Positional Perturbations Analysis for Micro-UAV Array With Relative Position-Based Formation

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***Abstract*— Recently, micro-UAV swarm-based antenna arrays have been exploited as a novel solution to support sensing and communication applications. However, positional perturbations represent a challenge while deploying the micro-UAV swarm, which may cause deterioration of the UAV array spatial spectrum and limit the array aperture to coherently resolve a target. This letter studies the impacts of positional errors on the direction-of-arrival (DoA) estimation and spatial spectrum of the micro-UAV swarm-based array deployed according to the relative position-based formation, where the perturbations of one UAV may be inherited by its following UAVs. Additionally, we study the beamforming of the micro-UAV swarm-based antenna array either with and without the beamformers' power constraint. Moreover, asymptotic performance has been examined with massive number of micro-UAVs. Several important insights are gained, and are verified through numerical simulations.**

***Index Terms*— UAV swarm, position perturbation, DoA, radar.**

I. Introduction

THE applications of unmanned aerial vehicles (UAVs) in both military and civilian fields have been growing up rapidly over the last decade, i.e., aerial surveillance, remote sensing and rescue operations [1], [2]. Compared with the ground-based and satellite-based sensing, UAVs sensing offers significantly larger coverage than ground-based radars and can conduct finer-grid measurements in comparison with satellite-based radars.

The recent concept of micro-UAV swarm exploited the use of simple, inexpensive sensors distributed across multiple UAVs that are able to work cooperatively to perform large scale operations and tasks. Specifically, micro-UAV swarm can be considered as a large mobile aperture in the air to sense and detect targets with high resolution [3], [4]. By considering each UAV as an array element, a narrow beam pattern by the beamforming of micro-UAV swarm-based array can be achieved [5]. So far, there are two main-stream techniques for controlling the formation of UAV swarms, namely, GPS position-based and relative position-based approaches [6], [7]. However, due to inaccurate GPS position data, especially with rapid flight, it is difficult to carry out an effective formation control for a micro-UAV swarm. Hence, relative position-based approach draws much attentions compared with its GPS position based counterpart.

On the other hand, while relative position-based formation is theoretically possible, there are several critical challenges that need to be tackled first. Since micro-UAVs are incredibly sensitive to the airflow disturbance, it may easily let the micro-UAVs to deviate from their nominal positions. However, with the relative position-based formation, the positional error incurred with the aperture of one UAV can be inherited by its subsequent UAVs, which may greatly deteriorate the beam patterns.

As for the problem of direction-of-arrival (DoA) estimation in the presence of unknown noise, it has been widely studied in lots of literature [8]–[10]. Swindlehurst and Kailath [8] developed the perturbation analysis of the multiple signal classification (MUSIC) under the assumption of independent random perturbations to the array response. Wang [9] investigated the performance of planar phased-array antennas with mechanical errors and analyzed the loss of peak response and the broadening of mainlobe. Pesavento and Gershman [10] derived a deterministic maximum likelihood DoA estimator for multiple sources observed on the background of nonuniform white noise with an arbitrary diagonal covariance matrix. Besides, the problem of phase perturbation has also been studied in the topic of array self-calibration with sensor position errors [11]–[13]. Specifically, [11] proposed a method for DoA estimation in the presence of structured uncertainty in the array parametrization, where the first and second moments of the random perturbation parameters have been known first. Flanagan and Bell [12] considered the problem of array self-calibration and developed a more robust technique for calibration of large sensor perturbation errors which eliminates the small error approximation. The work in [13] described an analytical study of the effects of normally distributed random phase errors on synthetic array performance, which shows that the ratio of main-lobe to sidelobe power is most sensitive to phase errors. However, in all the previous studies, the system models cannot be directly applied to the proposed micro-UAV swarm, for they do not consider the inheritance of positional error from one antenna to another.

In this letter, based on the inheritable positional error model, we derived the expressions of spacial spectra for three classical DoA estimation methods, i.e., matched filter (MF), minimum variance distortionless response (MVDR), and MUSIC, and analyzed the peak power of beampatterns, spatial spectrum, and asymptotic performance under different antenna numbers with and without power constraint. Finally, the achieved results are directly applied to the micro-UAV swarm with uniform linear array (ULA) under both parallel and vertical perturbations with respect to the aperture.

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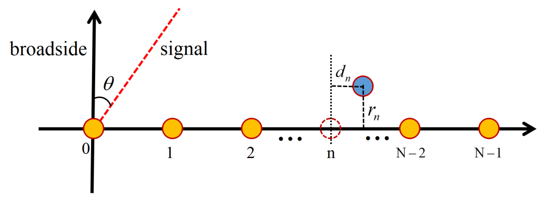


Fig. 1. Demonstration of micro-UAV array perturbation.

The main conclusions out of this work are listed as follows:

1) Positional errors do not generate DoA estimation bias in the expectation of DoA.
2) Position perturbations can deteriorate the spatial spectrum with both peak power and mainlobe width.
3) Spatial spectra evaluated by different estimation methods under perturbations asymptotically converge to the same result.
4) There is a trade-off between the peak power of spatial spectrum and mainlobe width on one hand, and the array gain and array mismatch loss on the other hand.

II. Signal Model

Consider a narrow-band signal from direction θ_0 in the far field impinges on an N-element linear array of micro-UAV swarm with relative position-based formation. Due to the influence of turbulence, we assume that the n-th UAV randomly deviates from its nominal location with an offset of d_n in parallel direction and r_n in vertical direction with respect to the aperture, as shown in Fig.1:

Then, the output from the perturbed array at time t can be modelled as:

$$\mathbf{x}(t) = \bar{\mathbf{a}}(\theta_0)s(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbb{E}|s(t)|^2 = \sigma_s^2$, with $\mathbb{E}(\cdot)$ stands for the expectation. The signal $\mathbf{n}(t)$ is a complex vector of additive white Gaussian noise satisfying $\mathbf{n}(t) \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I})$, where $\mathbf{I}$ refers to the identity matrix. The n-th element of $\bar{\mathbf{a}}$ is $\bar{a}_n = \exp\{jk_0((nd+d_n)\sin(\theta_0)+r_n\cos(\theta_0))\}$, $n = 0, \cdots, N-1$, $k_0 = 2\pi/\lambda_0$, λ_0 is signal wavelength and $d \triangleq \alpha\lambda_0$ is the inter-UAVs spacing, where α denotes a scalar coefficient.

It should be noted that the formation of the micro-UAV array is based on relative-position deployment, which implies that the positional errors are inheritable, i.e., $d_n = d_{n-1}+n_d$, $r_n = r_{n-1}+n_d$, $n = 0, \cdots, N-1$, where n_d denotes the positional noise of an independent UAV, which follows the Gaussian distribution with zero mean and variance of σ_d^2, i.e., $n_d \sim \mathcal{N}(0, \sigma_d^2)$. Then the n-th parallel and vertical positional errors with respect to the aperture satisfy:

$$d_n \sim \mathcal{N}(0, n\sigma_d^2), \quad r_n \sim \mathcal{N}(0, n\sigma_d^2). \tag{2}$$

By choosing a weight vector $\mathbf{w}$ as the beamformer, the output $y(t)$ can be expressed as:

$$y(t) = \mathbf{w}^H\bar{\mathbf{a}}(\theta_0)s(t) + \mathbf{w}^H\mathbf{n}(t), \tag{3}$$

where $(\cdot)^H$ denotes Hermitian operation, and the theoretical covariance matrix of the received signals can be written as:

$$\mathbf{R}_\mathbf{x} = \mathbb{E}_\mathbf{x}[\mathbf{x}(t)\mathbf{x}^H(t)] = \sigma_s^2\bar{\mathbf{a}}(\theta_0)\bar{\mathbf{a}}^H(\theta_0) + \sigma_n^2\mathbf{I}, \tag{4}$$

where $\mathbb{E}_\mathbf{x}(\cdot)$ stands for the expectation with respect to $\mathbf{x}$. Based on the property of joint characteristic function, it turns out that:

$$\mathbb{E}[\exp\{j(a_1X + a_2Y)\}] = \exp\{-\frac{1}{2}(a_1^2\sigma_X^2 + a_2^2\sigma_Y^2)\}, \tag{5}$$

where $X \sim \mathcal{N}(0, \sigma_X^2)$, $Y \sim \mathcal{N}(0, \sigma_Y^2)$, with a_1 and a_2 are real constants. By defining $\boldsymbol{\rho} \triangleq [d_0, \cdots, d_{N-1}, r_0, \cdots, r_{N-1}]$, $s_0 \triangleq \frac{1}{2}k_0^2\sigma_d^2$, we deduce the following formulation in advance:

$$\begin{aligned} F_{\theta_0}(\theta) &\triangleq \mathbb{E}_{\boldsymbol{\rho}}|\mathbf{a}^H(\theta)\bar{\mathbf{a}}(\theta_0)|^2 \\ &= \sum_{n=0}^{N-1}\mathbb{E}_{\boldsymbol{\rho}}[a_n^*(\theta)\bar{a}_n(\theta_0)] \cdot \sum_{m=0}^{N-1}\mathbb{E}_{\boldsymbol{\rho}}[\bar{a}_m^*(\theta_0)a_m(\theta)] \\ &\quad - \sum_{n=0}^{N-1}\mathbb{E}_{\boldsymbol{\rho}}[a_n^*(\theta)\bar{a}_n(\theta_0)] \cdot \mathbb{E}_{\boldsymbol{\rho}}[\bar{a}_n^*(\theta_0)a_n(\theta)] + N \\ &= \frac{1 + e^{-2Ns_0} - 2\cos(2\pi\alpha N(\sin\theta_0 - \sin\theta))e^{-Ns_0}}{1 + e^{-2s_0} - 2\cos(2\pi\alpha(\sin\theta_0 - \sin\theta))e^{-s_0}} \\ &\quad - \frac{1 - e^{-2Ns_0}}{1 - e^{-2s_0}} + N, \end{aligned} \tag{6}$$

where $\mathbf{a}(\theta) \in \mathbb{C}^N$ denotes the ideal steering vector in the direction of θ with the n-th element $a_n = \exp(jk_0\, nd\sin\theta_0)$. It can be easily verified that the maximum of the function $F_{\theta_0}(\theta)$ is achieved at $\theta = \theta_0$, i.e., $\max F_{\theta_0}(\theta) = F_{\theta_0}(\theta_0)$, which can be expressed as:

$$F_{\theta_0}(\theta_0) = \frac{(1 - e^{-Ns_0})^2}{(1 - e^{-s_0})^2} - \frac{1 - e^{-2Ns_0}}{1 - e^{-2s_0}} + N. \tag{7}$$

III. Performance Analysis With Power Constraint

In this section, we evaluate the impacts of positional errors on DoA estimations and spatial spectrum based on three classical estimation methods, namely, MF, MVDR and MUSIC. For fair comparison, all the beamformers are imposed with power constraints.

A. MF-Based DoA Estimation

The main idea of MF is to maximize the signal-to-noise ratio (SNR) for the received signals, which can be formulated as:

$$\max_{\mathbf{w}} \quad \text{SNR} = e_0\frac{\mathbb{E}_\rho|\mathbf{w}^H\bar{\mathbf{a}}(\theta_0)|^2}{||\mathbf{w}^H||^2} \tag{8}$$

where $e_0 = \sigma_s^2/\sigma_n^2$, and $||\cdot||$ denotes Euclidean norm. At the receiver, which has no idea about the position perturbations, has to search the peaks of spatial spectrum by the nominal steering vector, i.e., $\mathbf{w}_{\text{mf}}(\theta) = \frac{1}{\sqrt{N}}\mathbf{a}(\theta)$. Then, the spatial spectrum of MF is given by:

$$P_{\text{mf}}(\theta) = \mathbf{w}_{\text{mf}}^H(\theta)\mathbf{R}_\mathbf{x}\mathbf{w}_{\text{mf}}(\theta) = \frac{\sigma_s^2}{N}F_{\theta_0}(\theta) + \sigma_n^2. \tag{9}$$

According to the properties of $F_{\theta_0}(\theta)$, when $\theta = \theta_0$, the spatial spectrum goes to its maximum value. Hence, the estimated DoA is the same as the real direction:

$$\hat{\theta}_{\text{mf}} = \theta_0, \tag{10}$$

which implies that such positional errors do not lead to a bias for MF-based DoA estimation. However, this does not mean that the positional perturbations are harmless. In fact, (7) shows that even slight positional errors may lead to serious deterioration for spatial spectrum in terms of both peak power and mainlobe width, especially with large number of UAVs. This will be verified clearly in Section V.

Additionally, as the number of UAVs increases, the array gain grows up, and on the other hand, (2) shows that the perturbations variance linearly increases as well. Hence, there is a trade-off between array gain and mismatch loss caused by positional errors regarding the peak power of the spatial spectrum.

Lemma 1: *Suppose that $z = z(x)$ is a real differentiable scalar function of $x \in \mathbb{R}^+$, and its differentiation is denoted as $z' = dz/dx$, the function*

$$g(x, z) = \big(a(1 - e^{-s_0 x})^2 - b(1 - e^{-2s_0 x}) + x\big)/z(x), \quad (11)$$

where $a, b \in \mathbb{R}^+$, achieves a maximum value at:

$$x = -\frac{1}{s_0}\ln h(x, z(x)), \quad (12)$$

where $\ln h(x, z(x))$ is expressed as (14), as shown at the bottom of the page, which can be iteratively solved using:

$$x_{n+1} = -\frac{1}{s_0}\ln h(x_n, z(x_n)), \quad (13)$$

where x_n denotes the value of x at the end of the n-th iteration. □

In this Lemma, (12) can be verified by letting the differentiation of $g(x, z)$ with respect to x equals zero. According to Lemma 1, let $a = (1 - e^{-s_0})^{-2}$, $b = (1 - e^{-2s_0})^{-1}$, $z_{\text{mf}}(N) = N$ and $z'_{\text{mf}}(N) = 1$, then the maximum value of the peak power of MF-based spatial spectrum can be obtained at an UAV number equals $\arg\max_N g(N, z_{\text{mf}}(N))$, which can be iteratively solved and given by:

$$N_{n+1} = -\frac{1}{s_0}\ln h(N_n, z_{\text{mf}}(N_n)). \quad (15)$$

Until now, we have analyzed the unbiased performance of DoA estimation and the peak power of spatial spectrum for MF method.

B. MVDR-Based DoA Estimation

The MVDR beamformer for micro-UAV array with positional perturbations can be obtained through solving:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H(\theta)\mathbf{R_x}\mathbf{w}(\theta) \\ s.t. \quad & \mathbf{w}^H(\theta)\mathbf{a}(\theta) = \sqrt{N}, \end{aligned} \quad (16)$$

with the optimal solution given as:

$$\mathbf{w}_{\text{mvdr}}(\theta) = \frac{\sqrt{N}\mathbf{R_x}^{-1}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{R_x}^{-1}\mathbf{a}(\theta)}. \quad (17)$$

According to the Sherman-Morrison formula, we have

$$\begin{aligned} \mathbf{R_x}^{-1} &= (\sigma_n^2\mathbf{I} + \sigma_s^2\bar{\mathbf{a}}(\theta_0)\bar{\mathbf{a}}^H(\theta_0))^{-1} \\ &= \sigma_n^{-2}(\mathbf{I} + e_0\bar{\mathbf{a}}(\theta_0)\bar{\mathbf{a}}^H(\theta_0))^{-1} \\ &= \sigma_n^{-2}\Big(\mathbf{I} - \frac{e_0\bar{\mathbf{a}}(\theta_0)\bar{\mathbf{a}}^H(\theta_0)}{1 + e_0 N}\Big) \end{aligned} \quad (18)$$

and the "spatial spectrum" of MVDR can be deduced as:

$$\begin{aligned} P_{\text{mvdr}}(\theta) &= \frac{N}{\mathbb{E}_{\mathbf{x},\boldsymbol{\rho}}[\mathbf{a}^H(\theta)\mathbf{R_x}^{-1}\mathbf{a}(\theta)]} \\ &= \frac{1}{\sigma_n^{-2}\Big(1 - \frac{e_0}{N+N^2 e_0}F_{\theta_0}(\theta)\Big)}. \end{aligned} \quad (19)$$

Based on the properties of $F_{\theta_0}(\theta)$, $P_{\text{mvdr}}(\theta)$ can achieve its maximum value at:

$$\hat{\theta}_{\text{mvdr}} = \theta_0, \quad (20)$$

which also implies that the DoA estimation of MVDR is "unbiased" with the existence of positional perturbations. However, it can also be proved that both the peak power and mainlobe width are severely impacted by such positional errors, and there exists a trade-off between the array gain and mismatch loss. Nonetheless, the optimal value N is much smaller than the value obtained with MF-based DoA estimation, which implies the mismatch loss factor in MVDR-based DoA estimation is much stronger than that obtained with MF-based method. Let $z_{\text{mvdr}}(N) = N^2 + e_0^{-1}N$, $z'_{\text{mvdr}}(N) = 2N + e_0^{-1}$, then the maximum value of MVDR-based peak power with perturbations can be obtained as the UAV number equals to $\arg\max_N g(N, z_{\text{mvdr}}(N))$, which can be solved by:

$$N_{n+1} = -\frac{1}{s_0}\ln h(N_n, z_{\text{mvdr}}(N_n)). \quad (21)$$

The above is the contents of the unbiased performance of DoA estimation and the peak power of spatial spectrum for MVDR method.

C. MUSIC-Based DoA Estimation

By denoting $\mathbf{E_n}$ as the noise subspace, and it can be computed as:

$$\mathbf{E_n} = \mathbf{I} - \frac{1}{N}\bar{\mathbf{a}}(\theta)\bar{\mathbf{a}}^H(\theta), \quad (22)$$

which can be easily verified that $\mathbf{E_n}$ is orthogonal to $\bar{\mathbf{a}}(\theta)$. Also, we can find that

$$\mathbf{E_n}\mathbf{E_n}^H = \mathbf{I} - \frac{1}{N}\bar{\mathbf{a}}(\theta)\bar{\mathbf{a}}^H(\theta). \quad (23)$$

By searching the peak of spatial spectrum with $\mathbf{w}_{\text{music}}(\theta) = \frac{1}{\sqrt{N}}\mathbf{a}(\theta)$, the "spatial spectrum" of MUSIC with positional noise can be written as:

$$\begin{aligned} P_{\text{music}}(\theta) &= \frac{1}{\mathbb{E}_{\boldsymbol{\rho}}[\mathbf{w}_{\text{music}}^H(\theta)\mathbf{E_n}\mathbf{E_n}^H\mathbf{w}_{\text{music}}(\theta)]} \\ &= \frac{1}{1 - F_{\theta_0}(\theta)/N^2}. \end{aligned} \quad (24)$$

$$h(x, z) = \frac{a(s_0 z + z') - \sqrt{a^2(s_0 z + z')^2 - (a - b)(2s_0 z + z')((a - b + x)z' - z)}}{(2s_0 z + z')(a + b)} \quad (14)$$

It can still be verified that the MUSIC-based DoA estimation in the presence of UAVs positional errors is also "unbiased":

$$\hat{\theta}_{\text{music}} = \theta_0. \tag{25}$$

However, different from MF and MVDR cases, it can be shown that the MUSIC-based peak power is monotonically decreasing as the number of UAVs increases, which implies that mismatch loss is the dominant factor in the MUSIC-based spatial spectrum.

In conclusion, the positional errors do not lead to estimation bias for DoA estimations based on MF, MVDR and MUSIC algorithms, but deteriorate the peak powers and mainlobes in varying degrees.

IV. Asymptotic Performance Without Power Constraint

Without the beamformers' power constraint, we can focus on the study of asymptotic performance of normalized spatial spectrum.

Lemma 2: *As N grows up, the limitation shape of $F_{\theta_0}(\theta)$ is geometrically similar to the shape of*

$$F_0(\theta) = \frac{1}{1 + e^{-2s_0} - 2e^{-s_0}\cos(k_0 d(\sin\theta - \sin\theta_0))}, \tag{26}$$

which can be mathematically expressed as:

$$F_{\theta_0}(\theta) \dot{\sim} F_0(\theta), \tag{27}$$

where $\dot{\sim}$ denotes "asymptotically similar to".

Proof: When N is large,

$$F_{\theta_0}(\theta) \approx F_0(\theta) + C, \tag{28}$$

where $C = \frac{1}{e^{-2s_0}-1} + N$ is independent of θ. □

From (9), it is obvious that the normalized MF-based spatial spectrum $\hat{P}_{\text{mf}}(\theta)$ is similar to $F_{\theta_0}(\theta)$. Hence, the limitation of the normalized MF-based spectrum with positional noise meets:

$$\hat{P}_{\text{mf}}(\theta) \dot{\sim} F_0(\theta). \tag{29}$$

As for MVDR approach, it can be verified that:

$$P_{\text{mvdr}}(\theta) \dot{\sim} (N^2 + e_0^{-1}N)^{-1} \frac{1}{F_{\theta_0}^{-1}(\theta) - (N^2 + e_0^{-1}N)^{-1}}, \tag{30}$$

while

$$\lim_{N\to\infty} \frac{1}{F_{\theta_0}^{-1}(\theta) - (N^2 + e_0^{-1}N)^{-1}} = F_{\theta_0}(\theta). \tag{31}$$

Hence, the normalized spatial spectrum for MVDR $\hat{P}_{\text{mvdr}}(\theta)$ with positional errors also satisfies:

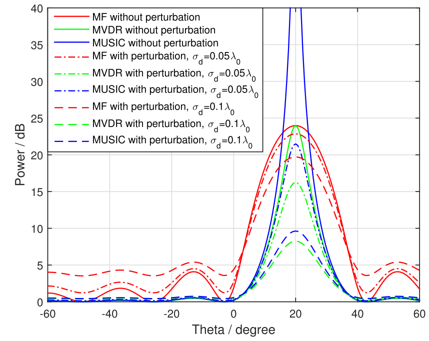
$$\hat{P}_{\text{mvdr}}(\theta) \dot{\sim} F_0(\theta). \tag{32}$$

Finally, as for the MUSIC approach, since

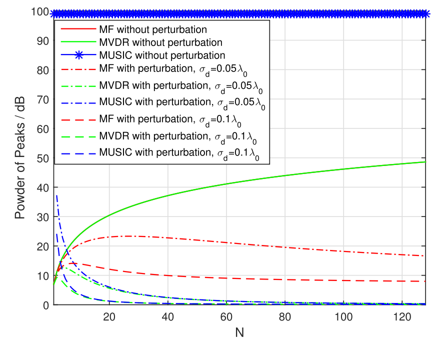
$$\frac{1}{1 - F_{\theta_0}(\theta)/N^2} = 1 + N^{-2} \frac{1}{F_{\theta_0}^{-1}(\theta) - N^{-2}}, \tag{33}$$

while

$$\lim_{N\to\infty} \frac{1}{F_{\theta_0}^{-1}(\theta) - N^{-2}} = F_{\theta_0}(\theta). \tag{34}$$



(a) Spatial spectra, N=10.



(b) Peak powers v.s. N, where the peak powers of MF and MVDR without perturbation are overlapped.

Fig. 2. Spatial spectra with power constraints.

As a result, the normalized MUSIC-based spatial spectrum $\hat{P}_{\text{music}}(\theta)$ with position perturbations also follows

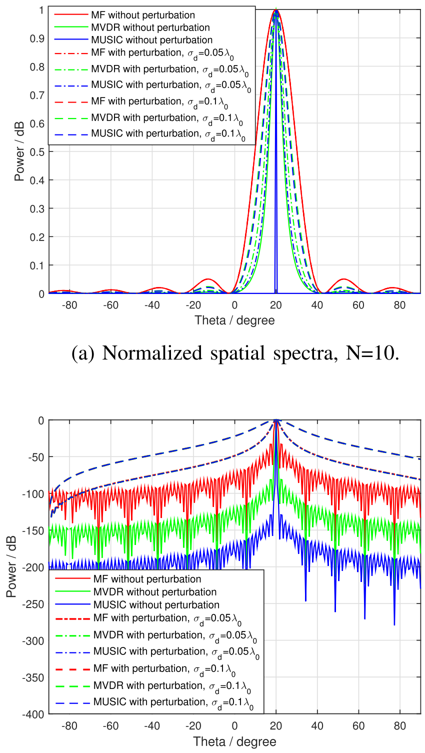
$$\hat{P}_{\text{music}}(\theta) \dot{\sim} F_0(\theta). \tag{35}$$

To sum up, the limitations of normalized spatial spectra based on MF, MVDR and MUSIC under positional noise all asymptotically converge to the same shape $F_0(\theta)$.

V. Numerical Analysis

In this part, we numerically evaluate the performance of the spatial spectrum with and without beamformer power constraint. Assume that the DoA is $\theta_0 = 20°$ and $\sigma_s^2 = \sigma_n^2 = 1$. For the case of power constraint-enabled beamformers, we compare the spatial spectrum of the MF, MVDR, and MUSIC approaches with and without positional perturbations. Assume that the number of UAVs $N = 10$, $\lambda_0 = 1$, $\alpha = 0.5$, then $d = 0.5\lambda_0$. To examine the impacts of DoA estimation from the basic perturbation variance, we compare two sets of $\sigma_d = 0.05\lambda_0,\ 0.1\lambda_0$. Besides, the number of trails for each experiment is 500.

Fig. 2a shows that, compared with the ideal spatial spectrum, the spatial spectrum with positional perturbations are all degraded in terms of both mainlobe width and peak power as a result of the positional error factors in the spatial spectrum formulations, i.e., e^{-s_0} and e^{-Ns_0}. However, e^{-Ns_0} implies that the loss factor is related to the number of UAVs. In order to have an insight on the peak power attenuation under different UAVs number, we draw the curves of peak power versus the number of UAVs N in the range from 1 to 128, as shown in Fig. 2b.



(a) Normalized spatial spectra, N=10.

(b) Asymptotical spatial spectra.

Fig. 3. Spatial spectra without power constraints.

Since the ideal MUSIC-based curve should have infinite peak power level, in Fig. 2b, we use dotted line marked with asterisks to refer to peak power of the MUSIC without perturbations in the top of the figure. Both Fig. 2a and Fig. 2b show that, the change of σ_d from $0.05\lambda_0$ to $0.1\lambda_0$ can lead to significant impacts on the spacial spectrum, especially at large number of antennas.

In Fig. 2b, it is obvious that, compared with the peak power with absence of perturbations, the peak powers of all the three approaches with the presence of perturbations significantly degraded as the number of UAVs increases, especially for the MVDR and MUSIC approaches, which go down very fast with even a small positional variance with only few UAVs. In fact, since the variance of positional noise is proportional to the number of UAVs, as N becomes larger, the matching between the real steering vector and the beamformer is degraded, leading to more serious peak power loss. In addition, we notice that the peak powers of MVDR and MUSIC based spatial spectrum increase first and then decrease as the number of UAVs grows up, which verifies that there exists a trade-off between the array gain and array mismatch loss. Theoretically, the "optimal point" is determined by DoA and the perturbation variance, which can be a reference to decide the number of UAVs in practical scenarios.

With the case of no power constraint, by setting $N = 10$, we normalized all the spatial spectrum values with respect to their maximum values, as shown in Fig. 3a. Apparently, compared with the mainlobes of the absence of perturbations scenario, the mainlobes of MVDR and MUSIC with perturbations become wider. Additionally, to verify the limited performance predicted in IV, we simulate the normalized spatial spectrum given that the number of UAVs N equals 128 in Fig. 3b. It is clear that the spatial spectrum curves of the three approaches have converged to the same shape, which verify the conclusions in Section IV. Similarly, we could conclude that for a slight change of σ_d with small number of antennas, the impacts on the spatial spectrum can be ignored, however, with large number of antennas, the impact becomes more significant.

VI. Conclusion

The impacts of positional perturbations on the DoA estimation and spatial spectrum of micro-UAV swarm-based array deployed by relative position-based formation are analyzed. Theoretically and numerically, it is proven that not all the terms of the performance of spatial spectrum become better as the number of UAVs increases. It turns out that as the number of UAVs increases, the gain of the phased array increases, while the mismatch of array elements also increases, which leads to a trade-off between the peak power of spatial spectrum and mainlobe width. Additionally, it is proven that the normalized spatial spectrum based on different DoA estimation approaches asymptotically converge to the same shape.

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