$$\begin{cases} \dot{V}_{L_1} = j\omega L_{11}\dot{I}_{S_1} + j\omega L_m \dot{I}_{S_2} \\ \dot{V}_{L_2} = j\omega L_m \dot{I}_{S_1} + j\omega L_{22}\dot{I}_{S_2} \end{cases}$$

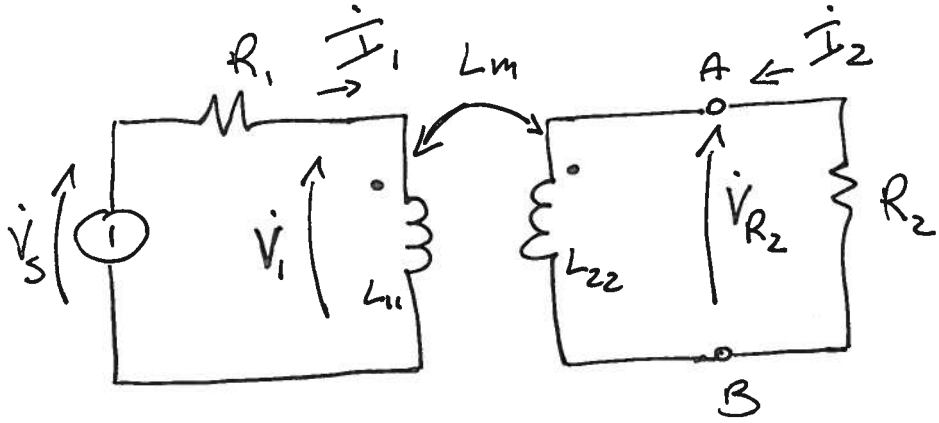
$$\dot{V}_1 = R_1\dot{I}_{S_1} + \dot{V}_{L_1} = (R_1 + j\omega L_{11})\dot{I}_{S_1} + j\omega L_m \dot{I}_{S_2}$$

$$\dot{V}_2 = R_2\dot{I}_{S_2} + \dot{V}_{L_2} = j\omega L_m \dot{I}_{S_1} + (R_2 + j\omega L_{22})\dot{I}_{S_2}$$

$$\dot{V}_1 = 12,5 + j6 \text{ V}$$

$$\dot{V}_2 = 15 - j16,4 \text{ V}$$

ES 49



$\dot{V}_S = 2,5 + j2,5$ V

$X_{11} = 6\,\Omega$

$X_{22} = 12\,\Omega$

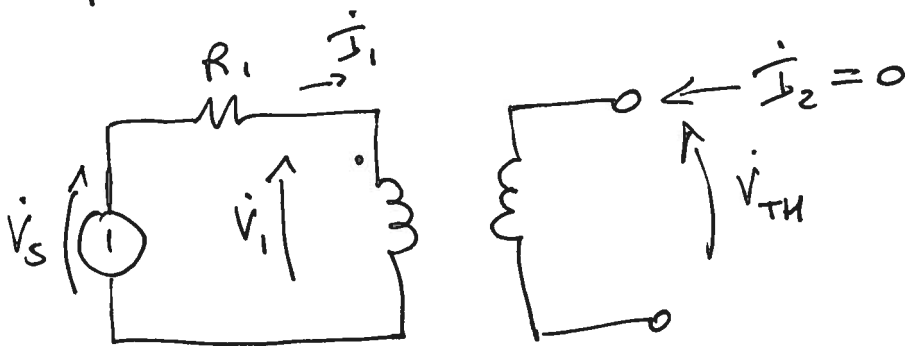
$X_m = 8\,\Omega$

$R_1 = 8\,\Omega$

$R_2 = 12\,\Omega$

Calcolare i parametri del circuito equivalente di Thévenin ai morsetti A-B e la Tensione $\dot{V}_{R_2}$ ai capi del resistore R_2.

$\dot{V}_{TH}$



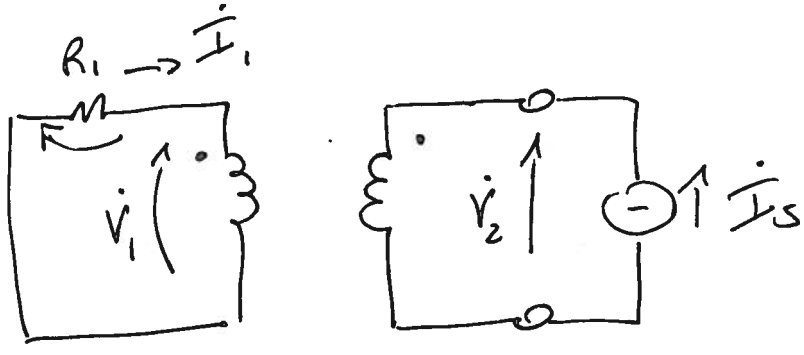
$$\dot{V}_1 = j X_{11} \dot{I}_1$$

$$\dot{V}_S = R_1 \dot{I}_1 + j X_{11} \dot{I}_1 = (R_1 + j X_{11}) \dot{I}_1$$

$$\dot{I}_1 = \frac{\dot{V}_S}{R_1 + j X_{11}}$$

$$\dot{V}_2 = \dot{V}_{TH} = j X_m \dot{I}_1 = j X_m \frac{\dot{V}_S}{R_1 + j X_{11}} = -0{,}4 + j\,2{,}8\ V$$

Z_{TH}



$$\dot{V}_1 = -R_1 \dot{I}_1$$

$$\begin{cases} \dot{V}_1 = j X_{11} \dot{I}_1 + j X_m \dot{I}_S \\ \dot{V}_2 = j X_m \dot{I}_1 + j X_{22} \dot{I}_S \end{cases}$$

$$(-R_1 - jX_{11})\dot{I}_1 = jX_m \dot{I}_S$$

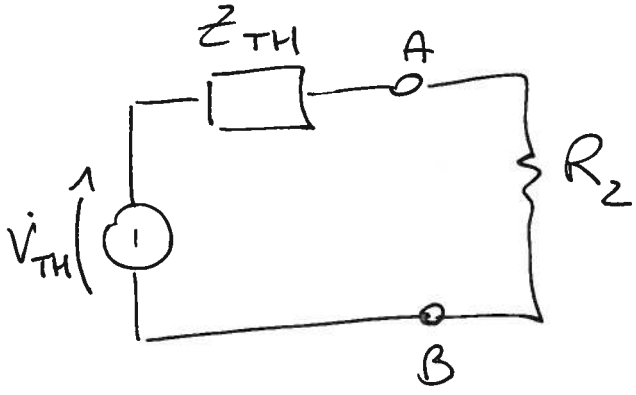
$$\dot{V}_2 = jX_m \frac{jX_m}{-R_1 - jX_{11}} \dot{I}_S + jX_{22}\dot{I}_S$$

$$= \left[\frac{X_m^2}{R_1 + jX_{11}} + jX_{22}\right]\dot{I}_S$$

$$Z_{TH} = \frac{\dot{V}_2}{\dot{I}_S} = \frac{X_m^2 + jX_{22}(R_1 + jX_{11})}{R_1 + jX_{11}} =$$

$$= \frac{X_m^2 - X_{11}X_{22} + jR_1X_{22}}{R_1 + jX_{11}}$$

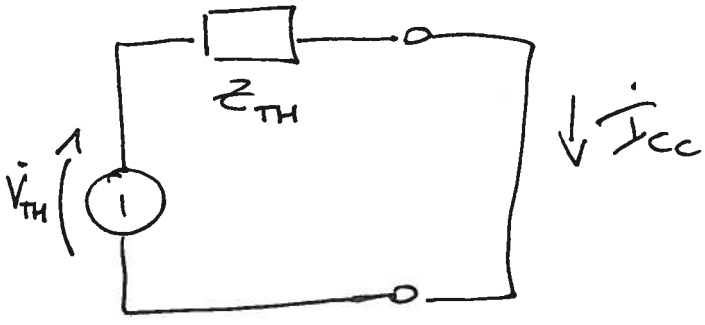
$$= 5{,}12 + j\,8{,}16\ \Omega$$



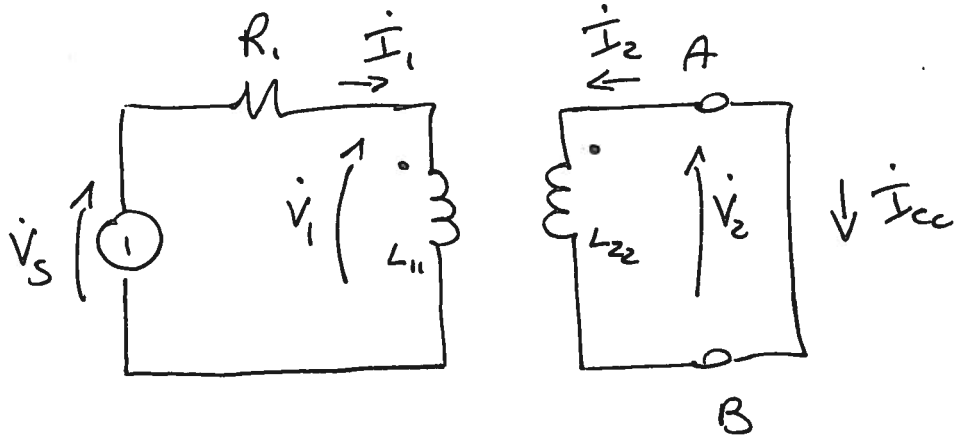
$$\dot{V}_{R_2} = \dot{V}_{TH}\frac{R_2}{Z_{TH} + R_2} = 0{,}534 + j\,1{,}708\ V$$

$\underline{Z}_{TH}$ → calcolata usando la corrente di corto circuito $\dot{I}_{cc}$

Infatti



$$\dot{V}_{TH} = \underline{Z}_{TH}\,\dot{I}_{cc} \Rightarrow \underline{Z}_{TH} = \frac{\dot{V}_{TH}}{\dot{I}_{cc}}$$



$$\dot{I}_2 = -\dot{I}_{cc} \quad , \quad \dot{V}_1 = \dot{V}_S - R_1\,\dot{I}_1$$

$$\dot{V}_2 = 0$$

$$\begin{cases} \dot{V}_1 = j X_{11}\,\dot{I}_1 - j X_m\,\dot{I}_{cc} \\ 0 = j X_m\,\dot{I}_1 - j X_{22}\,\dot{I}_{cc} \end{cases}$$

$$\dot{I}_1 = \frac{X_{22}}{X_m}\,\dot{I}_{cc} \qquad \dot{V}_S = (R_1 + j X_{11})\,\dot{I}_1 - j X_m\,\dot{I}_{cc}$$

$$\dot{V}_S = \left[(R_1 + j X_{11}) \frac{X_{22}}{X_m} - j X_m \right] \dot{I}_{cc}$$

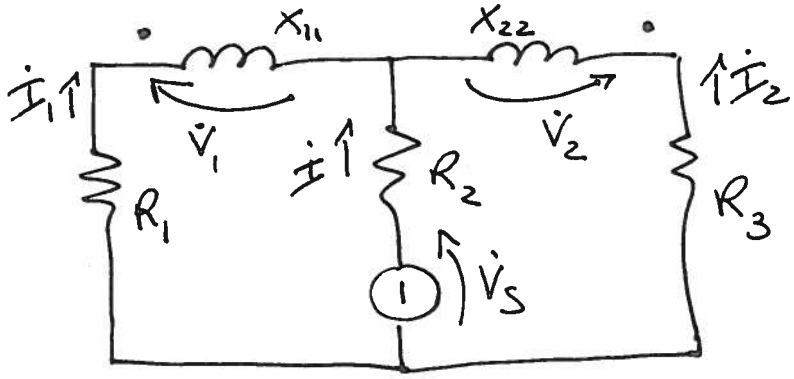
$$\dot{I}_{cc} = \frac{X_m \dot{V}_S}{X_{22}(R_1 + j X_{11}) - j X_m^2}$$

$$\mathcal{Z}_{TH} = \frac{\dot{V}_{TH}}{\dot{I}_{cc}} = \frac{j X_m \dot{V}_S}{R_1 + j X_{11}} \quad \frac{X_{22}(R_1 + j X_{11}) - j X_m^2}{X_m \dot{V}_S}$$

$$= \frac{j X_{22}(R_1 + j X_{11}) + X_m^2}{R_1 + j X_{11}}$$

$$= \frac{X_m^2 - X_{11} X_{22} + j R_1 X_{22}}{R_1 + j X_{11}}$$

ES 51



$\dot{V}_S = 10V$

$R_1 = R_3 = 1\Omega$

$R_2 = 2\Omega$

$X_{11} = X_{22} = 2\Omega$

$X_m = 1\Omega$

$\dot{I} = ?$

$$\begin{cases} \dot{V}_1 = jX_{11}\dot{I}_1 + jX_m\dot{I}_2 \\ \dot{V}_2 = jX_m\dot{I}_1 + jX_{22}\dot{I}_2 \end{cases}$$

$$\dot{I} = -(\dot{I}_1 + \dot{I}_2)$$

$$\dot{V}_S - R_2\dot{I} + \dot{V}_1 + R_1\dot{I}_1 = 0$$

$$\dot{V}_S - R_2\dot{I} + \dot{V}_2 + R_3\dot{I}_2 = 0$$

$$\dot{V}_1 = R_2\dot{I} - R_1\dot{I}_1 - \dot{V}_S = jX_{11}\dot{I}_1 + jX_m\dot{I}_2$$

$$\dot{V}_2 = R_2\dot{I} - R_3\dot{I}_2 - \dot{V}_S = jX_m\dot{I}_1 + jX_{22}\dot{I}_2$$

$$\dot{V}_S + (R_1 + R_2 + jX_{11})\dot{I}_1 + (R_2 + jX_m)\dot{I}_2 = 0$$

$$\dot{V}_S + (R_2 + jX_m)\dot{I}_1 + (R_2 + R_3 + jX_{22})\dot{I}_2 = 0$$

$$(3 + j2)\dot{I}_1 + (2 + j)\dot{I}_2 = -10$$

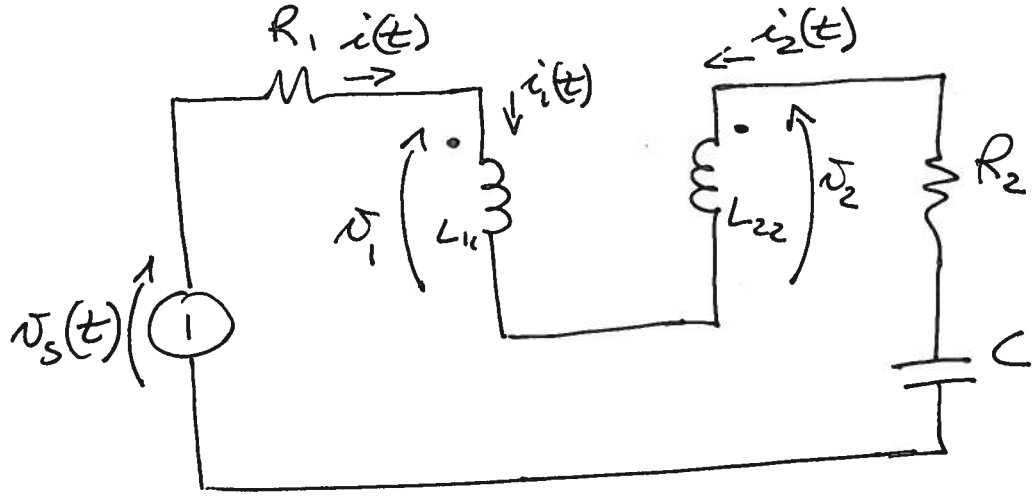
$$(2 + j)\dot{I}_1 + (3 + j2)\dot{I}_2 = -10$$

Risolvendo

$$\dot{I}_1 = \dot{I}_2 = -1{,}471 + j\,0{,}8824 \text{ A}$$

$$\dot{I} = 2{,}941 - j\,1{,}765 \text{ A}$$

ES 52



$v_s(t) = 100\sqrt{2}\cos(\omega t)$ V

$\omega = 1000$ rad/s

$R_1 = 2\,\Omega$ $\quad L_{11} = 3$ mH

$R_2 = 3\,\Omega$ $\quad L_{22} = 2$ mH

$C = \frac{1}{5}$ mF $\quad L_m = 2$ mH

$i(t) = ?$

$X_C = -5\,\Omega$ $\quad X_{22} = 2\,\Omega$ $\quad \dot{V}_S = 100$ V

$X_{11} = 3\,\Omega$ $\quad X_m = 2\,\Omega$

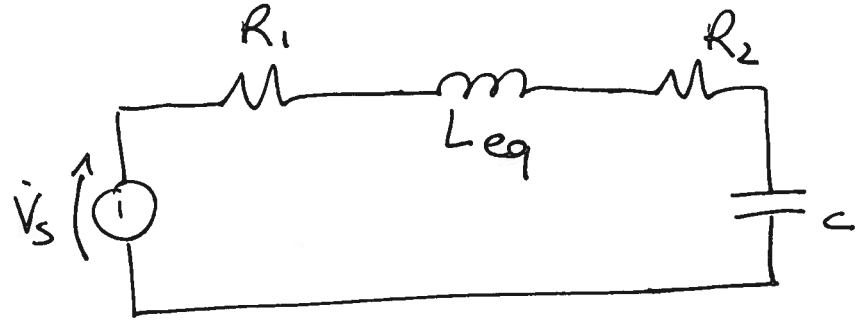
$\dot{I} = \dot{I}_1 = -\dot{I}_2$

$$\dot{V}_S = R_1\dot{I} + \dot{V}_1 - \dot{V}_2 + (R_2 + jX_C)\dot{I}$$

$$\begin{cases} \dot{V}_1 = jX_{11}\dot{I}_1 + jX_m\dot{I}_2 = j(X_{11} - X_m)\dot{I} \\ \dot{V}_2 = jX_m\dot{I}_1 + jX_{22}\dot{I}_2 = j(X_m - X_{22})\dot{I} \end{cases}$$

$$\dot{V}_S = \left[R_1 + R_2 + j\left(X_{11} - 2X_m + X_{22}\right) + jX_C\right]\dot{I}$$

Il mutuo induttore, infatti, è in serie controverse.
Il circuito potrebbe essere ridisegnato così:



$$X_{eq} = X_{11} + X_{22} - 2X_m = 1\,\Omega$$

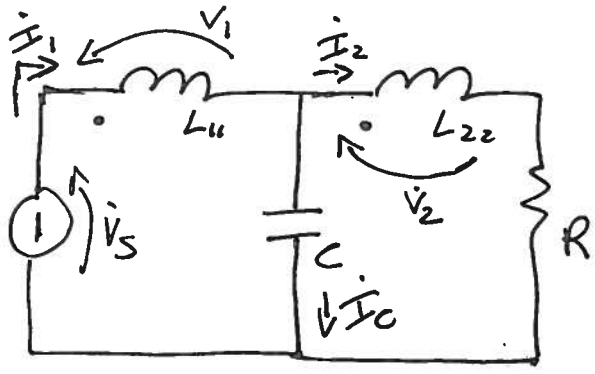
Risolvendo

$$\dot{I} = 15{,}62 \exp(j\,0{,}675)\ A$$

$$i(t) = \mathrm{Re}\left\{\sqrt{2}\,\dot{I}\exp(j\omega t)\right\}\ A$$

$$= 22{,}086 \cos(\omega t + 0{,}675)\ A$$

ES 53



$V_S = 10\,V$ rms

$L_{11} = 3H$

$L_{22} = 2H$

$L_m = 1\,H$ $\quad C = \frac{1}{6}\,F$

$R = 4\,\Omega$

$\omega = 3\,\frac{rad}{s}$

Calcolare le potenze attive e le potenze reattive erogate dal generatore di tensione.

$X_{11} = 9\,\Omega \qquad X_m = 3\,\Omega$

$X_{22} = 6\,\Omega \qquad X_C = -2\,\Omega$

$$\begin{cases} \dot{V}_1 = j X_{11} \dot{I}_1 + j X_m \dot{I}_2 \\ \dot{V}_2 = j X_m \dot{I}_1 + j X_{22} \dot{I}_2 \end{cases}$$

$$\dot{I}_C = \dot{I}_1 - \dot{I}_2$$

$$\dot{V}_C = j X_C \dot{I}_C$$

$$\begin{cases} \dot{V}_C - \dot{V}_2 - R\dot{I}_2 = 0 \\ \dot{V}_C + \dot{V}_1 - \dot{V}_S = 0 \end{cases}$$

$$\begin{cases} j X_C \dot{I}_C - j X_m \dot{I}_1 - j X_{22} \dot{I}_2 - R\dot{I}_2 = 0 \\ j X_C \dot{I}_C + j X_{11} \dot{I}_1 + j X_m \dot{I}_2 - \dot{V}_S = 0 \end{cases}$$

$$\begin{cases} j(X_c - X_m)\dot{I}_1 - [R + j(X_c + X_{22})]\dot{I}_2 = 0 \\ j(X_c + X_{11})\dot{I}_1 + j(X_m - X_c)\dot{I}_2 - \dot{V}_s = 0 \end{cases}$$

$$\dot{I}_2 = \frac{j(X_c - X_m)}{R + j(X_c + X_{22})}\dot{I}_1 = \alpha \dot{I}_1$$

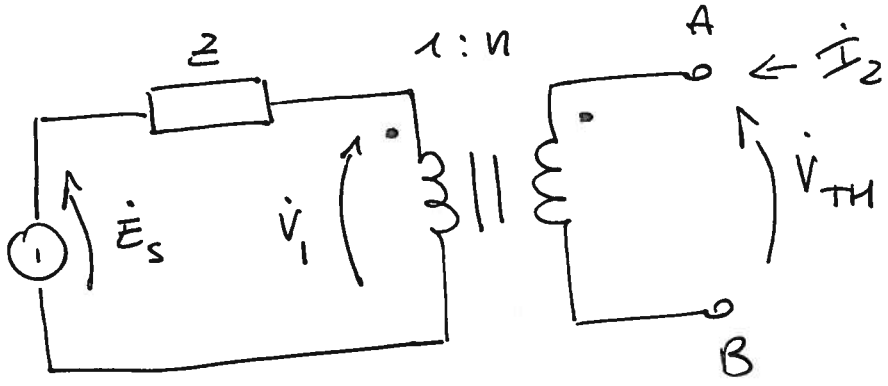
$$\dot{I}_1 = \frac{\dot{V}_s}{j(X_{11} + X_c) + j\alpha(X_m - X_c)} = 0,903 - j\,0,848\ A$$

$$\bar{S} = \dot{V}_s \dot{I}_1^* = 9,03 + j\,8,48\ VA$$

$$P_{V_s} = 9,03\ W$$

$$Q_{V_s} = 8,48\ VAr$$

ES 54

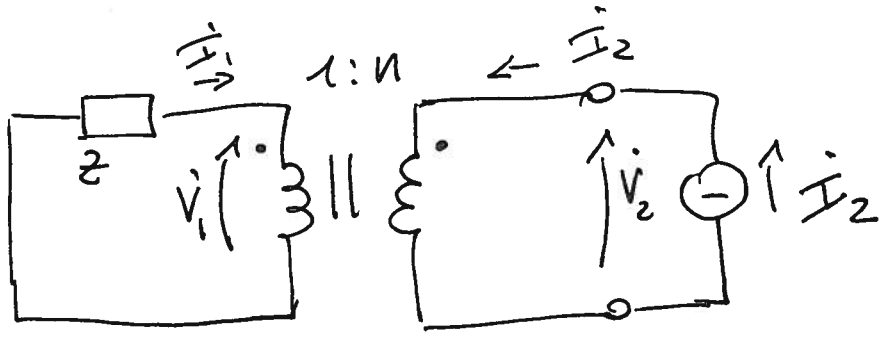


Determinare l'equivalente di Thévenin del bipolo di figura

$$\dot{I}_2 = 0 \qquad \dot{I}_1 = -n\dot{I}_2 = 0 \Rightarrow \dot{V}_1 = \dot{E}_S$$

$$\dot{V}_2 = n\dot{V}_1 = n\,\dot{E}_S = \dot{V}_{TH}$$

Impedenza equivalente

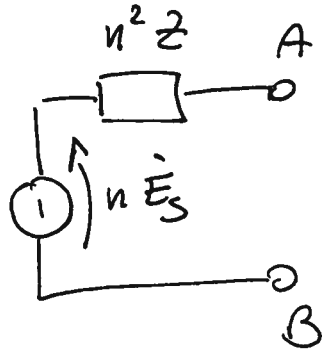


$$\dot{I}_1 = -n\dot{I}_2$$

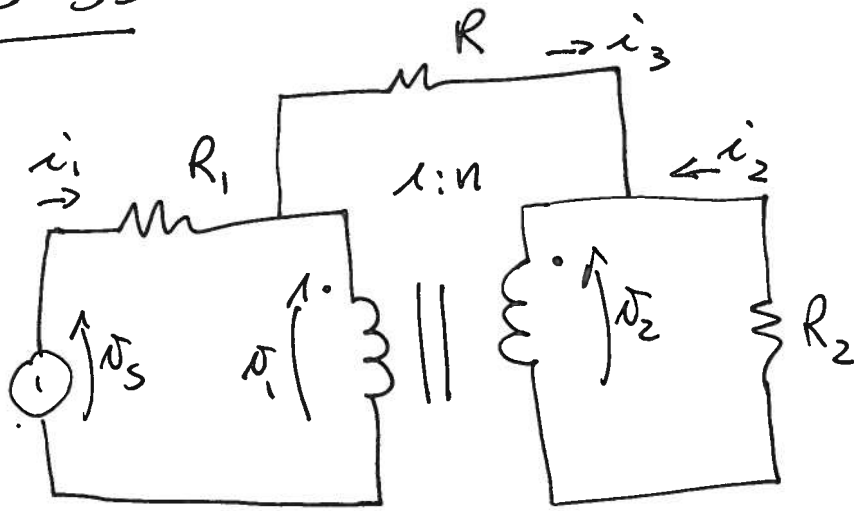
$$\dot{V}_1 = -Z\dot{I}_1 = nZ\dot{I}_2$$

$$\dot{V}_2 = n\dot{V}_1 = n^2 Z\dot{I}_2$$

$$Z_{TH} = \frac{\dot{V}_2}{\dot{I}_2} = n^2 Z$$



ES 55



Ricavare i valori di v_1, v_2, i_1 e i_2

$i_3 = 0$

per cui

$$\begin{cases} v_s = R_1 i_1 + v_1 \\ v_2 = -R_2 i_2 \end{cases} \qquad \begin{cases} v_2 = n v_1 \\ i_2 = -\frac{1}{n} i_1 \end{cases}$$

$$n v_1 = \frac{R_2}{n} i_1 \Rightarrow v_1 = \frac{R_2 i_1}{n^2}$$

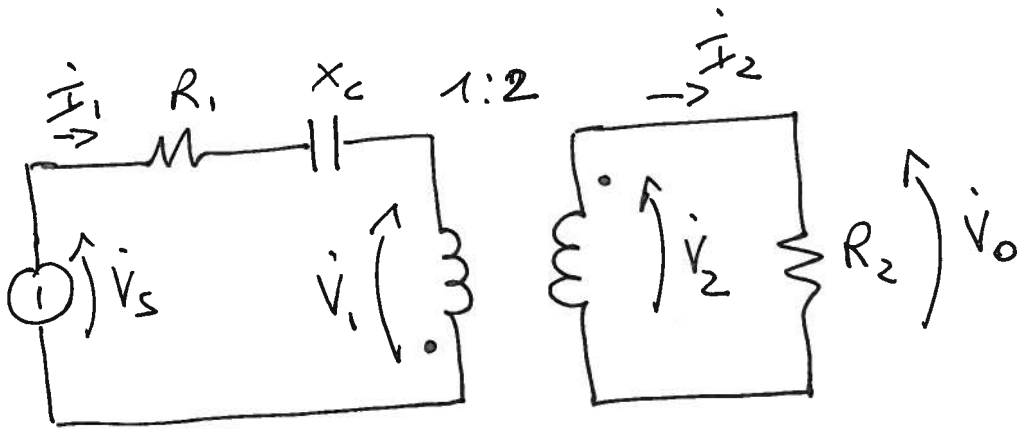
$$v_s = R_1 i_1 + \frac{R_2}{n^2} i_1 \Rightarrow i_1 = \frac{n^2 v_s}{R_2 + n^2 R_1}$$

$$v_1 = \frac{R_2 v_s}{R_2 + n^2 R_1}$$

$$i_2 = -\frac{n v_s}{R_2 + n^2 R_1}$$

$$v_2 = \frac{n R_2 v_s}{R_2 + n^2 R_1}$$

ES 56



$\dot{V}_S = 120\,V$

$R_1 = 4\,\Omega \qquad X_C = -6\,\Omega$

$R_2 = 20\,\Omega$

Calcolare $\dot{I}_1$, $\dot{V}_0$, $\bar{S}_{V_S}$

$$\begin{cases} \dot{I}_2 = -\frac{1}{n}\dot{I}_1 \\ \dot{V}_2 = -n\,\dot{V}_1 \end{cases}$$

$$\dot{V}_1 = \frac{R_2}{n^2}\dot{I}_1$$

$$\dot{V}_1 + (R_1 + jX_C)\dot{I}_1 = \dot{V}_S$$

$$\dot{V}_S - (R_1 + jX_C)\dot{I}_1 = \frac{R_2}{n^2}\dot{I}_1$$

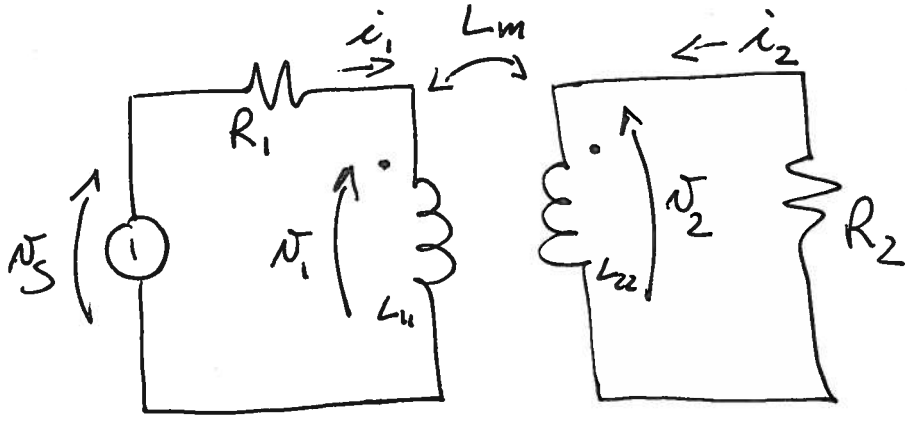
$$\dot{I}_1 = \frac{\dot{V}_S}{R_1 + \frac{R_2}{n^2} + jX_C} = 9{,}231 + j\,6{,}154\ A$$

$$\dot{I}_2 = -\frac{1}{\eta}\dot{I}_1 = -4{,}615 - j\,3{,}077\ A$$

$$\dot{V}_0 = R_2\,\dot{I}_2 = -92{,}31 - j\,61{,}54\ V$$

$$\overline{S}_{V_S} = \dot{V}_S\,\dot{I}_1^{*} = 1107 - j\,738{,}4\ VA$$

ES 57



$v_S(t) = 100\sqrt{2}\cos(100t)\ V$

$R_1 = 2\,\Omega$
$R_2 = 200\,\Omega$
$L_{11} = 0{,}4\ H$
$L_{22} = 1{,}6\ H$
$L_m = 0{,}6\ H$

Determinare il valore della corrente $i_1(t)$ erogata dal generatore di Tensione $v_S(t)$.

$v_S(t) \rightarrow \dot{V}_S = 100\ V \qquad \omega = 100\ \frac{rad}{s}$

$X_{11} = \omega L_{11} = 40\,\Omega$

$X_{22} = \omega L_{22} = 160\,\Omega$

$X_m = \omega L_m = 60\,\Omega$

$$\begin{cases} \dot{V}_1 = j X_{11} \dot{I}_1 + j X_m \dot{I}_2 \\ \dot{V}_2 = j X_m \dot{I}_1 + j X_{22} \dot{I}_2 \end{cases}$$

$$\begin{cases} \dot{V}_S - R_1 \dot{I}_1 - \dot{V}_1 = 0 \\ \dot{V}_2 + R_2 \dot{I}_2 = 0 \end{cases}$$

$$j X_m \dot{I}_1 + j X_{22} \dot{I}_2 + R_2 \dot{I}_2 = 0$$

$$\dot{I}_2 = - \frac{j X_m}{R_2 + j X_{22}} \dot{I}_1 = \alpha \dot{I}_1$$

$$\alpha = -0,1463 - j\,0,1829$$

$$\dot{V}_S - R_1 \dot{I}_1 - j X_{11} \dot{I}_1 + j \alpha X_m \dot{I}_1 = 0$$

$$\dot{I}_1 = \frac{\dot{V}_S}{R_1 + j(X_{11} + \alpha X_m)} = 1,135 - j\,2,7313 \ A$$

$$= 2,9578 \exp(-j\,1,1769) \ A$$

$$i_1(t) = \mathrm{Re}\left\{ \sqrt{2}\, \dot{I}_1 \exp(j\omega t) \right\} = \cancel{4,183 \cos(100t) \ A}$$

$$= 4,183 \cos(100t - 1,1769) \ A$$