Supplementary Information for "*A Multitask Network Robustness Analysis System Based on the Graph Isomorphism Network Model*"

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Table. S1: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on BA (Barabási–Albert) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

BA (*: 23, ≈: 1)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.012 (1)	0.039 (2, *)	0.070 (4, *)	0.043 (3, *)
		RND	0.016 (1)	0.020 (2, *)	0.046 (4, *)	0.027 (3, *)
	Connectivity Robustness	TAR	0.024 (1)	0.047 (2, *)	0.076 (4, *)	0.050 (3, *)
		RND	0.016 (1)	0.025 (2, *)	0.120 (4, *)	0.026 (3, *)
Directed	Controllability Robustness	TAR	0.014 (1)	0.025 (2, *)	0.092 (4, *)	0.043 (3, *)
		RND	0.015 (1)	0.025 (2, *)	0.077 (4, *)	0.030 (3, *)
	Connectivity Robustness	TAR	0.024 (1)	0.059 (2, *)	0.065 (4, *)	0.046 (3, *)
		RND	0.017 (1)	0.020 (2, ≈)	0.120 (4, *)	0.034 (3, *)

Table. S2: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on EH (extreme homogeneous) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

EH (*: 19, ≈: 5)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.009 (1)	0.011 (2, ≈)	0.054 (4, *)	0.045 (3, *)
		RND	0.008 (1)	0.009 (2, ≈)	0.020 (4, *)	0.017 (3, *)
	Connectivity Robustness	TAR	0.016 (1)	0.106 (2, *)	0.138 (4, *)	0.057 (3, *)
		RND	0.016 (1)	0.023 (3, *)	0.121 (4, *)	0.019 (2, *)
Directed	Controllability Robustness	TAR	0.011 (1)	0.010 (2, ≈)	0.118 (4, *)	0.073 (3, *)
		RND	0.010 (1)	0.010 (2, ≈)	0.050 (4, *)	0.029 (3, *)
	Connectivity Robustness	TAR	0.013 (1)	0.111 (3, *)	0.118 (4, *)	0.058 (2, *)
		RND	0.015 (1)	0.016 (2, ≈)	0.126 (4, *)	0.023 (3, *)

Table. S3: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on ER (Erdös-Rényi) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

ER (*: 21, ≈: 3)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.009 (1)	0.020 (2, *)	0.060 (4, *)	0.047 (3, *)
		RND	0.010 (1)	0.011 (2, ≈)	0.022 (4, *)	0.013 (3, *)
	Connectivity Robustness	TAR	0.022 (1)	0.087 (3, *)	0.130 (4, *)	0.083 (2, *)
		RND	0.016 (1)	0.023 (3, *)	0.114 (4, *)	0.022 (2, *)
Directed	Controllability Robustness	TAR	0.014 (1)	0.018 (2, *)	0.074 (4, *)	0.052 (3, *)
		RND	0.013 (1)	0.019 (2, *)	0.048 (4, *)	0.030 (3, *)
	Connectivity Robustness	TAR	0.025 (1)	0.091 (3, *)	0.117 (4, *)	0.052 (2, *)
		RND	0.017 (1)	0.018 (2, ≈)	0.118 (4, *)	0.020 (3, ≈)

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Table. S4: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on QS (q-snapback) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

QS (*: 19, ≈: 5)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.009 (1)	0.017 (2, *)	0.028 (4, *)	0.021 (3, *)
		RND	0.009 (1)	0.010 (2, ≈)	0.020 (4, *)	0.012 (3, *)
	Connectivity Robustness	TAR	0.017 (1)	0.088 (3, *)	0.102 (4, *)	0.030 (2, *)
		RND	0.016 (1)	0.023 (3, *)	0.117 (4, *)	0.020 (2, *)
Directed	Controllability Robustness	TAR	0.013 (1)	0.016 (2, ≈)	0.055 (4, *)	0.029 (3, *)
		RND	0.015 (1)	0.017 (2, ≈)	0.055 (4, *)	0.019 (3, *)
	Connectivity Robustness	TAR	0.023 (1)	0.110 (4, *)	0.100 (3, *)	0.035 (2, *)
		RND	0.021 (1)	0.021 (1, ≈)	0.122 (3, *)	0.021 (1, ≈)

Table. S5: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on RH (random hexagon) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

RH (*: 20, ≈: 4)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.008 (1)	0.019 (2, *)	0.034 (4, *)	0.027 (3, *)
		RND	0.010 (1)	0.012 (2, *)	0.022 (4, *)	0.014 (3, *)
	Connectivity Robustness	TAR	0.021 (1)	0.080 (3, *)	0.089 (4, *)	0.045 (2, *)
		RND	0.015 (1)	0.023 (2, *)	0.115 (4, *)	0.024 (3, *)
Directed	Controllability Robustness	TAR	0.012 (1)	0.014 (2, ≈)	0.059 (4, *)	0.033 (3, *)
		RND	0.012 (1)	0.013 (2, ≈)	0.045 (4, *)	0.018 (3, *)
	Connectivity Robustness	TAR	0.025 (1)	0.084 (3, *)	0.085 (4, *)	0.046 (2, *)
		RND	0.018 (1)	0.017 (2, ≈)	0.118 (4, *)	0.020 (3, ≈)

Table. S6: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on RT (random triangle) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

RT (*: 22, ≈: 2)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.010 (1)	0.026 (2, *)	0.045 (4, *)	0.030 (3, *)
		RND	0.011 (1)	0.014 (2, *)	0.025 (4, *)	0.016 (3, *)
	Connectivity Robustness	TAR	0.023 (1)	0.065 (3, *)	0.088 (4, *)	0.050 (2, *)
		RND	0.018 (1)	0.029 (3, *)	0.116 (4, *)	0.024 (2, *)
Directed	Controllability Robustness	TAR	0.011 (1)	0.023 (2, *)	0.079 (4, *)	0.039 (3, *)
		RND	0.014 (1)	0.020 (2, *)	0.050 (3, *)	0.020 (2, *)
	Connectivity Robustness	TAR	0.027 (1)	0.073 (3, *)	0.079 (4, *)	0.048 (2, *)
		RND	0.020 (1)	0.023 (2, ≈)	0.121 (4, *)	0.024 (3, ≈)

Table. S7: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on SF (generic scale-free) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS; a † denotes that the corresponding method is statistically superior to GIN-MAS; while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

SF (*: 17, ≈: 6, †: 1)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.041 (1)	0.105 (3, *)	0.209 (4, *)	0.039 (2, ≈)
		RND	0.020 (1)	0.050 (3, *)	0.123 (4, *)	0.025 (2, ≈)
	Connectivity Robustness	TAR	0.038 (2)	0.022 (1, †)	0.053 (4, ≈)	0.035 (2, ≈)
		RND	0.026 (1)	0.043 (3, *)	0.119 (4, *)	0.035 (2, *)
Directed	Controllability Robustness	TAR	0.010 (1)	0.045 (3, *)	0.162 (4, *)	0.021 (2, *)
		RND	0.014 (1)	0.045 (3, *)	0.130 (4, *)	0.019 (2, ≈)
	Connectivity Robustness	TAR	0.024 (1)	0.045 (2, ≈)	0.044 (4, *)	0.043 (3, *)
		RND	0.030 (1)	0.045 (2, *)	0.141 (4, *)	0.054 (3, *)

Table. S8: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on SW-NW (Newman–Watts small-world) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

SW-NW (*: 22, ≈: 2)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.008 (1)	0.011 (2, *)	0.018 (3, *)	0.021 (4, *)
		RND	0.008 (1)	0.009 (2, ≈)	0.017 (4, *)	0.011 (3, *)
	Connectivity Robustness	TAR	0.016 (1)	0.089 (3, *)	0.090 (4, *)	0.026 (2, *)
		RND	0.016 (1)	0.022 (3, *)	0.124 (4, *)	0.021 (2, *)
Directed	Controllability Robustness	TAR	0.011 (1)	0.017 (2, *)	0.047 (4, *)	0.022 (3, *)
		RND	0.010 (1)	0.015 (2, *)	0.034 (3, *)	0.015 (2, *)
	Connectivity Robustness	TAR	0.019 (1)	0.098 (4, *)	0.087 (3, *)	0.029 (2, *)
		RND	0.016 (1)	0.019 (2, ≈)	0.125 (4, *)	0.022 (3, *)

Table. S9: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on SW-WS (Watts–Strogatz small-world) networks. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, while an approximation '≈' indicates no statistical difference between the corresponding method and GIN-MAS, according to the Kruskal-Wallis H-test.

SW-WS (*: 21, ≈: 3)			GIN-MAS	LFR	RP	SPP
Undirected	Controllability Robustness	TAR	0.008 (1)	0.013 (2, *)	0.029 (4, *)	0.022 (3, *)
		RND	0.008 (1)	0.009 (2, ≈)	0.016 (4, *)	0.011 (3, *)
	Connectivity Robustness	TAR	0.016 (1)	0.091 (3, *)	0.092 (4, *)	0.025 (2, *)
		RND	0.015 (1)	0.023 (3, *)	0.122 (4, *)	0.021 (2, *)
Directed	Controllability Robustness	TAR	0.012 (1)	0.016 (2, *)	0.039 (4, *)	0.027 (3, *)
		RND	0.011 (1)	0.012 (2, ≈)	0.033 (4, *)	0.017 (3, *)
	Connectivity Robustness	TAR	0.020 (1)	0.097 (4, *)	0.088 (3, *)	0.028 (2, *)
		RND	0.017 (1)	0.019 (2, ≈)	0.123 (4, *)	0.021 (3, *)

Table. S10: Prediction errors obtained by GIN-MAS, LFR, RP, and SPP, on the 'Out-of-Range' (OR) networks. The training data are drawn from the synthetic networks of sizes $N \in [700, 1300]$, while the test data are from that of sizes $N \in [200, 700)$ and $N \in (700, 1800]$. An integer in parentheses indicates the rank of the method for robustness measure; an asterisk '*' denotes that the corresponding method is statistically inferior to GIN-MAS, using the Kruskal-Wallis H-test.

				Range of Tested Network Sizes N									
				$[200, 300]$	$[300, 400]$	$[400, 500]$	$[500, 600]$	$[200, 700]$	$[1300, 1400]$	$[1400, 1500]$	$[1500, 1600]$	$[1600, 1700]$	$[1700, 1800]$
Undirected	Controllability Robustness	TAR	GIN-MAS	0.020 (1)	0.017 (1)	0.016 (1)	0.014 (1)	0.014 (1)	0.012 (1)	0.013 (1)	0.013 (1)	0.013 (1)	0.014 (1)
			LFR	0.133 (3, *)	0.143 (3, *)	0.115 (3, *)	0.095 (3, *)	0.061 (3, *)	0.034 (3, *)	0.043 (3, *)	0.052 (3, *)	0.059 (3, *)	0.066 (3, *)
			RP	0.317 (4, *)	0.253 (4, *)	0.193 (4, *)	0.131 (4, *)	0.080 (4, *)	0.068 (4, *)	0.078 (4, *)	0.089 (4, *)	0.102 (4, *)	0.112 (4, *)
			SPP	0.079 (2, *)	0.072 (2, *)	0.052 (2, *)	0.044 (2, *)	0.038 (2, *)	0.033 (2, *)	0.035 (2, *)	0.038 (2, *)	0.043 (2, *)	0.047 (2, *)
		RND	GIN-MAS	0.025 (1)	0.020 (1)	0.017 (1)	0.014 (1)	0.013 (1)	0.009 (1)	0.009 (1)	0.009 (1)	0.009 (1)	0.009 (1)
			LFR	0.070 (3, *)	0.064 (3, *)	0.052 (3, *)	0.040 (3, *)	0.027 (3, *)	0.018 (2, *)	0.021 (3, *)	0.025 (3, *)	0.029 (3, *)	0.032 (3, *)
			RP	0.178 (4, *)	0.129 (4, *)	0.086 (4, *)	0.061 (4, *)	0.044 (4, *)	0.042 (4, *)	0.047 (4, *)	0.052 (4, *)	0.057 (4, *)	0.060 (4, *)
			SPP	0.032 (2, *)	0.033 (2, *)	0.026 (2, *)	0.024 (2, *)	0.021 (2, *)	0.018 (3, *)	0.019 (2, *)	0.023 (2, *)	0.027 (2, *)	0.031 (2, *)
	Connectivity Robustness	TAR	GIN-MAS	0.044 (1)	0.032 (1)	0.028 (1)	0.025 (1)	0.023 (1)	0.021 (1)	0.022 (1)	0.022 (1)	0.023 (1)	0.024 (1)
			LFR	0.231 (3, *)	0.216 (3, *)	0.191 (3, *)	0.160 (3, *)	0.110 (3, *)	0.081 (3, *)	0.093 (3, *)	0.107 (3, *)	0.116 (3, *)	0.125 (3, *)
			RP	0.499 (4, *)	0.421 (4, *)	0.352 (4, *)	0.257 (4, *)	0.131 (4, *)	0.108 (4, *)	0.129 (4, *)	0.149 (4, *)	0.165 (4, *)	0.184 (4, *)
			SPP	0.121 (2, *)	0.099 (2, *)	0.079 (2, *)	0.059 (2, *)	0.051 (2, *)	0.046 (2, *)	0.052 (2, *)	0.057 (2, *)	0.064 (2, *)	0.071 (2, *)
		RND	GIN-MAS	0.037 (1)	0.024 (1)	0.023 (1)	0.021 (1)	0.020 (1)	0.015 (1)	0.015 (1)	0.015 (1)	0.015 (1)	0.014 (1)
			LFR	0.086 (3, *)	0.067 (3, *)	0.059 (3, *)	0.051 (3, *)	0.039 (3, *)	0.026 (2, *)	0.030 (3, *)	0.032 (2, *)	0.036 (2, *)	0.038 (2, *)
			RP	0.273 (4, *)	0.225 (4, *)	0.192 (4, *)	0.160 (4, *)	0.132 (4, *)	0.131 (4, *)	0.138 (4, *)	0.146 (4, *)	0.152 (4, *)	0.158 (4, *)
			SPP	0.053 (2, *)	0.047 (2, *)	0.039 (2, *)	0.031 (2, *)	0.026 (2, *)	0.027 (3, *)	0.029 (2, *)	0.034 (3, *)	0.038 (3, *)	0.043 (3, *)
Directed	Controllability Robustness	TAR	GIN-MAS	0.025 (1)	0.020 (1)	0.017 (1)	0.015 (1)	0.014 (1)	0.01 (1)	0.011 (1)	0.011 (1)	0.011 (1)	0.011 (1)
			LFR	0.168 (3, *)	0.150 (3, *)	0.111 (3, *)	0.084 (3, *)	0.047 (3, *)	0.022 (2, *)	0.026 (2, *)	0.031 (2, *)	0.035 (2, *)	0.039 (2, *)
			RP	0.383 (4, *)	0.323 (4, *)	0.256 (4, *)	0.191 (4, *)	0.123 (4, *)	0.099 (4, *)	0.111 (4, *)	0.124 (4, *)	0.133 (4, *)	0.144 (4, *)
			SPP	0.101 (2, *)	0.081 (2, *)	0.065 (2, *)	0.051 (2, *)	0.046 (2, *)	0.041 (3, *)	0.044 (3, *)	0.046 (3, *)	0.052 (3, *)	0.056 (3, *)
		RND	GIN-MAS	0.026 (1)	0.022 (1)	0.018 (1)	0.016 (1)	0.015 (1)	0.011 (1)	0.011 (1)	0.010 (1)	0.010 (1)	0.01 (1)
			LFR	0.104 (3, *)	0.080 (3, *)	0.070 (3, *)	0.061 (3, *)	0.040 (3, *)	0.020 (2, *)	0.023 (2, *)	0.026 (3, *)	0.028 (2, *)	0.031 (2, *)
			RP	0.219 (4, *)	0.189 (4, *)	0.149 (4, *)	0.113 (4, *)	0.078 (4, *)	0.071 (4, *)	0.081 (4, *)	0.090 (4, *)	0.100 (4, *)	0.108 (4, *)
			SPP	0.068 (2, *)	0.058 (2, *)	0.040 (2, *)	0.032 (2, *)	0.027 (2, *)	0.021 (3, *)	0.023 (3, *)	0.025 (2, *)	0.030 (3, *)	0.034 (3, *)
	Connectivity Robustness	TAR	GIN-MAS	0.048 (1)	0.036 (1)	0.031 (1)	0.028 (1)	0.026 (1)	0.021 (1)	0.020 (1)	0.021 (1)	0.020 (1)	0.020 (1)
			LFR	0.267 (3, *)	0.218 (3, *)	0.166 (3, *)	0.132 (3, *)	0.104 (3, *)	0.086 (3, *)	0.091 (3, *)	0.096 (3, *)	0.101 (3, *)	0.107 (3, *)
			RP	0.381 (4, *)	0.319 (4, *)	0.261 (4, *)	0.197 (4, *)	0.111 (4, *)	0.101 (4, *)	0.115 (4, *)	0.127 (4, *)	0.142 (4, *)	0.154 (4, *)
			SPP	0.148 (2, *)	0.122 (2, *)	0.083 (2, *)	0.064 (2, *)	0.054 (2, *)	0.043 (2, *)	0.044 (2, *)	0.048 (2, *)	0.055 (2, *)	0.061 (2, *)
		RND	GIN-MAS	0.036 (1)	0.028 (1)	0.026 (1)	0.023 (1)	0.022 (1)	0.017 (1)	0.016 (1)	0.016 (1)	0.016 (1)	0.016 (1)
			LFR	0.085 (3, *)	0.072 (3, *)	0.060 (3, *)	0.048 (3, *)	0.035 (3, *)	0.022 (2, *)	0.022 (2, *)	0.024 (2, *)	0.024 (2, *)	0.025 (2, *)
			RP	0.247 (4, *)	0.203 (4, *)	0.178 (4, *)	0.157 (4, *)	0.135 (4, *)	0.135 (4, *)	0.141 (4, *)	0.149 (4, *)	0.157 (4, *)	0.164 (4, *)
			SPP	0.059 (2, *)	0.058 (2, *)	0.045 (2, *)	0.037 (2, *)	0.032 (2, *)	0.030 (3, *)	0.031 (3, *)	0.036 (3, *)	0.042 (3, *)	0.047 (3, *)

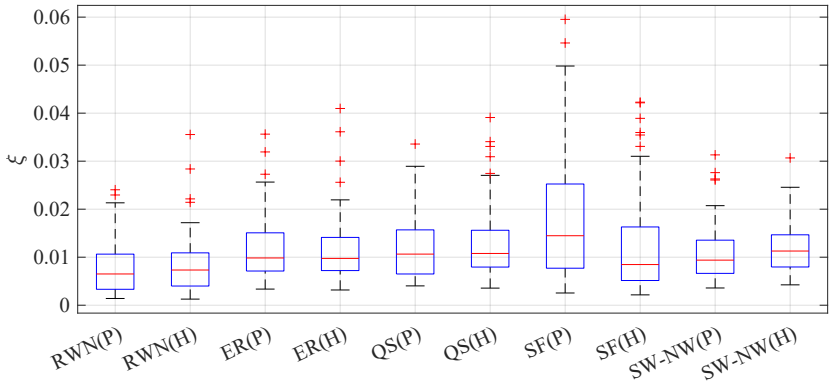

Fig. S1: Boxplot for the prediction errors obtained by GIN-MAS. In the 'pure' mode (P), the training and test datasets are from either pure real-world networks (RWN) or synthetic networks (ER, QS, SF, or SW-NW). In the 'hybrid' mode (H), the training dataset consists of both real-world networks and synthetic networks.

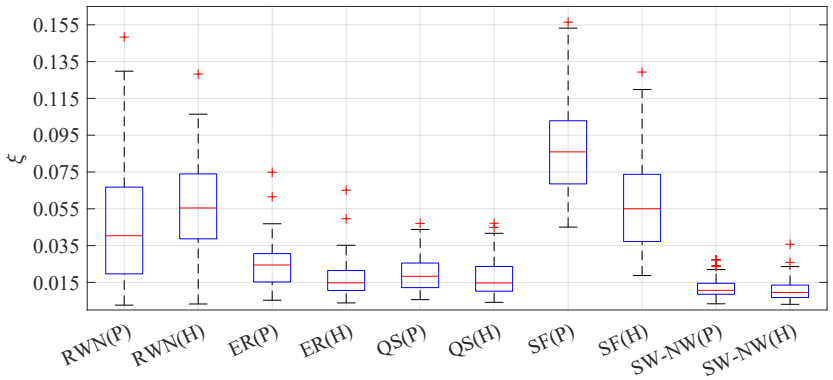

Fig. S2: Boxplot for the prediction errors obtained by LFR. In the 'pure' mode (P), the training and test datasets are from either pure real-world networks (RWN) or synthetic networks (ER, QS, SF, or SW-NW). In the 'hybrid' mode (H), the training dataset consists of both real-world networks and synthetic networks.

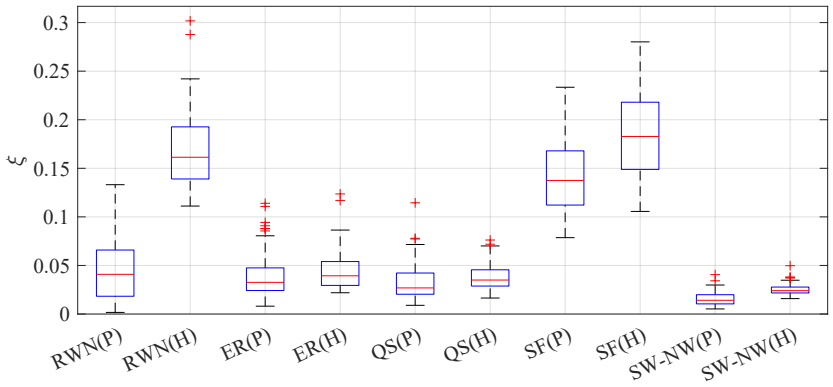

Fig. S3: Boxplot for the prediction errors obtained by RP. In the 'pure' mode (P), the training and test datasets are from either pure real-world networks (RWN) or synthetic networks (ER, QS, SF, or SW-NW). In the 'hybrid' mode (H), the training dataset consists of both real-world networks and synthetic networks..

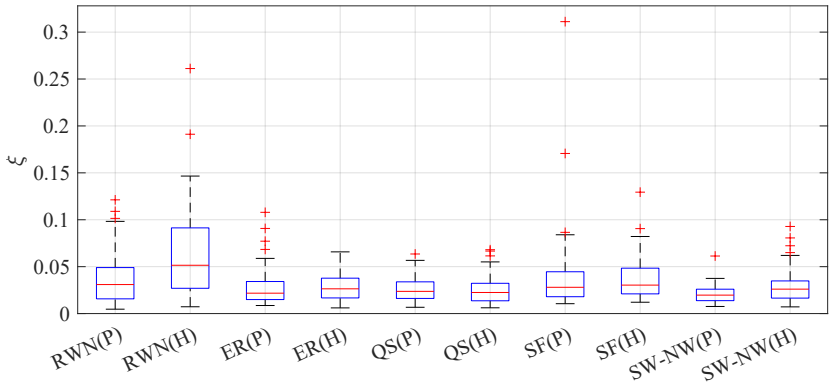

Fig. S4: Boxplot for the prediction errors obtained by SPP. In the 'pure' mode (P), the training and test datasets are from either pure real-world networks (RWN) or synthetic networks (ER, QS, SF, or SW-NW). In the 'hybrid' mode (H), the training dataset consists of both real-world networks and synthetic networks..