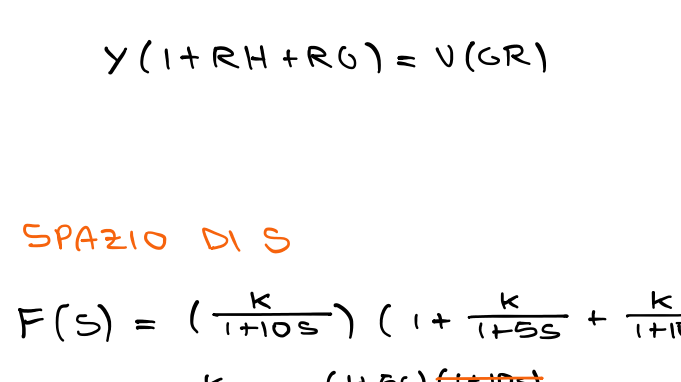
$G_{RH} = \frac{R}{1+RH}$ — RETROAZIONE NEGATIVA

$G_{RHG} = \frac{RG}{1+RH}$ — SERIE

$G_{AGGREGATO} = F = \dfrac{\frac{RG}{1+RH}}{1+\frac{RG}{1+RH}}$ — RETROAZIONE NEGATIVA

$F = \frac{RG}{1+RH} \cdot \left(\frac{1+RH+RG}{1+RH}\right)^{-1} = \frac{RG}{1+RH+RG}$

OPPURE



$y = G(y_1)$
$y_1 = R(U - y - y_2)$
$y_2 = H(y_1)$

$y_1 = R(U - y - H(y_1)) = R(U) - R(y) - RH(y_1)$

$y_1(1+RH) = R(U-y)$

$y = \frac{GR(U-y)}{1+RH} = (1+RH)y = GRU - GRy$

$y(1+RH+RG) = U(GR)$

SPAZIO DI S

$F(s) = \left(\frac{K}{1+10s}\right)\left(1 + \frac{K}{1+5s} + \frac{K}{1+10s}\right)^{-1} =$

$= \frac{K}{1+10s} \cdot \frac{(1+5s)(1+10s)}{1+15s+50s^2+K+10s+K+5s}$

$= \frac{(5K)s + K}{50s^2 + (30+15K)s + (1+2K)}$

STABILITA'

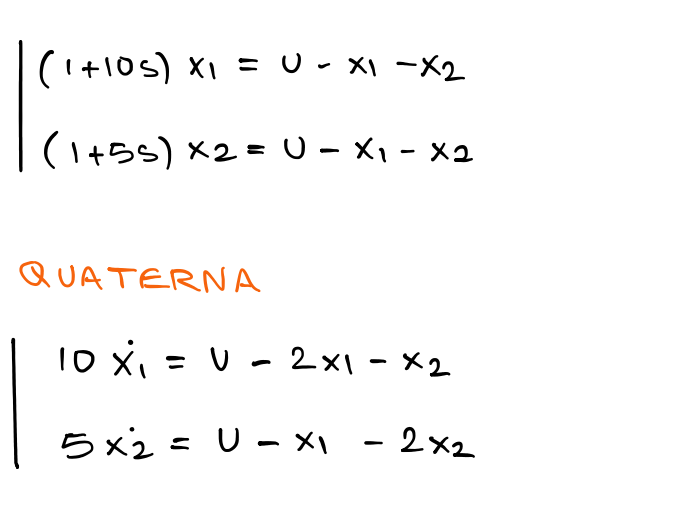
SISTEMA STABILE ⟷ ∀ POLI s_i DI $F(s)$: $Re(s_i) < 0$

$D(s) = 0 \rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-(30+15K) \pm \sqrt{(30+15K)^2 - 4(50)(1+2K)}}{100} =$

ECCETERA...

RAPPRESENTAZIONE DI STATO (K=1)



$\dot{x}_1 = \dot{x}_2 = U - x_1 - x_2$ POICHE' $K = +1$

$x_1 = G(U - x_1 - x_2)$

$x_2 = H(U - x_1 - x_2)$

$G^{-1}x_1 = U - x_1 - x_2$
$H^{-1}x_2 = U - x_1 - x_2$

$(1+10s)x_1 = U - x_1 - x_2$
$(1+5s)x_2 = U - x_1 - x_2$

QUATERNA

$10\dot{x}_1 = U - 2x_1 - x_2$
$5\dot{x}_2 = U - x_1 - 2x_2$

$$A = \begin{vmatrix} -\frac{2}{10} & -\frac{1}{10} \\ -\frac{1}{5} & -\frac{2}{5} \end{vmatrix} \quad \begin{vmatrix} \frac{1}{10} \\ \frac{1}{5} \end{vmatrix} = b$$

$c^T = |1 \ \ 0| \qquad \frac{1}{10} = d$ IMPROPRIO

ARMA

$x_2 = \frac{U - x_1}{2+5s}$

$(2+10s)x_1 = U - \frac{U - x_1}{2+5s}$

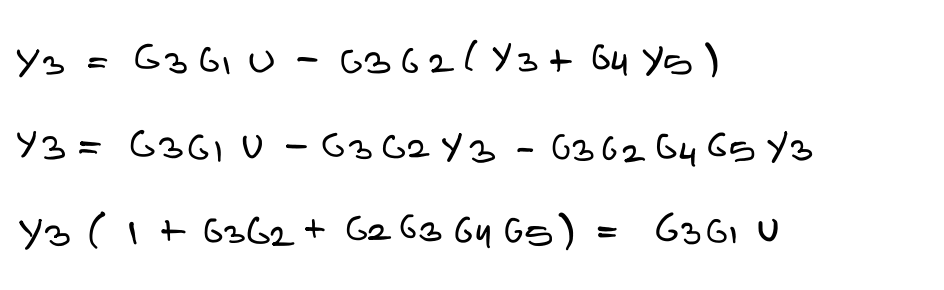
$(2+10s)x_1 = \frac{2U + 5sU - U + x_1}{2+5s}$

$(2+10s)(2+5s)x_1 = x_1 + (1+5s)U$

$(3+30s+50s^2)x_1 = (1+5s)U$

$(50s^2+30s+3)y = (5s+1)U$

CALCOLA F TRASFERIMENTO



$y_1 = G_1(U)$
$y_2 = G_2(y_3 + y_4)$
$y_3 = G_3(y_1 - y_2)$
$y_4 = G_4(y_5)$
$y_5 = y = G_5(y_3)$

$y_3 = G_3(G_1(U) - G_2(y_3 + y_4))$

$y_3 = G_3G_1U - G_3G_2(y_3 + G_4y_5)$

$y_3 = G_3G_1U - G_3G_2y_3 - G_3G_2G_4G_5y_3$

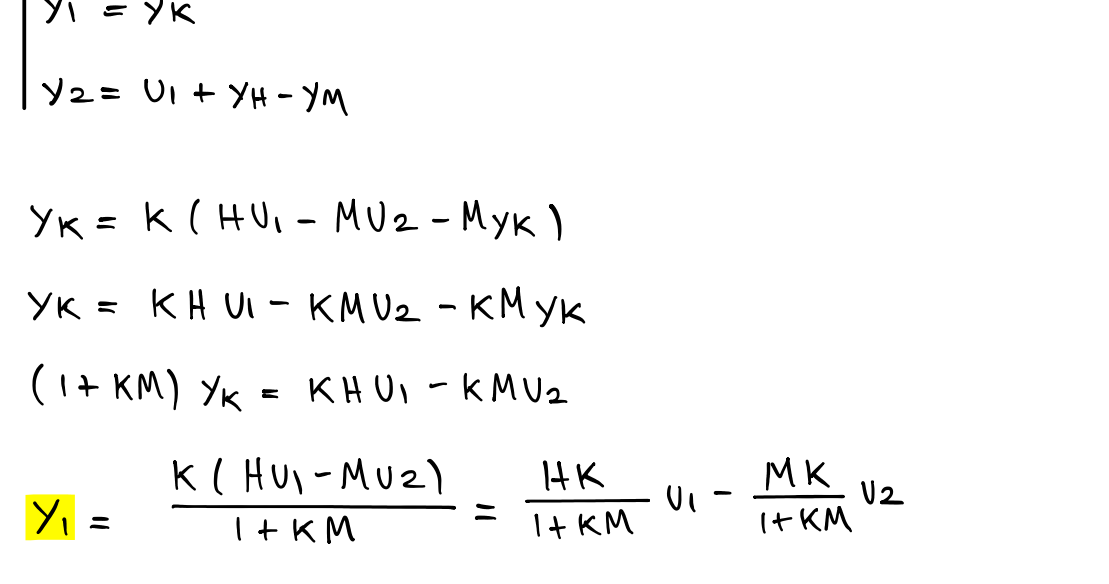
$y_3(1 + G_3G_2 + G_2G_3G_4G_5) = G_3G_1U$

$y_3 = \frac{G_3G_1U}{1+G_2G_3+G_2G_3G_4G_5}$

$y_5 = y = G_5(y_3) \longrightarrow y = \frac{G_1G_3G_5}{1+G_2G_3+G_2G_3G_4G_5} U$

OVVERO $F = \frac{G_1G_3G_5}{1+G_2G_3+G_2G_3G_4G_5}$

5 - CALCOLA F TRASFERIMENTO



$y_H = H(U_1)$
$y_M = M(U_2 + y_K)$
$y_K = K(y_H - y_M)$
$y_1 = y_K$
$y_2 = U_1 + y_H - y_M$

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$y_K = K(HU_1 - MU_2 - My_K)$

$y_K = KHU_1 - KMU_2 - KMy_K$

$(1+KM)y_K = KHU_1 - KMU_2$

$y_1 = \frac{K(HU_1 - MU_2)}{1+KM} = \frac{HK}{1+KM}U_1 - \frac{MK}{1+KM}U_2$

$y_2 = U_1 + HU_1 - MU_2 - My_K$

$y_2 = U_1(1+H) - MU_2 - \frac{MK(HU_1 - MU_2)}{1+KM}$

$y_2 = \frac{U_1 + KMU_1 + HU_1 + KMHU_1 - MU_2 - KM^2U_2 - MKHU_1 + M^2KU_2}{1+KM}$

$y_2 = \frac{(1+KM+H)U_1 - MU_2}{1+KM}$