AN ALGORITHM FOR COMPUTING THE DISCRETE RADON TRANSFORM WITH SOME APPLICATIONS

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ABSTRACT

The enormous growth in the application areas of the Radon Transform and the fact that digital computations are often required, has led to the development of the Discrete Radon Transform (DRT). This paper prôposes a method for computing the DRT by exploiting the converse of the Central Slice Theorem relating the Radon and Fourier Transforms, using the FFT algorithm. Simulation of the DRT algorithm is presented and some applications are discussed. The proposed algorithm can be extended for filtering in the Radon Space.

1. Introduction

The Radon transform (RT), apart from its key role in the reconstruction of multidimensional signals from their projections (Computer Assisted Tomography [12]), is a powerful tool in various image processing and machine vision applications [10], as it offers significant advantages for general image representation and manipulations. The Central Slice Theorem which relates the RT (spatial concentrations) of an image with its Fourier transform (FT)(spatial frequencies), reinforces the considerations underlying the use of the projection space as a 'Recognition Domain'. Significant features in the image can be detected in a sufficiently sized projection space [11]. Projection generation is being used to aid feature selection [8,11,14-17]. The Hough Transform (HT) [7], which is an algorithm for detecting straight line segments in pictures is a special case of the RT [5].

The computation of the RT is thus gaining importance. It is not possible to obtain the RT directly from discrete data. The RT for discrete data has been defined in a number of ways, each suitable to a particular application. Some require the interpolation of the available data values to a dense grid [3,18], and some other require a weighting of the discrete data [11,14]. An approximation to the RT for discrete data has been generally referred to in the literature as the DRT. However, strictly speaking, the DRT involves a *summation* of the discrete data along various straight lines in the plane of the grid. This invariably involves an interpolation of the available data.

In many applications, there is a need for simpler and fast methods for computing the RT. Recently, a fast algorithm has been proposed for the computation of the DRT by Beylkin [3] based on the formulation given by Scheibner [18]. However, the slope intercept form of a straight line used here introduces complications, as these parameters are not bound. Apart from requiring the storage of a large number of huge 'transform' matrices, this algorithm appears to be fast only for rectangular arrays with one of the dimensions being considerably small and hence suitable for seismic applications. Hinkle et al.[10] have considered simpler schemes, however, for a special purpose architecture.

A new approach to compute the DRT of a digital image using FFT algorithms based on the (converse of) Central Slice Theorem (CST) [12] is explained in the following sections. The emphasis has been to develop a method that is appreciably fast, of moderate cost, relatively simple and amenable for parallel implementation. The organization of the rest of the paper is as follows. The CST and its converse are reviewed in the following Sections. The DRT algorithm is developed in Section 4, followed by simulation results in Section 5. Some important applications are discussed in Section 6. Section 7 concludes the paper.

2. The Radon Transform

Let $f(x,y)$ represent a two dimensional (2-D) function on the plane and $F(u,v)$, its FT. The Radon Tansform of $f(x,y)$ is the integral of $f(x,y)$ along all possible straight lines in the plane containing the function, provided the line integral exists:

$$R\{f(x,y)\}=p(t,\theta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(t-x\cos\theta-y\sin\theta)\,dx\,dy \qquad (1)$$

The straight line is represented in its normal form',

$$t = x\cos\theta + y\sin\theta \qquad (2)$$

where, 't' is the radial distance and θ, the angle made by the normal. The term 'projection' at an angle θ refers to the RT integral (1) evaluated at θ. $p(t,\theta)$ will be written as $p_\theta(t)$ to emphasize that a 1-D

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function is considered, with θ as a parameter.

3. The Central Slice Theorem

The Central Slice Theorem states that the Fourier Transform of the projection at an angle θ (the RT along parallel lines at angle θ) denoted by $P_\theta(w)$, is a central slice of the 2-D Fourier Transform of the image $f(x,y)$, at the same angle, θ:

$$FT\{p_\theta(t)\} = P_\theta(w) = F(w\cos\theta, w\sin\theta) \qquad (3)$$

Conversely, the projection $p_\theta(t)$ may be obtained by using the inverse FT:

$$p_\theta(t)=F^{-1}\{P_\theta(w)\}=F^{-1}\{\ F(w\cos\theta,w\sin\theta)\ \} \qquad (4)$$

Equation (4) states that the inverse FT of the 'central' slice of the 2-D FT of the image $f(x,y)$ gives the projection of the image at the same angle. It is thus possible to fill the Radon Space by considering several equi-angular slices.

4. Discrete Radon Transform Algorithm

Definition: The Discrete Radon Transform (DRT) of a digital image $f(m,n)$ at an angle θ, is the sum of the values of $f(m,n)$ along the corresponding orthogonal directions.

As a consequence of the converse of the Central Slice Theorem (equation (4)) the DRT may be defined in terms of the DFT of the image, as follows:

The DRT of a digital image $f(m,n)$ at an angle θ, is the inverse DFT of the samples on the central slices of the 2-D DFT of $f(m,n)$, at the same angle θ.

The above definition provides the basis for the computation of the DRT from the DFT of $f(m,n)$. The advantage of such a method is efficiency, since FFT algorithms can be used to compute the DFT. Since the 2-D DFT corresponds to the samples of the (2-D) FT of the image on a Cartesian grid, it is required to map the samples onto a polar raster. A simple method is inverse distance linear interpolation. However, it is important to note that the accuracy of the technique depends on the type of interpolation used. The steps in the method are:

1) Compute the 2-D DFT $X(k,l)$ of the image $x(m,n)$:

$$X(k,l) = \sum_{m=1}^{N}\sum_{n=1}^{N} x(m,n)e^{-j(2\pi/N)(mk+ln)}, \quad k,l=1,2,..N \qquad (5)$$

This can be done by first computing the transforms of the rows (in parallel) & the transforms of the columns of the result (in parallel) using FFT algorithms.

2) Compute the central slices of the 2-D DFT. This requires a mapping of the samples of the FT from the Cartesian raster onto a polar raster. Inverse distance linear interpolation is used for illustration (for lines passing through the lattice points, interpolation is not required and the DRT is obtained by summing the corresponding data values):

$$XP_\theta(w) = \frac{\sum_{i=1}^{4} (\ X\ (\ k_i,l_i)/d_i(w,\theta)\ )}{\sum_{i=1}^{4} (\ 1\ /\ d_i(w,\theta)\ )} \qquad (6)$$

$XP_\theta(w)$ are the samples of the FT of $x(m,n)$ at angle θ and distances w (radial frequency) from the origin. $d_i(w,\theta)$ denote the distances between the point (w,θ) on the polar grid and its four nearest Cartesian neighbors. Fig. 1(a) shows the parameters for the definition of linear interpolation. Fig. 1(b) shows the polar raster indicating the set of points onto which the Cartesian samples are mapped. The distances d_is need be precomputed and stored only for points in the first quadrant, due to symmetry.

3) Perform the inverse FFT of the these slices to get the DRT- this can be done at all angles independently and hence in parallel.

<u>Remark</u>: In a local interpolation scheme such as the one described above, the error will be independent from point to point.

The above algorithm is efficient as FFT can be used to compute the DFT and is also amenable for parallel implementation. It is important to note that, suitability of an algorithm for parallel implementation is often a desirable feature, irrespective of the exact number of computations involved.

<u>Remark</u>: A similar approach for the computation of the RT has been considered by Murphy [13], however, in the context of linear feature enhancement in SAR images. Very recently, a transputer implementation has been developed for the same [9].

5. Simulation

Simulation results on a computer generated test image of size 64x64 are shown in figs. 2-4. Fig. 2 shows the image under consideration. Figs. 3(a)-(d) show the projections obtained by the weighted summation scheme using the 'pixel' assumption, as outlined by Barret [2]. The weights have been set equal to the lengths of intersection of the lines (2) with the pixels assumed around the Cartesian grid points, in order to closely approximate (1). Figs. 4(a)-(d) show those computed by the proposed DRT algorithm. It may be observed that at angles corresponding to the Fourier slices away from the axes, the projections computed by the proposed DRT algorithm do not compare well with those of fig. 3. This can be attributed to the following: (1) The points within the circle inside the Cartesian frequency raster are considered, which amounts to truncation (see fig. 1.1(b)). This effect becomes severe for slices far away from the axes. This may be minimized by considering all the uniformly spaced points along the slices so as to encompass the entire square raster. (2) As mentioned earlier, simple linear interpolation may not be sufficient to accurately estimate the polar samples, especially in the case of images with considerable high frequency content. This is a limitation of the proposed algorithm. However, a high degree of accuracy is not required in many applications.

6. Applications

The Hough Transform (HT) has been recognized in the literature as a special case of the RT. However, strictly speaking, it is the DRT which is closer to the HT than the RT. This can be appreciated from the fact that the DRT (and not the RT) of a binary image reduces to its HT. The conventional Hough technique is limited by slow speed and excessive memory requirements. In addition, as Suter[19] has pointed out, one of the deficiencies of the HT is a distortion in the shape and the location of the peaks in the parameter (Transform space, which may be eliminated by the use of the DRT. The proposed DRT algorithm can replace the Hough Transformation step in a straight line detection scheme, making it very efficient. Recently, D. Casacent and R. Krishnapuram [4] *have developed a technique which is an* extension of the HT, to detect and locate curves in images. This algorithm avoids the extensive pre-processing and high dimensionality of the Hough Space that is required in the recent methods in generalizing the Hough Transform [1]. The proposed algorithm serves as a fast and efficient (and parallel) implementation of the Hough scheme and curved *object recognition.*

The DRT algorithm described above may be extended for filtering in the Radon Space, as follows. The convolution theorem in the Radon Space [6] states that the RT of the convolution of a function f(x,y) with a filtering function h(x,y) is the convolution of their projections $p_\theta(t)$ and $h_\theta(t)$, respectively, for all θ:

$$RT\{f(x,y)**h(x,y)\}=p_\theta(t)*h_\theta(t)=p_\theta'(t) \qquad (7)$$

In the frequency domain:

$$P_\theta'(w) = H_\theta(w)P_\theta(w) \qquad (8)$$

Step-2 of the DRT algorithm may be *modified by introducing* $H_\theta(w)$ *as a* weighting factor to obtain the samples of the FT of the filtered function on a polar grid:

$$P_\theta'(w) = \frac{H_\theta(w) \sum_{i=1}^{4} X(k_i,l_i)/d_i(w,\theta)}{\sum_{i=1}^{4} (1 / d_i(w,\theta))} \qquad (9)$$

By the CST, $P'_\theta(w)$ forms a central slice of the (2-D) FT of the filtered image $f'(x,y)$, which may be obtained by direct Fourier inversion technique. Thus, 2-D linear shift-invariant filters can be realized by a set of decoupled 1-D filters, by working in the Radon Space. The scheme is illustrated in fig. 5. This technique is particularly useful for implementing 2-D circularly symmetric filters.

The technique outlined above suggests application to multi-directional filtering, a topic that has not received much attention. Remark: The proposed algorithm is amenable to transputer implementation [9].

7. Summary

An efficient algorithm for the computation of the DRT has been presented. Its application for the detection of straight line segments and extension for curved object recognition have been discussed. The algorithm is amenable for parallel implementation, a current trend in signal and image processing. A modification of the algorithm for filtering in the Radon Space has been proposed.

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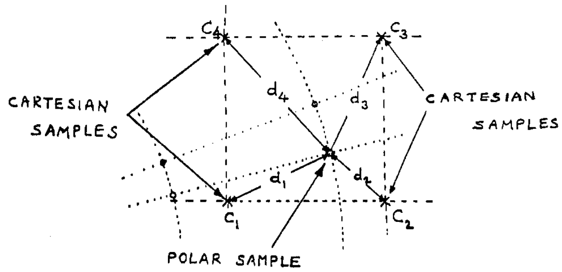

Fig. 1(a)

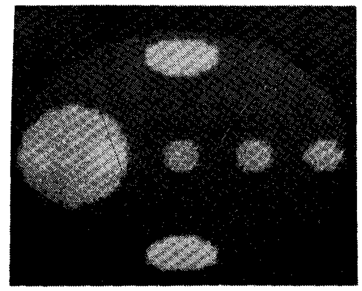

Fig. 2: The Test Image

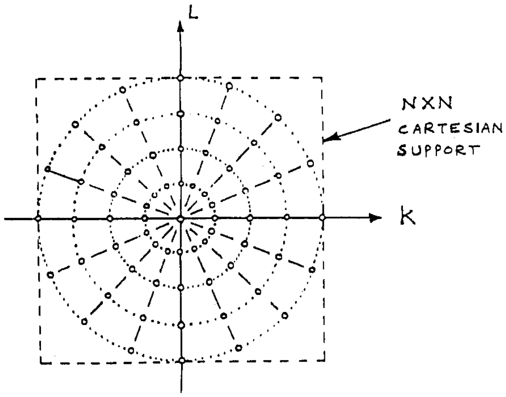

Fig. 1(b)

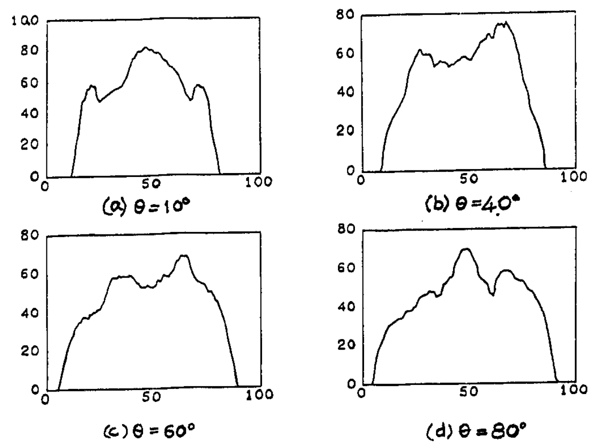

Fig. 3

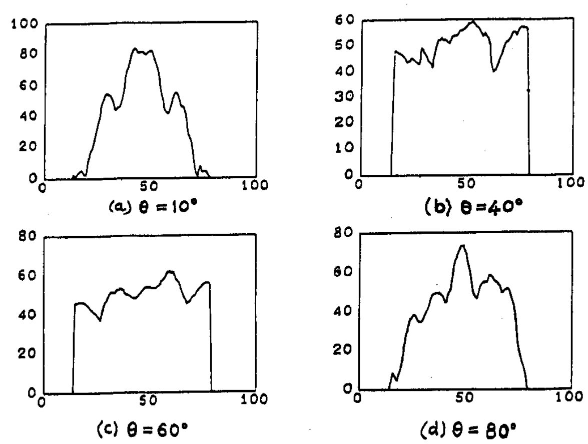

Fig. 4

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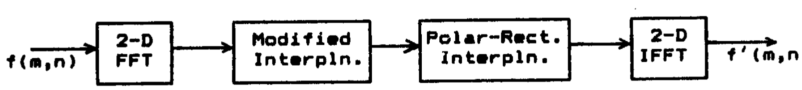

Fig.5