Logistic Regression

CS 4650 Section B "Natural Language Processing" Project 0
Georgia Tech, Fall 2024 (Instructor: Weicheng Ma)

In this assignment, we will walk you through the process of implementing logistic regression from scratch. You will also apply your implemented logistic regression model to a small dataset and predict whether a student will be admitted to a university. This dataset will allow you to visualize the data and debug more easily. You may find this documentation very helpful (although it uses Octave rather than Python).

This assignment also serves as a programming preparation test. We will use NumPy -- a popular Python package for scientific computing and implementing machine learning algorithms. It provides very good support for matrix and vector operations. You need to feel comfortable working with matrics, vectors, and tensors in order to complete all the programming projects in CS 4650.

To start, first make a copy of this notebook to your local drive, so you can edit it.

⚠ External Packages

> In this assignment, except NumPy and Matplotlib, no other external Python packages are allowed. You may use SciPy's `optimize` for gradient checking, but it is not allowed elsewhere.

0. Honor Code [1 points]

Honor Code: I hereby agree to abide the Georgia Tech's Academic Honor Code, promise that the submitted assignment is my own work, and understand that my code is subject to plagiarism detection.

Signature: *(double click on this block and type your name here)*

1. Setup - Importing Numpy and Matplotlib [Code provided - do not change]

Here we import Matplotlib for data visualization and NumPy for matrix operations.

```
# ======================================================================
# Run some setup code for this notebook. Don't modify anything in this cell.
# ======================================================================
```

```
=====

import sys
import numpy as np
import matplotlib.pyplot as plt

# This is a bit of magic to make matplotlib figures appear inline in
the notebook
# rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of
plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Reload external python modules
# http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-
ipython
%load_ext autoreload
%autoreload 2

# Check what version of Python is running
print(sys.version)
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
3.12.0 (tags/v3.12.0:0fb18b0, Oct  2 2023, 13:03:39) [MSC v.1935 64
bit (AMD64)]
```

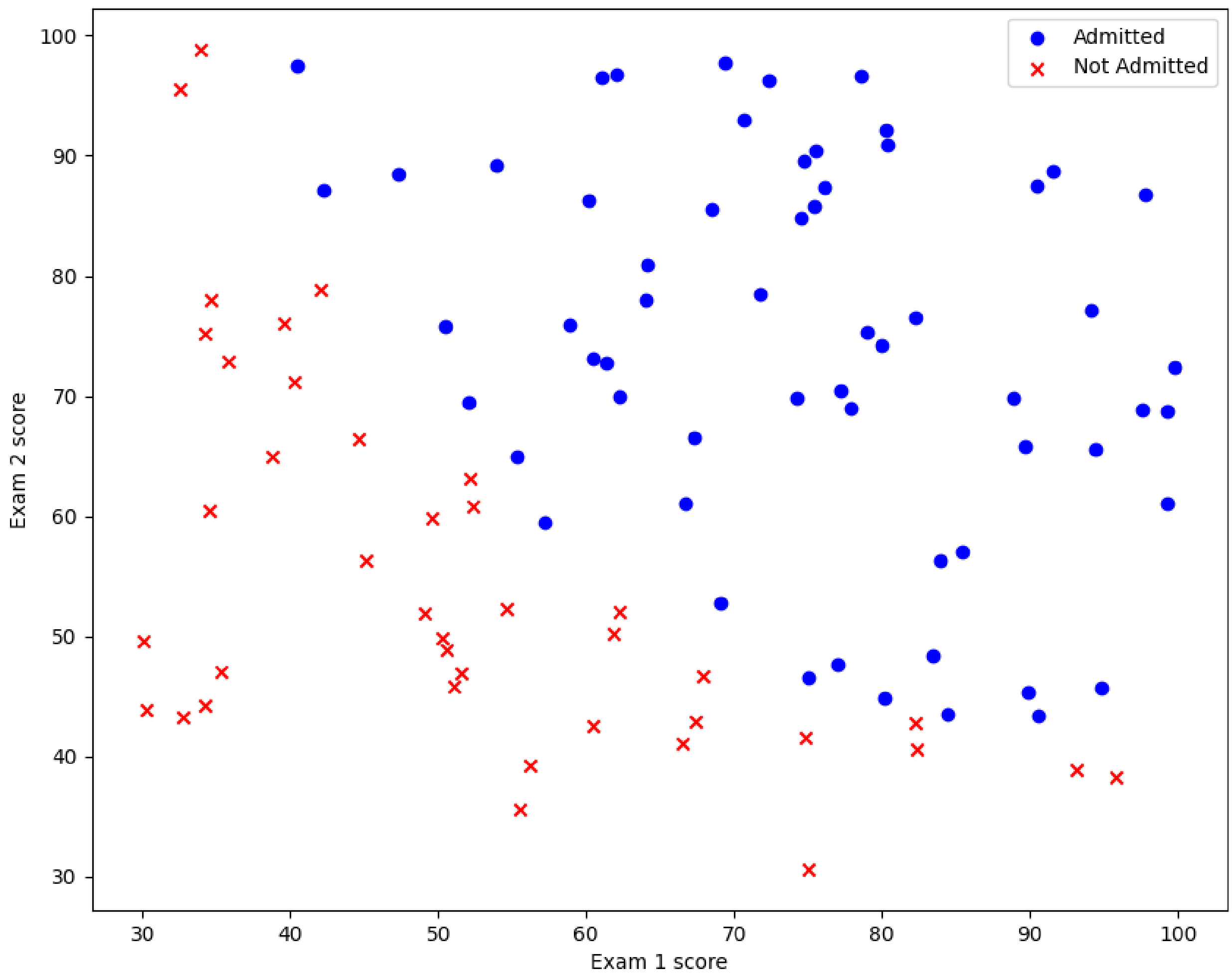
2. Visualizing the Data [Code provided - no need to change]

Go ahead download the data file (p0_data.txt), then upload to Google Colab using the files panel on the left (click the last icon on the menu).

Our dataset contains applicants' scores on two exams and the admission decisons for 100 students. This dataset will allow us to visualize in a 2D figure and showcase how the logistic regression algorithm works more intuitively.

```
# Load the dataset with NumPy
data = np.loadtxt('p0_data.txt', delimiter=',')

train_X = data[:, 0:2]
train_y = data[:, 2]

# Get the number of training examples and the number of features
m_samples, n_features = train_X.shape
print ("# of training examples = ", m_samples)
print ("# of features = ", n_features)
```

```
pos = np.where(train_y == 1)
neg = np.where(train_y == 0)
plt.scatter(train_X[pos, 0], train_X[pos, 1], marker='o', c='b')
plt.scatter(train_X[neg, 0], train_X[neg, 1], marker='x', c='r')
plt.xlabel('Exam 1 score')
plt.ylabel('Exam 2 score')
plt.legend(['Admitted', 'Not Admitted'])
plt.show()

# of training examples =  100
# of features =  2
```



3. Cost Function [5 points]

You're going to first implement the sigmoid function, then the cost function for (binary) logistic regression.

The sigmoid function is defined as $sigmoid(z) = \frac{1}{1+e^{-z}}$.

Note that, you are asked to use the NumPy package for vector and matrix operations. NumPy implements matrix-specific optimizations (compared to expensive Python datatypes, `for` loops, etc.) which dramatically improves performance, particularly in our later projects.

```
def sigmoid(z):
    """ Sigmoid function """

###################################################################
    # Compute the sigmoid function for the input here.
#

###################################################################
    s = None

    ### YOUR CODE HERE: be careful of the potential underflow or
overflow here
    s = 1/(1+np.exp(-z))


    ### END YOUR CODE

    return s

# Check your sigmoid implementation
z = np.array([[1, 2], [-1, -2]])
f = sigmoid(z)
print(f)

[[0.73105858 0.88079708]
 [0.26894142 0.11920292]]

def cost_function(theta, X, y):
    """ The cost function for logistic regression """

######################################################################
###############
    # Compute the cost given the current parameter theta on the
training data set (X, y)#

######################################################################
###############
    cost = None

    ### YOUR CODE HERE
    s = sigmoid(np.dot(X,theta))
    cost = (1/len(y)) * np.sum(-y * np.log(s)- (1-y) * np.log(1-s)) #
bce formula
```

```
    ### END YOUR CODE
    return cost

# Check your cost function implementation

t_X = np.array([[1, 2], [-1, -2]])
t_y = np.array([0, 1])
t_theta1 = np.array([-10, 10])
t_theta2 = np.array([10, -10])
t_c1 = cost_function(t_theta1, t_X, t_y)
t_c2 = cost_function(t_theta2, t_X, t_y)
print(t_c1)
print(t_c2)
```

```
10.000045398899701
4.539889921682063e-05
```

4. Gradient Computation [5 points]

Implement the gradient computations for logistic regression.

```
def gradient_update(theta, X, y):
    """ The gradient update for logistic regression"""
    ###############################
    # Compute the gradient update #
    ###############################
    grad = None

    ### YOUR CODE HERE
    s = sigmoid(np.dot(X, theta))
    grad = (1 / len(y)) * np.dot(X.T, (s - y))

    ### END YOUR CODE


    return grad

# Check your gradient computation implementation
t_X = np.array([[1, 2, 3], [-1, -2, -3]])
t_y = np.array([0, 1])
t_theta1 = np.array([-10, 10, 0])
t_theta2 = np.array([10, -10, 0])
t_g1 = gradient_update(t_theta1, t_X, t_y)
t_g2 = gradient_update(t_theta2, t_X, t_y)
print(t_g1)
print(t_g2)
```

```
[0.9999546  1.9999092  2.99986381]
[4.53978687e-05 9.07957374e-05 1.36193606e-04]
```

5. Gradient Checking [Code provided. Bonus 5 points if implemented from scratch]

You can use the code provided below to check the gradient of your logistic regression functions using SciPy package. Alternatively, you can implement the gradient checking from scratch by yourself (bonus 5 points).

Gradient checking is an important technique for debugging the gradient computation. Logistic regression is a relatively simple algorithm where it is straightforward to derive and implement its cost function and gradient computation. For more complex models, the gradient computation can be notoriously difficulty to debug and get right. Sometimes a subtly buggy implementation will manage to learn something that can look surprisingly reasonable, while performing less well than a correct implementation. Thus, even with a buggy implementation, it may not at all be apparent that anything is amiss.

```
# Check your gradient computation implementation
t_samples, t_features = 100, 10
t_X = np.random.randn(t_samples, t_features)
t_y = np.random.randint(2, size=t_samples)
t_theta = np.random.randn(t_features)

# ==============================================================
# If you decide to implement from scratch, please use the below
function:
#
# def check_grad(cost_function, gradient_update, theta, X, y):
#   grad = None
#
#   ### YOUR CODE HERE
#
#
#
#   ### END YOUR CODE
#
#   return grad
#
# print(f'Output of check_grad: {check_grad(cost_function,
gradient_update, t_theta, t_X, t_y)}')
# ==============================================================

# Check gradient using SciPy
from scipy import optimize
print(f'Output of check_grad: {optimize.check_grad(cost_function,
gradient_update, t_theta, t_X, t_y)}')
```

```
Output of check_grad: 1.0203406555960879e-07
```

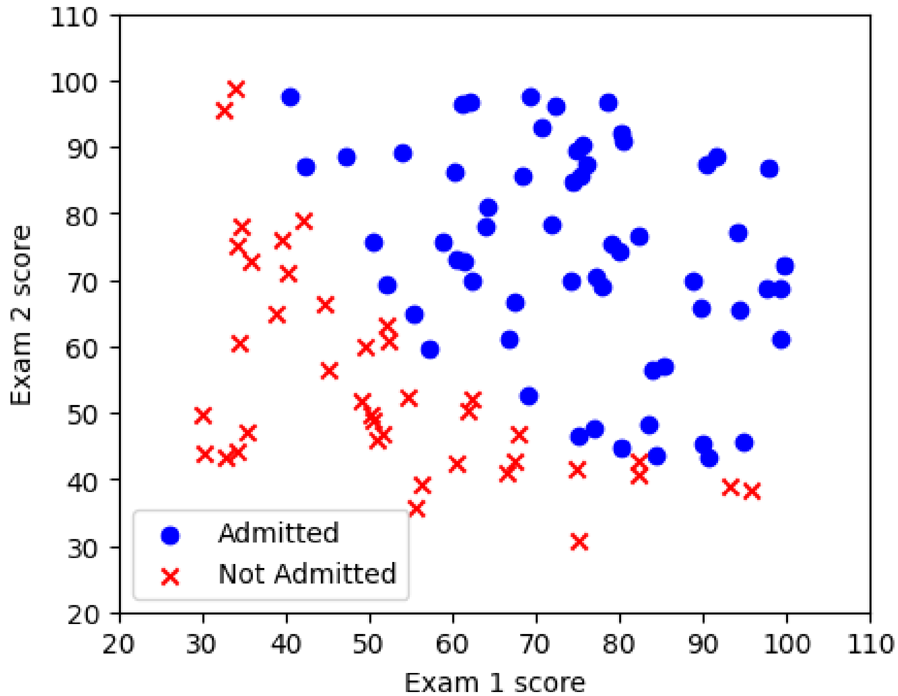
6. Gradient Descent and Decision Boundary [10 points]

Implement the batch gradient decent algorithm for logistic regression. For every `print_iterations` number of iterations, also visualize the decision boundary and observe how it changes during the training.

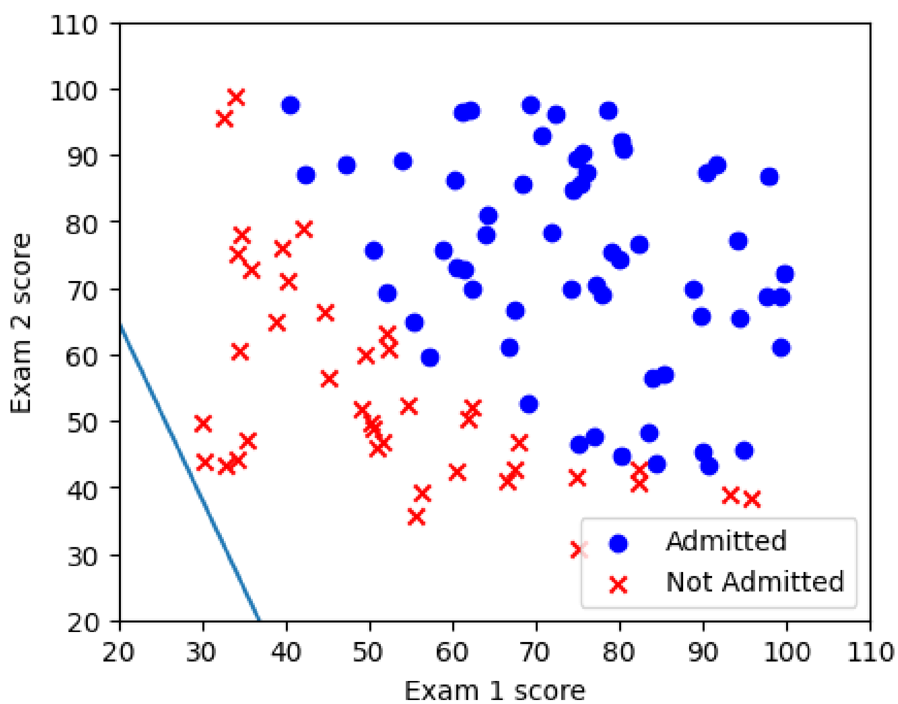
Note that, you will need to carefully choose the learning rate and the total number of iterations (you may want to use a small learning rate and a very large number of interations), given that the starter code does not include feature scaling (e.g., scale each feature by its maximum absolute value to convert feature value to $[-1,1]$ range -- in order to make this homework simple and easier for you to write code to visualize.

```
def gradient_descent(theta, X, y, alpha, max_iterations,
print_iterations):
    """ Batch gradient descent algorithm """
    #################################################################
    # Update the parameter 'theta' iteratively to minimize the cost #
    # Also visualize the decision boundary during learning          #
    #################################################################

    alpha *= m_samples
    iteration = 0

    ### YOUR CODE HERE:
    new_theta = theta
    X = np.hstack([np.ones((X.shape[0], 1)), X])

    ### END YOUR CODE



    while(iteration < max_iterations):
        iteration += 1

        ### YOUR CODE HERE: simultaneous update of partial gradients
        grad = gradient_update(new_theta, X, y)
        new_theta = new_theta - alpha * grad

        ### END YOUR CODE


        # For every print_iterations number of iterations
        if iteration % print_iterations == 0 or iteration == 1:
            cost = 0

            ### YOUR CODE HERE: calculate the cost
            ### IMPORTANT: The cost function is guaranteed to decrease
after
            ## every iteration of the gradient descent algorithm.
            cost = cost_function(new_theta, X, y)
```

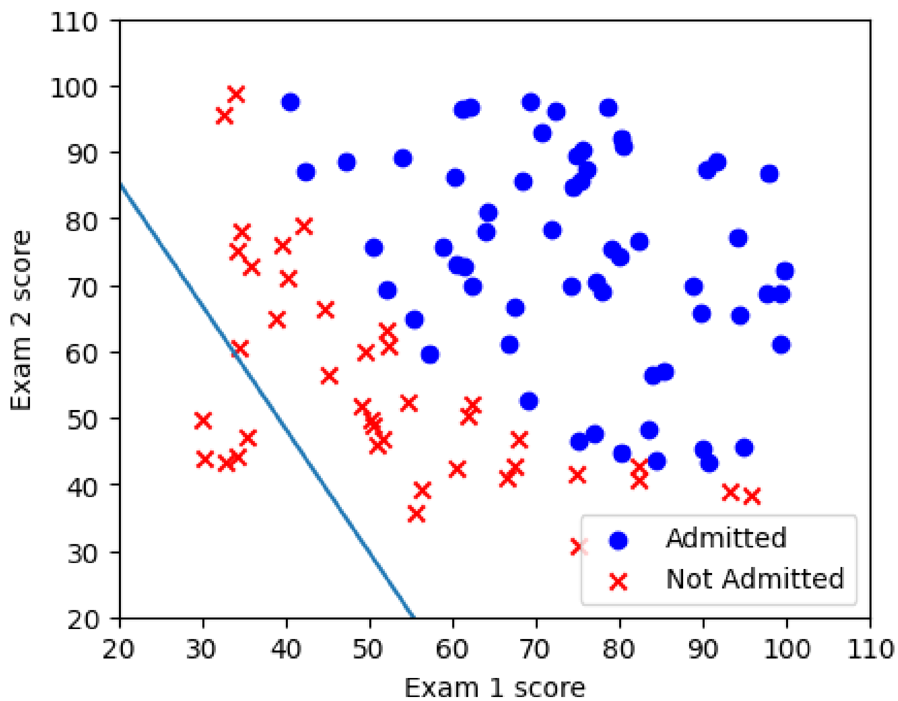
```
            ### END YOUR CODE

            print ("[ Iteration", iteration, "]", "cost =", cost)
            plt.rcParams['figure.figsize'] = (5, 4)
            plt.xlim([20,110])
            plt.ylim([20,110])

            pos = np.where(y == 1)
            neg = np.where(y == 0)

            plt.scatter(X[pos, 1], X[pos, 2], marker='o', c='b')
            plt.scatter(X[neg, 1], X[neg, 2], marker='x', c='r')
            plt.xlabel('Exam 1 score')
            plt.ylabel('Exam 2 score')
            plt.legend(['Admitted', 'Not Admitted'])
            t = np.arange(10, 100, 0.1)


            ### YOUR CODE HERE: plot the decision boundary
            decision = -(new_theta[0] + new_theta[1] * t) /
new_theta[2]
            plt.plot(t, decision, label = "Boundary")

            ### END YOUR CODE

            plt.show()

    return new_theta


### YOUR CODE HERE: initialize the parameters 'theta' to random
values;
### And set up learning rate, number of max iterations, number of
iterations for printing intermediate outputs

initial_theta = np.zeros(train_X.shape[1] + 1)
alpha_test = 0.00001
max_iter = 100000
print_iter = 10000


### END YOUR CODE


learned_theta = gradient_descent(initial_theta, train_X, train_y,
alpha_test, max_iter, print_iter)

[ Iteration 1 ] cost = 0.6982906893667754
```



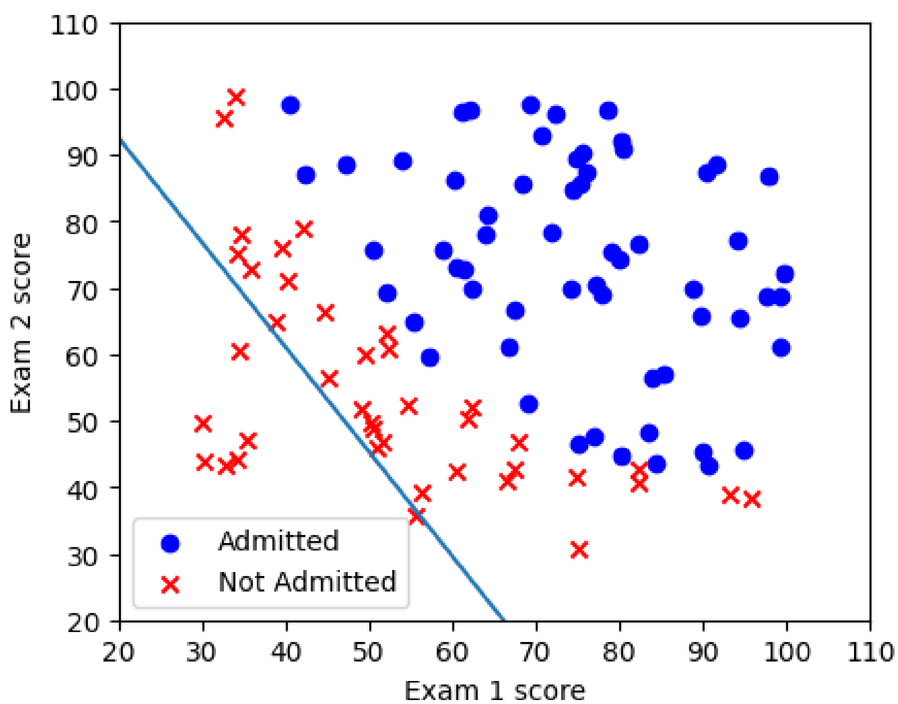

```
[ Iteration 10000 ] cost = 0.5850274988176747
```



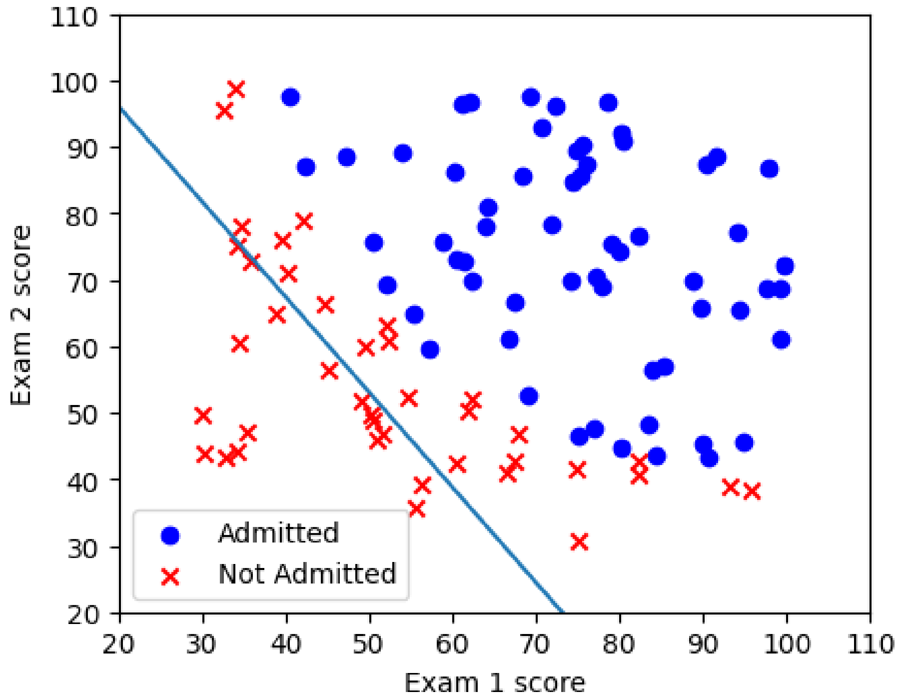

```
[ Iteration 20000 ] cost = 0.5472954636936675
```



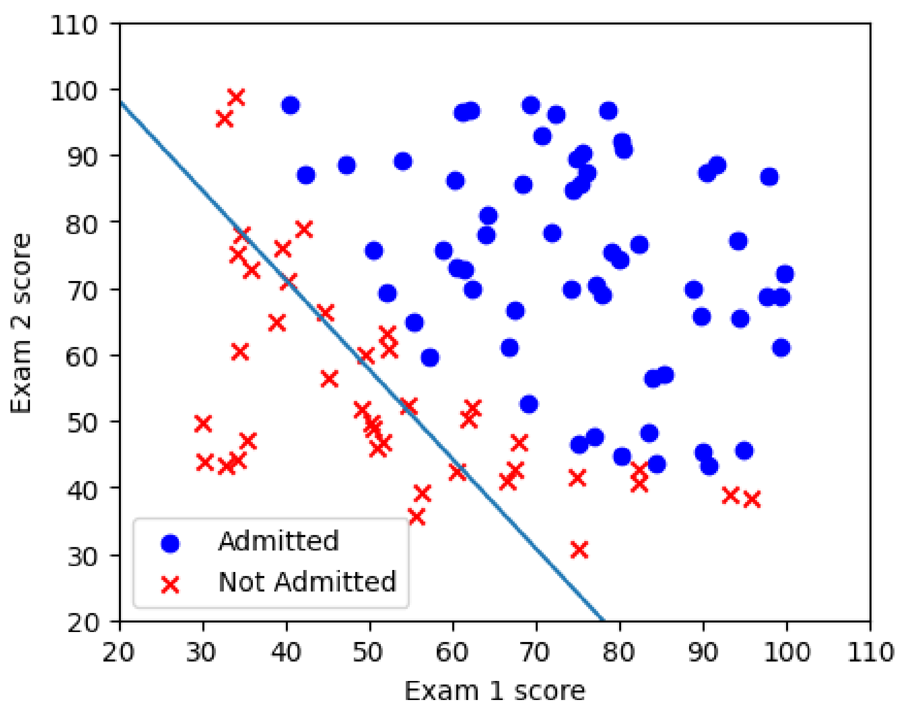

```
[ Iteration 30000 ] cost = 0.5154018233277201
```



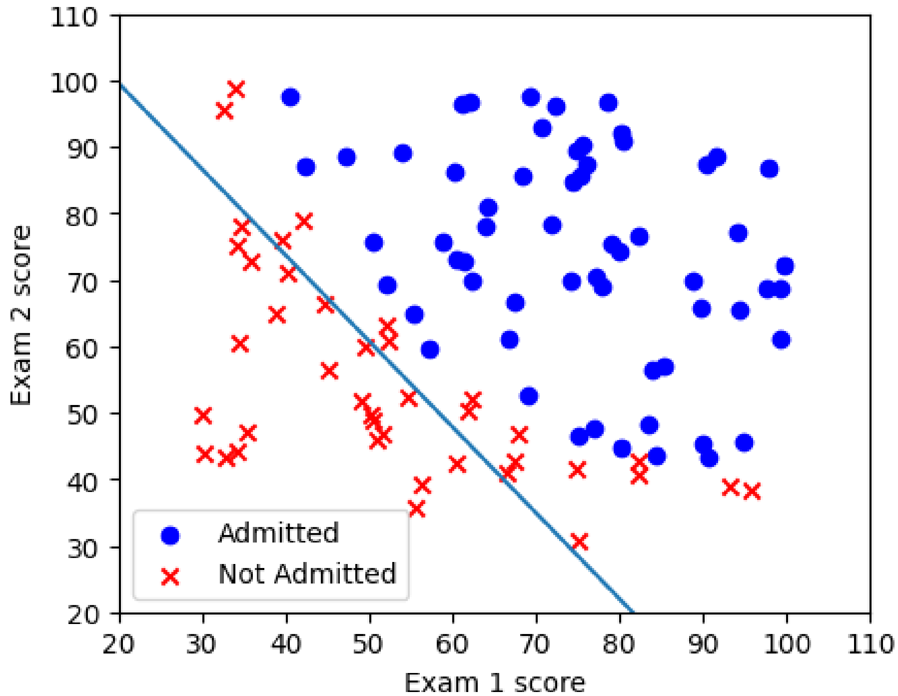

```
[ Iteration 40000 ] cost = 0.48829570918772613
```



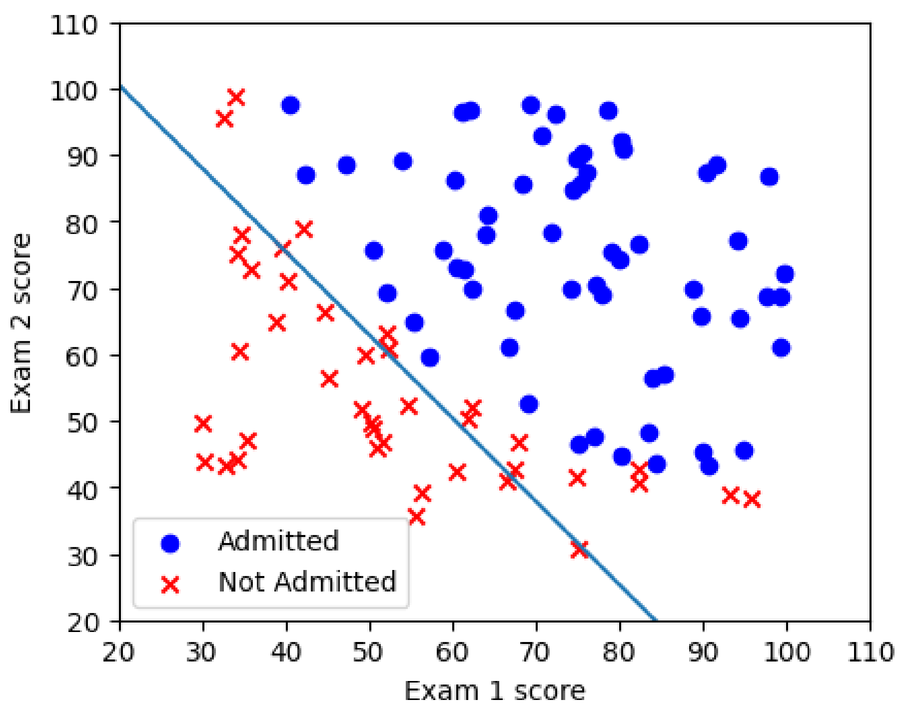

```
[ Iteration 50000 ] cost = 0.46510456912211945
```



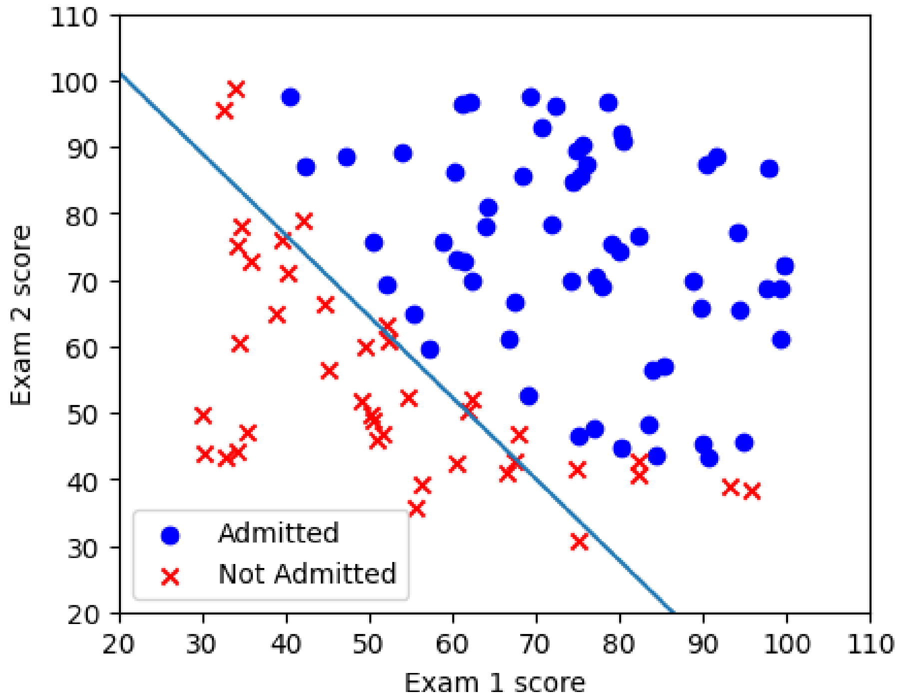

```
[ Iteration 60000 ] cost = 0.445118809496071
```



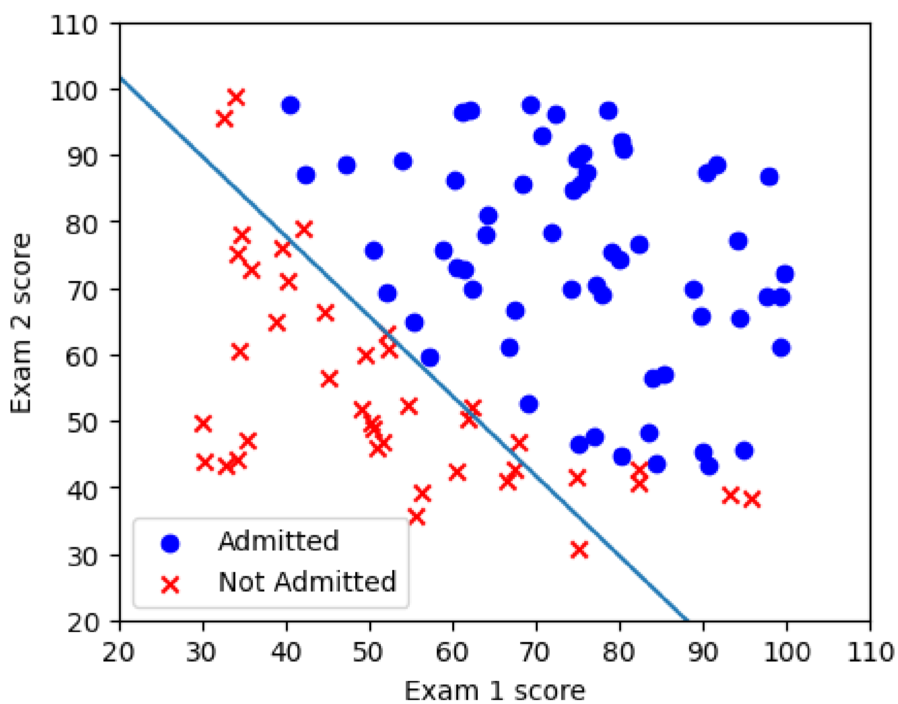

```
[ Iteration 70000 ] cost = 0.42776773553739156
```



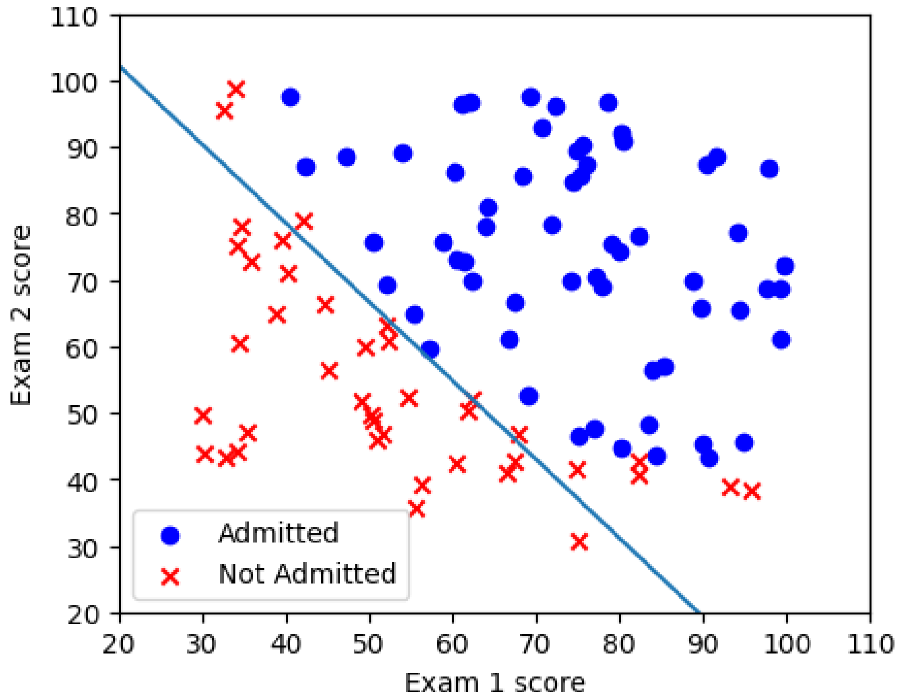

```
[ Iteration 80000 ] cost = 0.41259440170978545
```




```
[ Iteration 90000 ] cost = 0.39923293458511083
```




```
[ Iteration 100000 ] cost = 0.38738952271118804
```



6. Prediction [5 points]

Now that you learned the parameters of the model, you can use the model to predict whether a particular student will be admitted.

```
def predict(theta, X):
    """ Predict whether the label is 0 or 1 using learned logistic
regression parameters """

    ### YOUR CODE HERE:
    if X.shape[1] < theta.shape[0] - 1:
        padding = theta.shape[0] - 1 - X.shape[1]  # pad for missing
features
        X = np.pad(X, ((0, 0), (0, padding)), mode='constant')

    if X.shape[1] + 1 == theta.shape[0]: # bias
        X = np.hstack([np.ones((X.shape[0], 1)), X])


    probabilities = sigmoid(np.dot(X, theta))
    predicted_labels = probabilities >= 0.5 #50% chance

    ### END YOUR CODE


    # Convert an array of booleans 'predicted_labels' into an array of
0 or 1 intergers
    return probabilities, 1*predicted_labels
```

```
# Check your predication function implementation
t_X1 = np.array([[90, 90]])
t_X2 = np.array([[50, 60]])
t_X3 = np.array([[10, 50]])
print(predict(learned_theta, t_X1))
print(predict(learned_theta, t_X2))
print(predict(learned_theta, t_X3))

# Computer accuracy on the training dateset
t_prob, t_label = predict(learned_theta, train_X)
t_precision = t_label[np.where(t_label == train_y)].size /
float(train_y.size) * 100
print('Accuracy on the training set: %s%%' % round(t_precision,2))
```

```
(array([0.93706746]), array([1]))
(array([0.43627873]), array([0]))
(array([0.07948104]), array([0]))
Accuracy on the training set: 91.0%
```

7. Submit Your Homework

This is the end. Congratulations!

Now, follow the steps below to submit your homework in Gradescope:

1. Rename this ipynb file to `CS4650_p0_GTusername.ipynb`. Make sure all cells have been run.
2. Click on the menu 'File' --> 'Download' --> 'Download .py'.
3. Click on the menu 'File' --> 'Download' --> 'Download .ipynb'.
4. Download the notebook as a .pdf document. Ensure the output from step 5. Gradient Descent and Decision Boundary is captured. This question cannot be graded if the output from this cell is not captured.
5. Upload all 3 files to Gradescope. Double check the files start with `CS4650_p0_*`, capitalization matters.