sigmoid derivative

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$\sigma'(x) = \frac{d}{dx} \frac{1}{1 + e^{-x}}$$

quotient rule

$$= \frac{(1+e^{-x}) \cdot \overset{0}{\frac{d}{dx} 1} - 1 \cdot \overset{-e^{-x}}{\left(\frac{d}{dx} 1 + e^{-x}\right)}}{(1+e^{-x})^2}$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{(1+e^{-x}) - 1}{(1+e^{-x})^2}$$

$$= \frac{1}{(1+e^{-x})} \frac{(1+e^{-x}) - 1}{(1+e^{-x})}$$

$$= \frac{1}{(1+e^{-x})} \left(\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}} \right)$$

$$= \sigma(x) \cdot (1 - \sigma(x))$$

partial derivatives (multivariable calculus)

$$f(x,z,a,b) := y = (4ax^2 + a) + 3 + \sigma(z) + (\sigma(b)^2)$$

$$f'(x,z,a,b) = \frac{dy}{dx} (4ax^2 + a) + 3 + \sigma(z) + (\sigma(b)^2)$$

$$= 8ax$$

$$f'(x,z,a,b) = \frac{dy}{dz} (4ax^2 + a) + 3 + \sigma(z) + (\sigma(b)^2)$$

$$= \sigma(z) \cdot (1 - \sigma(z))$$

$$f'(x,z,a,b) = \frac{dy}{da} (4ax^2 + a) + 3 + \sigma(z) + (\sigma(b)^2)$$

$$= 4x^2 + 1$$

$$f'(x,z,a,b) = \frac{dy}{db} (4ax^2 + a) + 3 + \sigma(z) + (\sigma(b)^2)$$

chain rule

$$= 2\sigma(b) \cdot (\sigma(b) \cdot (1 - \sigma(b)))$$