Presentation of Fan Yimin

Infomation

This is the presentation section prepared by **Fan Yimin(PB17000047)** for the **Signals and Systems** course in USTC,2019 spring.

Topic

Homework

> 1.21 (d)(e)(f)
> 1.22 (d)(g)(h)

Basic forms of that kind of problems

1.Combination of different signals,especially the combination with **special signals**(*impulse signal,unit step signal,etc*).

> 1.21 (f)

2.Use of various signal operations,especially **shifting,scaling and reversal**.

> 1.21 (d) 1.22(d)(g)

3.Transform on the **time domain**.

> 1.22 (h)

4.Comprehensive problems.

> 1.21(e)
> (combination of signal reversal and properties of $u(t)$)

Basic ways of solving the problem

1. Make use of the properties of the special signals

eg:

$$x(t)\delta(t-t_0)=x(t_0)\delta(t-t_0)$$

$$for \quad t>t_0: \qquad x(t)u(t-t_0)=x(t)$$

$$for \quad t<t_0: \qquad x(t)u(t-t_0)=0$$

2. Make use of the signal operations

eg:

$$shifting, reversal, scaling$$

$$add/minus/times \quad a \quad signal/constant$$

$$\ldots$$

3. Transform on the time domain

eg:

$$t_0 \to x(t_0)$$

To find the figure of

$$x(f(t))$$

Consider

$$f(t^{'}) \to x(t_0) \quad and \quad f(t^{'}) = t_0$$

So we can find that

$$t^{'} = f^{-1}(t_0)$$

So we can show that given values at some points of a signal

$$x(t) : (t_1, x(t_1)),(t_2, x(t_2)), (t_3, x(t_3))...((t_n), x(t_n))$$

Signal

$$x(f(t))$$

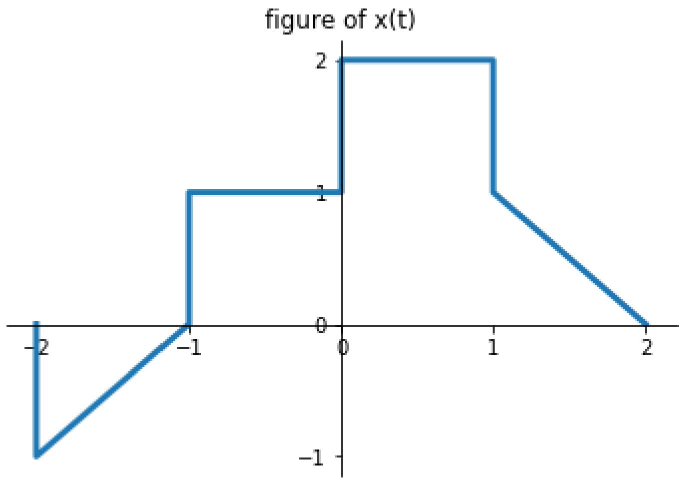
will have these points:

$$(f^{-1}(t_1), x(t_1)),(f^{-1}(t_2), x(t_2)), (f^{-1}(t_3), x(t_3))...((f^{-1}(t_n), x(t_n))$$

This method is useful only when:$f(t)$ is invertible $x(t)$ is continuous time signal However,we can still make use of it even when this condition doen not hold true(eg:1.22(h)).

1.21

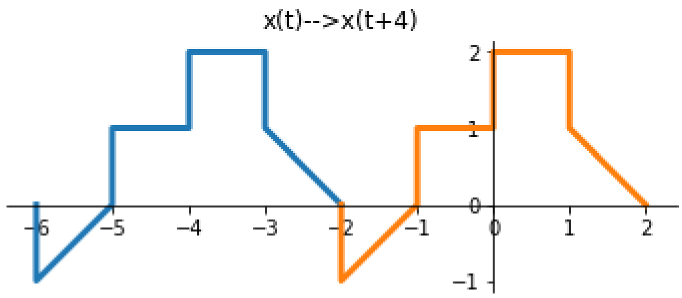
> A continuous time signal x(t)is shown in Figure P1.21.Sketch and label carefully each of the following signals



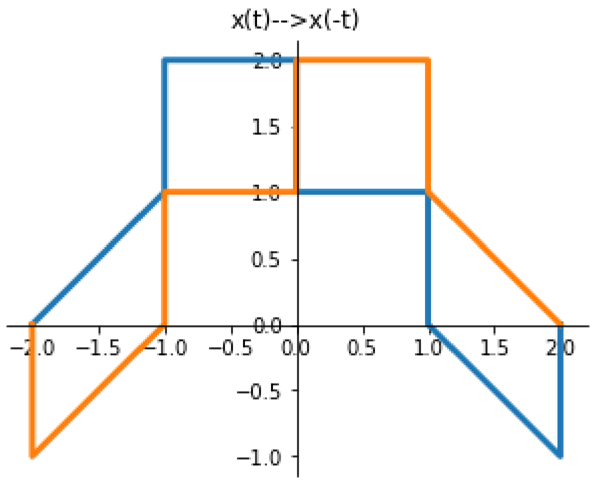
$$(d) \quad x(4 - t/2)$$

1.Using scaling,time reversal,shifting.
Basically,these three oprations will work as follows:

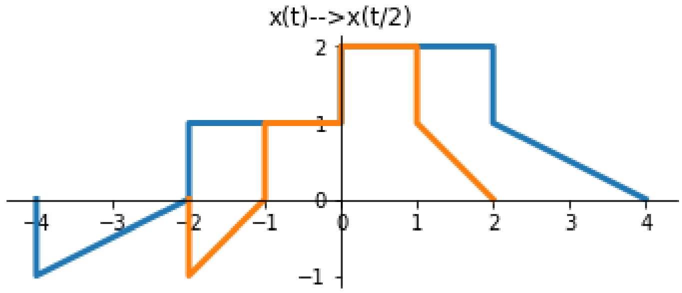
shifting



time reversal



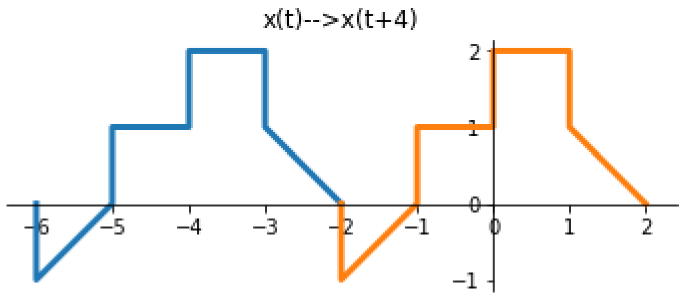
scaling

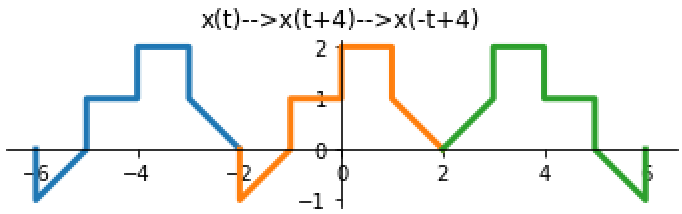


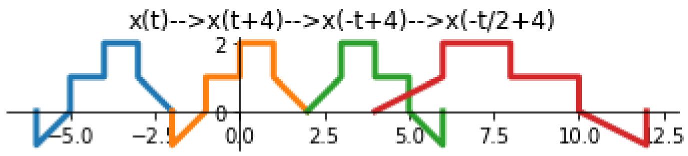
I will not do the 6 orders of these oprations one by one,which is tedious and have a lot in common with the *additional problems*,so I will just show you two of them.

> the yellow signal is the original signal
> the blue signal is the second signal
> the green signal is the third signal
> the red signal is the final signal

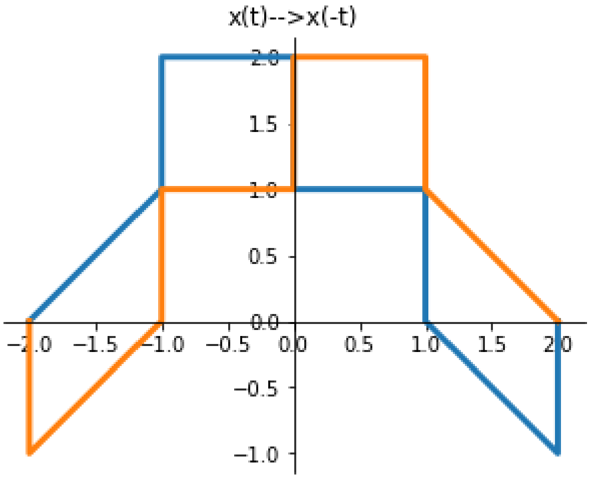
a.*shifting-->reversal-->scaling*

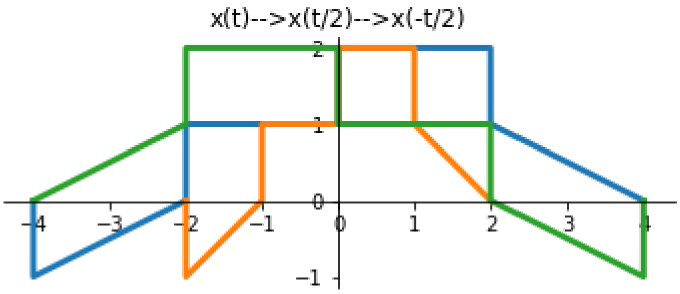


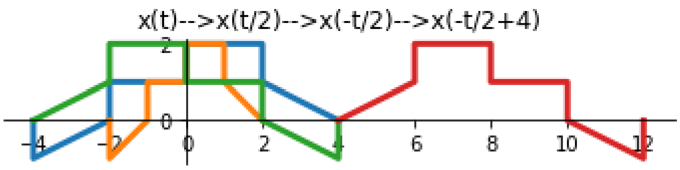




b. *reversal-->scaling-->shifting*







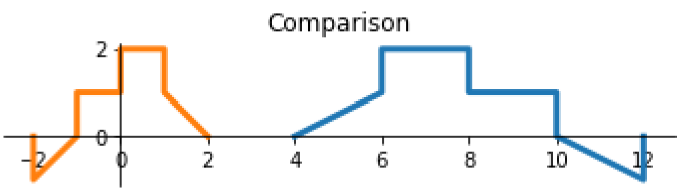
Personally,I think it is especially important to notice the differences between different orders of operations.

eg:

$$x(at+b) \quad a>0$$

If we do the shifting $+b$ first,then we just scale the shifted signal with $*a$,However,if we do the scaling $*a$ first,we must shift the scaled signal with $+(b/a)$,and this point confused me when I first learned it.

And the final result:



2.Using transformation on the time domain: From the method I introduced before,in this problem :

$$f(t)=4-t/2$$

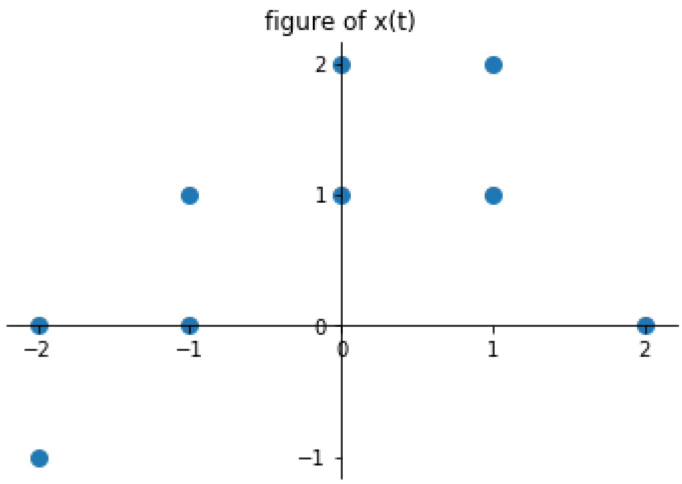
And of course:

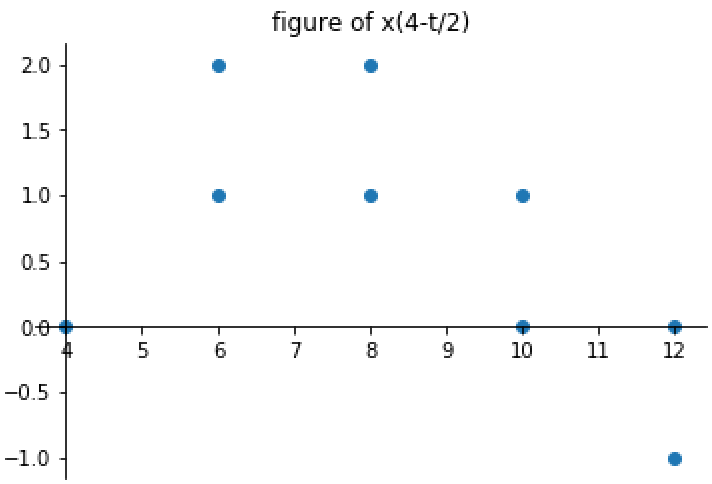
$$f^{-1}(t)=8-2t$$

Some points of x(t) is listed below(*Here I just choose some points that are easy to compute and are the turning points*):

t	$f^{-1}(t)$	$x(t)$
-2	12	0
-2	12	-1
-1	10	0
-1	10	1
0	8	1
0	8	2
1	6	1
1	6	2
2	4	0

Draw the scatter:



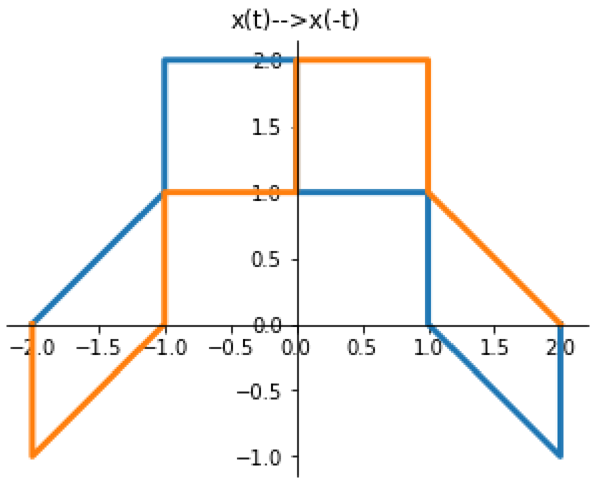


> Note that **mathematically**,this method is **not strict** and will not be helpful in many situations,however,when we do problems in our homework,it is a efficent way to reduce mistakes and help you examine the result of your signal operations.

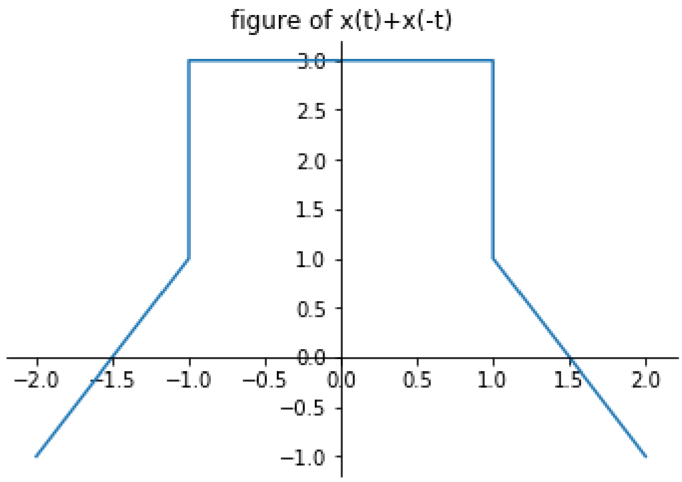
$(e) \quad [x(t) + x(-t)]u(t)$

Basically,this problem can be solved by using the signal operation(*reversal,add a signal*)and the properties of $u(t)$.

(a)find $x(-t)$by using time reversal

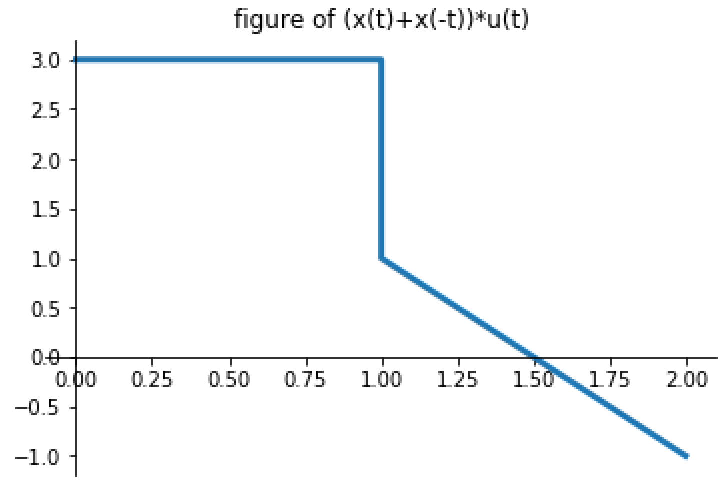


(b)adding $x(t)$ and $x(-t)$



It is obvoius that $x(t) + x(-t)$ *is an even signal for it is equal to the double of the even part of x(t)*

(c)times the signal $u(t)$,in fact it is cutting off the left part of $x(t) + x(-t)$



$(f) \quad x(t)(\delta(t+3/2)-\delta(t-3/2))$

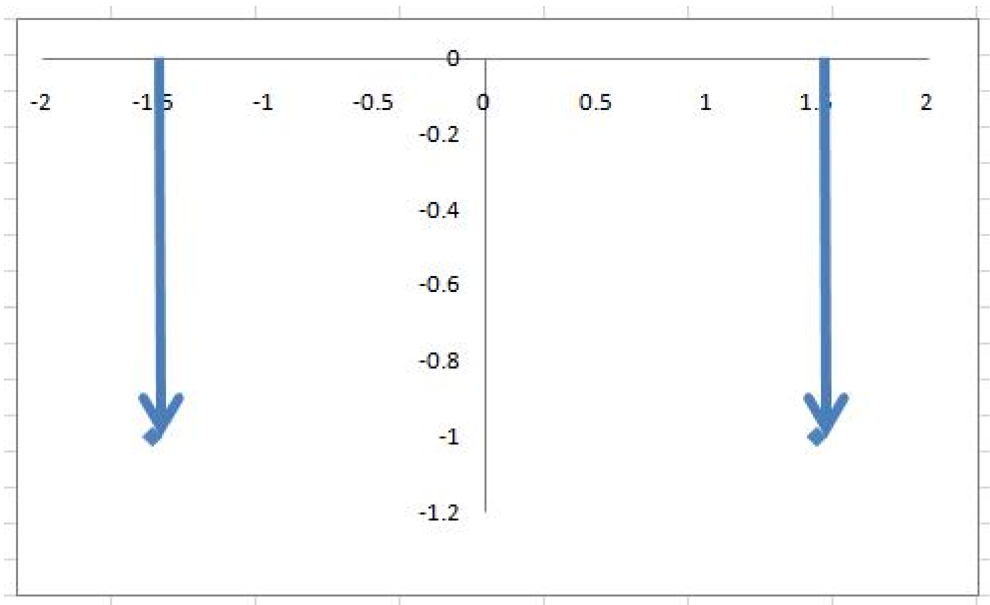
As we know :

$$x(t)\delta(t-t_0)=x(t_0)\delta(t-t_0)$$

So:

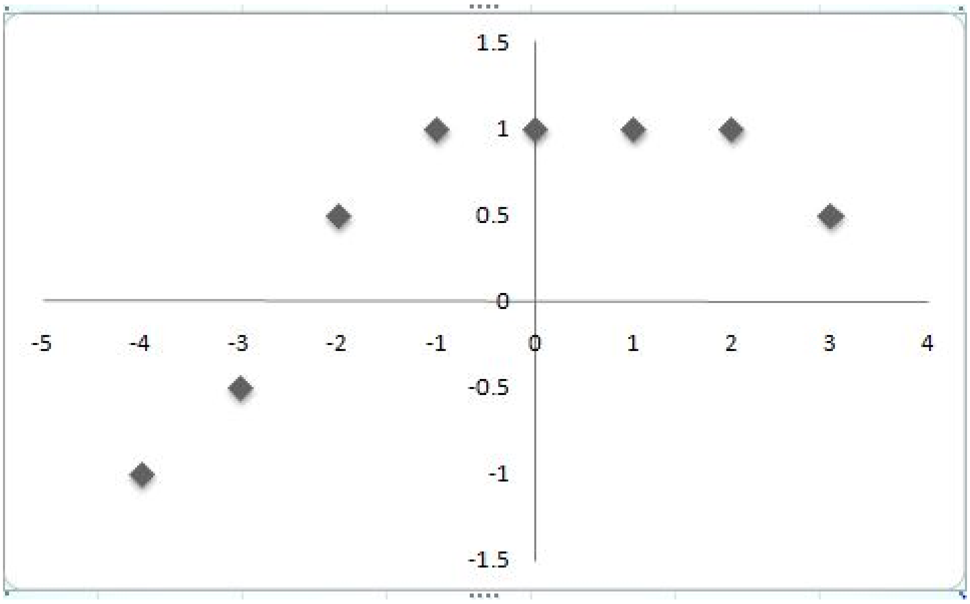
$$\begin{gathered}x(t)(\delta(t+3/2)-\delta(t-3/2))\\ =x(-3/2)\delta(t+3/2)-x(3/2)\delta(t-3/2))\\ =0 \quad when \quad t>3/2\\ =-x(3/2)\delta(0) \quad when \quad t=3/2\\ =0 \quad when \quad t>-3/2 \quad and \quad t<3/2\\ =x(-3/2)\delta(0) \quad when \quad t=-3/2\\ =0 \quad when \quad t<-3/2\end{gathered}$$

So the figure is shown as below



1.22

> A discrete-time signal is shown in Figure P1.22.Sketch and lable carefully each of the following signals.



$$(d) \quad x[3n+1]$$

In fact,by using the same signal operations,such as ***scaling,shifting and reversal***,we can process this discrete time signal as well.However,there may be some problems when you do these kind of this operations,because $x[3n+1]$ will only take values at integer time,which may cause a little confusion.And if we use the transformation,there will also be some problems because $f^{-1}(n)$ may not be integer. So I will show you another way of doing it.
Firstly,find the time domain when $x[n]$ is non-zero:

$$-4 < n < 3$$

Secondly,replace n with $f(n)$ and solve that inequation:

$$-4 < 3n+1 < 3$$

$$-5/3 < n < 2/3$$

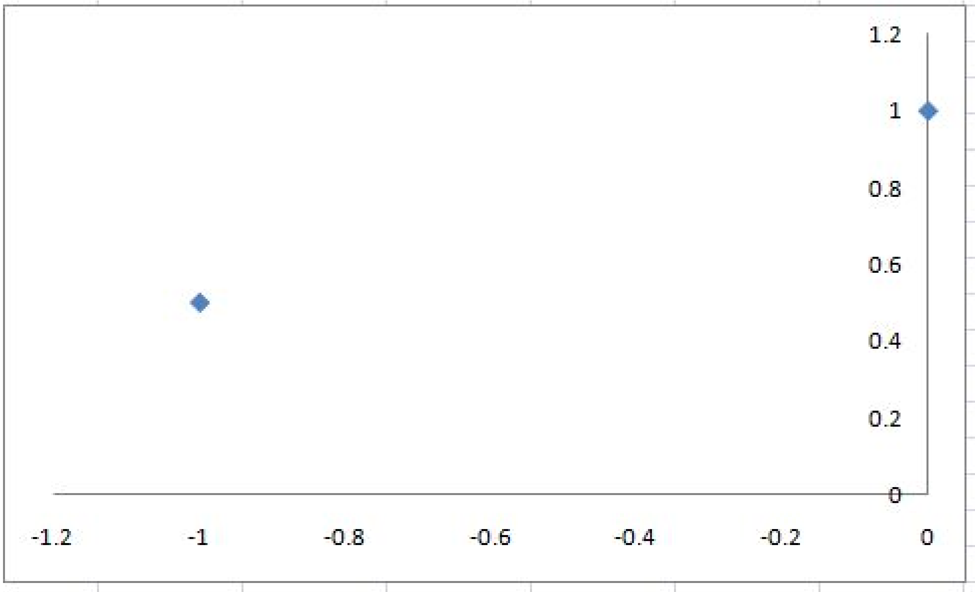
$$n = 0, -1$$

Thirdly,note that the soultion to that inequality is actuall is the non-zero time domain of $x[3n+1]$,so just compute the points:

$$n = 0, x[3n+1] = x[1] = 1$$

$$n = -1, x[3n+1] = x[-2] = 0.5$$

the figure is shown as below:



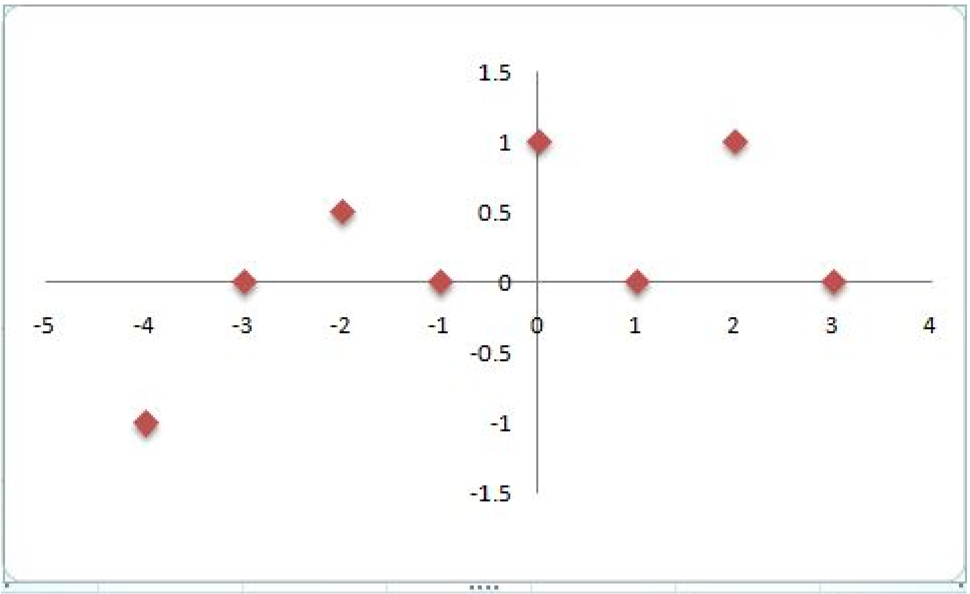
from that problem we can conclude a quick wan to draw the figure of $x[an+b]$ in general.

$(g)\frac{1}{2}x[n]+\frac{1}{2}(-1)^n x[n]$

We know that the figure depends on the value of n, for $(-1)^n$ is one when n is even and is minus one when n is odd.

$$\begin{aligned} &\frac{1}{2}x[n]+\frac{1}{2}(-1)^n x[n] \\ =&\frac{1}{2}x[n]+\frac{1}{2}x[n]=x[n] \quad n \quad is \quad even \\ =&\frac{1}{2}x[n]-\frac{1}{2}x[n]=0 \quad n \quad is \quad odd \end{aligned}$$

So this signal takes $x[n]$ when n is even and takes zero when n is odd.
The figure is shown as below:



$(h)x[(n-1)^2]$

Basically,we will use the same method as (d).And this method is especially useful for (h) because $(n-1)^2$ is not linear function so we can not use time shifting,reversal and scaling,and $(n-1)^2$ is not invertible,so we can not use the transformation on the time domain.
Firstly,find the time domain when $x[n]$ is non-zero:

$$-4 < n < 3$$

Secondly,replace n with $f(n)$ and solve that inequation:

$$-4 < (n-1)^2 < 3$$

$$-\sqrt{3} < n-1 < \sqrt{3}$$

$$-\sqrt{3}+1 < n < \sqrt{3}+1$$

$$n = 0,1,2$$

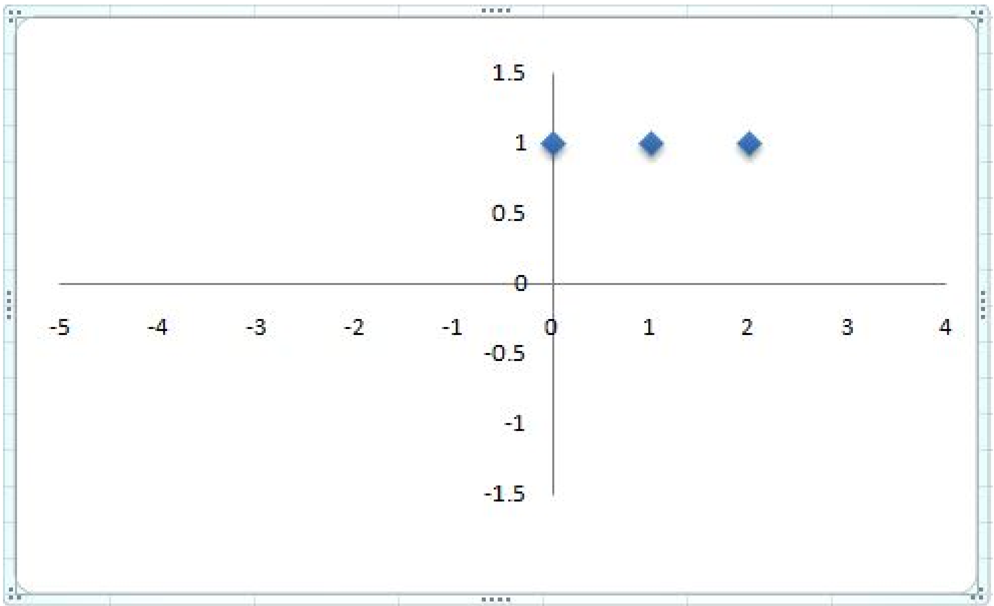
Thirdly,note that the soultion to that inequality is actuall is the non-zero time domain of $x[(n-1)^2]$,so just compute the points:

$$n=0, x_1[n] = x[(n-1)^2] = x[1] = 1$$

$$n=1, x_1[n] = x[(n-1)^2] = x[0] = 1$$

$$n=2, x_1[n] = x[(n-1)^2] = x[1] = 1$$

the figure is shown as below:



I have not thought of another way of solving that problem,if you have one,please tell me.

Sicere thanks for Prof. Li and Prof.Chen for their teaching and devotion!