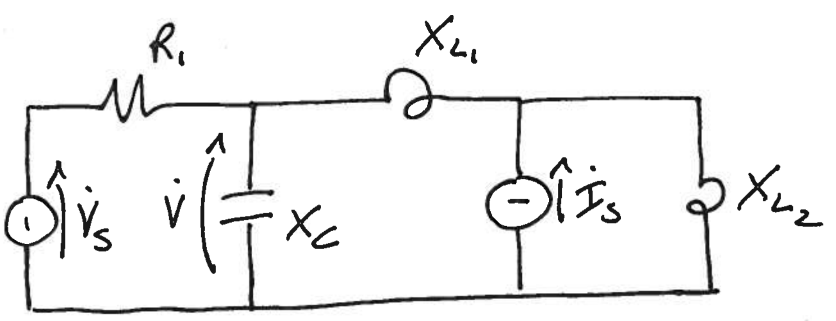
# ES 38

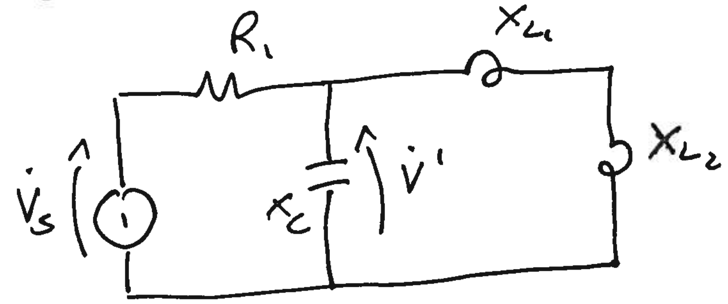



$\dot V = ?$

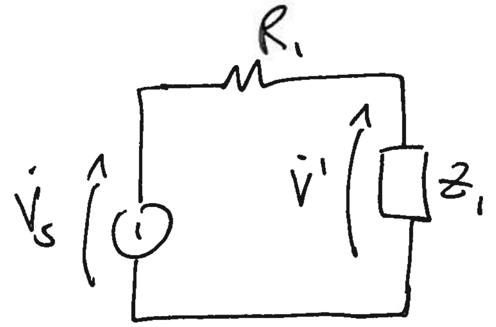
$\dot V_s = 10 \exp(j0)$ V
$\dot I_s = 10 \exp(j0)$ A
$R_1 = 2\,\Omega$
$X_C = -2\,\Omega$
$X_{L_1} = 1\,\Omega$
$X_{L_2} = 3\,\Omega$

Applichiamo il Teorema di sovrapposizione
(entrambi i generatori hanno la medesima pulsazione $\omega$)

$\dot I_s = 0$

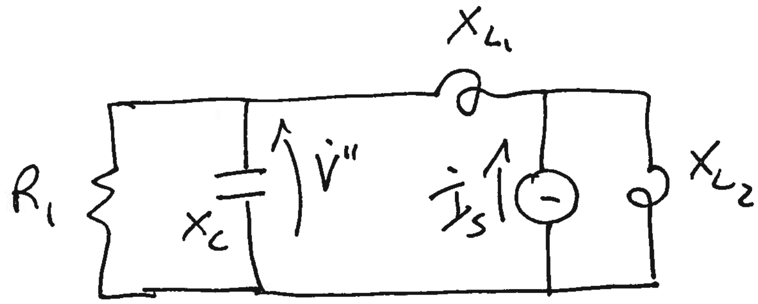



$$Z_1 = \frac{jX_C \, j(X_{L_1} + X_{L_2})}{j(X_C + X_{L_1} + X_{L_2})} = -j4\,\Omega$$

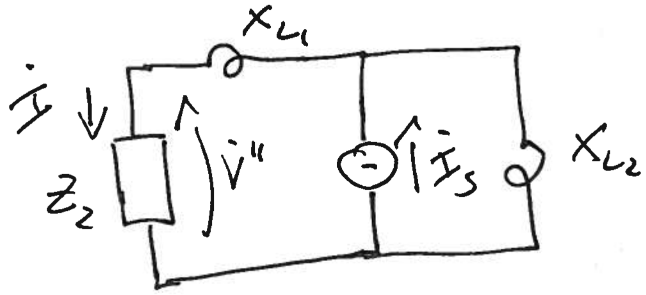



$$\dot V' = \dot V_s \frac{Z_1}{R_1 + Z_1} = 8 - j4 \text{ V}$$

$\dot{V}_S = 0$




$$Z_2 = \frac{j X_C R_1}{R_1 + j X_C} = 1 - j\ \Omega$$



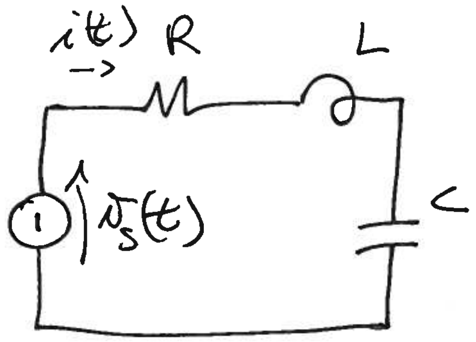

$$\dot{I} = \dot{I}_S \frac{\dfrac{1}{Z_2 + jX_{L_2}}}{\dfrac{1}{Z_2 + jX_{L_1}} + \dfrac{1}{jX_{L_2}}} = \dot{I}_S \frac{jX_{L_2}}{Z_2 + j(X_{L_1} + X_{L_2})}$$

$$= 9 + j3\ A$$

$$\dot{V}'' = Z_2 \dot{I} = 12 - j6\ V$$

$$\dot{V} = \dot{V}' + \dot{V}'' = 20 - j10\ V$$

# ES 39




$v_s(t) = \sqrt{2}\ 100 \cos(1000t)$ V

$R = 10\,\Omega$

$L = 20$ mH

$C = 100\ \mu$F

Verificare che le potenze complesse si conserva

$\dot{V}_s = 100$ V , $\omega = 1000\ \frac{rad}{s}$

$X_L = \omega L = 20\,\Omega$

$X_C = -\frac{1}{\omega C} = -10\,\Omega$

$$\dot{I} = \frac{\dot{V}_s}{R + j(X_L + X_C)} = 5 - j5\,A$$

$\dot{V}_R = R\dot{I} = 50 - j50$ V

$\dot{V}_L = jX_L\dot{I} = 100 + j100$ V

$\dot{V}_C = jX_C\dot{I} = -50 - j50$ V

$\bar{S}_{V_s} = \dot{V}_s \dot{I}^* = 500 + j500$ VA

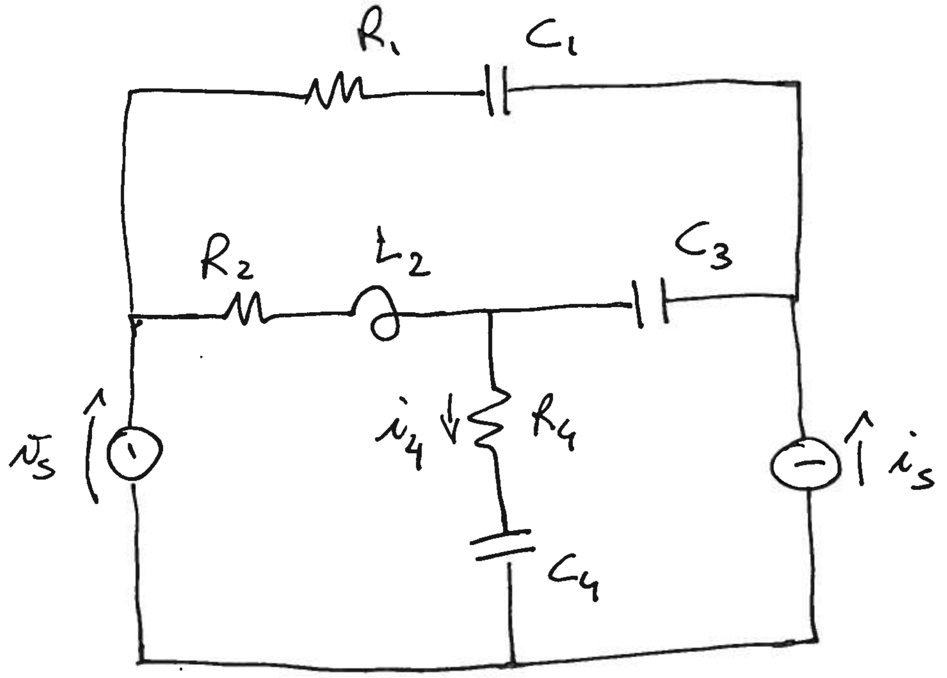
$\bar{S}_R = \dot{V}_R \dot{I}^* = 500$ VA

$\bar{S}_L = \dot{V}_L \dot{I}^* = j1000$ VA

$\bar{S}_C = \dot{V}_C \dot{I}^* = -j500$ VA

$\bar{S}_R + \bar{S}_L + \bar{S}_C = 500 + j500\ \text{VA} = \bar{S}_{V_s}$

# ES 40




$R_1 = R_2 = R_4 = 1\,\Omega$

$L_2 = 1\,H$

$C_1 = C_3 = C_4 = 1\,F$

$\omega_1 = 1\,rad/s$

$\omega_2 = 2\,rad/s$

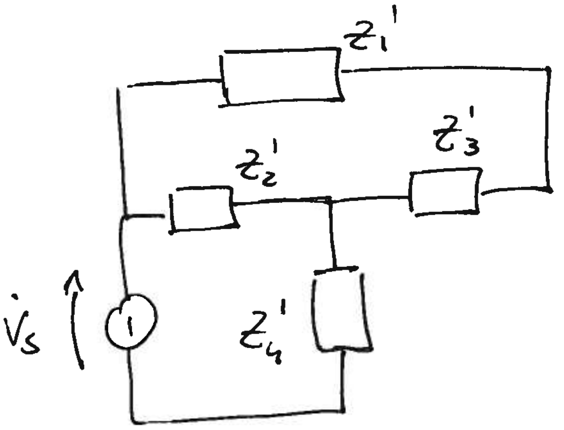
$$v_s(t) = 4\sqrt{2}\cos\left(\omega_1 t + \frac{\pi}{4}\right)\,V$$

$$i_s(t) = 5\cos\left(\omega_2 t + \frac{\pi}{2}\right)\,A$$

Calcolare la potenza assorbita da $R_4$.

$\boxed{i_s(t) = 0}$

$$v_s(t) \rightarrow \dot{V}_s = 4\exp\left(j\frac{\pi}{4}\right)\,\cancel{V} = 2\left(\sqrt{2} + j\sqrt{2}\right)\,V$$

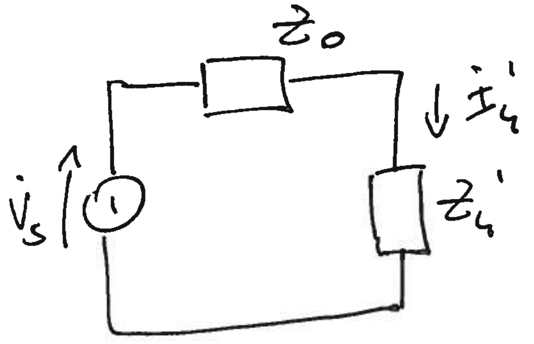



$$\dot{Z}_1' = R_1 - j\frac{1}{\omega_1 C_1} = 1 - j\,\Omega$$

$$\dot{Z}_2' = R_2 + j\omega_1 L_2 = 1 + j\,\Omega$$

$$\dot{Z}_3' = -j\frac{1}{\omega_1 C_3} = -j\,\Omega$$

$$\dot{Z}_4' = R_4 - j\frac{1}{\omega_1 C_4} = 1 - j\,\Omega$$




$$Z_0 = (Z_1' + Z_3') \parallel Z_2'$$

$$= \frac{Z_2''(Z_1' + Z_3')}{Z_1' + Z_2' + Z_3'} = \frac{7+j}{5}\ \Omega$$

$$\dot{I}_4' = \frac{\dot{V}_s}{Z_0 + Z_4'} = \frac{4+j4}{\sqrt{2}} \frac{1}{\frac{7+j}{5} + 1 - j}\ A = \frac{1+j2}{\sqrt{2}}\ A$$

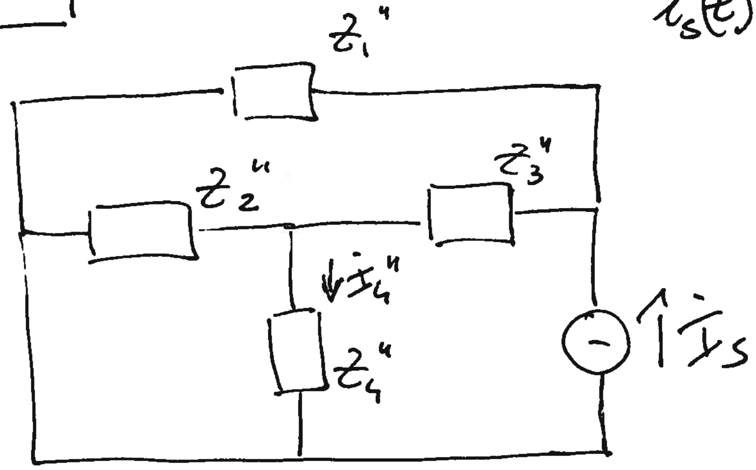
$$|\dot{I}_4'| = \frac{\sqrt{5}}{\sqrt{2}}$$

$$\arg(\dot{I}_4') = \operatorname{atan}\left(\frac{2}{1}\right) = 1{,}11\ \text{rad}$$

$$i_4'(t) = \sqrt{5}\cos(\omega_1 t + 1{,}11)$$

$v_s(t) = 0$

$$i_s(t) \rightarrow \dot{I}_s = j\frac{5}{\sqrt{2}}\ A$$

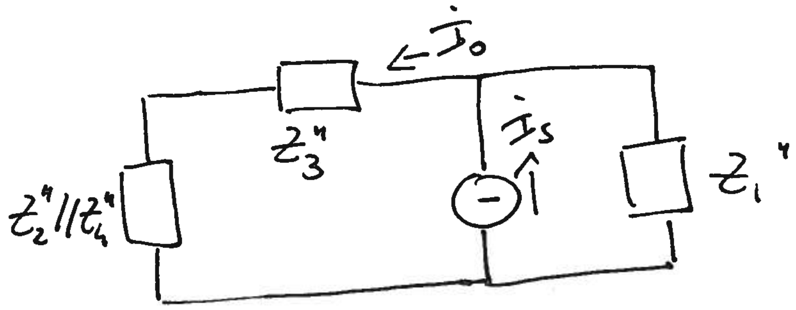



$$Z_1'' = R_1 - j\frac{1}{\omega_2 C_1} = 1 - j\frac{1}{2}\ \Omega$$

$$Z_2'' = R_2 + j\omega_2 L_2 = 1 + j2\ \Omega$$

$$Z_3'' = -j\frac{1}{\omega_2 C_3} = -j\frac{1}{2}\ \Omega$$

$$Z_4'' = R_4 - j\frac{1}{\omega_2 C_4} = 1 - j\frac{1}{2}\ \Omega$$




$$Z_0'' = Z_2''//Z_4'' + Z_3 = 1 - j\frac{1}{2}\ \Omega$$

$$\dot{I}_0'' = \dot{I}_S \frac{\frac{1}{Z_0''}}{\frac{1}{Z_0''} + \frac{1}{Z_1''}} = j\frac{5}{2\sqrt{2}}\ A$$

$$\dot{I}_4'' = \dot{I}_0' \frac{Z_2''}{Z_2'' + Z_4''} = \frac{-1 + j2}{\sqrt{2}}\ A$$

$$|\dot{I}_4''| = \frac{\sqrt{5}}{\sqrt{2}}\ A \qquad i_4''(t) = \sqrt{5}\cos(\omega_2 t + 2{,}03)$$

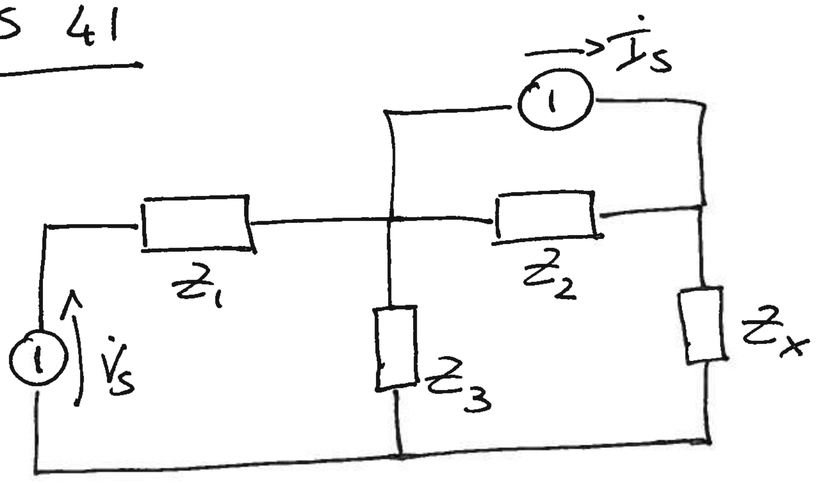
$$\arg(\dot{I}_4'') = 2{,}03\ \text{rad}$$

$$i_4(t) = \sqrt{5}\cos(\omega_1 t + 1{,}11) + \sqrt{5}\cos(\omega_2 t + 2{,}03)$$
$$= \sqrt{5}\left[\cos(t + 1{,}11) + \cos(2t + 2{,}03)\right]$$

$$P_4' = R_4|\dot{I}_4'|^2 = \frac{5}{2}\ W$$
$$P_4'' = R_4|\dot{I}_4''|^2 = \frac{5}{2}\ W$$

$$P_4 = P_4' + P_4'' = 5\ W$$

# ES 41




$\dot{V}_S = 10 V$

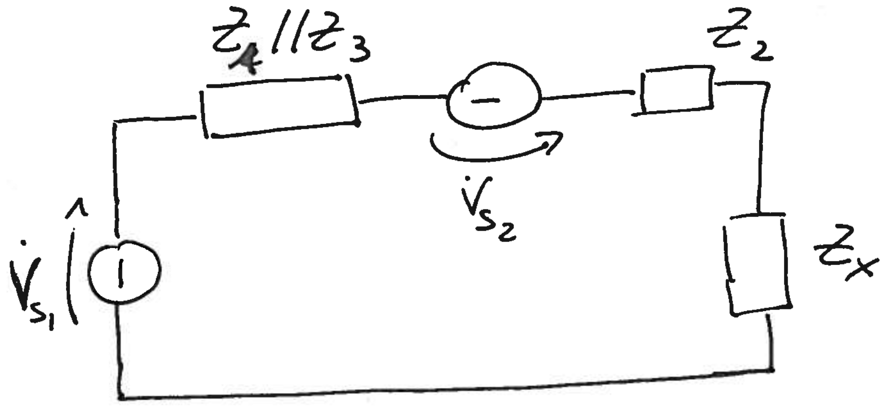
$\dot{I}_S = j2 A$

$Z_1 = 2 + j3 \Omega$

$Z_2 = 4 + j2 \Omega$

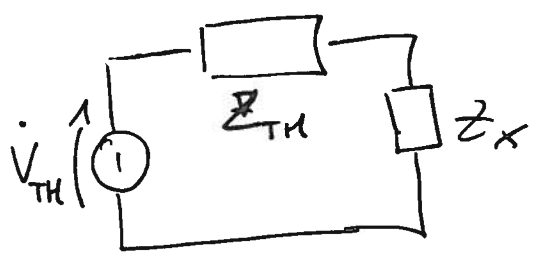
$Z_3 = 3 - j5 \Omega$

Determinare $Z_x$ in modo che essa assorba la massima potenza attiva ed il valore di tale potenza.




$$\dot{V}_{S_1} = \frac{\dot{V}_S}{Z_1} Z_1 // Z_3 = \dot{V}_S \frac{Z_3}{Z_1 + Z_3} = 8,6207 - j\,6,5517 \; V$$

$$\dot{V}_{S_2} = Z_2 \dot{I}_S = -4 + j8 \; V$$




$$\dot{V}_{TH} = \dot{V}_{S_1} + \dot{V}_{S_2} = 4,6207 + j\,1,4483 V$$

$$Z_{TH} = Z_2 + Z_1 // Z_3 = 7,6897 + j\,3,2759 \; \Omega$$

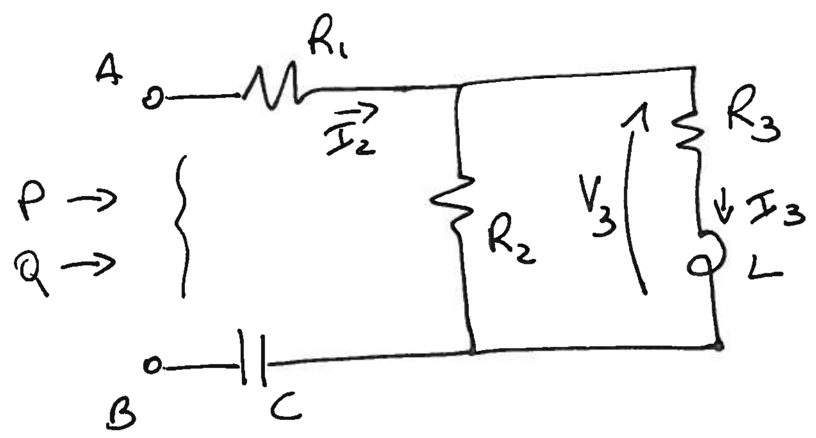
$$Z_x = Z_{TH}^* = 7,6897 - j3,2759\ \Omega$$

$$\dot{I}_x = \frac{\dot{V}_{TH}}{Z_{TH} + Z_{TH}^*} = \frac{\dot{V}_{TH}}{2\,\mathrm{Re}\{Z_{TH}\}} = 0,3004 + j0,0942$$

$$P = \mathrm{Re}\{Z_{TH}\}\,|\dot{I}_x|^2$$

$$= \frac{|\dot{V}_{TH}|^2}{4\,\mathrm{Re}\{Z_{TH}\}} = 0,7623\ W$$

## ES 42




$R_1 = 1\Omega$
$R_2 = 10\Omega$
$R_3 = 3\Omega$
$L = 0,02\,H$
$C = 1\,mF$
$f = 50\,Hz$
$I_3 = 10\,A$ rms

Potenza dissipata da $R_1$
P e Q assorbite ai morsetti A-B

$$P_3 = R_3 I_3^2 = 300\ W$$

$$Q_3 = X_L I_3^2 = 628,31\ VAr$$

$$V_3 = \sqrt{R_3^2 + X_L^2}\, I_3 = 69,62\,V$$

$Q_2 = Q_3$

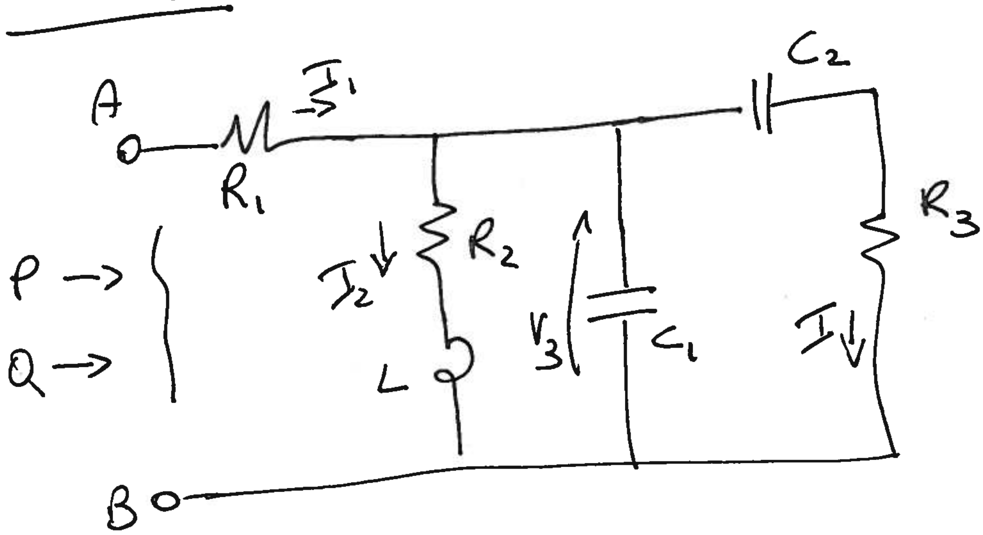
$$P_2 = P_3 + \frac{V_3^2}{R_2} = 784{,}78\ W$$

$$A_2 = \sqrt{P_2^2 + Q_2^2} \Rightarrow I_2 = \frac{A_2}{V_3} = 14{,}43\ A$$

$$P = P_2 + \underbrace{R_1 I_2^2}_{208{,}4\ W} = 993{,}2\ W$$

$$Q = Q_2 + X_c I_2^2 = -35{,}28\ VAr$$

## ES 43




$I = 20\ A\ rms$
$L = 50\ mH$
$C_1 = 30\ \mu F$
$C_2 = 50\ \mu F$
$R_1 = 5\ \Omega$
$R_2 = 10\ \Omega$
$R_3 = 3\ \Omega$
$f = 50\ Hz$

$P_{R_2} = ?\ ,\ P = ?\ ,\ Q = ?$

$P_3 = R_3 I^2 = 1200\ W$

$Q_3 = X_{c_2} I^2 = -25{,}465\ kVAr$

$$V_3 = \frac{\sqrt{P_3^2 + Q_3^2}}{I} = 1{,}2747\ kV$$

$$Q_2 = Q_3 + \frac{V_3^2}{X_{C_1}} = -40,778 \text{ kVAR}$$

$$P_2 = P_3$$

$$I_2 = \frac{V_3}{\sqrt{R_2^2 + X_L^2}} = 68,45 \text{ A}$$

$$P_1 = P_2 + R_2 I_2^2 = 48,058 \text{ kW}$$

$$Q_1 = Q_2 + X_L I_2^2 = 32,826 \text{ kVAR}$$

$$I_1 = \frac{\sqrt{P_1^2 + Q_1^2}}{V_3} = 45,65 \text{ A}$$

$$P = P_1 + R_1 I_1^2 = 58,481 \text{ kW}$$

$$Q = Q_1 = 32,826 \text{ kVAR}$$