Presentation of Fan Yimin

Infomation

This is the presentation section prepared by **Fan Yimin(PB17000047)** for the **Signals and Systems** course in USTC,2019 spring.

Topic

Homework

> 1.21 (d)(e)(f)
>
> 1.22 (d)(g)(h)

Basic forms of that kind of problems

1.Combination of different signals,especially the combination with **special signals**(*impulse signal,unit step signal,etc*).

> 1.21 (f)

2.Use of various signal operations,especially **shifting,scaling and reversal**.

> 1.21 (d) 1.22(d)(g)

3.Transform on the **time domain**.

> 1.22 (h)

4.Comprehensive problems.

> 1.21(e)
> (combination of signal reversal and properties of $u(t)$)

Basic ways of solving the problem

1. Make use of the properties of the special signals

eg:

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

$$for \quad t > t_0: \qquad x(t)u(t-t_0) = x(t)$$

$$for \quad t < t_0: \qquad x(t)u(t-t_0) = 0$$

2. Make use of the signal operations

eg:

$$shifting, reversal, scaling$$

$$add/minus/times \quad a \quad signal/constant$$

$$\dots$$

3. Transform on the time domain

eg:

$$t_0 \to x(t_0)$$

To find the figure of

$$x(f(t))$$

Consider

$$f(t^{'}) \rightarrow x(t_0) \quad and \quad f(t^{'}) = t_0$$

So we can find that

$$t^{'} = f^{-1}(t_0)$$

So we can show that given values at some points of a signal

$$x(t) : (t_1, x(t_1)),(t_2, x(t_2)), (t_3, x(t_3))...((t_n), x(t_n))$$

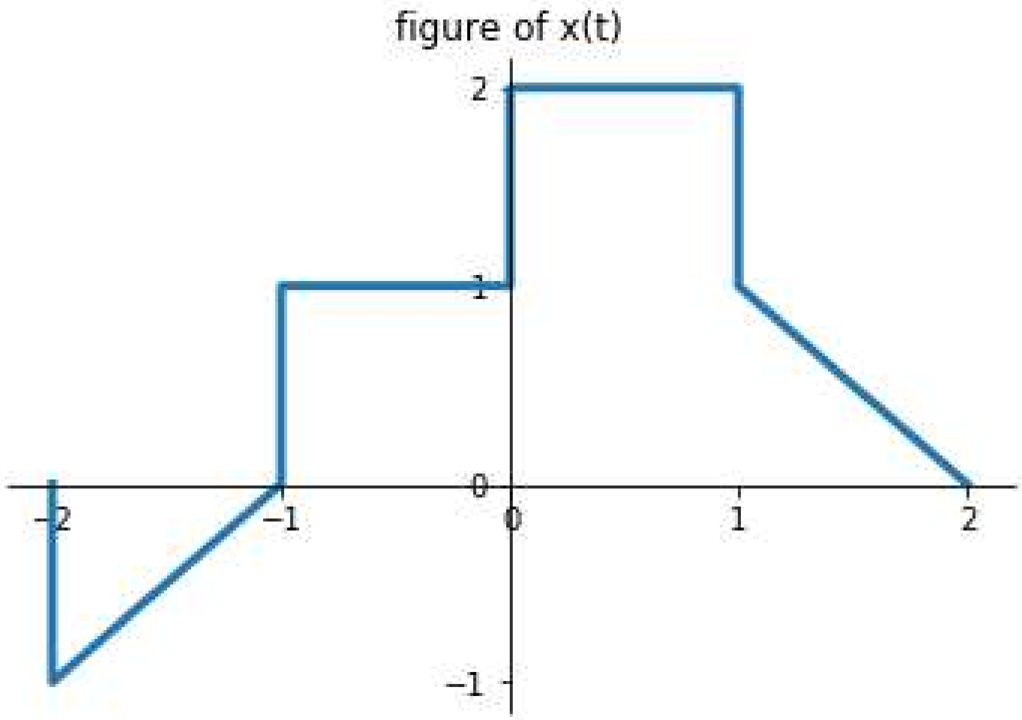
Signal $x(f(t))$ will have these points:

$$(f^{-1}(t_1), x(t_1)),(f^{-1}(t_2), x(t_2)), (f^{-1}(t_3), x(t_3))...((f^{-1}(t_n), x(t_n))$$

Condition :$f(t)$ **is invertible** $x(t)$ **is continuous time signal**

1.21

> A continuous time signal $x(t)$is shown in Figure P1.21.Sketch and label carefully each of the following signals

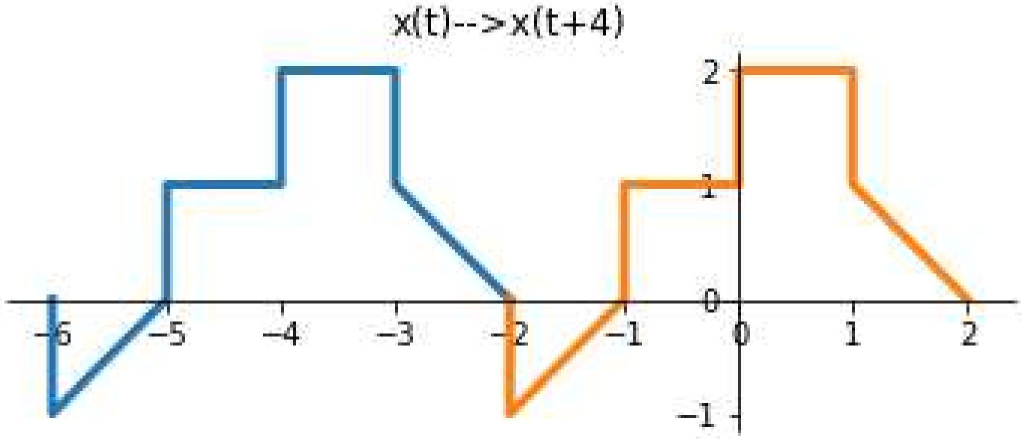


$$(d) \quad x(4-t/2)$$

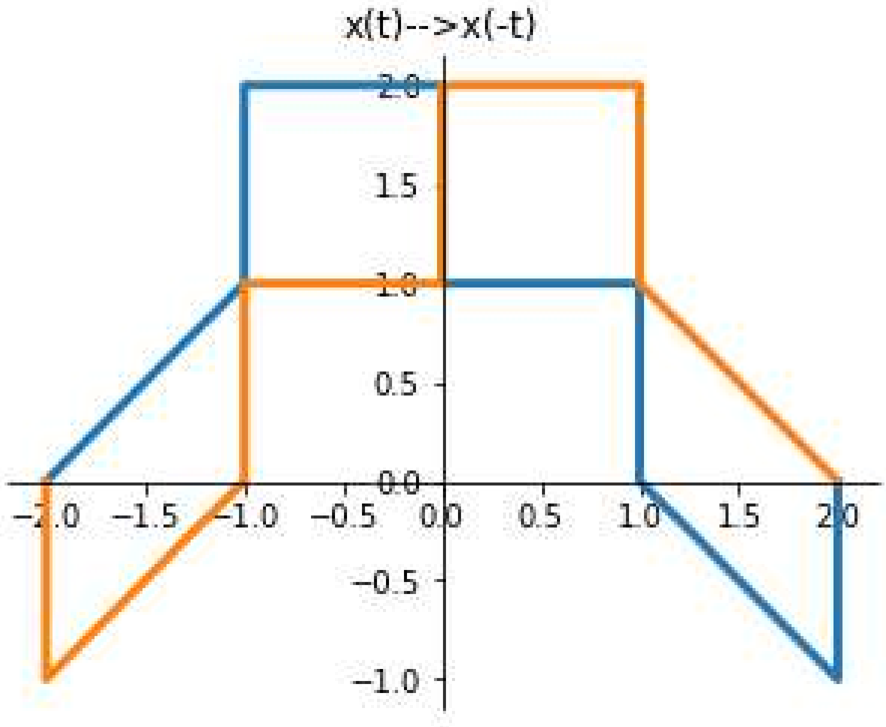
1.Using scaling,time reversal,shifting.

Basically,these three oprations will work as follows:

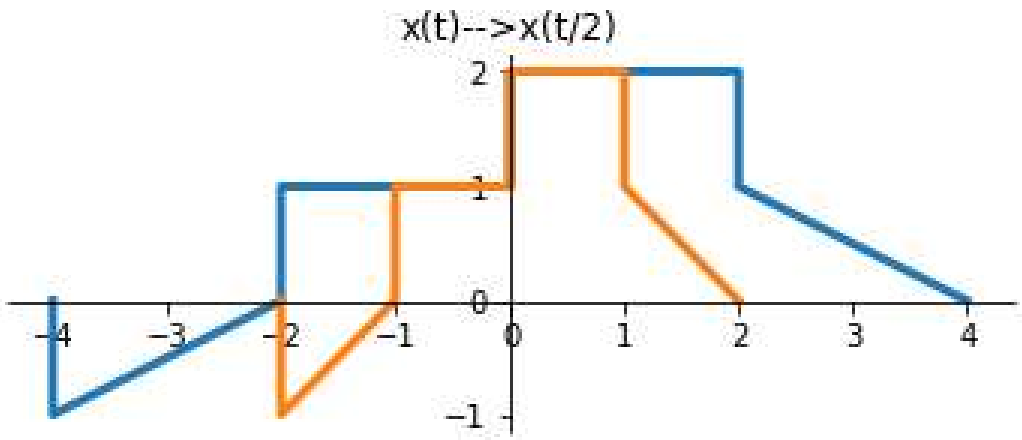
> shifting



time reversal



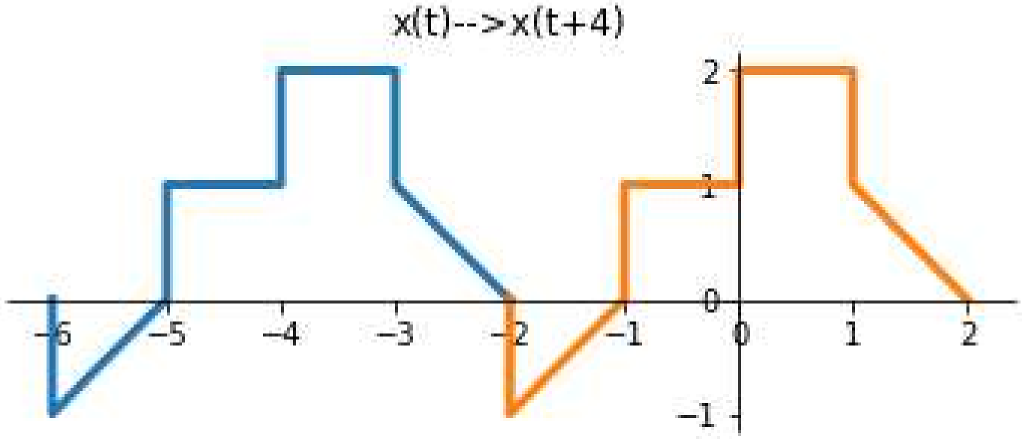
scaling

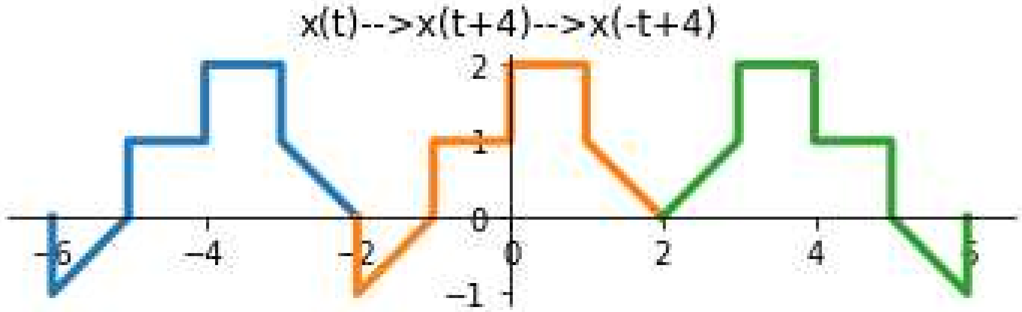


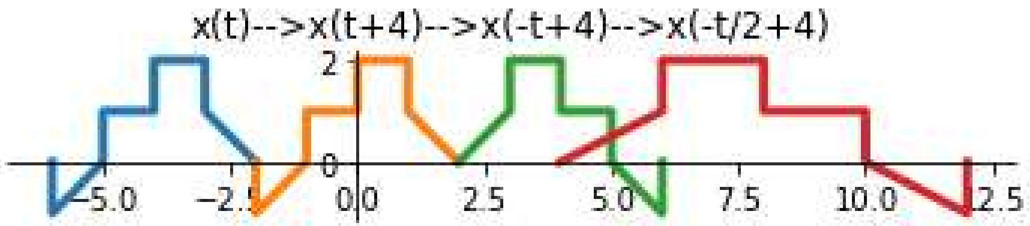
Two examples:

> the yellow signal is the original signal
> the blue signal is the second signal
> the green signal is the third signal
> the red signal is the final signal

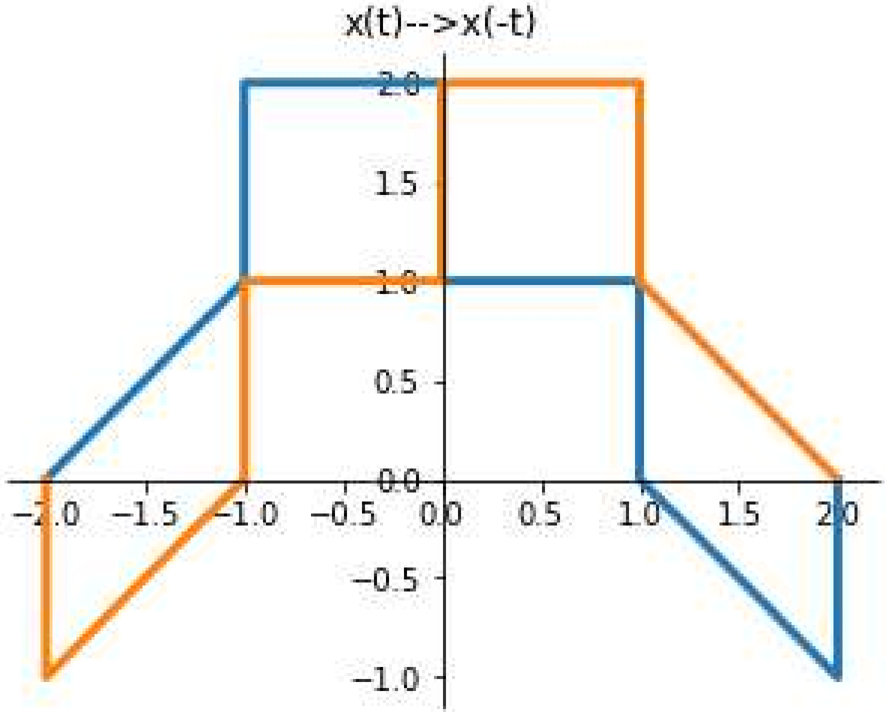
a.*shifting-->reversal-->scaling*

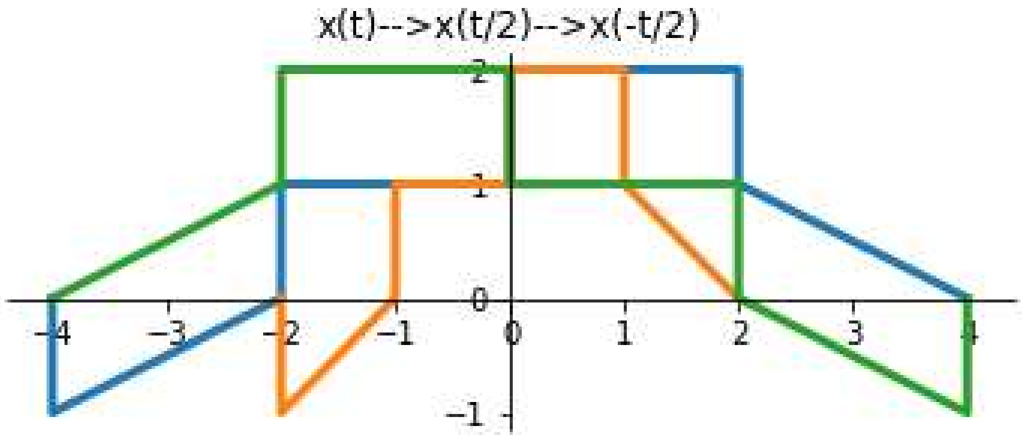


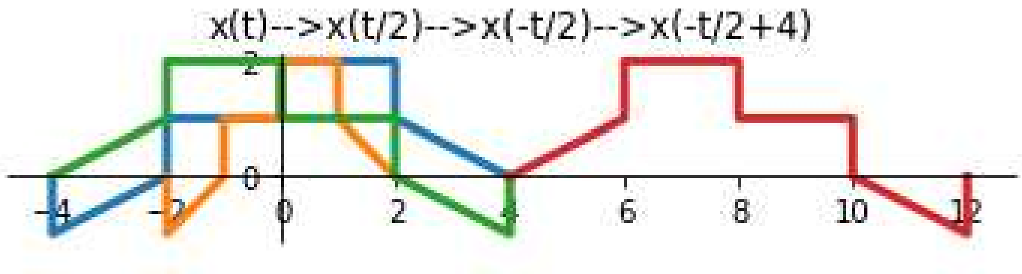




b. *reversal-->scaling-->shifting*


x(t)-->x(-t)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0


x(t)-->x(t/2)-->x(-t/2)
2
1
0
−1
−4 −3 −2 −1 0 1 2 3 4



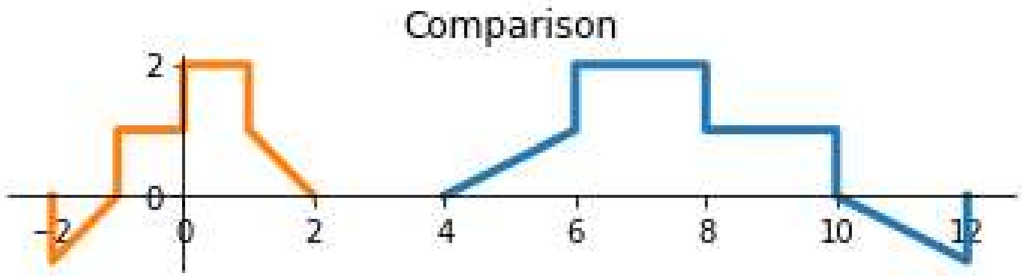
Personally,I think it is especially important to notice the differences between different orders of operations.

eg:

$$x(at+b) \quad a>0$$

Shifting first: $+b \rightarrow *a$
Scaling first: $*a \rightarrow +\frac{b}{a}$.

And the final result:



2.Using transformation on the time domain: From the method I introduced before,in this problem：

$$f(t) = 4 - \frac{t}{2}$$

And of course:

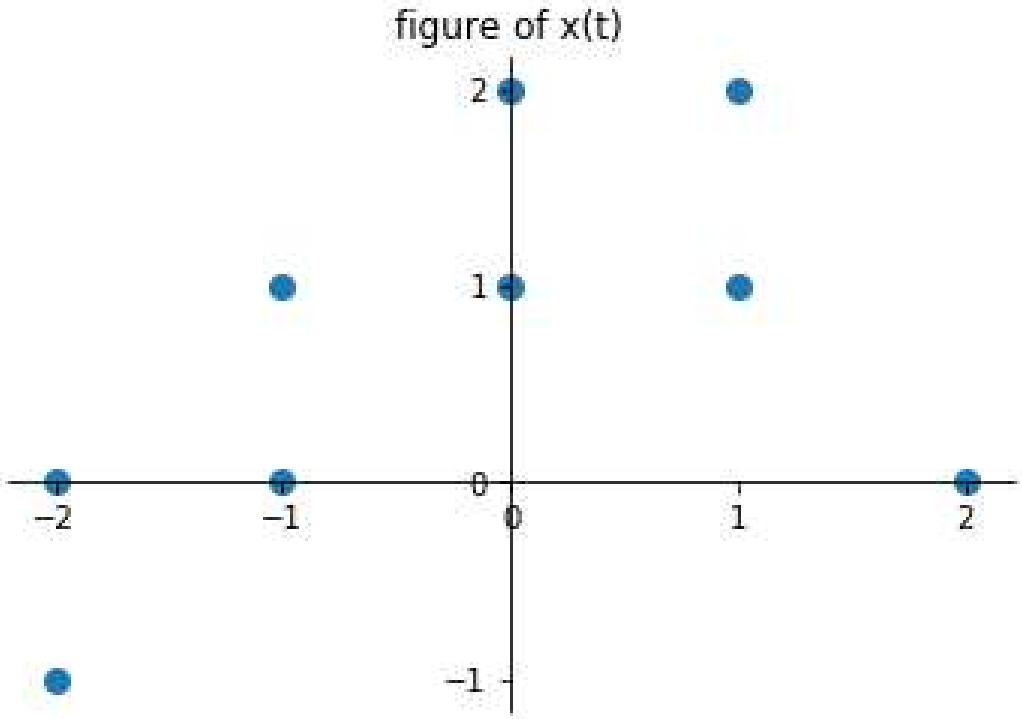
$$f^{-1}(t) = 8 - 2t$$

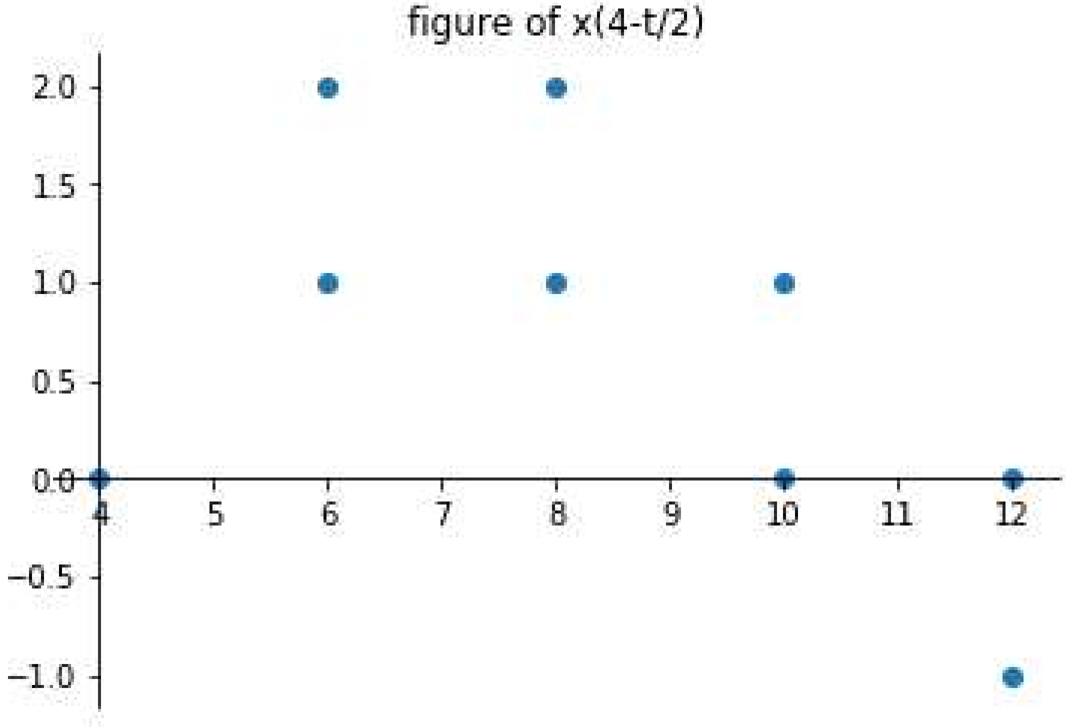
Some points of x(t) is listed below(*Here I just choose some points that are easy to compute and are the turning points*):

t	$f^{-1}(t)$	$x(t)$
-2	12	0
-2	12	-1
-1	10	0
-1	10	1
0	8	1
0	8	2
1	6	1
1	6	2

t	$f^{-1}(t)$	$x(t)$
2	4	0

Draw the scatter:

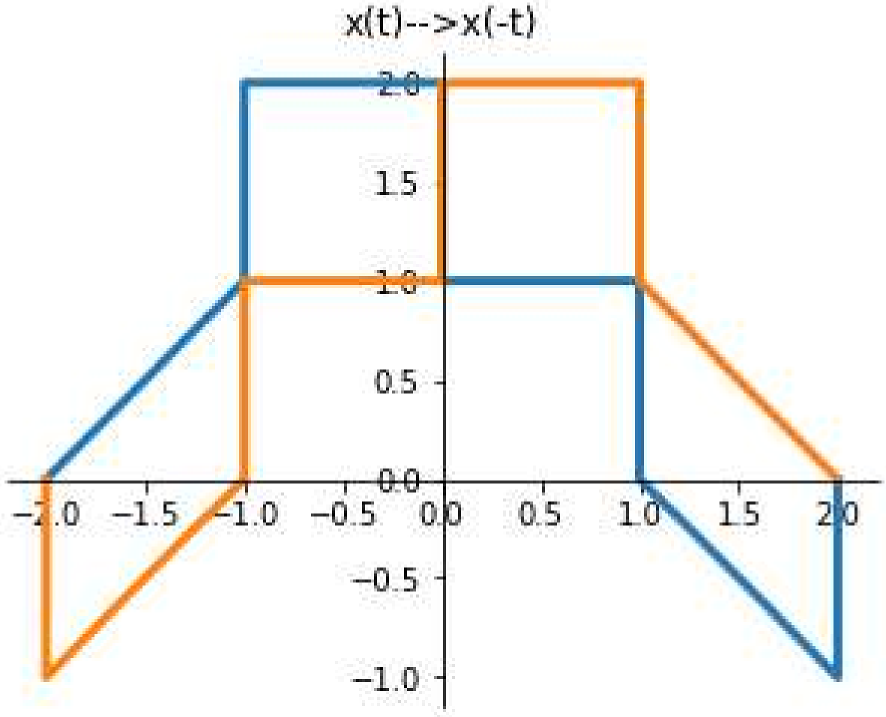




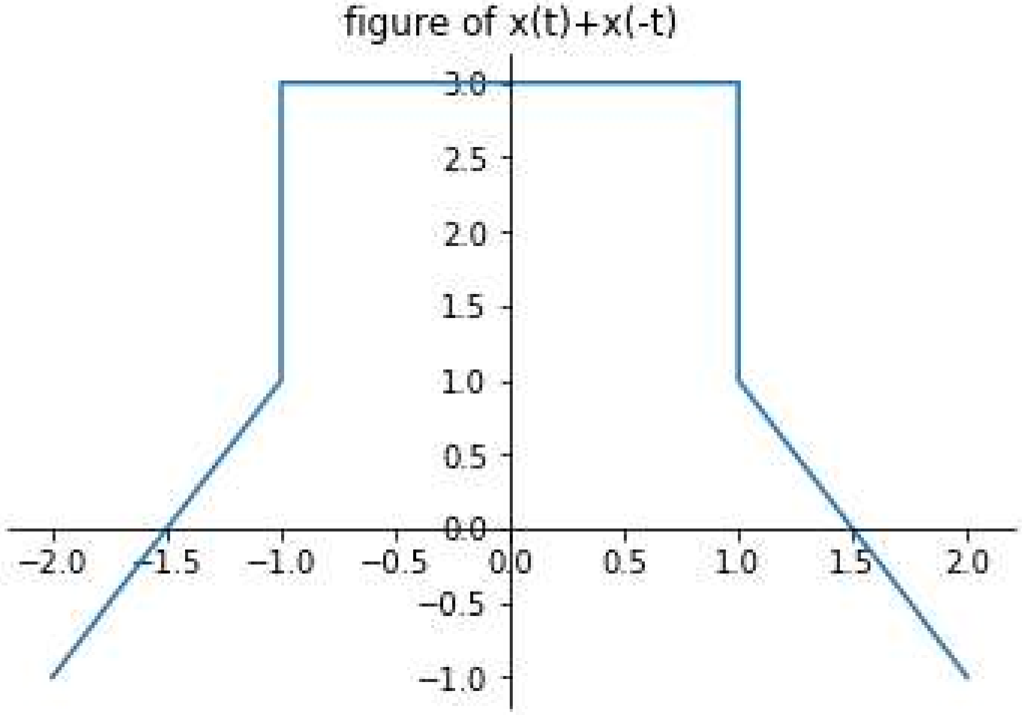
> Note that **mathematically**,this method is **not strict**
> but it will be **helpful** in many situations

$$(e) \quad [x(t)+x(-t)]u(t)$$

Basically,this problem can be solved by using the signal operation(*reversal,add a signal*)and the properties of $u(t)$.
(a)find $x(-t)$by using time reversal

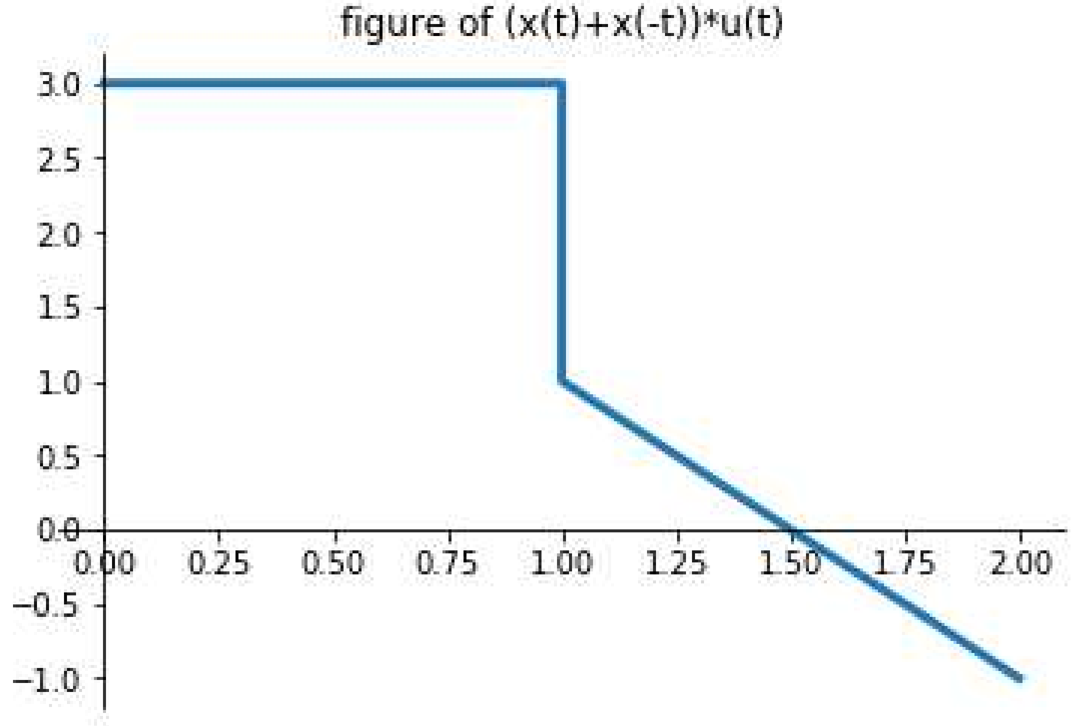


(b)adding $x(t)$ and $x(-t)$



$$x(t) + x(-t) = 2Even\{x(t)\}$$

(c)times the signal $u(t)$.
in fact it is cutting off **the left part** of $x(t) + x(-t)$



$$(f) \quad x(t)(\delta(t+3/2)-\delta(t-3/2))$$

As we know :

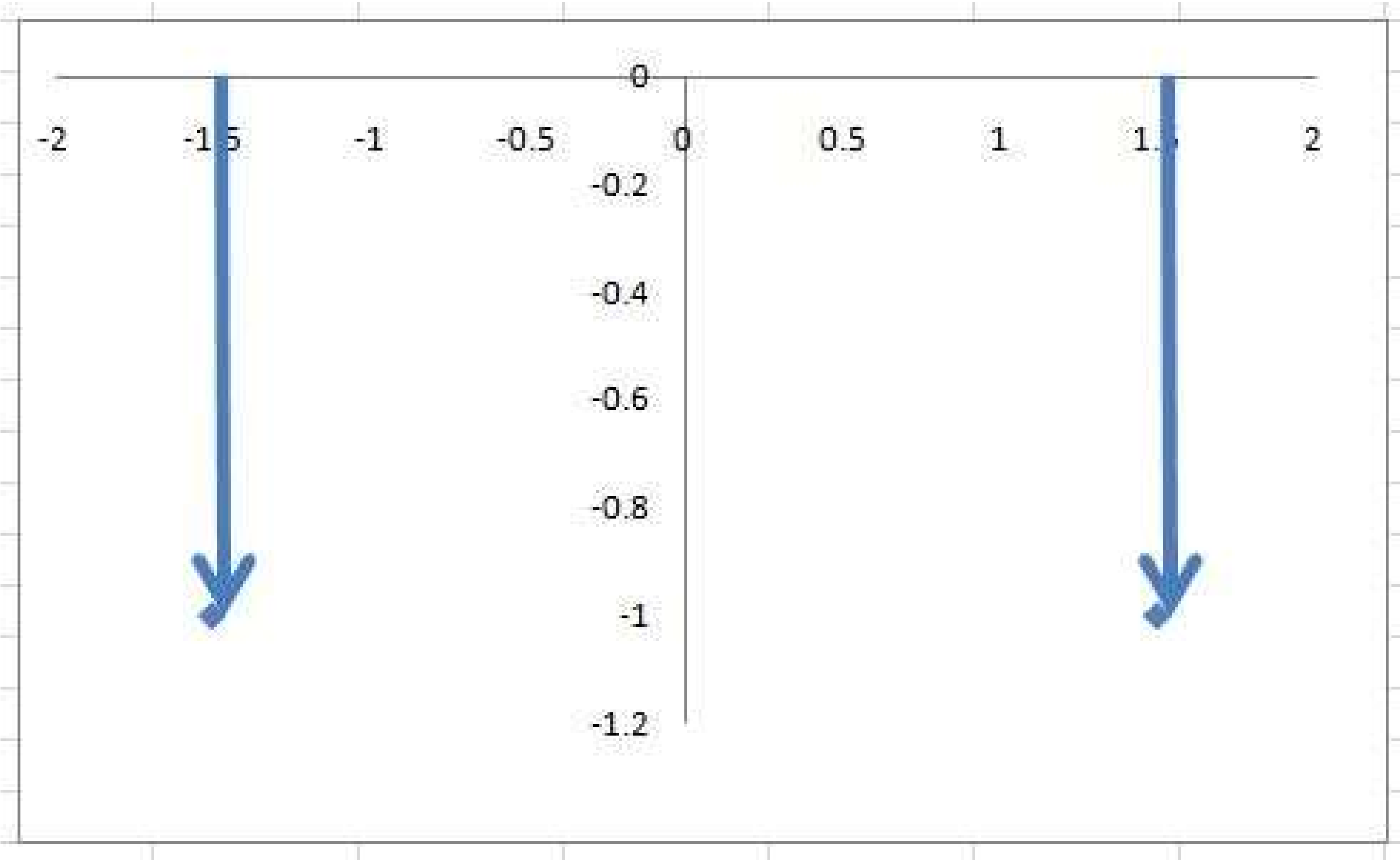
$$x(t)\delta(t-t_0)=x(t_0)\delta(t-t_0)$$

So:

$$x(t)(\delta(t+3/2)-\delta(t-3/2))$$

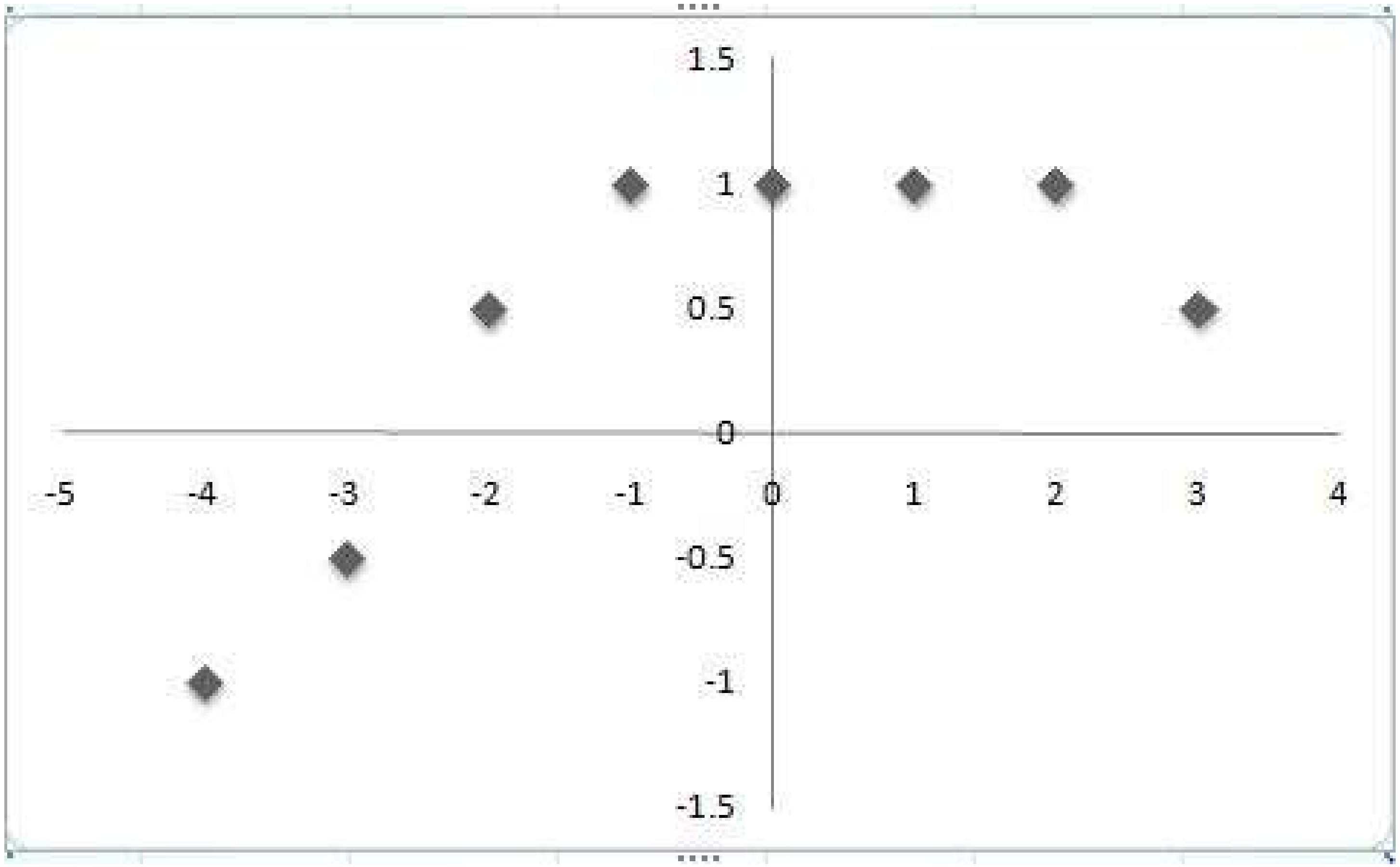
$$= x(-3/2)\delta(t+3/2) - x(3/2)\delta(t-3/2))$$
$$= -x(3/2)\delta(0) \quad when \quad t = 3/2$$
$$= +x(-3/2)\delta(0) \quad when \quad t = -3/2$$
$$(= 0 \quad otherwise)$$

So the figure is shown as below



1.22

> A discrete-time signal is shown in Figure P1.22.Sketch and lable carefully each of the following signals.



$$(d) \quad x[3n+1]$$

scaling,shifting and reversal$\rightarrow$ ***OK!***

transformation on the time domain$\rightarrow$ ***may cause problems***

Another way of doing it!

Firstly,find the time domain when $x[n]$ is non-zero:

$$-4 < n < 3$$

Secondly,replace n with $f(n)$ and solve that inequation:

$$-4 < 3n + 1 < 3$$

$$-5/3 < n < 2/3$$

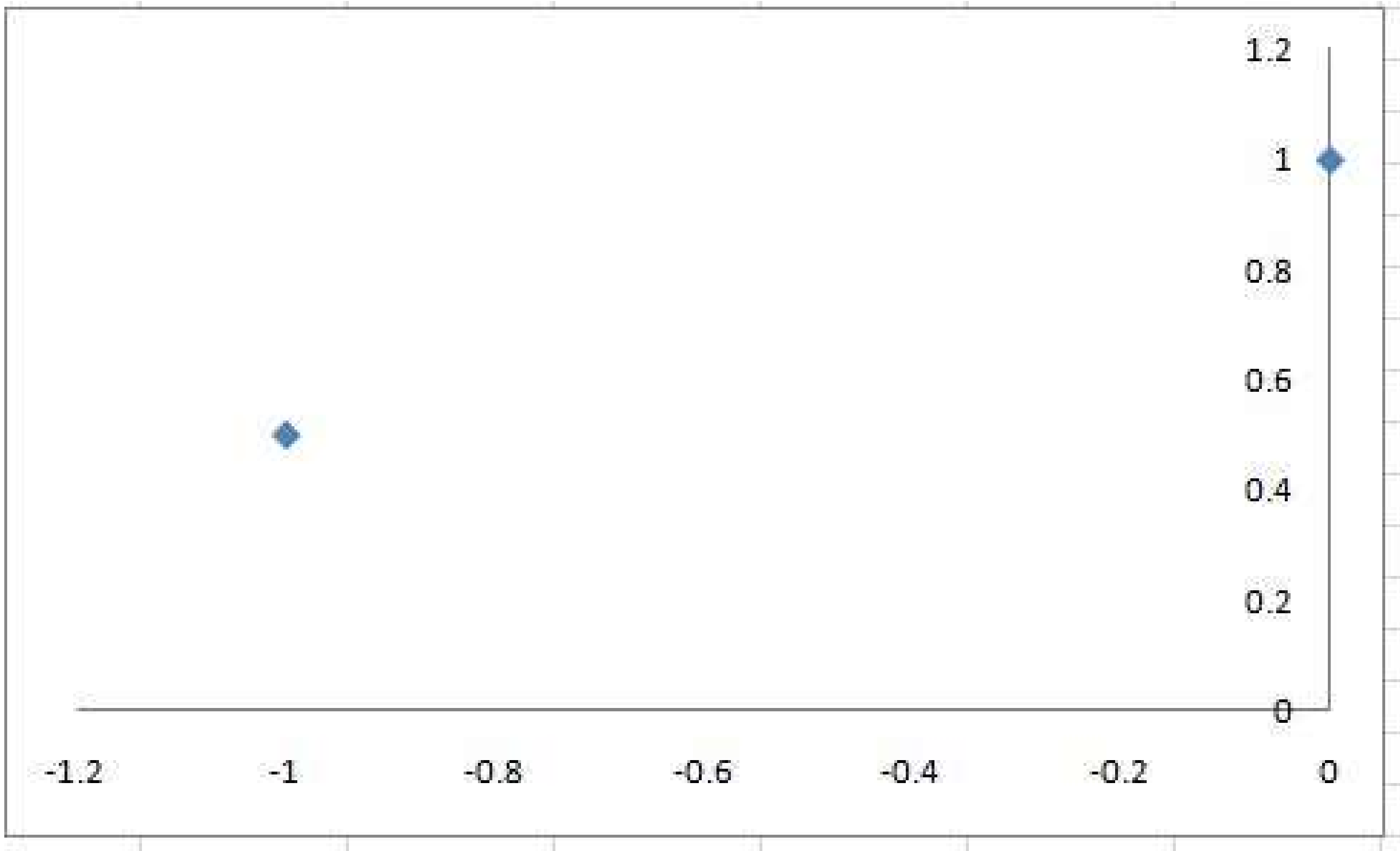
$$n = 0, -1$$

Thirdly,note that the soultion to that inequality is actuall is the non-zero time domain of $x[3n+1]$,so just compute the points:

$$n = 0, x[3n+1] = x[1] = 1$$

$$n = -1, x[3n+1] = x[-2] = 0.5$$

the figure is shown as below:



from that problem we can conclude a quick wan to draw the figure of $x[an+b]$ in general.

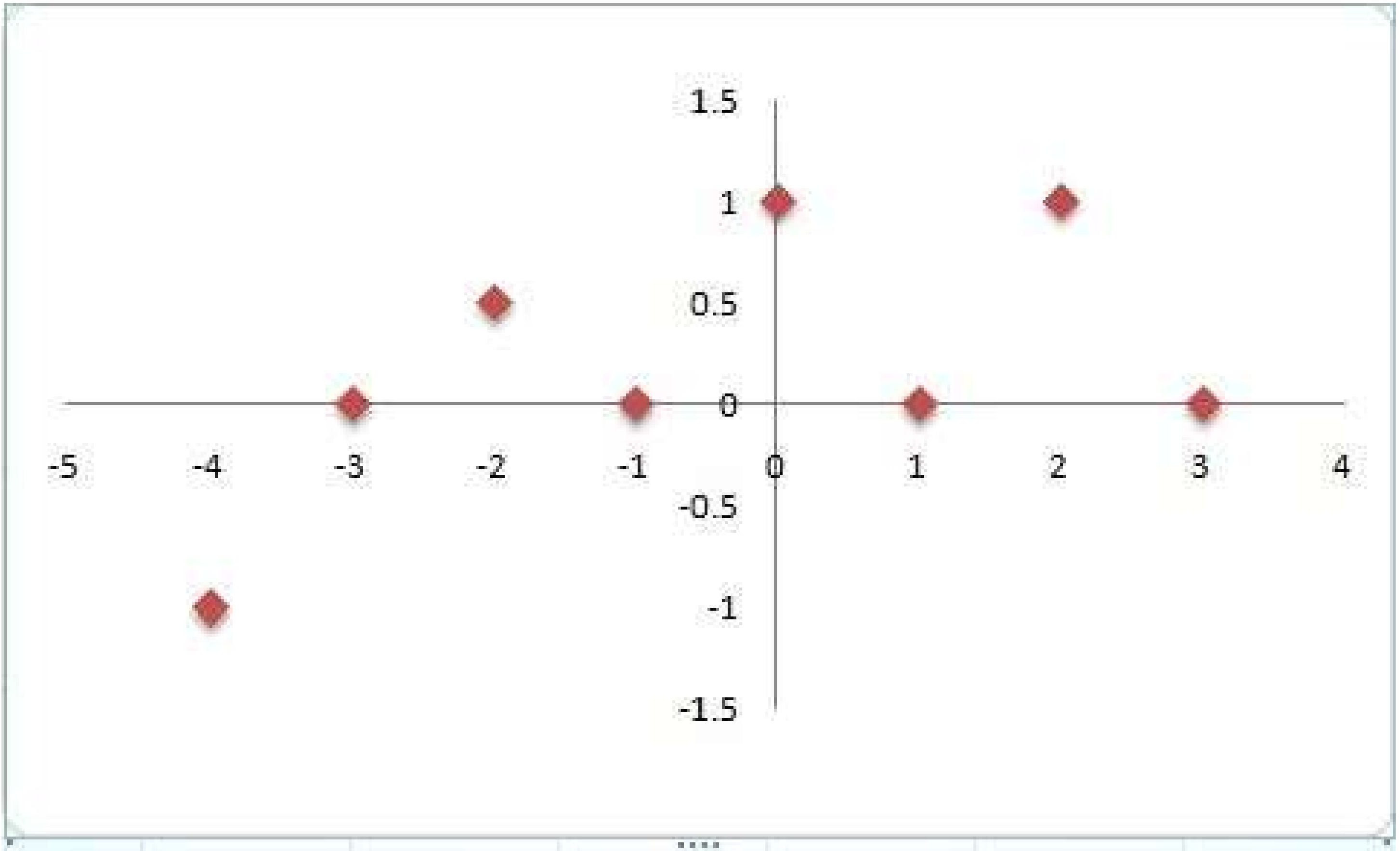
$$(g)\frac{1}{2}x[n]+\frac{1}{2}(-1)^n x[n]$$

be sure to ***consider the sign*** of $(-1)^n$

$$\begin{aligned}&\frac{1}{2}x[n]+\frac{1}{2}(-1)^n x[n]\\&=\frac{1}{2}x[n]+\frac{1}{2}x[n]=x[n]\quad n\quad is\quad even\\&=\frac{1}{2}x[n]-\frac{1}{2}x[n]=0\quad n\quad is\quad odd\end{aligned}$$

So this signal takes $x[n]$ when n is even and takes zero when n is odd.

The figure is shown as below:



$(h)x[(n-1)^2]$

The same method as (d).

Useful

a.$(n-1)^2$ is not linear function $\rightarrow$ can not use time shifting,reversal and scaling

b. $(n-1)^2$ is not invertible$\rightarrow$can not use the transformation on the time domain.

Firstly,find the time domain when $x[n]$ is non-zero:

$$-4 < n < 3$$

Secondly,replace n with $f(n)$ and solve that inequation:

$$-4 < (n-1)^2 < 3$$

$$-\sqrt{3} < n-1 < \sqrt{3}$$

$$-\sqrt{3}+1 < n < \sqrt{3}+1$$

$$n = 0, 1, 2$$

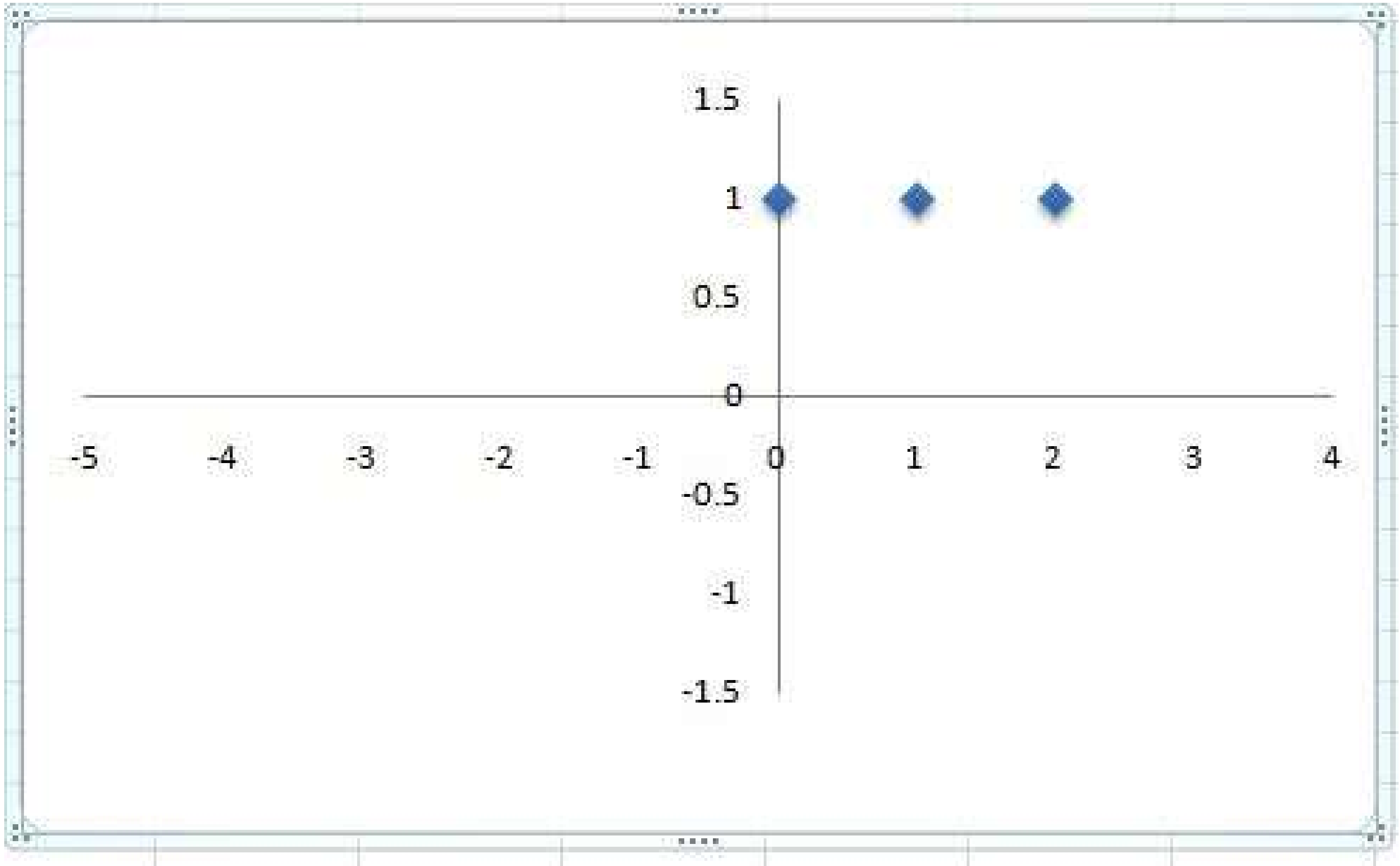
Thirdly,note that the soultion to that inequality is actuall is the non-zero time domain of $x[(n-1)^2]$,so just compute the points:

$$n = 0, x_1[n] = x[(n-1)^2] = x[1] = 1$$

$$n = 1, x_1[n] = x[(n-1)^2] = x[0] = 1$$

$$n = 2, x_1[n] = x[(n-1)^2] = x[1] = 1$$

the figure is shown as below:



I have not thought of another way of solving that problem,if you have one,please tell me.

Sincere thanks for Prof. Li and Prof.Chen for their teaching and devotion!