ACADEMIC REPORT

5G: Hybrid Precoding Design in Large-scale Antenna Arrays

Research Advisor

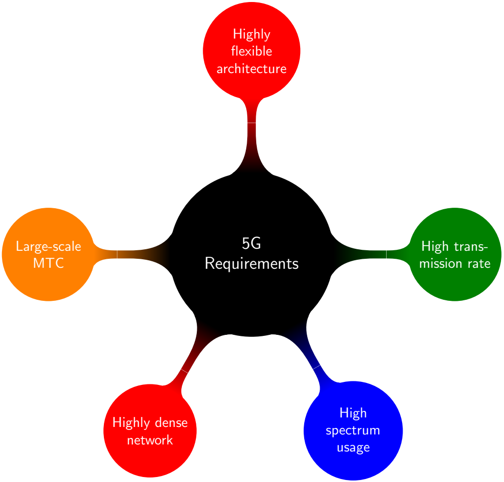
Prof. Suk Chan Kim

Speaker

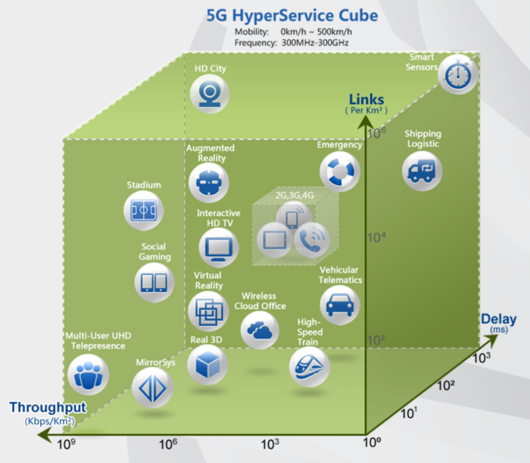
Yongpan Feng

March 9, 2018

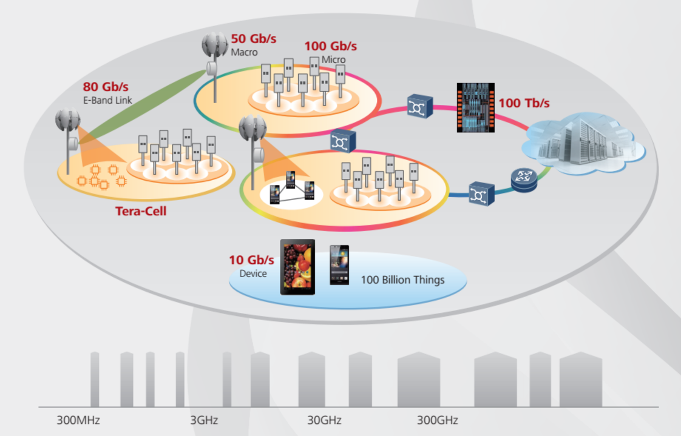
Overivew of 5G



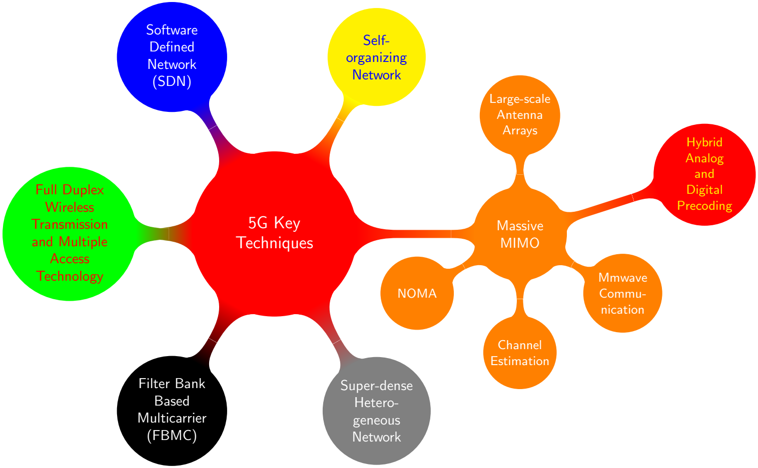
Overivew of 5G



Overivew of 5G



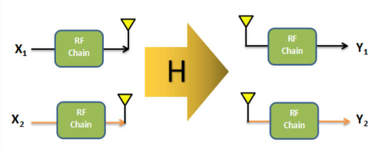
Overivew of 5G



Traditional Precoding

What is precoding and why we need it?

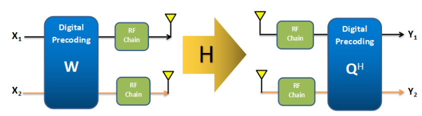
2x2 P2P MIMO



$$\mathbf{y} = \mathbf{H}\mathbf{x}$$

Precoding

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H \Rightarrow \mathbf{y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H x$$



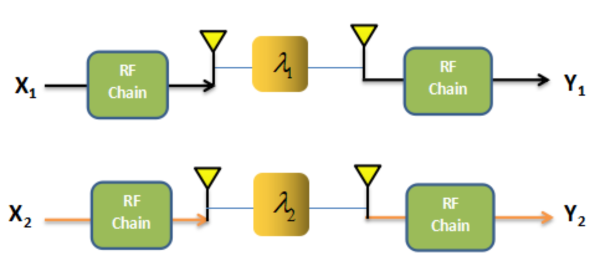
$$\mathbf{y} = \mathbf{Q}^H\mathbf{H}\mathbf{W}\mathbf{x} = \mathbf{Q}^H\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H\mathbf{W}\mathbf{x}$$

Traditional Precoding

Equivlent Model

Set $\mathbf{Q} = \mathbf{U}, \mathbf{V} = \mathbf{W}$, we have

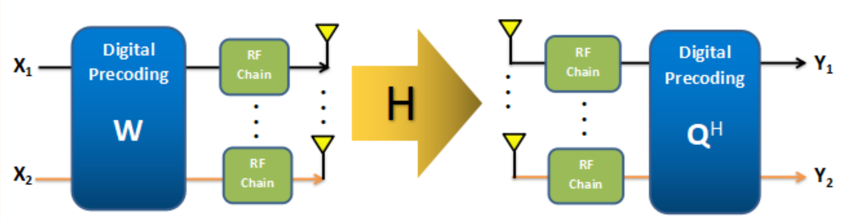
$$\mathbf{y} = \mathbf{\Sigma}\mathbf{x}.$$



Problem

What will happen when antenna number greatly increases ($10^2 \sim 10^3$ order)?
Array gain but also cost.

We note that conventional MIMO precoding requires a dedicated radio frequency (RF) chain for each antenna element, which is prohibitive cost and power consumption for massive MIMO.

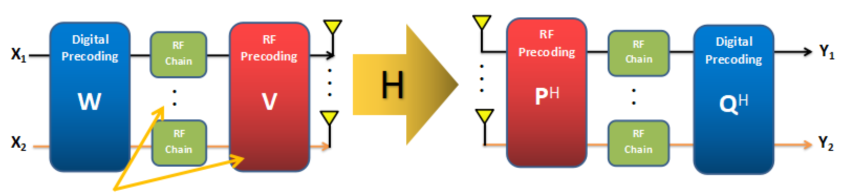


Note: a RF chain includes digital-to-analog conversion, signal mixing and power amplifying.

Hybrid Precoding

To solve this problem, the hybrid analog and digital precoding is proposed [Alkhateeb13, El14, Sohrabi16].

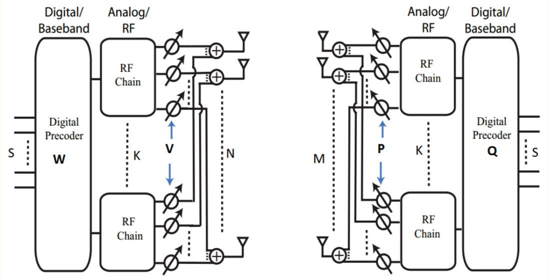
Hybrid Precoding



Hybrid Precoding

Requirement: RF Precoder should be realized only with phase-shifter!

Example: N × M P2P MIMO

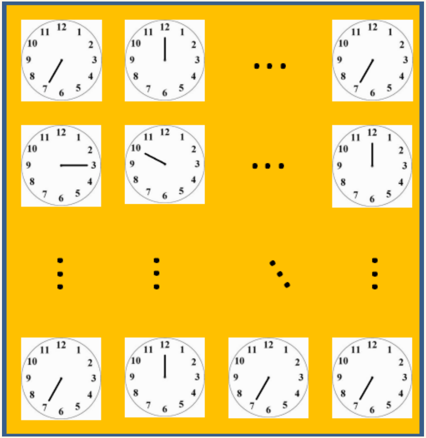


Note

- $S \ll K \ll N$
- $M \gg K \gg S$
- $|V_{ij}| = const$
- $|P_{ij}| = const$

Objective

RF precoder: phase-clock array



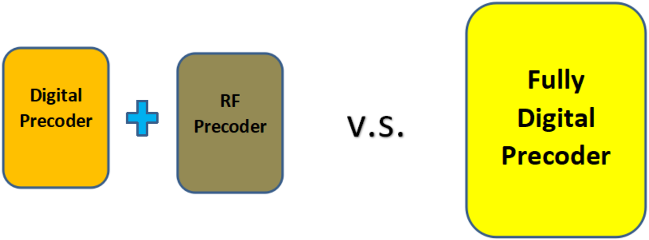
Objective

- Find the optimal phase combination of RF precoder
- Find the optimal digital precoder

Objective of Hybrid Precoding

Challenge

How to achieve or approach the spectrum efficiency using hybrid precoding scheme, or in other word, how to design digital precoder and analog precoder to approach the performance of fully digital precoder?



State of Art

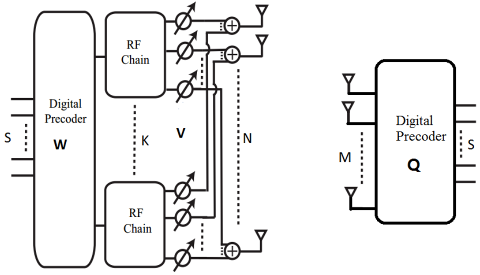
Others' Work

- Liang14 proposed a low-complexity RF precoding scheme, which has a poor performance
- Alkhateeb15 developed iterative algorithms based on Mathing Pursuit (MP), which only limited a certain channel model
- Sohrabi16 raised another iterative algorithm, however it is element-wise updating method, which greatly consume time.

System Model of P2P MIMO

Begin with the simplest case: Single Side Hybrid Precoding in P2P MIMO.

System Model



Note

- $S \ll K \ll N$
- $M \gg S$
- $|V_{ij}| = 1$

System Model of P2P MIMO

The outputs of the receiver can be modeled as

System Model

$$\hat{\mathbf{s}} = \mathbf{Q}^H \mathbf{H}\mathbf{V}\mathbf{W}\mathbf{s} + \mathbf{Q}^H \mathbf{z} \tag{1}$$

- $\mathbf{s} \in \mathbb{C}^{M\times 1}$ is the original signal vector
- $\mathbf{Q} \in \mathbb{C}^{M\times M}$ is the digital precoder of the receiver
- $\mathbf{H} \in \mathbb{C}^{M\times N}$ is the matrix of complex channel
- $\mathbf{z} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_M)$ denotes additive white Gaussian noise.

Problem of Hybrid Precoding

Assuming perfect knowledge of the channel matrix, the precoder design problem for the BS side can be written as

Hybrid Precoding Optimization Problem

$$\begin{aligned} \max_{\mathbf{W},\mathbf{V}} \quad & \log_2 \left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{V}\mathbf{W}\mathbf{W}^H\mathbf{V}^H\mathbf{H}^H\right| \\ s.t. \quad & |V_{n,m}| = 1, \forall n, m \\ & tr(\mathbf{V}\mathbf{W}\mathbf{W}^H\mathbf{V}^H) \leq P \end{aligned} \tag{2}$$

where P is the power budget for the transmitting system.

Problem of RF Precoding

We divide the above problem into two steps: RF precoder design and digital precoder design.

First, by initializing $\mathbf{W}\mathbf{W}^H = \gamma^2 \mathbf{I}$ [Sohrabi16], the RF precoder can be captured by

RF Precoder Design

$$\begin{aligned} \max_{\mathbf{V}} \quad & \log_2 \left| \mathbf{I} + \frac{\gamma^2}{\sigma^2} \mathbf{V}^H \mathbf{H}^H \mathbf{H} \mathbf{V} \right| \\ s.t. \quad & |V_{n,m}| = 1, \forall n, m \end{aligned} \tag{3}$$

where γ^2 is the power of $\mathbf{W}\mathbf{W}^H$

Solutions for RF Precoding

The above problem almost shares the same solution with

RF Precoder Design

$$
\begin{aligned}
\max_{\mathbf{V}} \quad & f = \log_2 |\mathbf{V}^H \mathbf{H}^H \mathbf{H} \mathbf{V}| \\
s.t. \quad & |V_{n,m}| = 1, \forall n, m
\end{aligned}
\tag{4}
$$

Definition

$$
\mathbf{V} \triangleq \exp(i\boldsymbol{\Phi}) \tag{5}
$$

Solutions for RF Precoding

We employ the direction of steepest descent of f in unconstrained space and then map the result to the feasible space. Formally, we named it as matrix complex exponential learning (MCXL), which extends from matrix exponential learning (MXL) [Mertikopoulos15].

RF Precoder Design

$$\mathbf{\Phi}_{n+1} = \mathbf{\Phi}_n + \gamma_n \frac{\mathbf{D}(\mathbf{\Phi}_n)}{||\mathbf{D}(\mathbf{\Phi}_n)||_F}, \tag{6}$$

$$\mathbf{V}_{n+1} = \exp(i\mathbf{\Phi}_{n+1})$$

where $\mathbf{D}(\mathbf{\Phi}) = \frac{\partial f(\mathbf{\Phi})}{\partial \mathbf{\Phi}}$

Definition

$$\begin{aligned}\mathbf{G}_f(\mathbf{\Phi}) &\triangleq \frac{\partial f(\mathbf{\Phi})}{\partial \mathbf{V}} \\ &= |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}|((\mathbf{V}^H\mathbf{H}^H \\ &\quad \mathbf{H}\mathbf{V})^{-1}\mathbf{V}^H\mathbf{H}^H\mathbf{H})^T \\ &= \alpha((\mathbf{H}\mathbf{V})^{-1}\mathbf{H})^T\end{aligned} \tag{7}$$

where $\alpha = |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}|$.

Solutions for RF Precoding

The derivative of the objective function with respect to the variable matrix $\mathbf{\Phi}$, denoted by $\mathbf{D}$ can be obtained as

derivative of $f(\mathbf{\Phi})$ with respect to $\mathbf{\Phi}$

$$
\begin{aligned}
D_{n,m} &\triangleq \frac{\partial f(\mathbf{\Phi})}{\partial \Phi_{n,m}} \\
&= tr\Big(\big(\frac{\partial f(\mathbf{\Phi})}{\partial \mathbf{V}}\big)^T \frac{\partial \mathbf{V}}{\partial V_{n,m}} \Big) \frac{\partial V_{n,m}}{\partial \Phi_{n,m}} \\
&= tr(\mathbf{G}^T \mathbf{P}_{n,m}) i V_{n,m} \\
&= G_{n,m} i V_{n,m}
\end{aligned}
\tag{8}
$$

Solutions for RF Precoding

Then, the directional matrix $\mathbf{D}$ can be obtained easily as follows,

Directional Matrix

$$\mathbf{D}(\mathbf{\Phi}) = i\mathbf{G} \circ \mathbf{V}, \tag{9}$$

where $\circ$ is Hadamard (elementwise) product.

Hybrid Precoding in P2P MIMO

Algorithm of RF Precoder

Require: $\mathbf{H}, \delta_0, \beta$

Initialze: $\mathbf{\Phi}_0 \in \mathbb{R}^{N \times M}$ is a random matrix

while $t \leq T_{threshold}$ and $||\mathbf{D}_t||_F^2 \geq \varepsilon$ **do**

  Calculate $\mathbf{V}_t = \mathbf{e}^{i\mathbf{\Phi}_t}$.

  Calculate $\mathbf{D}_t = i((\mathbf{HV})^{-1}\mathbf{H})^T \circ \mathbf{V}_t$.

  Initialize $\delta_t = \delta_0$.

  // Find the optimal step size.

  $F = f(\mathbf{\Phi}_t)$, $F_1 = f(\mathbf{\Phi}_t + \delta_t \frac{\Re \mathbf{D}_t}{||\mathbf{D}_t||_F^2})$

  while $F < F_1$ **do**

    $\delta_t = \beta \delta_t$,

    $F = F_1$,

    $F_1 = f(\mathbf{\Phi}_t + \delta_t \frac{\Re \mathbf{D}_t}{||\mathbf{D}_t||_F^2})$.

  end while

  Update $\mathbf{\Phi}_{t+1} = \mathbf{\Phi}_t + \delta_t \frac{\Re \mathbf{D}_t}{||\mathbf{D}_t||_F^2}$.

end while

Solutions for Digital Precoding (Waterfilling)

We define the effective channel as

Definition

$$\tilde{\mathbf{H}} = \mathbf{H}\mathbf{V}, \tag{10}$$

The digital precoder can be expressed as

Digital Precoder Design

$$\begin{aligned} \max_{\mathbf{W}} \quad & \log_2 \left| \mathbf{I} + \frac{1}{\sigma^2} \tilde{\mathbf{H}} \mathbf{W} \mathbf{W}^H \tilde{\mathbf{H}}^H \right| \\ s.t. \quad & tr(\mathbf{V}\mathbf{W}\mathbf{W}^H\mathbf{V}^H) \leq P \end{aligned} \tag{11}$$

Solutions for Digital Precoding (Waterfilling)

Solution

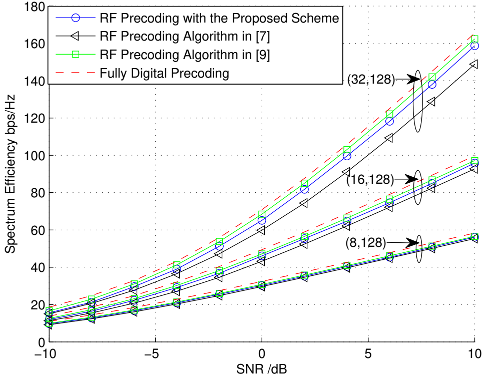
The problem has a well-known waterfilling solution as

$$\mathbf{W} = \mathbf{U}\boldsymbol{\Gamma} \tag{12}$$

where $\mathbf{U}$ is the set of right singular vectors of $\tilde{\mathbf{H}}$, $\boldsymbol{\Gamma}$ is a diagonal matrix.

Numerical Analysis

Simulations



7: Liang14

9: Sohrabi16

Numerical Analysis

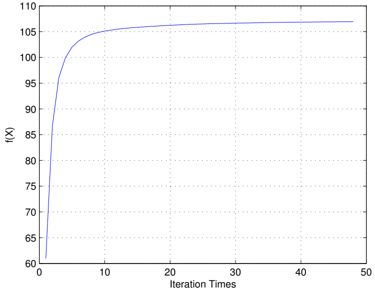
Then the comparisons of computational complexity between the proposed algorithm and the algorithm in [9] is shown as follows:

Computational Complexity Analysis

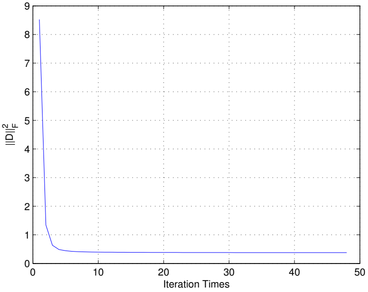
Algorithms	Computational Complexities
Proposed Algorithm	$\mathcal{O}(MN^2 + 2M^3)$
Algorithm in [9]	$\mathcal{O}(4M^2N^2 + 2M^3N + M^4)$

Numerical Analysis

Convergence Analysis



Convergence Analysis



Brief Derivative

Appendices 1

The details of the proof of equation (7) are shown as follows:

$$\begin{aligned}\partial f(\mathbf{\Phi}) &= |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}| tr((\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}\partial(\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})) \\ &= |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}| tr\big((\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}(\partial\mathbf{V}^H)\mathbf{H}^H\mathbf{H}\mathbf{V} \\ &\quad + (\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}\mathbf{V}^H\mathbf{H}^H\mathbf{H}\partial\mathbf{V}\big)\end{aligned} \tag{13}$$

First, the derivative is found with respect to the real part of $\mathbf{V}$

$$\begin{aligned}\frac{f(\mathbf{\Phi})}{\partial\Re\mathbf{V}} &= |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}| \cdot tr\Big(\frac{(\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}(\partial\mathbf{V}^H)\mathbf{H}^H\mathbf{H}\mathbf{V}}{\partial\Re\mathbf{V}} \\ &\quad + \frac{(\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}\mathbf{V}^H\mathbf{H}^H\mathbf{H}\partial\mathbf{V}}{\partial\Re\mathbf{V}}\Big) \\ &= |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}|\big(\mathbf{H}^H\mathbf{H}\mathbf{V}(\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1} \\ &\quad + ((\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}\mathbf{V}^H\mathbf{H}^H\mathbf{H})^T\big)\end{aligned} \tag{14}$$

Brief Derivative

Appendices 2

In addition, the derivative is found with respect to the imaginary part of $\mathbf{V}$

$$
\begin{aligned}
i\frac{f(\boldsymbol{\Phi})}{\partial\Im\mathbf{V}} &= |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}| \cdot tr\Big(i\frac{(\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}(\partial\mathbf{V}^H)\mathbf{H}^H\mathbf{H}\mathbf{V}}{\partial\Im\mathbf{V}} \\
&\quad + i\frac{(\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}\mathbf{V}^H\mathbf{H}^H\mathbf{H}\partial\mathbf{V}}{\partial\Im\mathbf{V}}\Big) \\
&= |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}|\big(\mathbf{H}^H\mathbf{H}\mathbf{V}(\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1} \\
&\quad - ((\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V})^{-1}\mathbf{V}^H\mathbf{H}^H\mathbf{H})^T\big)
\end{aligned} \tag{15}
$$

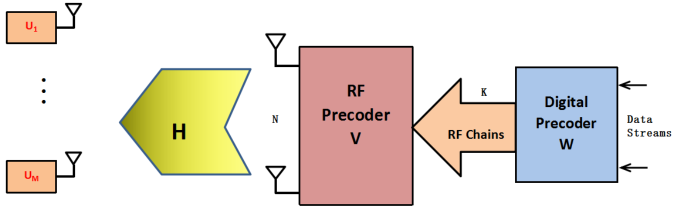
Hence, derivative yields

$$
\begin{aligned}
\frac{\partial f(\boldsymbol{\Phi})}{\partial\mathbf{V}} &= \frac{1}{2}\Big(\frac{\partial f(\boldsymbol{\Phi})}{\partial\Re\mathbf{V}} - i\frac{\partial f(\boldsymbol{\Phi})}{\partial\Im\mathbf{V}}\Big) \\
&= |\mathbf{V}^H\mathbf{H}^H\mathbf{H}\mathbf{V}|((\mathbf{H}\mathbf{V})^{-1}\mathbf{H})^T
\end{aligned} \tag{16}
$$

MU-MISO

Consider a narrowband downlink MU-MISO system as follows,

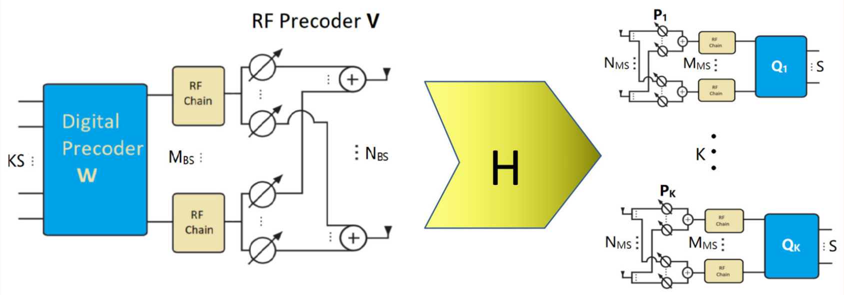
System Model



$$y_m = \mathbf{h}_m^T \mathbf{V} \mathbf{w}_m s_m + \mathbf{h}_m^T \sum_{l \neq m} \mathbf{V} \mathbf{w}_l s_l + z_m, \quad (17)$$

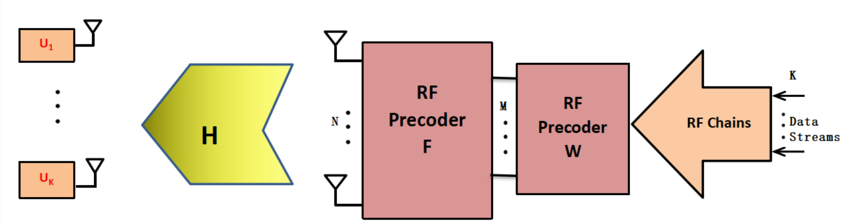
MU-MIMO

System Model



Double RF-Precoding Model

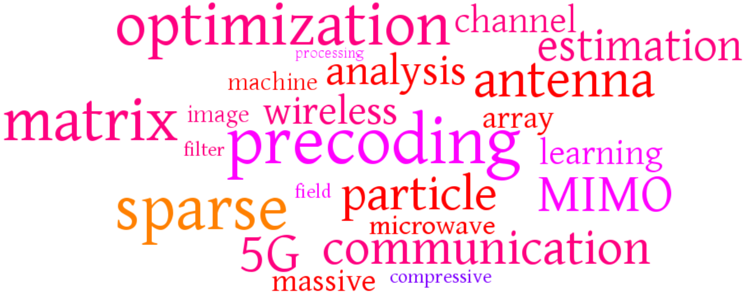
System Model



$$y_k = \mathbf{h}_k^T \mathbf{F} \mathbf{w}_k s_k + \mathbf{h}_k^T \sum_{l \neq k} \mathbf{F} \mathbf{w}_l s_l + z_k, \tag{18}$$

Cooperation

Research Interests



Email: fypan4study@gmail.com
Tel: +82 010-2749-9453