

# Convexity of Fairness-Aware Resource Allocation in Wireless Powered Communication Networks

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**Abstract**—This letter focuses on fairness-aware power and time allocation in wireless powered communication networks under the “harvest-then-transmit” protocol, where downlink wireless energy transfer is implemented at first and then uplink wireless information transfer takes place in a spectrum-sharing fashion. We aim to achieve the rate fairness of all users under three fairness criteria named max-min, proportional, and harmonic fairness. Although these problems are intractable in their original nonconvex formulations, all of them can be equivalently transformed to convex programming ones. As a result, the global optimal solution can be efficiently obtained via mature convex solvers. Simulation results verify the effectiveness of the proposed approach in terms of minimum-rate, geometric mean rate, and mean delay.

**Index Terms**—Fairness, power allocation, wireless energy transfer, wireless powered communication network.

## I. INTRODUCTION

CONTINUOUS demands for wireless data transfer put to the forefront the design of continuous energy supply for devices such as smart phones in mobile communication systems [1]. Recently, wireless powered communication networks (WPCNs) have received considerable attentions, where the required energy of user devices for information transfer is harvested by dedicated radio-frequency (RF) enabled wireless energy transfer. It has been pointed out that the harvested power by Powercast RF energy-harvester from RF signals at 915MHz at 0.6 and 11 meters away can reach 3.5 mW and 1  $\mu$ W, respectively [2].

In WPCNs, an energy and information transfer protocol named “harvest-then-transmit” has been proposed recently in [2] and has received increasing investigations. In this protocol, the time block for energy and information transfer is partitioned into two phases in sequence, that are downlink (DL) wireless energy transfer (WET) and uplink (UP) wireless information transfer (WIT). During the first phase, the hybrid access point (H-AP) broadcasts wireless energy over RF signals in the DL which is harvested by all users. Then, during the second phase, users transmit their information to the H-AP in the UL utilizing the harvested energy in the first phase. Under such a protocol in WPCNs, one of the key issues is to determine the lengths of the two phases and users’ transmit powers.

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For avoiding uplink co-channel interference, a plenty of works adopt the classical time-division-multiple-access (TDMA) technique to implement UL WIT. In [2], the network sum-rate maximization problem is discussed, where the decision variables are the time duration of the DL WET and the duration allocated to each user for the UL WIT. In [3], the problem of optimizing users’ performance and simultaneously minimizing transferred energy is tackled through the  $L$ -shaped decomposition method. The closed-form expression of the optimal dynamic time allocation is derived in [4] for system sum-rate maximization with a constraint on the total time. In [5], the time allocation and power allocation are jointly optimized for maximizing the system overall energy efficiency. All of the above mentioned works considered TDMA scheme in the UL WIT phase, which could result in a low spectrum efficiency and, more importantly, corresponds to a mathematically comfortable interference-free situation. There are also numerous works that suboptimally maximize the system sum-rate or the minimum-rate among all users by energy and/or information beamforming in multi-antenna WPCNs [6]–[9].

It is worth noting that the rate fairness among users has not been fully addressed in the sense of global optimality for WPCNs in the literature. Particularly, user unfairness issue is pretty severe in WPCNs caused by the doubly near-far phenomenon [2], i.e., users near the H-AP harvest more energy in the DL WET phase but require less energy in the UL WIT phase owing to the high-quality channel conditions, while far users harvest less energy in the DL WET phase but need more energy in the UL WIT phase due to the low-quality channel conditions. Those far users would be considerably starved of rate if user fairness is not taken into account.

This letter concentrates upon fairness-aware time and power allocation in a single-cell WPCN under the “harvest-then-transmit” protocol, where all users transmit their information simultaneously in the UL WIT phase. Specifically, three commonly utilized fairness criteria, named max-min, proportional and harmonic fairness [10], [11], are considered. Although the original formulated optimization problems are nonconvex due to the complicated co-channel interference and the coupled time and power variables, they all can be equivalently converted to convex ones by appropriate transformations. Hence, the globally optimal solution can be derived by interior-point methods efficiently.

## II. SYSTEM MODEL

Consider a single-cell WPCN consisting of one single-antenna H-AP and  $K$  single-antenna users. The H-AP located at the cell center has the ability to transfer energy in the DL and receive information in the UL. The “harvest-then-transmit” protocol is applied, i.e., the H-AP transmits energy to users in DL WET phase and receives information signals from users in UL WIT phase in sequence. All the energy and information transfers are operated over the same frequency band for obtaining high spectrum reuse. Slow-fading is considered, i.e., the DL and

UL channel conditions known at both transmitters and receivers are fixed during time duration of interest. Without loss of generality, we concentrate upon a unit length of time block in which the WET phase and the WIT phase occupy  $\tau_0$  and  $\tau_1$  amounts of time, respectively ( $\tau_0 + \tau_1 \leq 1$ ).

The DL channel power gain from the H-AP to user  $k$  is denoted by  $h_k$ . The transmit power of H-AP in the DL WET phase is fixed as  $P_0$ . With an assumption that the energy harvested due to the receiver noise is negligible comparing with the sufficiently large  $P_0$ , the amount of energy harvested by user  $k$  in the DL WET is given by

$$E_k = \eta_k h_k P_0 \tau_0 \quad (1)$$

where  $\eta_k$  is the energy harvesting efficiency at user  $k$  [1], [2]. In the UL WIT phase, all users transmit their information simultaneously. The transmit power of user  $k$  and the channel power gain from user  $k$  to the H-AP are represented by  $p_k$  and  $g_k$ , respectively. Utilize  $\mathbf{p} = (p_1, p_2, \dots, p_K)^T$  and  $\boldsymbol{\tau} = (\tau_0, \tau_1)^T$  to denote the power allocation and time allocation vectors, respectively, which are decision variables in this letter.

Consider that users' information are decoded successively in an increasing order of their indices. Then, the interference to user  $k$  ( $k = 1, 2, \dots, K-1$ ) only comes from the WIT of users  $k+1, k+2, \dots, K$  and, particularly, user  $K$  suffers no interference<sup>1</sup>. Therefore, the achievable rate of user  $k$  measured in nats/s/Hz can be expressed as

$$R_k(\mathbf{p}, \boldsymbol{\tau}) = \begin{cases} \tau_1 \ln \left( 1 + \frac{g_k p_k}{N_0 + \sum_{j=k+1}^K g_j p_j} \right), & k \neq K \\ \tau_1 \ln \left( 1 + \frac{g_K p_K}{N_0} \right), & k = K \end{cases} \quad (2)$$

where  $N_0$  is the power of the additive white Gaussian noise.

By defining  $\sigma_k = N_0/g_k$  and  $\alpha_{k,j} = g_j/g_k$ , we simplify the rate of user  $k$  as

$$R_k(\mathbf{p}, \boldsymbol{\tau}) = \tau_1 \ln(1 + p_k/(\sigma_k + I_k(\mathbf{p}))) \quad (3)$$

where  $I_k(\mathbf{p})$  denotes the normalized interference to user  $k$  given by

$$I_k(\mathbf{p}) = \begin{cases} \sum_{j=k+1}^K \alpha_{k,j} p_j, & k \neq K \\ 0, & k = K. \end{cases} \quad (4)$$

The total consumed power of user  $k$  in the WIT phase can be modeled as  $\xi_k p_k + P_k^c$ , where  $\xi_k$  and  $P_k^c$  are positive constants ( $\xi_k > 0, P_k^c > 0$ ) accounting respectively for the power amplifier and the circuit.

### III. PROBLEM FORMULATIONS AND TRANSFORMATIONS

This section discusses the formulations and convex reformulations of the power and time allocation issues for achieving max-min, proportional and harmonic fairness, separately.

#### A. Max-Min Fairness

In this subsection, our efforts are devoted to pursuing the max-min fairness by maximizing the minimum UL rate among all users. Max-min fairness can sufficiently improve the performance of users in the worst case and thus lead to a high level of fairness. For achieving the max-min fairness, we focus on solving the problem given in (5), where  $\mathbf{p}$  and  $\boldsymbol{\tau}$  are the

optimization variables,  $P_k^{\max}$  is the maximum allowed transmit power of user  $k$  and  $R_k^{\text{req}}$  denotes the minimum required rate of user  $k$ . The constraint (5f) implies that, for each user, the total amount of consumed energy in the WIT phase can not exceed its harvested energy in the WET phase. Users' quality-of-service (QoS) are guaranteed by the constraint (5g).

$$\max_{\mathbf{p}, \boldsymbol{\tau}} \quad \min_{k=1,2,\dots,K} \tau_1 \ln(1 + p_k/(\sigma_k + I_k(\mathbf{p}))) \quad (5a)$$

$$\text{s.t.} \quad p_k \geq 0, \forall k \quad (5b)$$

$$\tau_i \geq 0, i = 0, 1 \quad (5c)$$

$$p_k \leq P_k^{\max}, \forall k \quad (5d)$$

$$\tau_0 + \tau_1 \leq 1 \quad (5e)$$

$$\tau_1(\xi_k p_k + P_k^c) \leq \eta_k h_k P_0 \tau_0, \forall k \quad (5f)$$

$$\tau_1 \ln(1 + p_k/(\sigma_k + I_k(\mathbf{p}))) \geq R_k^{\text{req}}, \forall k \quad (5g)$$

Problem (5) has multi-dimensional variables  $\mathbf{p}$  and  $\boldsymbol{\tau}$  which are coupled in the objective function (5a), constraint (5f) and constraint (5g). Moreover, the co-channel interference greatly complicates the rate expression over power. As a result, it is substantially challenging to globally solve this problem. Nevertheless, actually, Problem (5) can be equivalently transformed to a convex programming as discussed in the sequel.

**Proposition 1:** If Problem (5) is feasible, the optimal  $p_k$  ( $k = 1, 2, \dots, K$ ) and  $\tau_i$  ( $i = 0, 1$ ) are all greater than zero.

**Proof:** The proof can be conducted by contradiction. If there exists any one optimal variable being zero, the optimal objective value would be zero. However, any feasible point with positive  $p_k$  for all  $k = 1, 2, \dots, K$  and positive  $\tau_i$  for all  $i = 0, 1$  gives raise to a positive objective value that is better than the optimal value. This is a contradiction and the proof can be completed. ■

According to Proposition 1, we can impose variable replacements  $p_k \triangleq \exp(y_k)$  for all  $k = 1, 2, \dots, K$  and  $\tau_i \triangleq \exp(t_i)$  for all  $i = 0, 1$ . In addition, we introduce an auxiliary variable  $x$  and add a constraint guaranteeing that the rates of all users are not less than  $\exp(x)$ . This further allows us to concentrate upon maximizing  $\exp(x)$  or equivalently maximizing  $x$ , i.e.,

$$\max_{\mathbf{y}, \mathbf{t}, x} \quad x \quad (6a)$$

$$\text{s.t.} \quad \exp(y_k) \leq P_k^{\max}, \forall k \quad (6b)$$

$$\exp(t_0) + \exp(t_1) \leq 1 \quad (6c)$$

$$(\xi_k \exp(y_k) + P_k^c) \exp(t_1) \leq \eta_k h_k P_0 \exp(t_0), \forall k \quad (6d)$$

$$\exp(t_1) \ln \left( 1 + \frac{\exp(y_k)}{\sigma_k + I_k(\mathbf{y})} \right) \geq R_k^{\text{req}}, \forall k \quad (6e)$$

$$\exp(t_1) \ln \left( 1 + \frac{\exp(y_k)}{\sigma_k + I_k(\mathbf{y})} \right) \geq \exp(x), \forall k \quad (6f)$$

where  $\mathbf{y}$  and  $\mathbf{t}$  are defined respectively as  $\mathbf{y} = (y_1, y_2, \dots, y_K)^T$  and  $\mathbf{t} = (t_0, t_1)^T$ , and  $I_k(\mathbf{y})$  is given by

$$I_k(\mathbf{y}) = \begin{cases} \sum_{j=k+1}^K \alpha_{k,j} \exp(y_j), & k \neq K \\ 0, & k = K. \end{cases} \quad (7)$$

**Proposition 2:** Problem (6) is convex.

**Proof:** It can be easily derived that the objective function is linear, and the constraints (6b) and (6c) are convex. In the following, we show that constraints (6d)–(6f) are all convex.

• Convexity illustration of (6d)

Taking the logarithms of both sides of (6d) with natural base, we have

$$\ln(\xi_k \exp(y_k + t_1) + P_k^c \exp(t_1)) \leq t_0 \ln(\eta_k h_k P_0). \quad (8)$$

<sup>1</sup>Notice that different decoding orders may result in different users' rates, which could influence the rate fairness of users. The optimization of the decoding order is not considered in this letter and it is left as a future work for further improving system performance.

The right-hand-side (RHS) is linear and the left-hand-side (LHS) is a log-sum-exp function which is convex [12].

- Convexity illustration of (6e)

Let us first consider the case  $k \neq K$ . The constraint (6e) can be easily rewritten as

$$\frac{\exp(y_k)}{\sigma_k + \sum_{j=k+1}^K \alpha_{k,j} \exp(y_j)} \geq \exp(R_k^{req} \exp(-t_1)) - 1 \quad (9)$$

which directly results in

$$\begin{aligned} \sigma_k \exp(-y_k) + \sum_{j=k+1}^K \alpha_{k,j} \exp(y_j - y_k) \\ \leq (\exp(R_k^{req} \exp(-t_1)) - 1)^{-1}. \end{aligned} \quad (10)$$

Taking the natural logarithm of both sides of (10), we have

$$\begin{aligned} \ln \left( \sigma_k \exp(-y_k) + \sum_{j=k+1}^K \alpha_{k,j} \exp(y_j - y_k) \right) \\ + \ln (\exp(R_k^{req} \exp(-t_1)) - 1) \leq 0. \end{aligned} \quad (11)$$

The first term in the LHS of (11) is a convex log-sum-exp function [12]. The second-order derivative of the second term in the LHS is computed as  $A(t_1)B(t_1)$ , where  $A(t_1)$  and  $B(t_1)$  are given by

$$A(t_1) = \frac{R_k^{req} \exp(R_k^{req} \exp(-t_1) - 1)}{(\exp(R_k^{req} \exp(-t_1)) - 1)^2} \quad (12)$$

$$B(t_1) = \exp(R_k^{req} \exp(-t_1)) - R_k^{req} \exp(-t_1) - 1. \quad (13)$$

It can be seen that  $A(t_1)$  is non-negative. Since  $f(z) = \exp(z) - z - 1$  is an increasing function for  $z \geq 0$ , we have  $f(z) \geq f(0) = 0$  for all  $z \geq 0$ . Replacing  $z$  by non-negative  $R_k^{req} \exp(-t_1)$ , we have  $B(t_1) \geq 0$ . That is to say, the second term in the LHS of (11) is convex [12]. As a result, the convexity of the constraint (6e) is established for  $k \neq K$ .

Now, let us examine the case  $k = K$ , in which the constraint (6e) can be transformed in a similar way as

$$\ln(\sigma_K) + \ln(\exp(R_K^{req} \exp(-t_1)) - 1) \leq y_K. \quad (14)$$

The convexity of (14) can be obtained from the convexity of (11), which implies that the constraint (6e) is convex for  $k = K$ .

- Convexity illustration of (6f)

The constraint (6f) can be rewritten as

$$\begin{aligned} \ln \left( \sigma_k \exp(-y_k) + \sum_{j=k+1}^K \alpha_{k,j} \exp(y_j - y_k) \right) \\ + \ln(\exp(\exp(x - t_1)) - 1) \leq 0, \forall k \neq K \end{aligned} \quad (15)$$

$$\ln(\sigma_K) + \ln(\exp(\exp(x - t_1)) - 1) \leq y_K, k = K. \quad (16)$$

The convexity of (15) and (16) can be easily proved based on the convexity of (11). Consequently, the constraint (6f) is convex. ■

## B. Proportional Fairness

Proportional fairness is widely applied in balancing user fairness and network sum-rate [13], [14]. Particularly, pursuing proportional fairness corresponds to maximizing the sum of users' utilities, where the utility of each user is defined as the logarithm of its achievable rate [10]. This leads to the optimization problem given as

$$\max_{\mathbf{p}, \boldsymbol{\tau}} \sum_{k=1}^K \ln \left( \tau_1 \ln \left( 1 + \frac{p_k}{\sigma_k + I_k(\mathbf{p})} \right) \right) \quad (17a)$$

$$\text{s.t.} \quad (5b) - (5g). \quad (17b)$$

Notice that the objective function (17a) can be viewed as  $K$  multiplied by the logarithm of the geometric mean rate. Hence, Problem (17) maximizes the geometric mean rate. Globally solving Problem (17) directly is a ticklish job due to its nonconvexity. However, the following analysis indicates that the global optimality of Problem (17) can actually be efficiently derived.

**Proposition 3:** If Problem (17) is feasible, the optimal  $p_k$  ( $k = 1, 2, \dots, K$ ) and  $\tau_i$  ( $i = 0, 1$ ) are all greater than zero.

*Proof:* The proof is similar to that of Proposition 1 and is omitted here. ■

According to Proposition 3, variable replacements  $p_k \triangleq \exp(y_k)$  and  $\tau_i \triangleq \exp(t_i)$  can be imposed for all  $k = 1, 2, \dots, K$  and  $i = 0, 1$ . In addition, we introduce an auxiliary variable  $x_k$  for user  $k$  and add a constraint guaranteeing that the rate of user  $k$  is not less than  $\exp(x_k)$ . Then, as the objective function is increasing in rates, the rate of user  $k$  in the objective function can be replaced by  $\exp(x_k)$ . This gives rise to an equivalent problem

$$\max_{\mathbf{y}, \mathbf{t}, \mathbf{x}} \sum_{k=1}^K x_k \quad (18a)$$

$$\text{s.t.} \quad (6b) - (6e) \quad (18b)$$

$$\exp(t_1) \ln \left( 1 + \frac{\exp(y_k)}{\sigma_k + I_k(\mathbf{y})} \right) \geq \exp(x_k) \quad (18c)$$

where  $\mathbf{x}$  is defined as  $\mathbf{x} = (x_1, x_2, \dots, x_K)^T$ .

**Proposition 4:** Problem (18) is convex.

*Proof:* The objective function is linear and the constraints are similar to the convex constraints of Problem (6). ■

## C. Harmonic Fairness

In this section, we aim to achieve the harmonic fairness among all users by maximizing the harmonic mean of all users' UL rates [10]. According to the definition of the harmonic mean, the object can be set as minimizing the sum of the inverse of user rates. Since the inverse of rate can be regarded as the delay for transmitting one-nat information, the aim of harmonic fairness can be interpreted as to minimizing the mean delay of all users in transmitting one-nat information [14].

The problem for achieving harmonic fairness is given by

$$\min_{\mathbf{p}, \boldsymbol{\tau}} \sum_{k=1}^K \left( \tau_1 \ln \left( 1 + \frac{p_k}{\sigma_k + I_k(\mathbf{p})} \right) \right)^{-1} \quad (19a)$$

$$\text{s.t.} \quad (5b) - (5g). \quad (19b)$$

Problem (19) has a complex nonconvex objective function as well as the intractable constraints, where multi-dimensional variables are coupled. As a result, it is quite challenging to obtain the global optimality. In the following, we devote ourself to an equivalent convex reformulation of Problem (19).

**Proposition 5:** If Problem (19) is feasible, the optimal  $p_k$  ( $k = 1, 2, \dots, K$ ) and  $\tau_i$  ( $i = 0, 1$ ) are all greater than zero.

*Proof:* The proof is similar to that of Proposition 1 and is omitted here. ■

Owing to the fact that the objective value is decreasing with respect to user rates, we can change the rate of user  $k$  to  $\exp(x_k)$  in the objective function and add a constraint  $R_k(\mathbf{p}, \boldsymbol{\tau}) \geq \exp(x_k)$  for all  $k$ , where  $x_k$  is an auxiliary variable for user  $k$ . Furthermore, according to Proposition 5, variable replacements  $p_k \triangleq \exp(y_k)$  and  $\tau_i \triangleq \exp(t_i)$  can be imposed. By such transformation, we obtain a problem that has the same constraints with Problem (18) but a different objective function, given as



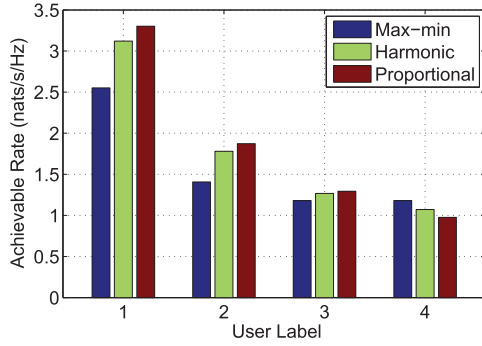


Fig. 1. Users' achievable rates under three kinds of fairness criteria.

$$\min_{\mathbf{y}, \mathbf{t}, \mathbf{x}} \sum_{k=1}^K \exp(-x_k) \quad (20a)$$

$$\text{s.t.} \quad (18b) - (18c) \quad (20b)$$

*Proposition 6:* Problem (20) is convex.

*Proof:* The objective function is convex and the constraints are the same as that of Problem (18). ■

#### IV. SIMULATION RESULTS

According to Section III, the feasibility of the problems considered in this letter can be easily checked by solving convex feasibility problems [12], [15]. Furthermore, these problems, if feasible, can be efficiently solved by convex solvers such as interior-point methods with great efficiency<sup>2</sup>.

In order to illustrate the network performance under different fairness criteria, we consider a WPCN with one H-AP and  $K = 4$  users, where the distance between the H-AP and user  $k$  ( $k = 1, 2, 3, 4$ ) is  $D_k = k \times 2.5$  (Unit:m). By such settings, user 1 is in the best position while user 4 is in the worst situation due to the doubly near-far phenomenon. For protecting users in bad conditions, users' information is decoded by an increasing order of user indices in the UL. With channel reciprocity, the channel power gains are modeled as  $h_k = g_k = 10^{-3} D_k^{-\alpha}$ , where  $\alpha$  is the pathloss exponent that is set as  $\alpha = 3$  in the simulation [2]. Other parameters are set as  $N_0 = -160$  dBm,  $P_0 = 1$ W,  $\eta_k = 1$ ,  $\xi_k = 1$ ,  $P_k^c = 0$ W,  $P_k^{\max} = 1$ W and  $R_k^{req} = 0.5$ nats/s/Hz for all users [2].

By the proposed convex formulations under three fairness criteria and interior-point methods, we derive the optimal solutions that result in the rate performance given in Fig. 1. It can be seen that the proportional fair resource allocation leads to a higher maximum-rate and a lower minimum-rate. This is because proportional fairness tries to not only maintain certain fairness among users but also pursue a high sum-rate, which results in the loss in the aspect of minimum-rate. Unlike the proportional fairness, harmonic fairness improves the rate in the worst case since harmonic mean is dominated by the minimum-rate of all users. Furthermore, owing to the fact that max-min fairness directly maximizes the minimum-rate, it derives the highest worst-case rate of all users.

We also present the mean delay in transmitting one-nat information with unit bandwidth, the minimum-rate and the geometric mean rate in Table I. It can be seen that the max-min, proportional and harmonic fairness based resource allocations lead to the best minimum-rate (denoted by Min-Rate in Table I),

<sup>2</sup>Notice that there may exist some users who have leftover harvested energy after a period of WET and WIT. The leftover energy could be exploited by user cooperation [16] which is not discussed here owing to space limitation.

TABLE I  
THE MINIMUM-RATE, GEOMETRIC MEAN RATE, AND MEAN DELAY

	Max-min	Proportional	Harmonic
<b>Min-Rate(nats/s/Hz)</b>	1.1807	0.9761	1.0717
<b>Geo-Mean(nats/s/Hz)</b>	1.4959	1.6719	1.6578
<b>Mean Delay(s)</b>	0.7031	0.6585	0.6509

geometric mean rate (denoted by Geo-Mean in Table I) and mean delay, respectively.

#### V. CONCLUSION

This letter considered power and time allocation issues in a single-cell WPCN for achieving three kinds of fairness named max-min, proportional and harmonic fairness. Through convex transformations and interior-point methods, we globally solved these problems that are nonconvex in their original forms. Simulation results confirmed the effectiveness of the proposed resource allocations under different fairness criteria in terms of minimum-rate, geometric mean rate and mean delay.

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