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On Proportional Fairness in Power Allocation for Two-Tone Spectrum-Sharing Networks

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***Abstract*—To efficiently trade off system sum rate and link fairness, power allocation in wireless spectrum-sharing networks often concentrates upon proportional fairness. The corresponding problem has been proved to be convex for a single tone but NP-hard for more than two tones. However, in the two-tone case, the complexity of the problem for achieving proportional fairness has not been addressed yet. In this paper, we prove that the issue of proportional-fairness optimization for the two-tone situation is NP-hard by reducing the problem of finding the maximum independent set in an undirected graph to it. Moreover, a computationally efficient algorithm is proposed to provide an efficient suboptimal solution. Simulation results are presented to illustrate the effectiveness of our proposal.**

***Index Terms*—Difference of two concave functions, interference, NP-hard, power allocation, proportional fairness, spectrum-sharing network.**

I. Introduction

The broadcast feature of wireless communications gives rise to serious cochannel interference, which significantly limits the system performance in wireless spectrum-sharing networks [1]. In such networks in which many links share a fixed number of frequency bands (tones), power allocation plays an important role in managing interference, increasing the network sum rate and enhancing link fairness [2].

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As an important resource allocation issue in wireless networks, power allocation for sum-rate maximization has received extensive attention. It has been shown by Luo and Zhang [3] that this problem is NP-hard for any fixed number of tones. Therefore, one can design either high-complexity algorithms to obtain global optimality [1], [4] or low-complexity approaches to attain local optimality [2], [5]–[7]. On the other hand, to achieve a high level of fairness, a common practice is to maximize the minimum rate of all links, which is known as max-min fairness. Luo and Zhang [3] have proved that this problem is convex when there is only one tone and is NP-hard when the number of tones is more than two [3]. In the case of two tones, the minimum-rate maximization problem can be transformed into a univariate optimization problem and is thus easy to tackle [3].

Due to the diversity of different channels, sum-rate maximization often prefers high-quality links and allocates more power to them. In other words, this has the effect of discriminating low-quality links and results in low fairness. On the other hand, to maximize the minimum rate of all links, more power should be assigned to low-quality links, and meanwhile, less power should be allocated to high-quality links so that the interference to other links can be maintained at a low level, which considerably limits the sum rate [8]. To make a trade-off between sum rate and link fairness, numerous works have been dedicated to maximizing the sum utility of all links whereby each link's utility is defined as the logarithm of its rate. This is referred to as proportional fairness [9], [10]. Luo and Zhang [3] have shown that this problem is convex when there is only one tone and is NP-hard when there are more than two tones. However, for the two-tone case, the complexity of this problem is still unknown.

This paper proves that the power allocation problem for sum-log-utility maximization in the two-tone case is NP-hard. In particular, we establish an equivalence between this problem and that of finding the maximum independent set in an undirected graph, which is known to be NP-hard. In general, it is impossible to offer a polynomial-time algorithm to achieve global optimality unless P = NP. To circumvent this issue, we devise a computationally efficient algorithm in this paper to find suboptimal points.

The rest of this paper is organized as follows. Section II describes the system model and formulates the power allocation problem. Section III proves the NP-hardness of the problem. In Section IV, an efficient algorithm is developed to handle the NP-hard problem. Simulation results are presented in Section V. In addition, the applications and the extensions of this work are discussed in Section VI. Finally, Section VII concludes this paper.

II. System Model and Problem Formulation

Consider a spectrum-sharing network, in which two frequency tones are available for K transmitter–receiver pairs (links). For achieving high spectrum efficiency, all K links are allowed to access the two tones. Let $h_{k,j}^n$ be the channel fading coefficient from the transmitter of link j to the receiver of link k at tone n.

A slow-fading Rayleigh channel is assumed. In particular, $h_{k,j}^n$ ($k, j = 1, 2, \ldots, K$; $n = 1, 2$) are independent and identically distributed with $h_{k,j}^n \sim \mathcal{CN}(0, 1)$ and remain constant during the period considered. Denote by x_k^n the complex Gaussian signal transmitted from link k at tone n. For an additive white Gaussian noise channel, the received signal of link k at tone n is

$$y_k^n = z_k^n + \sum_{j=1}^{K} h_{k,j}^n x_j^n,\ n = 1, 2,\ k = 1, 2, \ldots, K \tag{1}$$

where z_k^n is the complex Gaussian noise of link k at tone n distributed with zero mean and variance N_k^n, i.e., $z_k^n \sim \mathcal{CN}(0, N_k^n)$. The rate of link k at tone n, which is measured in nats per second per hertz, is given by the Shannon formula as

$$R_k^n(\boldsymbol{p}) = \ln\left(1 + \frac{\left|h_{k,k}^n\right|^2 p_k^n}{N_k^n + \sum_{j\neq k}\left|h_{k,j}^n\right|^2 p_j^n}\right)$$
$$n = 1, 2, \quad k = 1, 2, \ldots, K \tag{2}$$

where $\ln(x)$ is the natural logarithm of x, $p_k^n = \mathbb{E}\{|x_k^n|^2\}$ is the transmit power of link k at tone n, and $\boldsymbol{p} = (p_1^1, p_2^1, \ldots, p_K^1, p_1^2, p_2^2, \ldots, p_K^2)^T$ is the power allocation vector.

For ease of expression, we normalize the channel coefficients and denote the rate of link k at tone n as

$$R_k^n(\boldsymbol{p}) = \ln\left(1 + \frac{p_k^n}{\sigma_k^n + \sum_{j\neq k}\alpha_{k,j}^n p_j^n}\right)$$
$$n = 1, 2, \quad k = 1, 2, \ldots, K \tag{3}$$

where $\sigma_k^n = N_k^n/|h_{k,k}^n|^2$ is the normalized power of the background noise, and $\alpha_{k,j}^n = |h_{k,j}^n|^2/|h_{k,k}^n|^2$ is the normalized crosstalk coefficient from link j to link k at tone n. Now, we get the overall rate of link k as

$$R_k(\boldsymbol{p}) = R_k^1(\boldsymbol{p}) + R_k^2(\boldsymbol{p}). \tag{4}$$

The power allocation problem with the aim to achieve proportional fairness is formulated as

$$\max_{\boldsymbol{p}} \quad \sum_{k=1}^{K} \ln\left(R_k(\boldsymbol{p})\right) \tag{5a}$$
$$\text{s.t.} \quad p_k^1, p_k^2 \geq 0, k = 1, 2, \ldots, K \tag{5b}$$
$$p_k^1 + p_k^2 \leq P_k, k = 1, 2, \ldots, K \tag{5c}$$

where $\boldsymbol{p}$ is the optimization variable. Here, constraint (5c) indicates that the total transmit power of link k cannot exceed its budget P_k. We should point out that there is no constraint for guaranteeing the links' rates. However, the optimal rate of each link cannot be too small, because the objective value approaches negative infinity when the rate tends to zero. This model can be utilized for best-effort traffic, such as file transfer.

It can be seen that problem (5) involves a complex nonconvex objective function that creates great challenges in solving it. In fact, problem (5) is NP-hard in general cases, as will be shown in the following section.

III. Complexity Analysis

Here, we establish the NP-hardness of problem (5) by constructing a reduction from the maximum independent set (MIS) problem in an undirected graph to problem (5), where the MIS problem is known to be NP-hard.

Before the complexity analysis, it is necessary to present some preliminaries in graph theory. Thereafter, we first introduce the MIS problem as well as a special case of problem (5) and then reduce the former problem to the latter problem.

A. Preliminaries

The definitions of graphs and independent sets are given as follows [11].

Definition 1 (Graph): A graph $G = (\mathcal{V}, \mathcal{E})$ is composed of a finite set of vertices $\mathcal{V}$ and a finite set of edges $\mathcal{E}$, where each edge represents an unordered pair of vertices, i.e., $\mathcal{E} \subseteq \{(u, v) : u, v \in \mathcal{V}, u \neq v\}$.

Note that two nodes $u \in \mathcal{V}$ and $v \in \mathcal{V}$ $(u \neq v)$ are said to be connected if and only if $(u, v) \in \mathcal{E}$.

Definition 2 (Independent Set): An independent set of graph $G = (\mathcal{V}, \mathcal{E})$ is a set of vertices $\mathcal{S} \subseteq \mathcal{V}$ such that no two nodes in $\mathcal{S}$ are connected: for any $u, v \in \mathcal{S}$, $(u, v) \notin \mathcal{E}$.

For graph G, an MIS is an independent set of the largest possible size. The problem of finding such a set is called the MIS problem, which is an NP-hard optimization problem.

B. Problem Constructions

On one hand, let us establish a graph $G = (\mathcal{V}, \mathcal{E})$ in which the number of nodes $|\mathcal{V}|$ is equal to K, which is the same as the number of links in the considered network. Such a construction of graph implies that the nodes in G can be one-to-one mapped to the links in the network. In particular, node k in the graph corresponds to link k in the network.

On the other hand, with parameter settings $\alpha_{k,j}^2 = 1$, $\sigma_k^1 = 1$, $\sigma_k^2 = 4$, and $P_k = 1$ for k and j, we consider a special case of problem (5) given as

$$\max_{\boldsymbol{p}} \sum_{k=1}^{K} \ln\left(\ln\left(1 + \frac{p_k^1}{1 + \sum_{j\neq k}\alpha_{k,j}^1 p_j^1}\right) + \ln\left(1 + \frac{p_k^2}{4 + \sum_{j\neq k} p_j^2}\right)\right) \tag{6a}$$
$$\text{s.t.} \quad p_k^1, p_k^2 \geq 0, k = 1, 2, \ldots, K \tag{6b}$$
$$p_k^1 + p_k^2 \leq 1, k = 1, 2, \ldots, K \tag{6c}$$

where $\alpha_{k,j}^1$ for $k \neq j$ is set as

$$\alpha_{k,j}^1 = \alpha_{j,k}^1 = \begin{cases} \infty, & (k, j) \in \mathcal{E} \\ 0, & (k, j) \notin \mathcal{E}. \end{cases} \tag{7}$$

Notice that parameters $\alpha_{k,j}^1$ defined in (7) bridges graph G and problem (6). Particularly, each one of links k and j results in infinite interference to the other if nodes k and j in graph G are connected.

C. Problem Reduction

Here, we shall show that the MIS problem in graph G can be reduced to problem (6), i.e., a solution to problem (6) yields a solution to the MIS problem.

For ease of expression, the optimal solution to problem (6) is denoted by $\boldsymbol{p}^* = ({p_1^1}^*, {p_2^1}^*, \ldots, {p_K^1}^*, {p_1^2}^*, {p_2^2}^*, \ldots, {p_K^2}^*)^T$. Meanwhile, the optimal rate of link k is represented by R_k^*. We have the following proposition.

Proposition 1: The set $\mathcal{I} = \{k | {p_k^1}^* > 0\}$ is an independent set in graph G.

Proof: We prove this by contradiction. Suppose that there exist k and i such that ${p_k^1}^* > 0$, ${p_i^1}^* > 0$, and $(k, i) \in \mathcal{E}$. Since $\alpha_{k,i}^1 = \alpha_{i,k}^1 = \infty$, we have

$$R_k^1 = \ln\left(1 + \frac{{p_k^1}^*}{1 + \alpha_{k,i}^1 {p_i^1}^* + \sum\limits_{j\neq k, j\neq i}\alpha_{k,j}^1 {p_j^1}^*}\right) = 0$$

$$R_i^1 = \ln\left(1 + \frac{{p_i^1}^*}{1 + \alpha_{i,k}^1 {p_k^1}^* + \sum\limits_{j\neq i, j\neq k}\alpha_{i,j}^1 {p_j^1}^*}\right) = 0.$$

This implies that for all nodes in $\mathcal{I} = \{k|p_k^{1*} > 0\}$, only the isolated nodes in graph G get a positive rate at tone 1. Let $\mathcal{I}' \subset \mathcal{I}$ be the set composed of all the isolated nodes in G. Then, links in $\mathcal{I}'$ have a positive rate at tone 1, whereas links in $\mathcal{I} - \mathcal{I}'$ have a zero rate at tone 1. We arbitrarily select $|\mathcal{I} - \mathcal{I}'| - 1$ links in $\mathcal{I} - \mathcal{I}'$ and set their power values to zero. Then, the remaining link in $\mathcal{I} - \mathcal{I}'$ is not interfered by other links and will get a positive rate at tone 1. Thus, the objective value increases, which means that $\boldsymbol{p}^*$ is not optimal. This is a contradiction, and the proof is completed. □

Based on Proposition 1, the following proposition can be obtained.

Proposition 2: The set $\mathcal{I} = \{k|p_k^{1*} > 0\}$ is the same as the set $\{k|p_k^{1*} = 1\}$.

Proof: Suppose that $\mathcal{I} = \{k|p_k^{1*} > 0\}$ is given. According to Proposition 1, the rate of link k in $\mathcal{I}$ at tone 1 is $R_k^1 = \ln(1 + p_k^1)$. Then, the optimal power allocation can be determined by solving the problem given by

$$\max_{\boldsymbol{p}} \sum_{k\in\mathcal{I}} \ln\left(\ln\left(1+p_k^1\right) + \ln\left(1 + \frac{p_k^2}{4+\sum_{j\neq k} p_j^2}\right)\right)$$

$$+ \sum_{k\in\mathcal{V}-\mathcal{I}} \ln\left(\ln\left(1 + \frac{p_k^2}{4+\sum_{j\neq k} p_j^2}\right)\right) \tag{8a}$$

$$\text{s.t.} \quad p_k^1, p_k^2 \geq 0, k \in \mathcal{V} \tag{8b}$$

$$p_k^1 + p_k^2 \leq 1, k \in \mathcal{V} \tag{8c}$$

$$p_k^1 = 0, k \in \mathcal{V} - \mathcal{I}. \tag{8d}$$

We will prove in the following that for problem (8), each link in $\mathcal{I}$ has the optimal power of 1 at tone 1, i.e., $p_k^{1*} = 1$ for all $k \in \mathcal{I}$. In particular, for link $k \in \mathcal{I}$, the following two scenarios are considered.

- $p_k^{2*} = 0$. In this case, we can directly get $p_k^{1*} = 1$ because the increment of p_k^1 will monotonically increase link k's rate at tone 1 and induce no interference to other links, as can be seen from the objective function in (8).
- $p_k^{2*} > 0$. In this situation, we have $p_k^{1*} < 1$ according to (8c). Now, let us consider another power allocation $\boldsymbol{p}' = (p_1^{1*}, p_2^{1*}, \ldots, p_{k-1}^{1\ *}, 1, p_{k+1}^{1\ *}, \ldots, p_K^{1*}, p_1^{2*}, p_2^{2*}, \ldots, p_{k-1}^{2\ *}, 0, p_{k+1}^{2\ *}, \ldots, p_K^{2*})$. Accordingly, the rate of link k resulting from $\boldsymbol{p}'$ is denoted by R'_k. For all $i \in \mathcal{V} - \mathcal{I}$, we have

$$R_i^* = \ln\left(1 + \frac{p_i^{2*}}{4 + p_k^{2*} + \sum_{j\neq i, j\neq k} p_j^{2*}}\right)$$

$$< \ln\left(1 + \frac{p_i^{2*}}{4 + \sum_{j\neq i, j\neq k} p_j^{2*}}\right) = R'_i. \tag{9}$$

For all $i \in \mathcal{I}$ $(i \neq k)$, we obtain

$$R_i^* = \ln\left(1+p_i^{1*}\right) + \ln\left(1 + \frac{p_i^{2*}}{4 + p_k^{2*} + \sum_{j\neq i, j\neq k} p_j^{2*}}\right)$$

$$< \ln\left(1+p_i^{1*}\right) + \ln\left(1 + \frac{p_i^{2*}}{4 + \sum_{j\neq i, j\neq k} p_j^{2*}}\right) = R'_i. \tag{10}$$

For link k, it follows that

$$\begin{aligned} R_k^* &= \ln\left(1+p_k^{1*}\right) + \ln\left(1 + \frac{p_k^{2*}}{4+\sum_{j\neq k} p_j^{2*}}\right) \\ &\leq \ln\left(1+p_k^{1*}\right) + \ln\left(1 + \frac{p_k^{2*}}{4}\right) \\ &\overset{①}{\leq} \ln\left(\left(1+p_k^{1*}\right)\left(1 + \frac{1-p_k^{1*}}{4}\right)\right) \\ &= \ln\left(-\frac{1}{4}\left(p_k^{1*}\right)^2 + p_k^{1*} + \frac{5}{4}\right) < \ln 2 = R'_k \end{aligned} \tag{11}$$

where inequality ① comes from the power budget constraint $p_k^{1*} + p_k^{2*} \leq 1$. According to (9)–(11), we can see that the objective function value corresponding to $\boldsymbol{p}'$ is greater than that of $\boldsymbol{p}^*$. This is a contradiction, and the case $p_k^{2*} > 0$ does not exist.

The given analysis shows that $\mathcal{I} = \{k|p_k^{1*} > 0\} = \{k|p_k^{1*} = 1\}$. □

According to Proposition 1 and Proposition 2, the optimal power of each link at tone 1 is either 0 or 1, and the set $\mathcal{I} = \{k|p_k^{1*} = 1\}$ is an independent set in graph G. Now, we can get the following proposition.

Proposition 3: Given the independent set $\mathcal{I} = \{k|p_k^{1*} = 1\}$ in graph G, the optimal value of problem (6) is

$$|\mathcal{I}| \ln(\ln(2)) + (K - |\mathcal{I}|) \ln\left(\ln\left(\frac{K - |\mathcal{I}| + 4}{K - |\mathcal{I}| + 3}\right)\right). \tag{12}$$

Proof: According to the property of set $\mathcal{I}$, we have $p_k^{1*} = 1$ and $p_k^{2*} = 0$ for $k \in \mathcal{I}$ and $p_k^{1*} = 0$ for $k \in \mathcal{V} - \mathcal{I}$. Therefore, we only need to determine p_k^{2*} for $k \in \mathcal{V} - \mathcal{I}$. The optimization of p_k^{2*} for $k \in \mathcal{V} - \mathcal{I}$ can be formulated as

$$\max_{p_k^2} \sum_{k\in\mathcal{V}-\mathcal{I}} \ln\left(\ln\left(1 + \frac{p_k^2}{4+\sum_{j\in\mathcal{V}-\mathcal{I}, j\neq k} p_j^2}\right)\right) \tag{13a}$$

$$\text{s.t.} \quad 0 \leq p_k^2 \leq 1, k \in \mathcal{V} - \mathcal{I}. \tag{13b}$$

Although the given problem has a nonconcave objective function, it can be transformed into a convex optimization problem [3]. To get the explicit expression of the optimal solution, we decompose this problem as

$$\max_{c} \quad f(c) \tag{14a}$$

$$\text{s.t.} \quad 0 \leq c \leq K - |\mathcal{I}| \tag{14b}$$

where $f(c)$ is given by

$$f(c) = \max_{p_k^2} \sum_{k\in\mathcal{V}-\mathcal{I}} \ln\left(\ln\left(1 + \frac{p_k^2}{4+\sum_{j\in\mathcal{V}-\mathcal{I}, j\neq k} p_j^2}\right)\right) \tag{15a}$$

$$\text{s.t.} \quad 0 \leq p_k^2 \leq 1, k \in \mathcal{V} - \mathcal{I} \tag{15b}$$

$$\sum_{k\in\mathcal{V}-\mathcal{I}} p_k^2 = c. \tag{15c}$$

For problem (15), we can transform the objective function into

$$\sum_{k\in\mathcal{V}-\mathcal{I}} \ln\left(\ln\left(\frac{4+c}{4+c-p_k^2}\right)\right). \tag{16}$$

The function in (16) is concave over the optimization variable because the second-order derivative of $\ln(\ln((4+c)/(4+c-p_k^2)))$ with respect to p_k^2 is

$$\left(\ln\left(\frac{4+c}{4+c-p_k^2}\right)-1\right)\left(\left(4+c-p_k^2\right)\ln\left(\frac{4+c}{4+c-p_k^2}\right)\right)^{-2} \quad (17)$$

which is less than zero for $0 \leq c \leq K - |I|$ and $0 \leq p_k^2 \leq 1$. Therefore, problem (15) has a convex form, and we can utilize Karush–Kuhn–Tucker (KKT) conditions to obtain its global optimality. That is, λ_k, μ_k, ν, and p_k^2, for all $k \in \mathcal{V} - \mathcal{I}$ satisfy the KKT conditions given in (18)–(21), (15b), and (15c).

$$\lambda_k, \mu_k \geq 0 \quad (18)$$

$$\lambda_k p_k^2 = 0 \quad (19)$$

$$\mu_k\left(1 - p_k^2\right) = 0 \quad (20)$$

$$\left(4+c-p_k^2\right)^{-1}\ln^{-1}\left(\frac{4+c}{4+c-p_k^2}\right)+\lambda_k-\mu_k+\nu=0. \quad (21)$$

As a result, $p_k^{2*} = c/(K - |\mathcal{I}|)$ for all $k \in \mathcal{V} - \mathcal{I}$ is the primal optimal point, and $(\lambda_k^*, \mu_k^*, \nu^*)$ for all $k \in \mathcal{V} - \mathcal{I}$ is the dual optimal point, where $\lambda_k^* = \mu_k^* = 0$, and ν^* is given by

$$\nu^* = -\left(4+\frac{K-|\mathcal{I}|-1}{K-|\mathcal{I}|}c\right)^{-1} \times \ln^{-1}\left(\frac{(K-|\mathcal{I}|)(4+c)}{4(K-|\mathcal{I}|)+(K-|\mathcal{I}|-1)c}\right). \quad (22)$$

Then, we get

$$f(c)=(K-|\mathcal{I}|)\ln\left(\ln\left(\frac{(K-|\mathcal{I}|)c+4(K-|\mathcal{I}|)}{(K-|\mathcal{I}|-1)c+4(K-|\mathcal{I}|)}\right)\right). \quad (23)$$

Since $f(c)$ is a monotonically increasing function of c, the optimal solution and the maximum value of problem (14) are $c^* = K - |\mathcal{I}|$ and $(K - |\mathcal{I}|)\ln(\ln((K - |\mathcal{I}| + 4)/(K - |\mathcal{I}| + 3)))$, respectively. In addition, because the rate of each link in $\mathcal{I}$ is $\ln 2$, the proof is finished. □

Utilizing Proposition 3, we characterize the complexity of problem (5) in the following theorem.

Theorem 1: Problem (6) is NP-hard.

Proof: According to Proposition 3, the optimization of problem (6) is equivalent to

$$\max_{\mathcal{I}} \quad |\mathcal{I}|\ln(\ln(2)) + (K-|\mathcal{I}|)\ln\left(\ln\left(\frac{K-|\mathcal{I}|+4}{K-|\mathcal{I}|+3}\right)\right) \quad (24a)$$

$$\text{s.t.} \quad \mathcal{I} \text{ is an independent set in graph } G. \quad (24b)$$

Notice that the derivative of the objective function with respect to $|\mathcal{I}|$ is

$$\ln^{-1}\left(\frac{K-|\mathcal{I}|+4}{K-|\mathcal{I}|+3}\right)\cdot\frac{K-|\mathcal{I}|}{(K-|\mathcal{I}|+4)(K-|\mathcal{I}|+3)} + \ln\left(\ln 2\cdot\ln^{-1}\left(\frac{K-|\mathcal{I}|+4}{K-|\mathcal{I}|+3}\right)\right) \quad (25)$$

which is greater than zero because $1 < (K-|\mathcal{I}|+4)/(K-|\mathcal{I}|+3) \leq 4/3$ for $1 \leq |\mathcal{I}| < K$. This indicates that the objective function of problem (24) is increasing in $|\mathcal{I}|$ for $1 \leq |\mathcal{I}| \leq K$. As a result, the optimal solution to problem (24) is the size of the MIS in graph G. Because the MIS problem in a graph is NP-hard, so is problem (6). □

According to Theorem 1, a special case of problem (5) is NP-hard, which gives rise to the NP-hardness of its general cases. Therefore, we cannot globally solve problem (5) in polynomial time in the global optimal sense unless P = NP. To this end, we shall devise a computationally efficient algorithm that is able to provide a suboptimal but efficient solution to this problem in the following section.

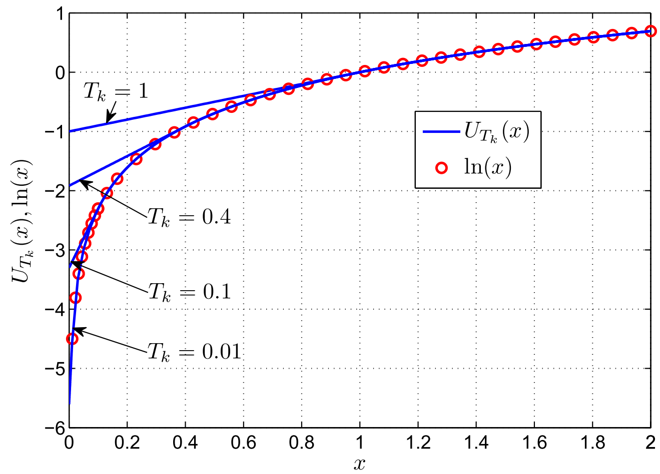

Fig. 1. Curves of $U_{T_k}(x)$ and $\ln(x)$.

IV. Proposed Algorithm to Solve Problem (5)

Due to the negative infinite value of the logarithm function at zero, it is known that each link has a positive rate under the optimal power allocation of problem (5). Given that the optimal rate of link k is at least T_k ($T_k > 0$), we define a utility function $U_{T_k}(x)$ as

$$U_{T_k}(x) = \begin{cases} \ln(x), & x \geq T_k \\ \frac{1}{T_k}x + (\ln(T_k) - 1), & 0 \leq x < T_k. \end{cases} \quad (26)$$

As shown in Fig. 1, $U_{T_k}(x)$ has the same concave curve with $\ln(x)$ when $x \geq T_k$ and corresponds to a line tangent to $\ln(x)$ at $(T_k, \ln(T_k))$ when $0 \leq x < T_k$. Apparently, $U_{T_k}(x)$ is concave for $x \geq 0$.

Then, the following problem:

$$\max_{\boldsymbol{p}} \quad \sum_{k=1}^{K} U_{T_k}(R_k(\boldsymbol{p})) \quad (27a)$$

$$\text{s.t.} \quad p_k^1, p_k^2 \geq 0, k = 1, 2, \ldots, K \quad (27b)$$

$$p_k^1 + p_k^2 \leq P_k, k = 1, 2, \ldots, K \quad (27c)$$

is equivalent to problem (5) in the sense that the solutions to these two problems are identical. The reason is as follows. Slater's condition is satisfied for both problems in (27) and (5), i.e., there exists a feasible point in the relative interior of the feasible set for these two problems. Therefore, the optimal solutions to each of these two problems must satisfy its own KKT conditions [12]. Since the KKT equations of problem (5) are the same as those of problem (27) when the optimal rate of link k is greater than T_k, the optimal solution to problem (5) is also the solution to problem (27).

With the definitions of $\alpha_{k,k}^n = 1$ for all $k = 1, 2, \ldots, K$ and $n = 1, 2$, we now rewrite the rate of link k as

$$R_k(\boldsymbol{p}) = \ln\left(\frac{\sigma_k^1 + \sum_j \alpha_{k,j}^1 p_j^1}{\sigma_k^1 + \sum_{j\neq k}\alpha_{k,j}^1 p_j^1}\right) + \ln\left(\frac{\sigma_k^2 + \sum_j \alpha_{k,j}^2 p_j^2}{\sigma_k^2 + \sum_{j\neq k}\alpha_{k,j}^2 p_j^2}\right)$$

$$= g_k(\boldsymbol{p}) - h_k(\boldsymbol{p}) \quad (28)$$

where $g_k(\boldsymbol{p})$ and $h_k(\boldsymbol{p})$ are concave functions with respect to $\boldsymbol{p}$ and defined as

$$g_k(\boldsymbol{p}) = \ln\left(\sigma_k^1 + \sum_j \alpha_{k,j}^1 p_j^1\right) + \ln\left(\sigma_k^2 + \sum_j \alpha_{k,j}^2 p_j^2\right) \qquad (29)$$

and

$$h_k(\boldsymbol{p}) = \ln\left(\sigma_k^1 + \sum_{j\neq k} \alpha_{k,j}^1 p_j^1\right) + \ln\left(\sigma_k^2 + \sum_{j\neq k} \alpha_{k,j}^2 p_j^2\right) \qquad (30)$$

respectively.

Theorem 2: Function $f_k(\boldsymbol{p})$ given by

$$f_k(\boldsymbol{p}) = U_{T_k}(R_k) + {T_k}^{-1} h_k(\boldsymbol{p}) \qquad (31)$$

is concave.

Proof: Since $U_{T_k}(R_k)$ is concave, we have

$$\begin{aligned} U_{T_k}(R_k) &= \inf_{\theta\geq 0}\left\{U_{T_k}(\theta) + U'_{T_k}(\theta)(R_k - \theta)\right\} \\ &= \inf_{\theta\geq 0}\left\{U_{T_k}(\theta) - \theta U'_{T_k}(\theta) + U'_{T_k}(\theta)\left(g_k(\boldsymbol{p}) - h_k(\boldsymbol{p})\right)\right\}. \end{aligned} \qquad (32)$$

Substituting this into (31), we get

$$\begin{aligned} f_k(\boldsymbol{p}) &= \inf_{\theta\geq 0}\left\{U_{T_k}(\theta) - \theta U'_{T_k}(\theta) \right. \\ &\qquad \left. + U'_{T_k}(\theta)\left(g_k(\boldsymbol{p}) - h_k(\boldsymbol{p})\right)\right\} + {T_k}^{-1} h_k(\boldsymbol{p}) \\ &= \inf_{\theta\geq 0}\left\{U_{T_k}(\theta) - \theta U'_{T_k}(\theta) + U'_{T_k}(\theta) g_k(\boldsymbol{p}) \right. \\ &\qquad \left. + \left(T_k^{-1} - U'_{T_k}(\theta)\right) h_k(\boldsymbol{p})\right\}. \end{aligned} \qquad (33)$$

According to the definition of $U_{T_k}(x)$ in (26), we obtain $U'_{T_k}(\theta) > 0$ and $T_k^{-1} - U'_{T_k}(\theta) \geq 0$ for all $\theta \geq 0$. Therefore, $f_k(\boldsymbol{p})$ is the infimum of a set of concave functions, which indicates that $f_k(\boldsymbol{p})$ is concave [12], [13]. □

Based on Theorem 2, we express the objective function of problem (27) as

$$\sum_{k=1}^{K} U_{T_k}\left(R_k(\boldsymbol{p})\right) = g(\boldsymbol{p}) - h(\boldsymbol{p}) \qquad (34)$$

where $g(\boldsymbol{p})$ and $h(\boldsymbol{p})$ are both concave functions given, respectively, by

$$g(\boldsymbol{p}) = \sum_{k=1}^{K} f_k(\boldsymbol{p}) = \sum_{k=1}^{K} U_{T_k}(R_k) + \sum_{k=1}^{K} T_k^{-1} h_k(\boldsymbol{p}) \qquad (35)$$

$$h(\boldsymbol{p}) = \sum_{k=1}^{K} T_k^{-1} h_k(\boldsymbol{p}). \qquad (36)$$

The objective function of problem (27) is now the difference of two concave (D.C.) functions. As a result, for a given T_k, we can utilize an iterative algorithm to obtain a suboptimal solution to problem (27). Specifically, given the power allocation vector $\boldsymbol{p}^\zeta$ in the ζth iteration, we approximate $h(\boldsymbol{p})$ by utilizing its first-order Taylor expansion, i.e., $h(\boldsymbol{p}^\zeta) + \triangledown h^T(\boldsymbol{p}^\zeta)(\boldsymbol{p} - \boldsymbol{p}^\zeta)$, and optimize $\boldsymbol{p}$ in the $(\zeta+1)$th iteration. In particular, $\boldsymbol{p}^{\zeta+1}$ can be achieved by solving the following problem:

$$\max_{\boldsymbol{p}} \quad g(\boldsymbol{p}) - \left(h(\boldsymbol{p}^\zeta) + \triangledown h^T(\boldsymbol{p}^\zeta)(\boldsymbol{p} - \boldsymbol{p}^\zeta)\right) \qquad (37a)$$

$$\text{s.t.} \quad p_k^1, p_k^2 \geq 0, k = 1, 2, \ldots, K \qquad (37b)$$

$$p_k^1 + p_k^2 \leq P_k, k = 1, 2, \ldots, K. \qquad (37c)$$

Note that problem (37) is convex and, thus, can be efficiently solved by mature tools such as the interior-point methods.

Now, the detailed algorithm to solve problem (27) for a given T_k is presented in Algorithm 1 in which ζ is the iteration index, $\boldsymbol{p}^\zeta$ is the power allocation after the ζth iteration, and ε is the maximum tolerance for terminating the algorithm. It is worth emphasizing that this iterative algorithm converges to a local optimal solution to problem (27) that is no worse than the starting point $\boldsymbol{p}^0$, which can be proved by the similar approaches in [14] and [15].

Algorithm 1 Solving problem (27) for a given T_k

Input: $\boldsymbol{p}^0$, ε, T_k for all k
Output: $\boldsymbol{p}^\zeta$
Initialization: Set $\zeta = 0$
repeat
  Solve problem (37) and get its optimal solution $\boldsymbol{p}^*$;
  $\zeta = \zeta + 1$;
  $\boldsymbol{p}^\zeta = \boldsymbol{p}^*$;
until $|\boldsymbol{p}^\zeta - \boldsymbol{p}^{\zeta-1}| \leq \varepsilon$

As previously mentioned, problem (27) is equivalent to problem (5) only if T_k is smaller than the rate of link k under the optimal power allocation. Therefore, for guaranteeing the equivalence between the two problems, T_k should not be too large. On the other hand, a too small value of T_k will result in a slow convergence of Algorithm 1. Let us go back to the definitions of $g(\boldsymbol{p})$ in (35) and $h(\boldsymbol{p})$ in (36). A small T_k makes $T_k^{-1}\sum_{k=1}^{K} h_k(\boldsymbol{p})$ the dominant part of $g(\boldsymbol{p})$, and therefore, $g(\boldsymbol{p})$ is quite close to $h(\boldsymbol{p})$. Then, problem (37) can be approximated as

$$\max_{\boldsymbol{p}} \quad h(\boldsymbol{p}) - \left(h(\boldsymbol{p}^\zeta) + \triangledown h^T(\boldsymbol{p}^\zeta)(\boldsymbol{p} - \boldsymbol{p}^\zeta)\right) \qquad (38a)$$

$$\text{s.t.} \quad p_k^1, p_k^2 \geq 0, k = 1, 2, \ldots, K \qquad (38b)$$

$$p_k^1 + p_k^2 \leq P_k, k = 1, 2, \ldots, K. \qquad (38c)$$

Since $h(\boldsymbol{p})$ is concave, the optimal solution to problem (38) is $\boldsymbol{p}^\zeta$. This indicates that the step size of the update of $\boldsymbol{p}^\zeta$ is very small. As a result, T_k should be appropriately initialized and carefully updated when it gets too large or too small.

Algorithm 2 Proposed power allocation algorithm

Initialization: Set $\boldsymbol{p}^0$, ε, μ and T_k for all k
Set $\varpi = 0$;
repeat
  Set Flag $= 1$ and $\varpi = \varpi + 1$;
  Get power allocation $\boldsymbol{p}$ by calling Algorithm 1;
  Calculate R_k under power allocation $\boldsymbol{p}$ for all k;
  for $k = 1 : K$
    if $T_k \geq R_k$
      $T_k = T_k/2$;
      Flag $= 0$;
    else if $T_k < R_k/2$
      $T_k = (1 - \mu^\varpi)T_k + \mu^\varpi R_k$;
    end if
  end for
  $\boldsymbol{p}^0 = \boldsymbol{p}$;
until Flag $= 1$

The overall iterative algorithm for solving problem (5) is provided in Algorithm 2, where ϖ is the number of times for calling Algorithm 1.

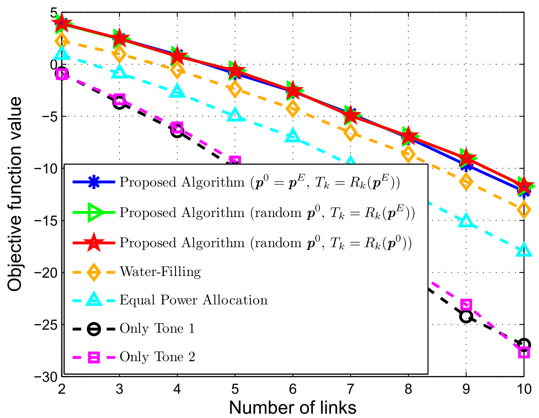

Fig. 2. Comparison of the objective function values under different methods.

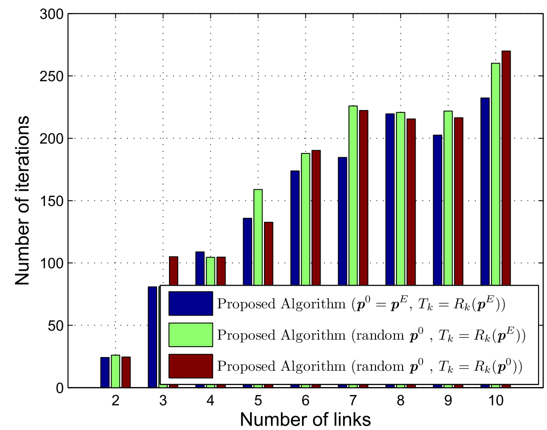

Fig. 3. Average number of iterations in the proposed method versus number of links.

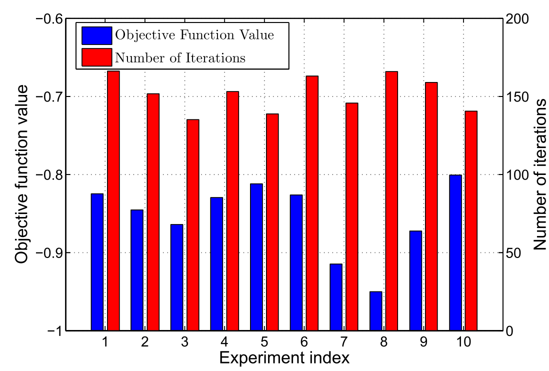

Fig. 4. Objective function values and numbers of iterations under different initial power allocations.

In practice, T_k is initialized as the rate of link k under a given feasible power allocation. Notice that in the ninth line, T_k is divided in half if R_k is less than or equal to T_k. Moreover, to prevent T_k from getting too small, T_k is increased according to $T_k = (1 - \mu^{\varpi})T_k + \mu^{\varpi} R_k$ if T_k is less than half of R_k, where $\mu \in (0, 1)$ is a small positive scalar for the stability of the algorithm.

Based on the given discussions, Algorithm 2 will finally locate a local optimal power allocation of problem (27), guaranteeing that $R_k > T_k$ for all k, which is also local optimal to problem (5).

V. Simulation Results

Here, we present the performance of our proposed algorithm, where, without loss of generality, the noise power is set to be $N_k^n = 0.1\,\mu$W, and the power budget is $P_k = 1$ mW for all links. Let $\boldsymbol{p}^E = (0.5, 0.5, \ldots, 0.5)^T$ mW be the equal power allocation, i.e., the power of each link at each tone is 0.5 mW. Parameters ε and μ are set as $\varepsilon = 10^{-3}$ and $\mu = 0.6$ in the proposed algorithm. For evaluating the influences of different initializations of $\boldsymbol{p}^0$ and T_k, we simulate the proposed algorithm by three different initializations: 1) $\boldsymbol{p}^0 = \boldsymbol{p}^E$ and $T_k = R_k(\boldsymbol{p}^E)$, where $R_k(\boldsymbol{p}^E)$ is the rate of link k under the power allocation $\boldsymbol{p}^E$; 2) $\boldsymbol{p}^0$ is randomly initialized and $T_k = R_k(\boldsymbol{p}^E)$; and 3) $\boldsymbol{p}^0$ is randomly initialized and $T_k = R_k(\boldsymbol{p}^0)$. For the purpose of comparison, we also present simulation results of the classical waterfilling power allocation for each link under an assumption that the power of every other link at each tone is 0.5 mW. Moreover, we consider the special case that only one tone can be utilized, which corresponds to a convex optimization problem as shown in [3].

For different numbers of links, we randomly pick up 100 channel realizations and give the averaged objective function values of problem (5) under different power allocation approaches in Fig. 2. It can be observed that as the number of links increases, the cochannel interference become serious, and thus, the objective function value decreases for all approaches. Specifically, the proposed algorithm outperforms the other power allocation approaches for any number of links. In addition, different initializations of $\boldsymbol{p}^0$ and T_k in the proposed algorithm are not critical to the performance. Moreover, the result indicates that the utilization of two tones can significantly improve the overall network utility compared with the utilization of only one tone.

We also evaluate the complexity of the proposed algorithm in terms of the number of iterations required to solve problem (37). Fig. 3 presents the average number of iterations for different numbers of links. It can be seen that the devised approach may converge to a local optimal point within 270 iterations. This complexity is relatively low because even solving a one-tone sum-rate maximization problem requires around 150 similar iterations for a ten-link case [5], [16]. Since the problem considered in this paper has double optimization variables and a more complex objective function, this slight increase in complexity is affordable. It can also be observed that the required number of iterations is not much sensitive to different initializations of $\boldsymbol{p}^0$ and T_k.

Now, we focus on the influence of different initial points on the performance in the proposed algorithm. In particular, considering a five-link case, we randomly pick up 100 channel realizations and carry out ten experiments, where in each experiment, we randomly generate an initial point $\boldsymbol{p}^0$ and set T_k as $T_k = R_k(\boldsymbol{p}^0)$ to implement the proposed algorithm in the 100 realizations. Then, the average objective function value of problem (5) and the number of iterations are given in Fig. 4. It can be seen that the maximum difference of objective function values in different experiments is about 0.15, and the maximum difference of the iteration number is around 40. This implies that the proposed algorithm is insensitive to the initialization of $\boldsymbol{p}^0$ in terms of both the objective value and the number of iterations.

VI. Applications and Extensions

Here, we discuss the practical applications of this work and the extensions of the proposed algorithm.

A. Applications

The proposed power allocation method under a two-tone spectrum-sharing model has numerous applications. Here, we take a scenario of heterogeneous cellular networks (HetNets) as a representative to illustrate the applications.

Consider a downlink HetNet consisting of one macrocell and $K-1$ picocells in which each cell involves a base station (BS) located at the center of the cell. As such, "cell" and "BS" are interchangeably used here. Without loss of generality, we denote cell 1 as the macrocell and cells $2, 3, \ldots, K$ as the picocells. We assume that each BS serves only one user, and specifically, user k is served by cell k. Let us concentrate on a unit duration of time in which the first fraction of time with length τ_1 is the muting period, and the remainder part with length τ_2 is the normal period ($\tau_1 + \tau_2 = 1$). Notice that the macrocell is muting to protect small-cell users in the muting period, whereas all BSs are transmitting data to their users in the normal period. For simplicity, the muting period and the normal period are referred to as period 1 and period 2, respectively. The achievable rate of user k in period n ($n=1, 2$) is given by

$$R_k^n(\boldsymbol{p}) = \tau_n \ln\left(1 + \frac{p_k^n}{\sigma_k^n + \sum_{j=1, j\neq k}^K \alpha_{k,j}^n p_j^n}\right) \tag{39}$$

where p_k^n is the transmit power of BS k in period n, $\boldsymbol{p} = (p_1^1, p_2^1, \ldots, p_K^1, p_1^2, p_2^2, \ldots, p_K^2)^T$ is the power vector, σ_k^n is the normalized noise power of user k in period n, and $\alpha_{k,j}^n$ is the normalized interference factor from BS j to user k in period n. Now, we get the mean rate of user k in the considered unit duration given by

$$R_k(\boldsymbol{p}) = \tau_1 R_k^1(\boldsymbol{p}) + \tau_2 R_k^2(\boldsymbol{p}). \tag{40}$$

In this scenario, when τ_n are fixed for $n = 1, 2$, the power allocation issue for achieving proportional fairness can be formulated by the programming, i.e.,

$$\max_{\boldsymbol{p}} \quad \sum_{k=1}^K \ln\left(\tau_1 R_k^1(\boldsymbol{p}) + \tau_2 R_k^2(\boldsymbol{p})\right) \tag{41a}$$

$$\text{s.t.} \quad p_k^1, p_k^2 \geq 0, k = 1, 2, \ldots, K \tag{41b}$$

$$p_1^1 = 0 \tag{41c}$$

$$\tau_1 p_k^1 + \tau_2 p_k^2 \leq Q_k, k = 1, 2, \ldots, K \tag{41d}$$

where Q_k is the power budget of BS k. It can be seen that problem (41) is the same as problem (5), except for the coefficients involved in the users' rates in two periods and the linearly transformed constraints (41c) and (41d). As a result, the complexity analysis and the proposed algorithm are valid in this situation.

B. Extensions

The proposed power allocation algorithm can be easily extended to a general N-tone case.[1] Particularly, in an N-tone scenario, the achievable rate of link k can be expressed as

$$\begin{aligned} R_k(\boldsymbol{p}) &= \sum_{n=1}^N \ln\left(1 + \frac{p_k^n}{\sigma_k^n + \sum_{j\neq k} \alpha_{k,j}^n p_j^n}\right) \\ &= \sum_{n=1}^N \ln\left(\frac{\sigma_k^n + \sum_j \alpha_{k,j}^n p_j^n}{\sigma_k^n + \sum_{j\neq k} \alpha_{k,j}^n p_j^n}\right) \\ &= g_k(\boldsymbol{p}) - h_k(\boldsymbol{p}) \end{aligned} \tag{42}$$

[1]Notice that the corresponding problem is convex when $N = 1$, which has been proved by Luo and Zhang in [3].

where $g_k(\boldsymbol{p})$ and $h_k(\boldsymbol{p})$ are given by

$$g_k(\boldsymbol{p}) = \sum_{n=1}^N \ln\left(\sigma_k^n + \sum_j \alpha_{k,j}^n p_j^n\right) \tag{43}$$

$$h_k(\boldsymbol{p}) = \sum_{n=1}^N \ln\left(\sigma_k^n + \sum_{j\neq k} \alpha_{k,j}^n p_j^n\right) \tag{44}$$

respectively. Apparently, both $g_k(\boldsymbol{p})$ and $h_k(\boldsymbol{p})$ are concave functions. As a result, only with replacements of $g_k(\boldsymbol{p})$ defined in (29) by (43) and $h_k(\boldsymbol{p})$ defined in (30) by (44) can the proposed method be applied directly.

VII. Conclusion

In this paper, we have proved that the power allocation problem, which is modeled as the sum-log-utility maximization in two-tone spectrum-sharing networks, is NP-hard. As a consequence, it is difficult to solve this problem in polynomial time. By formulating the objective function in a D.C. form, we devised a fast converging iterative algorithm that suboptimally solves the problem. An application of this work in HetNets and the extension of the proposed power allocation algorithm to general N-tone cases are also presented. Simulation results show that the proposed algorithm can efficiently lead to a satisfactory suboptimal solution.

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