Precoding for RadComm Systems Based on Hybrid Antenna Arrays

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***Abstract*—Integrated wireless communications and radar detections have so far mainly relied on phased-array (PA) antennas with only considering the beampatterns in angle domain. By applying the frequency diverse array (FDA) into radar and communications (RadComm) systems, a compact dual-functional structure of joint PA and FDA is designed. Based on the built joint PA and FDA channel models, different beamformer designs are proposed by jointly optimizing the signal-to-interference-noises of the communications and radar. Numerical simulations demonstrate the performances of both communications and the radar with the proposed approaches are over than that of the conventional PA RadComm systems.**

***Index Terms*—RadComm, frequency divers array, joint PA and FDA**

I. Introduction

DUE to the massive production of communications devices and the limited frequency spectrum resource, the problem of shared spectrum for RadComm systems has been proposed for a long time. At first, the Cognitive Radios (CRs) [1]–[3] using opportunistic approaches have been considered for sharing spectrum technology, in which the communications system transmits its signals when space and frequency spectra are not occupied by radar. This method makes it possible to share spectrum by using spectrum sensing [4]–[6], geolocation databases [7], or both of them in the form of radio environment maps. Although it seems that such a scheme is not difficult to implement, it does not allow the radar and communications to work simultaneously.

On the other hand, the pioneering work named null space projection (NSP) was proposed [8] for the coexistence of MIMO radar and communications, in which the designed radar beamformer is in the null space of the interference channel between the radar and base station (BS) so that no cross interference can be imposed on the communications and radar systems. Based on this approach, [9]–[12] concentrate on investigating different trade-offs between the performance of radar and communications by relaxing the zero-forcing precoder as the projection matrix. However, these works all employ the traditional phased array (PA) for the radar detection, which cannot determine the angle and range of the target at the same time, also have to pay the cost of impulse generator. Besides, limited by the hardware, they are difficult to transmit ultra wide band (UWB) impulse through the traditional PA.

II. Dual-functional RadComm System Model

In this section, we propose the scheme of the joint PA and FDA dual-functional system and establish the model of RadComm system.

A. System Model

The system model of RadComm is illustrated in Fig. 1. In this system, the joint PA and FDA RadComm transmitter (TX) with N-elements array communicates with the mobile station (MS) equipped with M-elements array through the PA, and detects the target through the FDA, simultaneously.

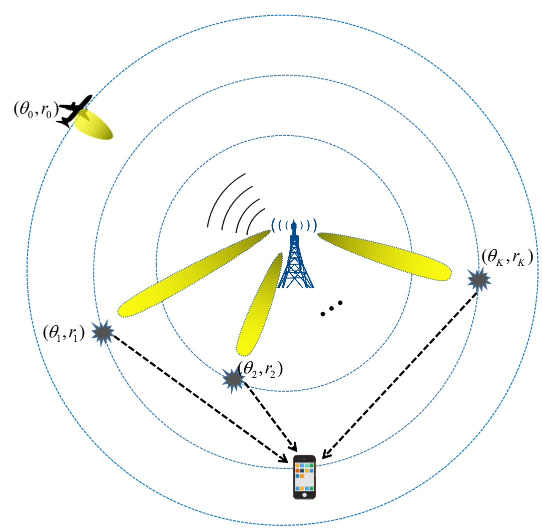

Fig. 1: System Model

1) Channel model: We assume the K clusters located at $\{(\theta_k, r_k), k = 1, 2, \cdots, K\}$, where θ_k and r_k are the angle and range of the kth cluster, while the target are located at (θ_0, r_0), where θ_0 and r_0 is the angle and range of the target.

We first consider the interference channel from the FDA radar to the MS. The electromagnetic wave rays with L various frequencies transmitted from the N-elements FDA radar are reflected to the M-elements MS by K clusters. It should be noted is that each antenna at the MS can receive all the L-frequencies waves from every cluster. Under this physical model, we build the channel model from FDA to PA as,

$$\mathbf{H}_{\text{PF}} = \sum_{k=1}^{K} \alpha_k \Big(\sum_{l=1}^{L} \mathbf{a}_{\text{PA}}(\phi_k, \lambda_l) \Big) \mathbf{a}_{\text{FDA}}^H(\theta_k, r_k, t) \tag{1}$$

where ϕ_k denotes the angle of the kth cluster w.r.t the MS, λ_l is the wavelength of the l-th carrier frequency at the TX side, α_k is the complex gain of the kth path, and $\mathbf{a}_{\text{PA}}(\phi_k, \lambda_l)$ as well as $\mathbf{a}_{\text{FDA}}(\theta_k, r_k, t)$ are the antenna array steering vectors of PA and FDA, respectively.

If ULA is employed, $\mathbf{a}_{\text{FDA}}(\theta, r, t)$ can be written as

$$\mathbf{a}_{\text{FDA}}(\theta, r, t) = \frac{1}{\sqrt{N}} [e^{j2\pi\psi_0}, e^{j2\pi\psi_1}, \cdots, e^{j2\pi\psi_{N-1}}]^T, \tag{2}$$

where $\psi_n = n\frac{d}{\lambda_0}\sin\theta - \frac{\Delta f_n}{c_0} r + \Delta f_n t$, and d is the distance between any neighbour antenna elements.

The PA steering vector $\mathbf{a}_{\text{PA}}(\phi, \lambda)$ can be formulated as

$$\begin{aligned}\mathbf{a}_{\text{PA}}(\phi, \lambda) =& \frac{1}{\sqrt{MN}} [1, e^{j\frac{2\pi}{\lambda} d \sin\phi}, \cdots, \\ & e^{j(M-1)\frac{2\pi}{\lambda} d \sin\phi}]^T.\end{aligned} \tag{3}$$

For simplicity, the channel can be written into a more compact form

$$\mathbf{H}_{\text{PF}} = \mathbf{A}_{\text{PA}} \mathbf{\Lambda} \mathbf{A}_{\text{FDA}}^H, \tag{4}$$

where

$$\mathbf{\Lambda} = \text{diag}\{\alpha_1, \alpha_2, \cdots, \alpha_K\}, \tag{5}$$

$$\mathbf{A}_{\text{FDA}} = [\mathbf{a}_{\text{FDA}}(\theta_1, r_1, t),\ \mathbf{a}_{\text{FDA}}(\theta_2, r_2, t), \cdots, \mathbf{a}_{\text{FDA}}(\theta_K, r_K, t)], \tag{6}$$

$$\mathbf{A}_{\text{PA}} = \sum_{l=1}^{L} [\mathbf{a}_{\text{PA}}(\phi_1, \lambda_l),\ \mathbf{a}_{\text{PA}}(\phi_2, \lambda_l),\ \cdots,\ \mathbf{a}_{\text{PA}}(\phi_K, \lambda_l)]. \tag{7}$$

Then, we study the channel from the PA TX to the MS. Assuming the wavelength of the carrier frequency of the PA is λ_C, which is approximated to the radar wavelength, then the path loss can be considered as the same as $\mathbf{H}_{\text{PF}}$. Hence, the channel from the PA TX to the MS can be expressed as follows

$$\begin{aligned}\mathbf{H}_{\text{PP}} &= \sum_{k=1}^{K} \alpha_k \mathbf{b}_{\text{MS}}(\phi_k) \mathbf{b}_{\text{BS}}^H(\theta_k) \\ &= \mathbf{B}_{\text{MS}} \mathbf{\Lambda} \mathbf{B}_{\text{BS}}^H\end{aligned} \tag{8}$$

where

$$\mathbf{b}_{\text{MS}}(\phi_k) = \frac{1}{\sqrt{M}} [1, e^{j\frac{2\pi}{\lambda_C} d \sin\phi_k}, \cdots, e^{j(M-1)\frac{2\pi}{\lambda_C} d \sin\phi_k}]^T, \tag{9}$$

$$\mathbf{b}_{\text{BS}}(\theta_k) = \frac{1}{\sqrt{N}} [1, e^{j\frac{2\pi}{\lambda_C} d \sin\theta_k}, \cdots, e^{j(N-1)\frac{2\pi}{\lambda_C} d \sin\theta_k}]^T, \tag{10}$$

$$\mathbf{B}_{\text{MS}} = [\mathbf{b}_{\text{MS}}(\phi_1),\ \mathbf{b}_{\text{MS}}(\phi_2), \cdots, \mathbf{b}_{\text{MS}}(\phi_K)], \tag{11}$$

$$\mathbf{B}_{\text{BS}} = [\mathbf{b}_{\text{BS}}(\theta_1),\ \mathbf{b}_{\text{BS}}(\theta_2), \cdots, \mathbf{b}_{\text{BS}}(\theta_K)]. \tag{12}$$

2) Radar Detection: As what we talked previously, compared with conventional PA radar, the FDA radar employs different frequencies for each element. Hence, the bandwidths of the FDA can be far wider than that of the PA radar and communications system. Besides, the discrete FDA impulse beampatterns also reduce the overlapped time with communications. We denote the overlapped rate of bandwidths and time of the FDA radar and communications by μ. Then the l-th sample of received signal at the RX can be written by

$$\begin{aligned}y_R(l) =& \alpha_0 \sqrt{pP_0} \mathbf{a}_{\text{FDA}}^H(\theta_0, r_0, t) \mathbf{w} s_0(l) \\ &+ \beta_0 \mu \sqrt{(1-p)P_0} \mathbf{b}_{\text{BS}}^H(\theta_R) \mathbf{W} \mathbf{s}(l) + n_R(l),\end{aligned} \tag{13}$$

where $\mathbf{w} \in \mathbb{C}^N$ with $||\mathbf{w}|| = 1$ is the weights of the radar signal, $\mathbf{W} \in \mathbb{C}^{N \times N}$ with $||\mathbf{W}|| = 1$ is the digital precoder of the communications signals, α_0 is the path gain from the radar TX to the radar RX through the target, β_0 is the path gain from the communications TX directly to the radar RX, P_0 is the total system power and $p \in [0, 1]$ denotes the power split ratio of the radar power over the total power, θ_R denotes the angle of the radar RX w.r.t TX, and $n_R(l) \sim \mathcal{CN}(0, \sigma_R^2)$ is the additive Gaussian white noise (AWGN) with zero-mean and variance σ_R^2.

Since $\mathbb{E}[\mathbf{s}(l)\mathbf{s}^H(l)] = \mathbf{I}$ and $\mathbb{E}[s_0(l)s_0^*(l)] = 1$, the SINR at the received radar can be expressed as

$$\text{SINR}_{\text{T}} = \frac{pP_0|\alpha_0|^2 \mathbf{w}^H \mathbf{a}_{\text{FDA}}(\theta_0, r_0) \mathbf{a}_{\text{FDA}}^H(\theta_0, r_0) \mathbf{w}}{(1-p)P_0|\mu\beta_0|^2 \mathbf{b}_{\text{BS}}^H(\theta_R) \mathbf{W}\mathbf{W}^H \mathbf{b}_{\text{BS}}(\theta_R) + \sigma_R^2}. \tag{14}$$

3) Communications: The main task of the communications part is that the TX has to transmitted its signals to the MS with high SINR. According to the channel model in this section, the received signal at the MS can be formulated as

$$\mathbf{y}_{\text{MS}}(l) = \sqrt{(1-p)P_0} \mathbf{H}_{\text{PP}} \mathbf{W} \mathbf{s}(l) + \sqrt{pP_0} \mu \mathbf{H}_{\text{PF}} \mathbf{w} s_0(l) + \mathbf{n}_M(l), \tag{15}$$

where $\mathbf{n}_M$ is the vector of independent and identically distributed (i.i.d) AWGN of $\mathcal{CN}(\mathbf{0}, \sigma_M^2 \mathbf{I}_M)$, where the $\mathbf{I}_M$ is the identity matrix.

Hence, the SINR at the MS can be expressed as

$$\text{SINR}_{\text{MS}} = \frac{(1-p)P_0 tr(\mathbf{H}_{\text{PP}} \mathbf{W}\mathbf{W}^H \mathbf{H}_{\text{PP}}^H)}{pP_0\mu^2 \mathbf{w}^H \mathbf{H}_{\text{PF}}^H \mathbf{H}_{\text{PF}} \mathbf{w} + \sigma_M^2}. \tag{16}$$

Then, the maximum achieved rate between the MS and BS assuming Gaussian signaling is

$$R_P = \log_2(1 + \frac{(1-p)P_0 tr(\mathbf{H}_{\text{PP}} \mathbf{W}\mathbf{W}^H \mathbf{H}_{\text{PP}}^H)}{pP_0\mu^2 \mathbf{w}^H \mathbf{H}_{\text{PF}}^H \mathbf{H}_{\text{PF}} \mathbf{w} + \sigma_M^2}). \tag{17}$$

III. Joint PA and FDA beamforming

In the RadComm system, our task, on the one hand, is to guarantee the high detection probability of the target, and on the other hand, to maximize the quality of the communications between the MS and BS. In other words, we can formulate this problem as a multiple objective optimization: maximizing the SINRs of the received signals for both radar and communications, which can be expressed by

$$
\begin{aligned}
\max_{\mathbf{w},\mathbf{W}} \quad & \text{SINR}_{\text{T}},\ \text{SINR}_{\text{MS}} \\
s.t. \quad & ||\mathbf{w}|| = 1, \\
& ||\mathbf{W}|| = 1.
\end{aligned} \tag{18}
$$

In fact, to maximize the SINRs of radar and communications, it needs to enhance the power to the directions of target and MS (or the clusters) by the TX of FDA and PA respectively and reduce its power to the interference directions.

A. Ratio-Maximizing Approach

Hence, an intuitive strategy to tackle this problem is breaking the problem of (18) into the following two sub-problems:

$$
\begin{aligned}
\max_{\mathbf{w}} \quad & \frac{\mathbf{w}^H \mathbf{A}(\theta_0, r_0)\mathbf{w}}{\mathbf{w}^H \mathbf{H}_{\text{PF}}^H \mathbf{H}_{\text{PF}} \mathbf{w}} \\
s.t. \quad & ||\mathbf{w}|| = 1,
\end{aligned} \tag{19a}
$$

$$
\begin{aligned}
\max_{\mathbf{W}} \quad & \frac{tr(\mathbf{H}_{\text{PP}} \mathbf{W}\mathbf{W}^H \mathbf{H}_{\text{PP}}^H)}{\mathbf{b}_{\text{BS}}^H(\theta_R)\mathbf{W}\mathbf{W}^H \mathbf{b}_{\text{BS}}(\theta_R)} \\
s.t. \quad & ||\mathbf{W}|| = 1,
\end{aligned} \tag{19b}
$$

where $\mathbf{A}(\theta_0, r_0) = \int_{\Omega} \mathbf{a}_{\text{FDA}}(\theta, r, t)\mathbf{a}_{\text{FDA}}^H(\theta, r, t) d\theta$, in which the Ω is the angle domain of FDA beampattern towards the target, which can be expressed as

$$
\Omega \triangleq \{\theta | \theta_0 - \nu \le \theta \le \theta_0 + \nu\}, \tag{20}
$$

where ν denotes the half beampattern width in angle domain. **Proposition 1**: *If $\mathbf{A}$ is positive-definite and Hermitian matrix, the optimization problem of (26) can be recast to find the eigenvector corresponding to the largest generalized eigenvalues as follows*

$$
\mathbf{w}^{opt} : \mathbf{A}\mathbf{w}^{opt} = \lambda_{max} \mathbf{H}_{PF}^H \mathbf{H}_{PF} \mathbf{w}^{opt} \tag{21}
$$

where λ_{max} and $\mathbf{w}_F^{opt}$ denote the maximum generalized eigenvalue and the corresponding generalized eigenvector, respectively. □

According to Proposition 1, the optimal beamformer for the FDA radar can be directly obtained.

Until now, the first sub-problem of (19a) has been solved. We are now at a position to solve the second sub-problem of (19b). For simplicity, we define the objective function as

$$
h(\mathbf{W}) = \frac{tr(\mathbf{H}_{\text{PP}} \mathbf{W}\mathbf{W}^H \mathbf{H}_{\text{PP}}^H)}{\mathbf{b}_{\text{BS}}^H(\theta_R)\mathbf{W}\mathbf{W}^H \mathbf{b}_{\text{BS}}(\theta_R)}. \tag{22}
$$

We first deduce the gradient of $h(\mathbf{W})$ w.r.t $\mathbf{W}$, then employ the steepest method to update $\mathbf{W}$ and normalize it at each iteration to meet the power constraint.

With several straightforward derivations, the gradient of $h(\mathbf{W})$ w.r.t $\mathbf{W}$ can be expressed as

$$
\begin{aligned}
\nabla h(\mathbf{W}) = \frac{2}{||\mathbf{W}^H \mathbf{b}_{\text{BS}}||^4} \Big(& \mathbf{H}_{\text{PP}}^H \mathbf{H}_{\text{PP}} \mathbf{W} \mathbf{b}_{\text{BS}}^H(\theta_R) \mathbf{W}\mathbf{W}^H \mathbf{b}_{\text{BS}} \\
& - tr(\mathbf{H}_{\text{PP}} \mathbf{W}\mathbf{W}^H \mathbf{H}_{\text{PP}}^H) \mathbf{b}_{\text{BS}} \mathbf{b}_{\text{BS}}^H \mathbf{W} \Big).
\end{aligned} \tag{23}
$$

According to the steepest method, we update $\mathbf{W}$ by

$$
\mathbf{W}_{k+1} = \mathbf{W}_k + \delta \nabla h(\mathbf{W}_k). \tag{24}
$$

where $\mathbf{W}_k$ denotes the precoding matrix at the k-th iteration.

To meet the power constraint, we normalize $\mathbf{W}$ by

$$
\mathbf{W}_{k+1} = \mathbf{W}_{k+1} / ||\mathbf{W}_{k+1}||. \tag{25}
$$

B. QCQP Approach

When considering the small interference of the RadComm, there is an alternative way to solve the problem (18), which can be expressed as

$$
\begin{aligned}
\max_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{A}(\theta_0, r_0)\mathbf{w} \\
s.t. \quad & \mathbf{w}^H \mathbf{H}_{\text{PF}}^H \mathbf{H}_{\text{PF}} \mathbf{w} \le I_R \\
& ||\mathbf{w}|| = 1,
\end{aligned} \tag{26a}
$$

$$
\begin{aligned}
\max_{\mathbf{W}} \quad & tr(\mathbf{H}_{\text{PP}} \mathbf{W}\mathbf{W}^H \mathbf{H}_{\text{PP}}^H) \\
s.t. \quad & \mathbf{b}_{\text{BS}}^H(\theta_R)\mathbf{W}\mathbf{W}^H \mathbf{b}_{\text{BS}}(\theta_R) \le I_M \\
& ||\mathbf{W}|| = 1.
\end{aligned} \tag{26b}
$$

where I_R, I_M are pre-values of the thresholds of interference powers for radar and communications, respectively.

In fact, both (26a) and (26b) are belong to quadratically constrained quadratic programming (QCQP) problem, which is a category of NP-hard problems. Nevertheless, an approximate solution of this problem can be obtained by a semi-definite relaxation approach. According to the relaxation steps, the beamfomer of $\mathbf{w}$ and $\mathbf{W}$ can be obtained through interior-point algorithms. To ensure the satisfaction of power constraints, we normalize the obtained approximate beamformers at each iteration.

IV. Numerical Analysis

In this section, numerical results are provided to demonstrate the performance of the proposed joint PA and FDA beamforming by comparing with the traditional PA beamforming. Unless otherwise explicitly stated, we use the following default values for the system parameters. The TX is equipped with a ULA of $N = 32$ antennas. For the communications part, we consider the MS is equipped with a ULA of $M = 4$ receiving antennas. While for the radar part, the main carrier frequency is set to $f_0 = 10$ GHz, the basic frequency offset $f_d = 5$ MHz, the number of elements in the frequency set $L = 5$.

Now, we evaluate the performance of the SINR_{T} and SINR_{MS} w.r.t different power split ratio p and total power P_0. In this simulation, we assume the received noise variances of the MS and radar RX are both equal to 0.1.

To compare the performances of different algorithms and schemes, we separately simulate the performances of the conventional PA method, the joint PA and FDA with different solutions of (19) and (26), and the joint PA and FDA with separated deployment using the algorithm of (26), where half antenna elements are used for the communications and the rest are used for the radar. In this simulation, the total power P_0 is

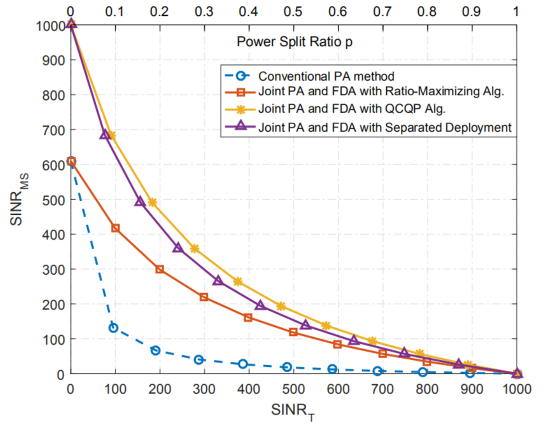

Fig. 2: Joint optimization with $M = 16, P_0 = 100$.

set to 100, and the power split ratio ranges from 0 to 1 with the step of 0.1.

Fig. 2 depicts that the performances of different methods with 16 antennas. The simulations results show that the joint SINR_T and SINR_MS of the proposed FOR-FDA is distinctly better than that of the conventional PA. This is because, on the one hand, that the frequency bandwidths of FDA can be much broader than that of PA, and on the other hand, the overlapped resource rate of radar FDA signals and communication data is fewer than that of the conventional PA scheme. Due to the solution of Ratio-Maximizing algorithm is unstable, the average performances is a little lower than that of the QCQP algorithm. For the separated deployment, due to its few degrees of freedom (DoF) for both communications and radar, the performance is not superior to the shared deployment scheme, although the separated one may generate less interference power budget.

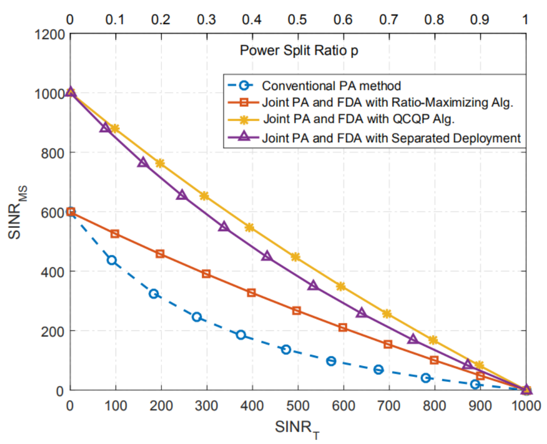

Fig. 3: Joint optimization with $M = 32, P_0 = 100$.

Fig. 3 demonstrates the joint communications and radar optimization with antenna number of 32, while other parameters maintain the same as the previous simulation. Compared with Fig. 2, the curves in Fig. 3 tend to straight lines, which means that with the same power split ratio, Fig. 3 can get higher SINRs for both communications and radar simultaneously than that of Fig. 2. In other words, with more antennas, the RadComm system can harvest more array gains and hence get extra SINRs gains.

V. Conclusion

Comparing with the traditional RadComm system with PA, whose beampattern is only angle-dependent, this work, based on the FDA technology, has proposed a joint PA and FDA RadComm system, whose beampattern is associated with angle, range and time. To obtain the optimal performance for communications and radar, we jointly optimized their SINRs under different power split ratios. Numerical simulations show that the harvested SINR is significantly promoted by the proposed methods compared with the conventional PA scheme.

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