On Convexity of Fairness-Aware Energy-Efficient Power Allocation in Spectrum-Sharing Networks

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***Abstract*—The optimization of energy efficiency (EE) in spectrum-sharing networks has received increasing attention. While much effort has been devoted to the network-side overall EE, the fairness of EE among users has not been adequately concerned. Due to the complicated interference between different users, the global optimality of power allocation problems for achieving proportional, harmonic, and max–min fair EE with guarantees on users' quality of service has not been fully addressed. By introducing auxiliary variables, changing variables, and transforming the objective and constraint functions, we convert the proportional, harmonic as well as max–min fairness-aware problems to convex ones. As a result, mature algorithms, such as interior-point methods, can be applied to achieve the global optimality with considerable efficiency.**

***Index Terms*—Energy efficiency, fairness, harmonic, max-min, proportional, power allocation, spectrum-sharing network.**

I. Introduction

THE INFORMATION communications technology (ICT) generates around 2%-4% of all the Carbon dioxide emissions by human, of which the 0.2% is produced by wireless networks [1], [2]. Due to the environmental considerations as well as the economic concerns, lots of efforts have been devoted to green communications with the main interest in energy efficiency (EE) measured in nats/bits per Joule [3], [4]. In spectrum-sharing networks, co-channel interference is a key factor that limits the system performance. As an important approach in managing interference, power allocation plays a prominent role in improving EE [3], [5].

The complicated inter-link interference in spectrum-sharing networks introduces a formidable challenge in optimizing EE globally. Therefore, various methodologies have been put forward to deal with the EE optimization problem in an interference-free environment or with an assumption that the level of the interference is known a priori [5]–[11]. With interference taken into account, a number of suboptimal approaches have been devised [5], [7], [10], [12], [13]. Most of these works focus on the overall network-side EE but have not yet sufficiently addressed the fairness of EE among users.

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One of the widely adopted fairness criterion is the max-min fairness. With the aim to maximize the minimum EE of all users, an iterative algorithm with local optimality guarantees has been proposed in [5], where a convex optimization is implemented in each iteration. Considering interference-free uplink OFDMA systems, the subcarrier assignment and power allocation problem for achieving max-min fairness has been addressed in [14]. In multi-cell multi-user systems, an iterative beamforming algorithm is proposed in [15] for achieving max-min fairness of EE. Besides, numerous works focus on overall EE maximization with a constraint that the ratios of rates between users are fixed [2], [16]. However, none of these works is able to optimally solve the fairness-aware EE optimization problems in spectrum-sharing networks in the presence of interferences.

The proportional, max-min and harmonic fairness are three representative fairness criteria, all of which have been discussed in [17] from the perspective of users' achievable rates. However, the proportional fair EE (PFEE), max-min fair EE (MFEE) and harmonic fair EE (HFEE) among all users have not been fully addressed. Here, the PFEE can be obtained by maximizing the sum-utility of all users, where the utility of each user is defined as the logarithm of its EE. The MFEE pays close attention to the EE of the user in the worst-case and attempt to improve it as much as possible [5], [14]. Maximizing the harmonic mean of all users' EEs leads to the HFEE. Nevertheless, the PFEE, MFEE and HFEE involve non-convex objective functions, posing a big challenge in the power optimization.

This work devotes to globally optimizing the power allocation problems for achieving PFEE, MFEE and HFEE, with constraints that guarantee users' quality of service (QoS). As a matter of fact, although these three problems are non-convex in their primal formulations, they can be eventually converted to the convex optimization issues, by appropriately introducing auxiliary variables, changing variables and converting the objective and constraint functions. Based on the obtained convexity property, a power allocation strategy is derived.

The contributions are threefold. First, we point out the convexity property that the considered problems have in potential. Second, the proposed method can lead to the global optimal power allocations. Third, low complexity is required in the devised approach.

II. System Model

Consider a spectrum-sharing network, where there are K transceiver pairs sharing one common frequency channel. A transceiver pair is also referred to as a user. For achieving high spectrum-efficiency, all users are allowed to access the channel.

Assume that perfect channel state information is available at both the transmitter and receiver.

Let x_k be the circularly symmetric complex Gaussian signal transmitted from user k and $h_{k,j}$ be the Gaussian channel from the transmitter of user j to the receiver of user k. For an additive white Gaussian noise (AWGN) channel, the received signal of user k is represented as

$$y_k = z_k + \sum\nolimits_{j=1}^{K} h_{k,j} x_j \tag{1}$$

where z_k is the Gaussian noise of user k.

Denote by $\boldsymbol{p} = (p_1, p_2, \cdots, p_K)^T$ the power vector, where $p_k = |x_k|^2$ is the transmit power of user k. According to Shannon's formula, the achievable rate of user k measured in nats/s/Hz can be expressed as

$$R_k(\boldsymbol{p}) = \ln\left(1 + \frac{|h_{k,k}|^2 p_k}{|z_k|^2 + \sum_{j\neq k} |h_{k,j}|^2 p_j}\right), \forall k \tag{2}$$

which, after a normalization, becomes

$$R_k(\boldsymbol{p}) = \ln\left(1 + \frac{p_k}{\sigma_k + \sum_{j\neq k} \alpha_{k,j} p_j}\right), \forall k \tag{3}$$

where $\ln(x)$ is the logarithm with natural base, $\alpha_{k,j} = |h_{k,j}|^2/|h_{k,k}|^2$ is the normalized crosstalk interference factor from user j to user k and $\sigma_k = |z_k|^2/|h_{k,k}|^2$ is the normalized noise power.

A practical power consumption model is considered. Particularly, the power consumption of user k is

$$P_k(\boldsymbol{p}) = A_k p_k + B_k \tag{4}$$

where $A_k > 0$ and $B_k \geq 0$ are constants determined by the power amplifier and circuit [5]. Denote by $p_k^{\max}$ the power budget of user k, that is to say, the transmit power of user k could not be greater than $p_k^{\max}$. The EE of user k, defined as the ratio of its rate to its total consumed power and measured in nats/Joule/Hz, is a function of $\boldsymbol{p}$ given by

$$\eta_k(\boldsymbol{p}) = R_k(\boldsymbol{p})/P_k(\boldsymbol{p}). \tag{5}$$

Instead of maximizing the sum-EE, i.e., $\sum_{k=1}^{K} \eta_k(\boldsymbol{p})$, which is widely addressed in the literature, this work is dedicated to achieving various kinds of fairness of EE among users by optimizing power allocation.

III. Problem Formulations and Transformations

In this section, the power allocation problems with the aims of proportional, max-min and harmonic fairness of EE are formulated. We shall show that although they are nonconvex in the primal forms, all of them can be equivalently converted to convex issues.

A. PFEE Issue

Power allocation for PFEE corresponds to the solution to the problem given by

$$\max_{\boldsymbol{p}} \quad \sum\nolimits_{k=1}^{K} \ln\left(\eta_k(\boldsymbol{p})\right) \tag{6a}$$
$$\text{s.t.} \quad R_k(\boldsymbol{p}) \geq T_k, \forall k \tag{6b}$$
$$0 \leq p_k \leq p_k^{\max}, \forall k \tag{6c}$$

where T_k ($T_k \geq 0$) denotes the minimum required rate of user k. Here, the constraint (6b) guarantees the QoS of all users from the perspective of achievable rates. Although the constraints in Problem (6) are all linear, the objective function (6a) is a complicated non-concave function. This indicates that directly solving this problem is challenging. In the sequel, we will transform it to an equivalent convex problem.

At first, we notice that each user must have a positive optimal power. Otherwise, the objective value would be negative infinity. Therefore, p_k can be replaced by $\exp(y_k)$, where y_k is an introduced variable. Additionally, due to the fact that the objective function is increasing in each user's EE, we can impose a constraint $\eta_k(\boldsymbol{p}) \geq \exp(x_k)$ and replace $\eta_k(\boldsymbol{p})$ by $\exp(x_k)$ in the objective function, where x_k is another introduced auxiliary variable. This leads to

$$\max_{\boldsymbol{y},\boldsymbol{x}} \quad \sum\nolimits_{k=1}^{K} x_k \tag{7a}$$
$$\text{s.t.} \quad \ln\left(1 + \frac{\exp(y_k)}{\sigma_k + \sum_{j\neq k} \alpha_{k,j} \exp(y_j)}\right) \geq T_k, \forall k \tag{7b}$$
$$\exp(y_k) \leq p_k^{\max}, \forall k \tag{7c}$$
$$\frac{\ln\left(1 + \frac{\exp(y_k)}{\sigma_k + \sum_{j\neq k} \alpha_{k,j} \exp(y_j)}\right)}{A_k \exp(y_k) + B_k} \geq \exp(x_k), \forall k \tag{7d}$$

where $\boldsymbol{y} = (y_1, y_2, \cdots, y_K)^T$ and $\boldsymbol{x} = (x_1, x_2, \cdots, x_K)^T$ are the optimization variables.

Proposition 1: Problem (7) is convex.

Proof: It is obvious that the objective function (7a) is linear and constraint (7c) is convex. In the following, we prove that both constraints (7b) and (7d) are convex. To this end, we write (7b) as

$$\exp(y_k)\Big/\left(\sigma_k + \sum\nolimits_{j\neq k} \alpha_{k,j} \exp(y_j)\right) \geq \exp(T_k) - 1. \tag{8}$$

Taking the nature logarithm of (8) leads to

$$\ln\left(\exp(T_k) - 1\right) + \ln\left(\sigma_k + \sum\nolimits_{j\neq k} \alpha_{k,j} \exp(y_j)\right) \leq y_k. \tag{9}$$

Since a log-sum-exp function is convex [18], the second term on the left-hand side (LHS) of (9) is convex. It follows that (7b) is convex. Let us now examine the constraint (7d), which can be expressed as

$$\exp(y_k)\Big/\left(\sigma_k + \sum\nolimits_{j\neq k} \alpha_{k,j} \exp(y_j)\right) \geq \exp\left(A_k \exp(x_k + y_k) + B_k \exp(x_k)\right) - 1. \tag{10}$$

It can be rewritten as

$$\sigma_k \exp(-y_k) + \sum\nolimits_{j\neq k} \alpha_{k,j} \exp(y_j - y_k) \leq \left(\exp\left(A_k \exp(x_k + y_k) + B_k \exp(x_k)\right) - 1\right)^{-1}. \tag{11}$$

Taking the nature logarithm of both sides of (11), we get

$$\ln\left(\sigma_k \exp(-y_k) + \sum\nolimits_{j\neq k} \alpha_{k,j} \exp(y_j - y_k)\right) + \ln\left(\exp\left(A_k \exp(x_k + y_k) + B_k \exp(x_k)\right) - 1\right) \leq 0. \tag{12}$$

The first term on the LHS of (12) is a log-sum-exp function and thereby convex. Consequently, we only need to prove that the function $f(x_k, y_k)$ is jointly convex over x_k and y_k, which is given as

$$f(x_k, y_k) = \ln\left(\exp\left(A_k \exp(x_k + y_k) + B_k \exp(x_k)\right) - 1\right). \tag{13}$$

To this end, we need to determine the first- and second-order partial derivatives of $f(x_k, y_k)$. In particular, the partial derivatives of $f(x_k, y_k)$ with respective to x_k and y_k are

$$\frac{\partial f(x_k, y_k)}{\partial x_k} = \frac{\exp(A_k \exp(x_k + y_k) + B_k \exp(x_k))}{\exp(A_k \exp(x_k + y_k) + B_k \exp(x_k)) - 1} \times (A_k \exp(x_k + y_k) + B_k \exp(x_k)) \tag{14}$$

$$\frac{\partial f(x_k, y_k)}{\partial y_k} = \frac{\exp(A_k \exp(x_k + y_k) + B_k \exp(x_k))}{\exp(A_k \exp(x_k + y_k) + B_k \exp(x_k)) - 1} \times A_k \exp(x_k + y_k). \tag{15}$$

The Hessian matrix of $f(x_k, y_k)$ is computed as

$$\mathbf{H}^f = \begin{bmatrix} H_{11}^f & H_{12}^f \\ H_{21}^f & H_{22}^f \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 f(x_k,y_k)}{\partial x_k^2} & \frac{\partial^2 f(x_k,y_k)}{\partial x_k \partial y_k} \\ \frac{\partial^2 f(x_k,y_k)}{\partial y_k \partial x_k} & \frac{\partial^2 f(x_k,y_k)}{\partial y_k^2} \end{bmatrix} \tag{16}$$

where the entries in the above matrix are the second-order partial derivatives of $f(x_k, y_k)$. With a notation F given by

$$F = A_k \exp(x_k + y_k) + B_k \exp(x_k) > 0 \tag{17}$$

the elements of $\mathbf{H}^f$ are given as

$$H_{11}^f = \frac{F \exp(F)}{(\exp(F) - 1)^2} (\exp(F) - F - 1) \tag{18}$$

$$H_{12}^f = H_{21}^f = \frac{A_k \exp(F) \exp(x_k + y_k)}{(\exp(F) - 1)^2} (\exp(F) - F - 1) \tag{19}$$

$$H_{22}^f = \frac{A_k \exp(F) \exp(x_k + y_k)}{(\exp(F) - 1)^2} \times (\exp(F) - A_k \exp(x_k + y_k) - 1). \tag{20}$$

As a result, the determinant of $\mathbf{H}^f$ is

$$\det \mathbf{H}^f = \frac{A_k \exp^2(F) \exp(x_k + y_k)}{(\exp(F) - 1)^3} \times (\exp(F) - F - 1)(F - A_k \exp(x_k + y_k)). \tag{21}$$

According to the definition of F, we have $F - A_k \exp(x_k + y_k) \geq 0$. Moreover, due to the monotonically increasing property of function $h(F) = \exp(F) - F - 1$ and $F > 0$, we obtain $h(F) > h(0) = 0$. Then, it is apparent that $\det \mathbf{H}^f \geq 0$. In addition, since $H_{11}^f > 0$, $\mathbf{H}^f$ is positive semidefinite and therefore $f(x_k, y_k)$ is jointly convex over x_k and y_k [18]. ■

B. MFEE Issue

Power allocation for MFEE can be achieved by solving the following problem

$$\max_{\boldsymbol{p}} \quad \min_{k=1,2,\cdots,K} \eta_k(\boldsymbol{p}) \tag{22a}$$

$$\text{s.t.} \quad R_k(\boldsymbol{p}) \geq T_k, \forall k \tag{22b}$$

$$0 \leq p_k \leq p_k^{\max}, \forall k. \tag{22c}$$

The difficulty in tackling the above problem lies in the complicated non-concave objective function (22a). Note that this problem is identical to that in [5]. However, only suboptimal solutions can be obtained in [5] by utilizing an iterative procedure. We are now going to show that Problem (22) can be converted to a globally solvable convex problem.

To maximize the minimum EE of all users, we impose a constraint that the EEs of all users are greater than a common expression $\exp(\tau)$, where τ is an auxiliary variable. This in turn allows us to focus on maximizing $\exp(\tau)$, or equivalently maximizing τ, that is

$$\max_{\boldsymbol{y},\tau} \quad \tau \tag{23a}$$

$$\text{s.t.} \quad \ln\left(1 + \frac{\exp(y_k)}{\sigma_k + \sum_{j \neq k} \alpha_{k,j} \exp(y_j)}\right) \geq T_k, \forall k \tag{23b}$$

$$\exp(y_k) \leq p_k^{\max}, \forall k \tag{23c}$$

$$\frac{\ln\left(1 + \frac{\exp(y_k)}{\sigma_k + \sum_{j \neq k} \alpha_{k,j} \exp(y_j)}\right)}{A_k \exp(y_k) + B_k} \geq \exp(\tau), \forall k \tag{23d}$$

where y_k results from the variable transformation $p_k := \exp(y_k)$. Notice that such a variable replacement is quite reasonable because the optimal p_k must be positive for all k. Otherwise, the minimum EE among all users would be zero, which is not optimal.

Proposition 2: Problem (23) is convex.

Proof: This problem has a linear objective function and the same convex constraints as Problem (7). As a result, the proof is similar to that of Proposition 1, and thereby ignored herein. ■

C. HFEE Issue

According to the definition of harmonic fairness in [17], we formulate the HFEE power allocation problem given by

$$\min_{\boldsymbol{p}} \quad \sum_{k=1}^{K} (\eta_k(\boldsymbol{p}))^{-1} \tag{24a}$$

$$\text{s.t.} \quad R_k(\boldsymbol{p}) \geq T_k, \forall k \tag{24b}$$

$$0 \leq p_k \leq p_k^{\max}, \forall k. \tag{24c}$$

Similarly, this problem has an objective function which is still quite difficult to tackle. However, it can actually be turned to a convex issue as presented as follows.

It can be easily obtained that the optimal p_k must be positive for all k. Otherwise, the objective function value would be infinite. By using the variable replacement $p_k := \exp(y_k)$ and introducing a new variable x_k, we convert Problem (24) to

$$\min_{\boldsymbol{y},\boldsymbol{x}} \quad \sum_{k=1}^{K} \exp(x_k) \tag{25a}$$

$$\text{s.t.} \quad \ln\left(1 + \frac{\exp(y_k)}{\sigma_k + \sum_{j \neq k} \alpha_{k,j} \exp(y_j)}\right) \geq T_k, \forall k \tag{25b}$$

$$\exp(y_k) \leq p_k^{\max}, \forall k \tag{25c}$$

$$\frac{1}{\exp(x_k)} \leq \frac{\ln\left(1 + \frac{\exp(y_k)}{\sigma_k + \sum_{j \neq k} \alpha_{k,j} \exp(y_j)}\right)}{A_k \exp(y_k) + B_k}, \forall k. \tag{25d}$$

Proposition 3: Problem (25) is convex.

Proof: We only need to show the convexity of the constraint (25d), which can be easily transformed to

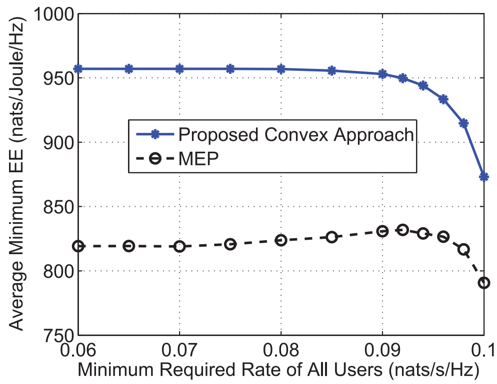

Fig. 1. Minimum EE achieved by the proposed method and the MEP algorithm in [5] under different rate requirements.

$$\exp(y_k)\Big/\Big(\sigma_k+\sum\nolimits_{j\neq k}\alpha_{k,j}\exp(y_j)\Big) \geq \exp\left(A_k\exp(y_k-x_k)+B_k\exp(-x_k)\right)-1. \tag{26}$$

It further gives rise to

$$\ln\Big(\sigma_k\exp(-y_k)+\sum\nolimits_{j\neq k}\alpha_{k,j}\exp(y_j-y_k)\Big) + \ln\left(\exp\left(A_k\exp(y_k-x_k)+B_k\exp(-x_k)\right)-1\right)\leq 0. \tag{27}$$

According to the definition of $f(x_k, y_k)$ in (13), the second term on the LHS of (27) is actually $f(-x_k, y_k)$, which is convex due to the convexity of $f(x_k, y_k)$ as proved in Proposition 1 [18]. In addition, since the first term is a convex log-sum-exp function, the constraint in (27) is convex and thus the proof is completed. ■

IV. Optimal Algorithm and Numerical Results

According to Section III, all the three considered fairness-aware EE optimization problems are formulated in convex forms. Therefore, convex solvers such as interior-point methods can be utilized to find the global optimal solutions with great efficiency [18]. In Algorithm 1, the proposed power allocation strategy is summarized.

Algorithm 1. Optimal power allocation strategy

1: *Given*. Fairness criterion
2: *Do*.
- Formulate the problem in a convex form according to the analysis in Section III;
- Implement interior-point methods to solve it.

Owning to the space limitation, we take the MFEE problem as a representative to verify the superiority of the proposed method. Consider there are $K = 10$ users in the network, where the channel coefficients $h_{k,j}$ are independent and identically distributed with $\mathcal{CN}(0, 0.1)$. Some other parameter settings are $p_k^{\max} = 1m\text{W}$, $A_k = 1$, $B_k = 0.1m\text{W}$ and $|z_k|^2 = 1\mu\text{W}$ for all users. By randomly picking up 50 channel realizations that result in nonempty feasible set under the QoS requirements, Fig. 1 presents the average minimum EE under the proposed method and the MEP algorithm in [5]. It can be observed that the proposed method greatly improves the minimum EE. Moreover, the average running time of our approach and the MEP algorithm are 0.60s and 266.18s, respectively, which confirms the low complexity of the devised strategy.

V. Conclusion

With QoS guarantees of all users in terms of data rates, this letter studied three power optimization problems for achieving proportional, max-min and harmonic fairness of EE among all users, respectively, in spectrum-sharing networks. These problems are not convex in their primary forms. By elaborate transformation, all of them are converted to convex problems and then globally solved by interior-point methods.

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