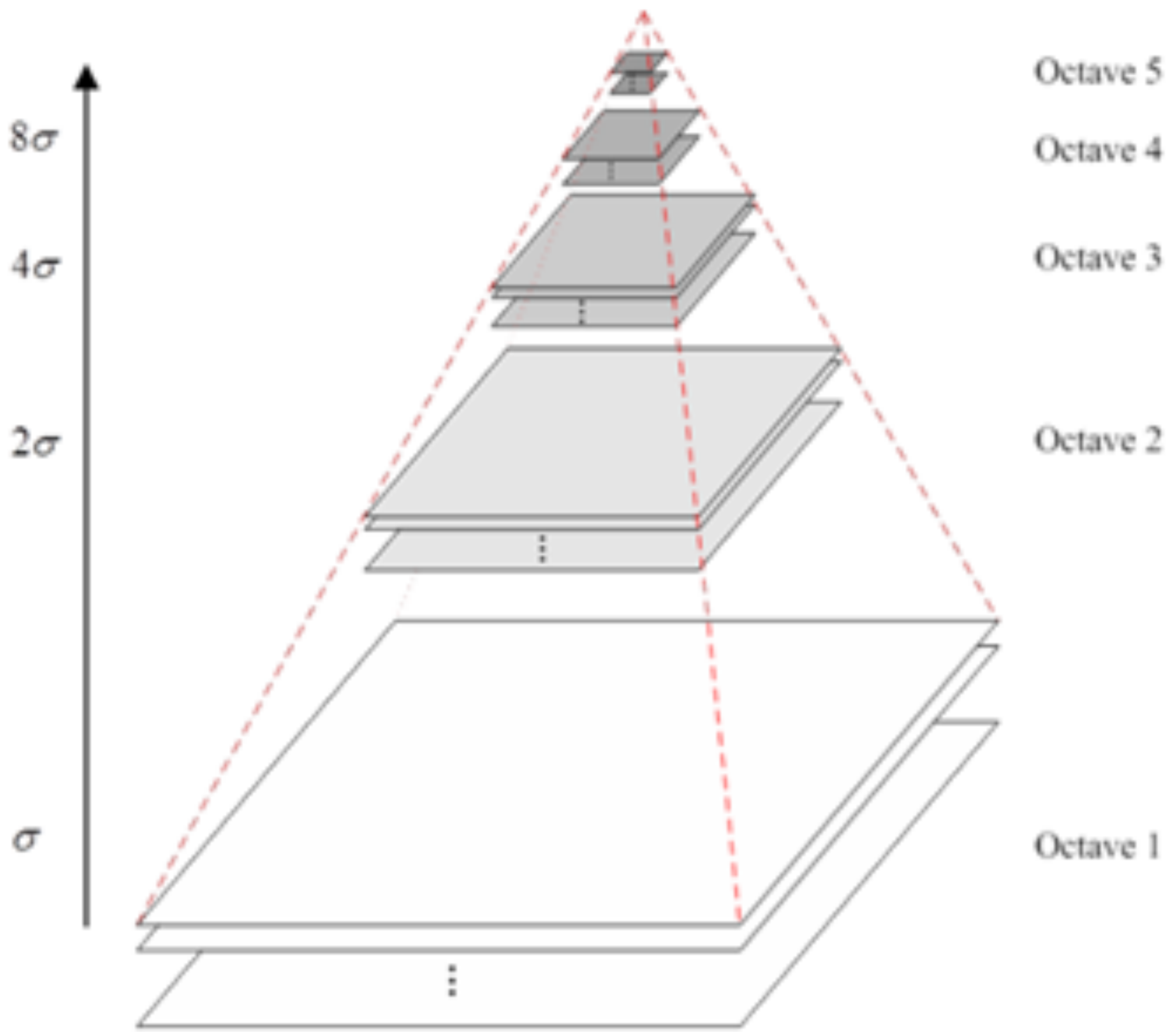
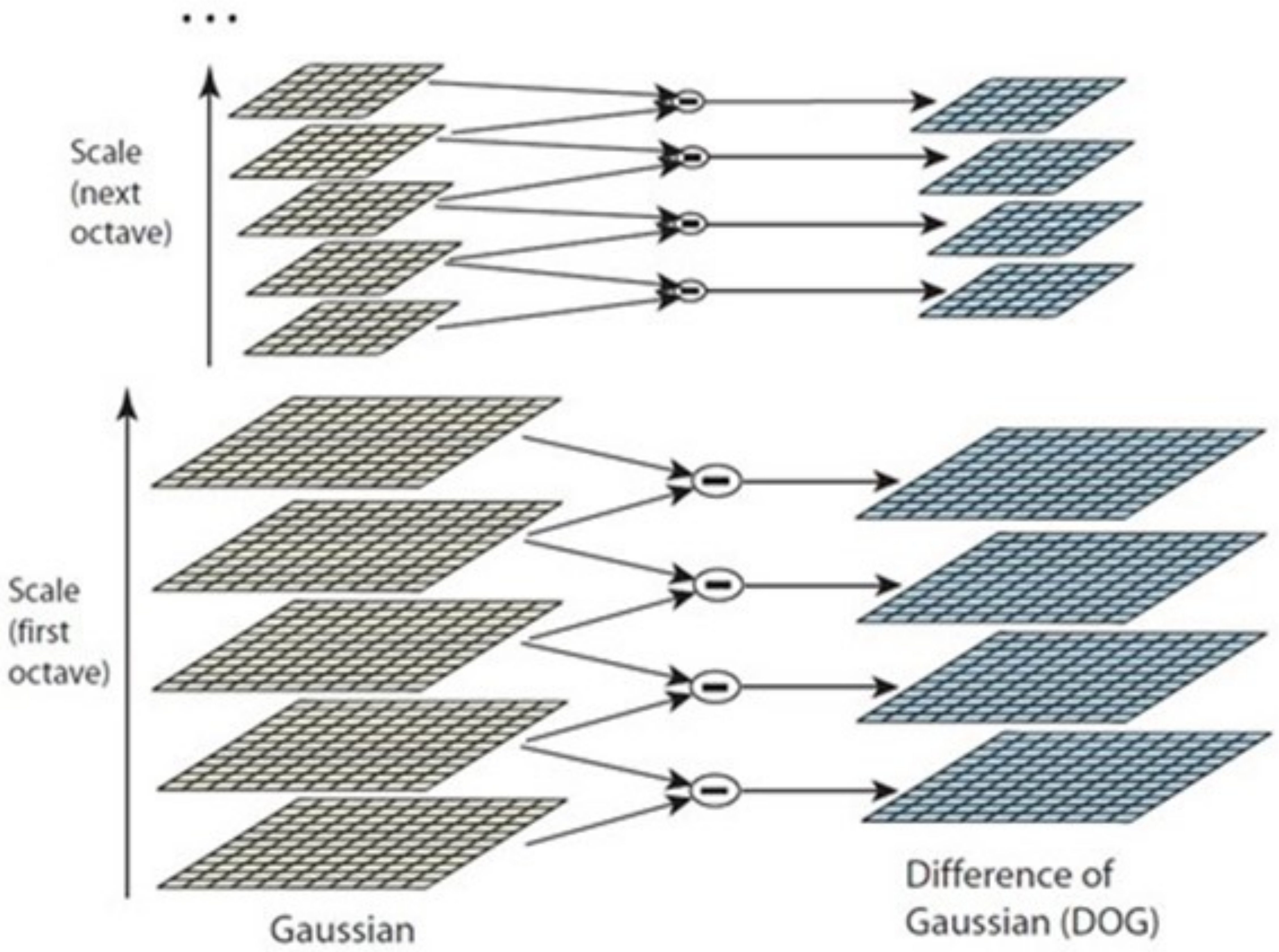
edge response of DoG

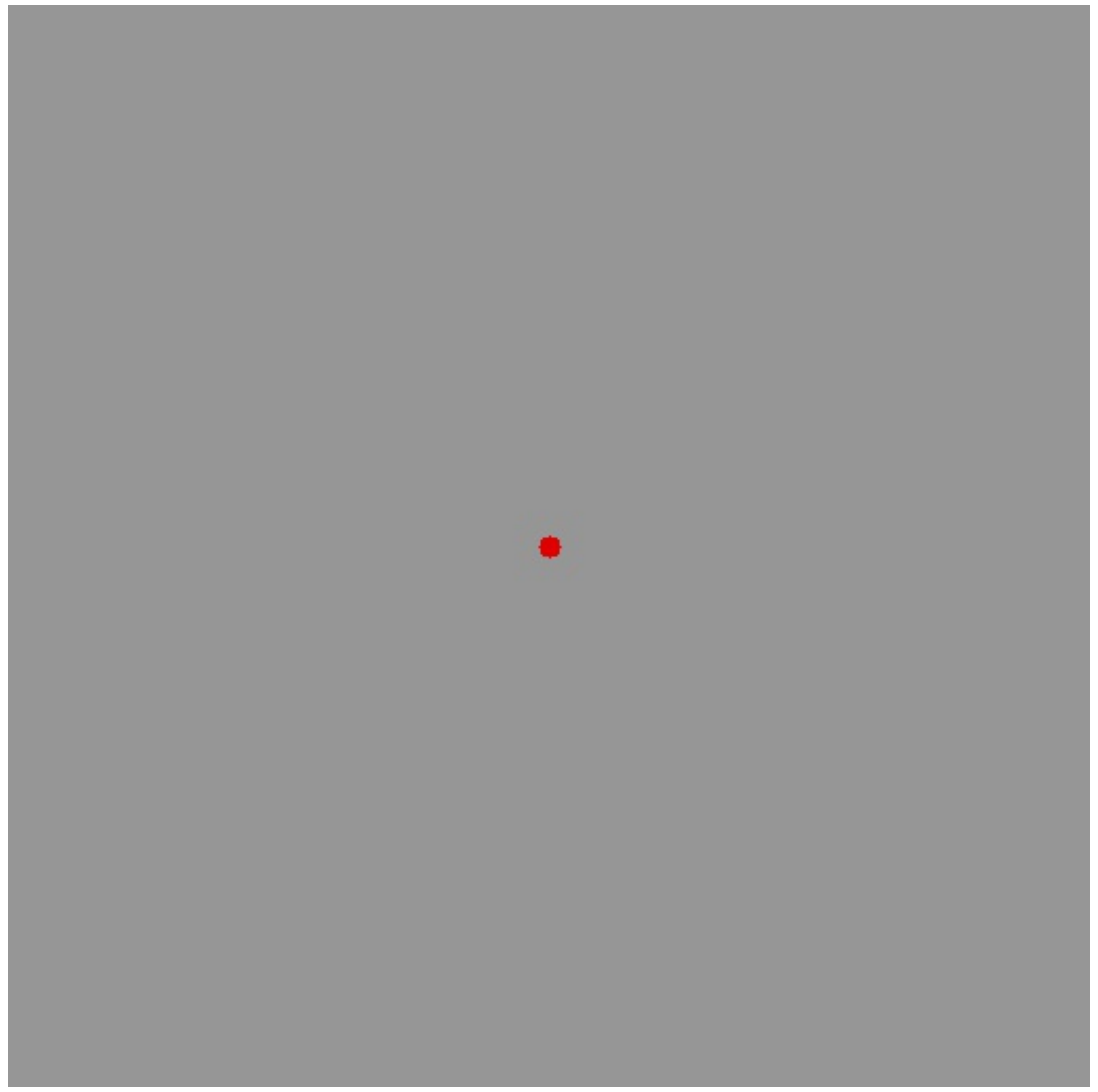
By: 会飞的吴克

高斯差分金字塔（DoG）：


Octave 5
Octave 4
Octave 3
Octave 2
Octave 1
8σ
4σ
2σ
σ

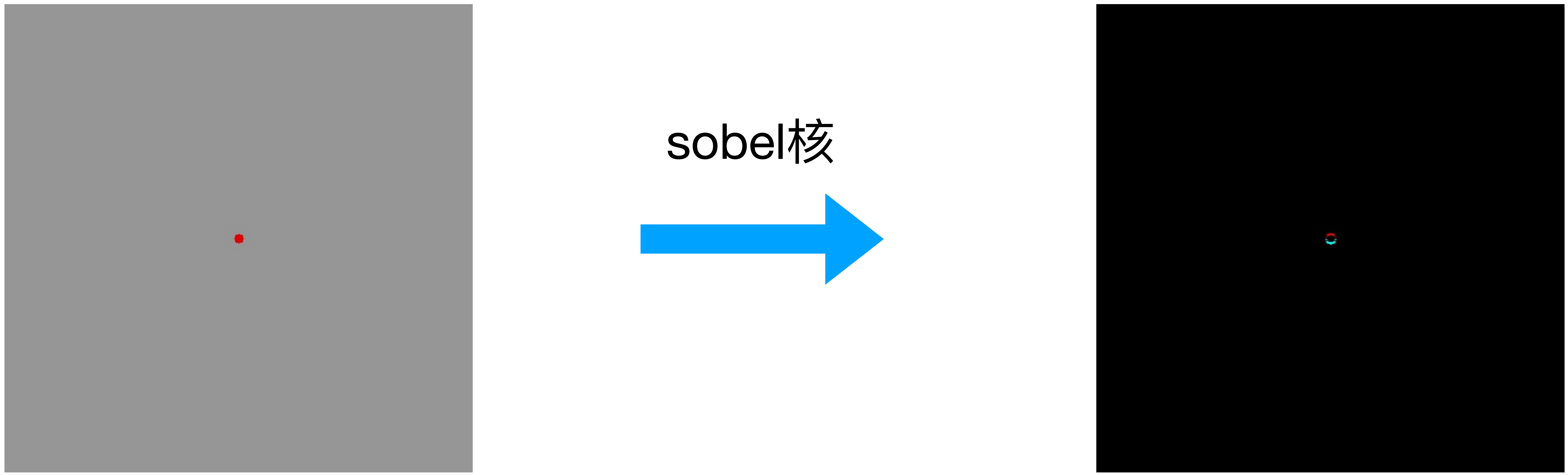

Scale (next octave)
Scale (first octave)
Gaussian
Difference of Gaussian (DOG)

Low pass:

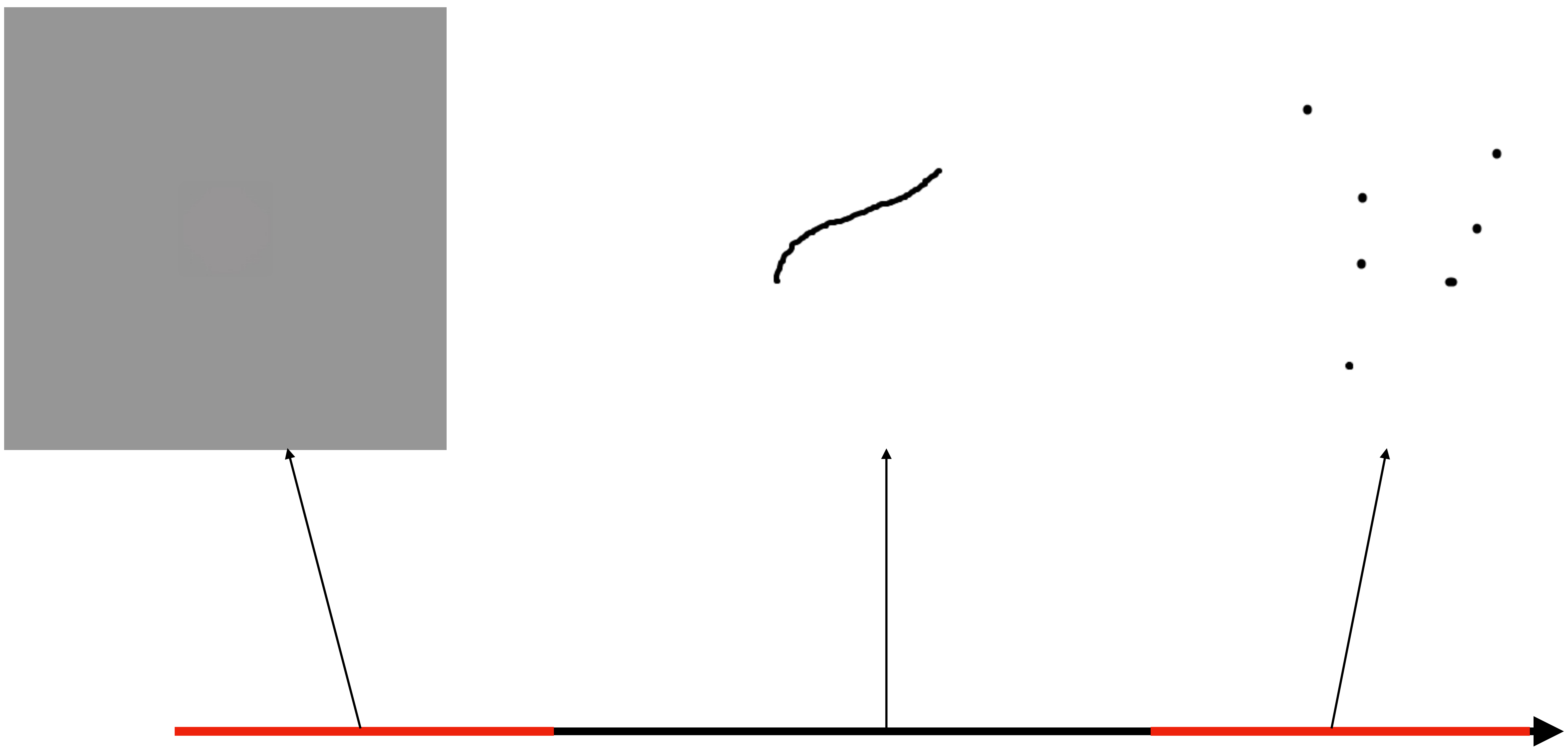


109x109
sigma=100
的高斯核

High pass:



Band pass:



边缘效应的去除

$$\boldsymbol{H}(x,y)=\begin{bmatrix} D_{xx}(x,y) & D_{xy}(x,y) \\ D_{xy}(x,y) & D_{yy}(x,y) \end{bmatrix}$$

$$\mathrm{Tr}(\boldsymbol{H})=D_{xx}+D_{yy}=\alpha+\beta$$

$$\mathrm{Det}(\boldsymbol{H})=D_{xx}D_{yy}-(D_{xy})^2=\alpha\beta$$

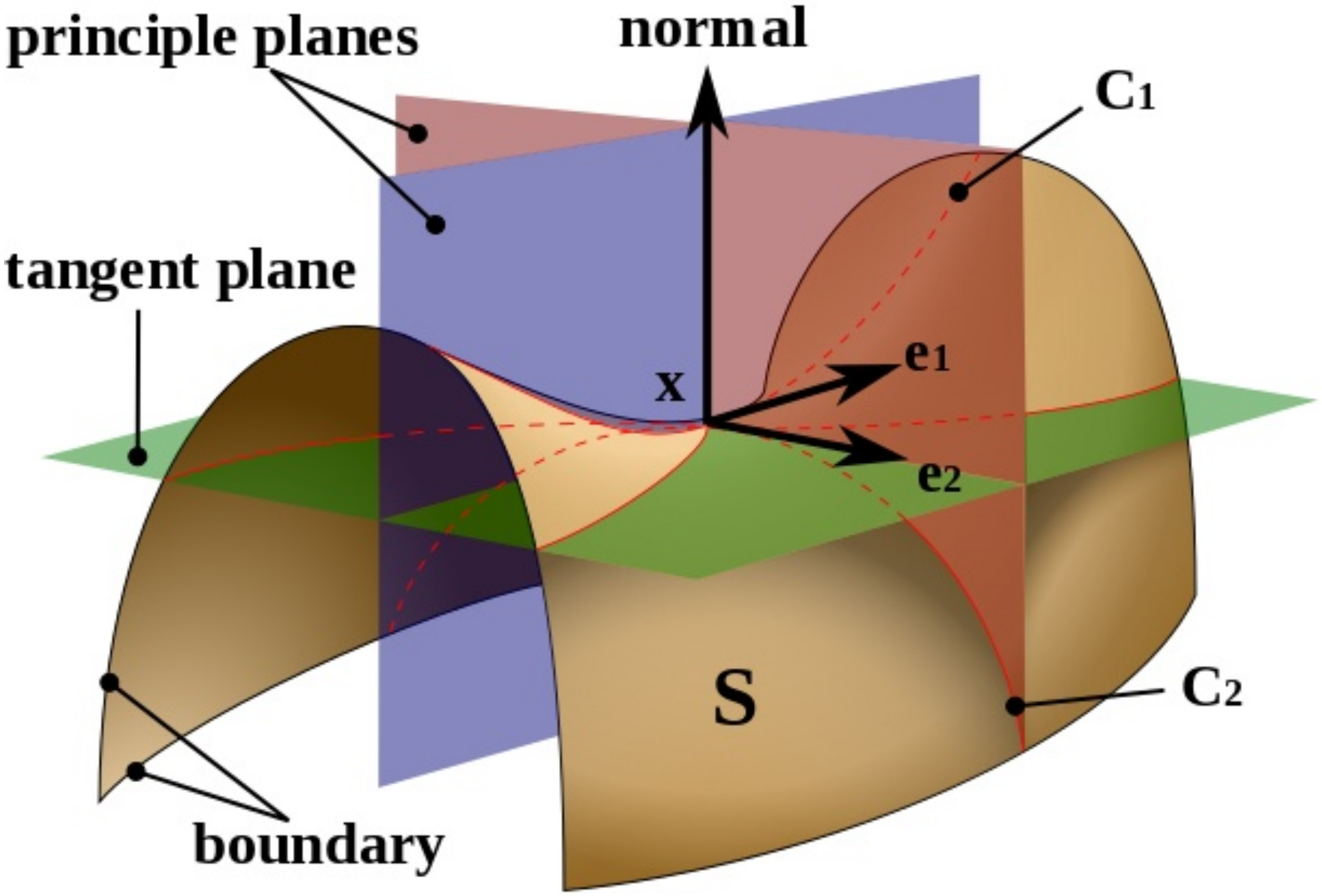
其中：$\alpha>\beta$； 且 $\alpha=\gamma\beta$

若 $\mathrm{Det}(\boldsymbol{H})<0$ 舍去点X

$$\frac{\mathrm{Tr}(\boldsymbol{H})^2}{\mathrm{Det}(\boldsymbol{H})}=\frac{(\alpha+\beta)^2}{\alpha\beta}=\frac{(\gamma\beta+\beta)^2}{\gamma\beta^2}=\frac{(\gamma+1)^2}{\gamma}$$

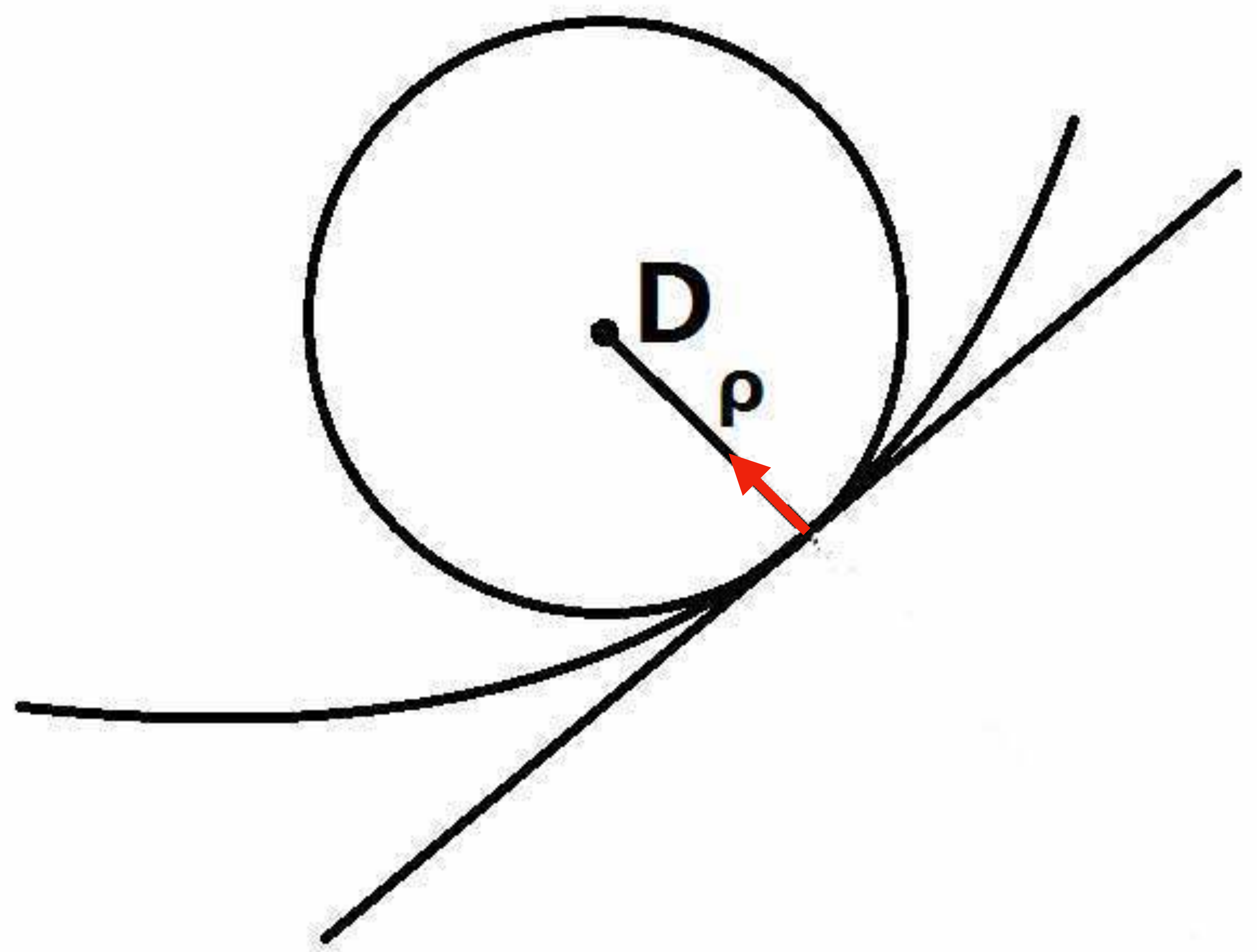
若不满足 $\frac{\mathrm{Tr}(\boldsymbol{H})}{\mathrm{Det}(\boldsymbol{H})}<\frac{(\gamma+1)^2}{\gamma}$ 舍去点X (建议 γ取 10.0)

主曲率和边：


principle planes
normal
C1
tangent plane
x
e1
e2
S
C2
boundary

曲率的定义:单位切向量对弧长的变化率

曲率是一个向量!



设有参数曲线C$(x_1(t), x_2(t), \dots x_n(t))$

则点 t_0 处曲率为:

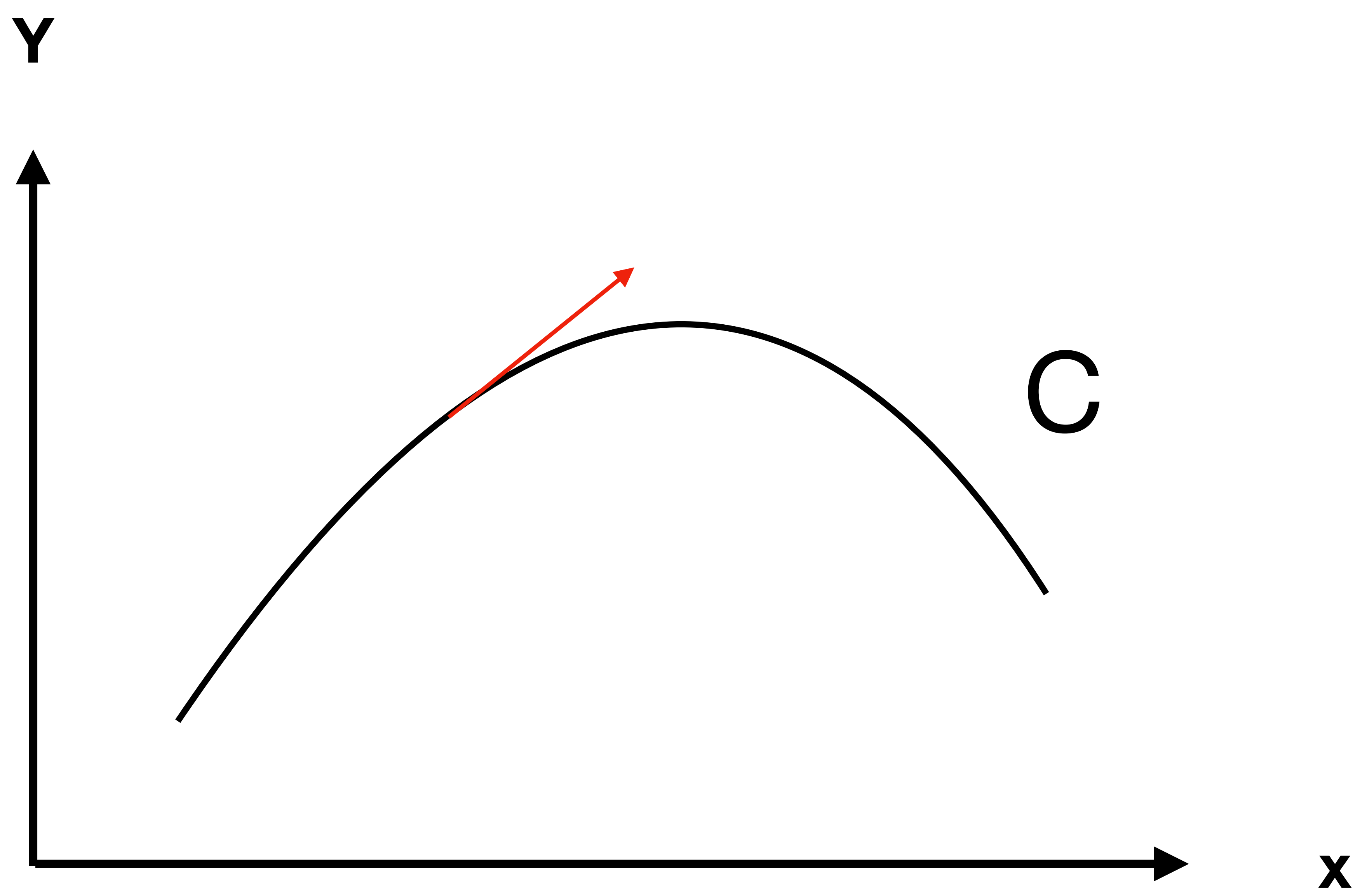
$$C''(s)|_{t=t_0} = \frac{\partial}{\partial s}\frac{\partial C}{\partial s}|_{t=t_0}$$

其中s为 t_0 处的弧长函数:

$$s(t) = \int_{t_0}^{t} \sqrt{(x_1')^2 + (x_2')^2 + \cdots + (x_n')^2}\, dt$$

$$\frac{\partial \alpha}{\partial s} = K = \frac{|y''|}{\left(1+y'^2\right)^{\frac{3}{2}}}$$

曲率的定义:单位切向量对弧长的变化率



$$\frac{\partial \vec{C}}{\partial s} = \frac{\partial \vec{C}}{\partial t} \Big/ \frac{\partial s}{\partial t} = \frac{\vec{v}}{|v|}$$ =单位切向量

因此，曲率= $\frac{\partial}{\partial s}\frac{\partial \vec{C}}{\partial s} = \vec{C}''(s)$

曲率公式的推导:

考虑单位切向量 $\frac{\partial \vec{C}}{\partial s} = \frac{\partial \vec{C}}{\partial t}\frac{\partial t}{\partial s} = \vec{C}'\frac{\partial t}{\partial s}$

故 $\frac{\partial t}{\partial s} = \frac{1}{|\vec{C}'|}$

曲率 $= \frac{\partial \frac{\partial \vec{C}}{\partial s}}{\partial s} = \frac{\partial \frac{\partial \vec{C}}{\partial s}}{\partial t}\frac{\partial t}{\partial s} = \frac{1}{|\vec{C}'|}\frac{\partial \frac{\partial \vec{C}}{\partial s}}{\partial t} = \frac{1}{|\vec{C}'|}\frac{\partial}{\partial t}(\vec{C}'\frac{1}{|\vec{C}'|})$

$$= \frac{1}{|\vec{C}'|}\left(\vec{C}''\frac{1}{|\vec{C}'|} + \vec{C}'\left(\frac{1}{|\vec{C}'|}\right)'\right) = \frac{1}{|\vec{C}'|^4}(\vec{C}''(\vec{C}'\cdot\vec{C}') - \vec{C}'\frac{|\vec{C}'|'}{|\vec{C}'|^2}\cdot|\vec{C}'|^3)$$

$$= \frac{1}{|\vec{C}'|^4}(\vec{C}''(\vec{C}'\cdot\vec{C}') - \vec{C}'(\vec{C}'\cdot\vec{C}''))$$

二维情况的验证： $\frac{1}{\left|\vec{C}'\right|^4}\left(\vec{C}''\left(\vec{C}'\cdot\vec{C}'\right)-\vec{C}'\left(\vec{C}'\cdot\vec{C}''\right)\right)$

对于任意二维曲线y=f(x),其参数曲线C(x,y),代入上式得:

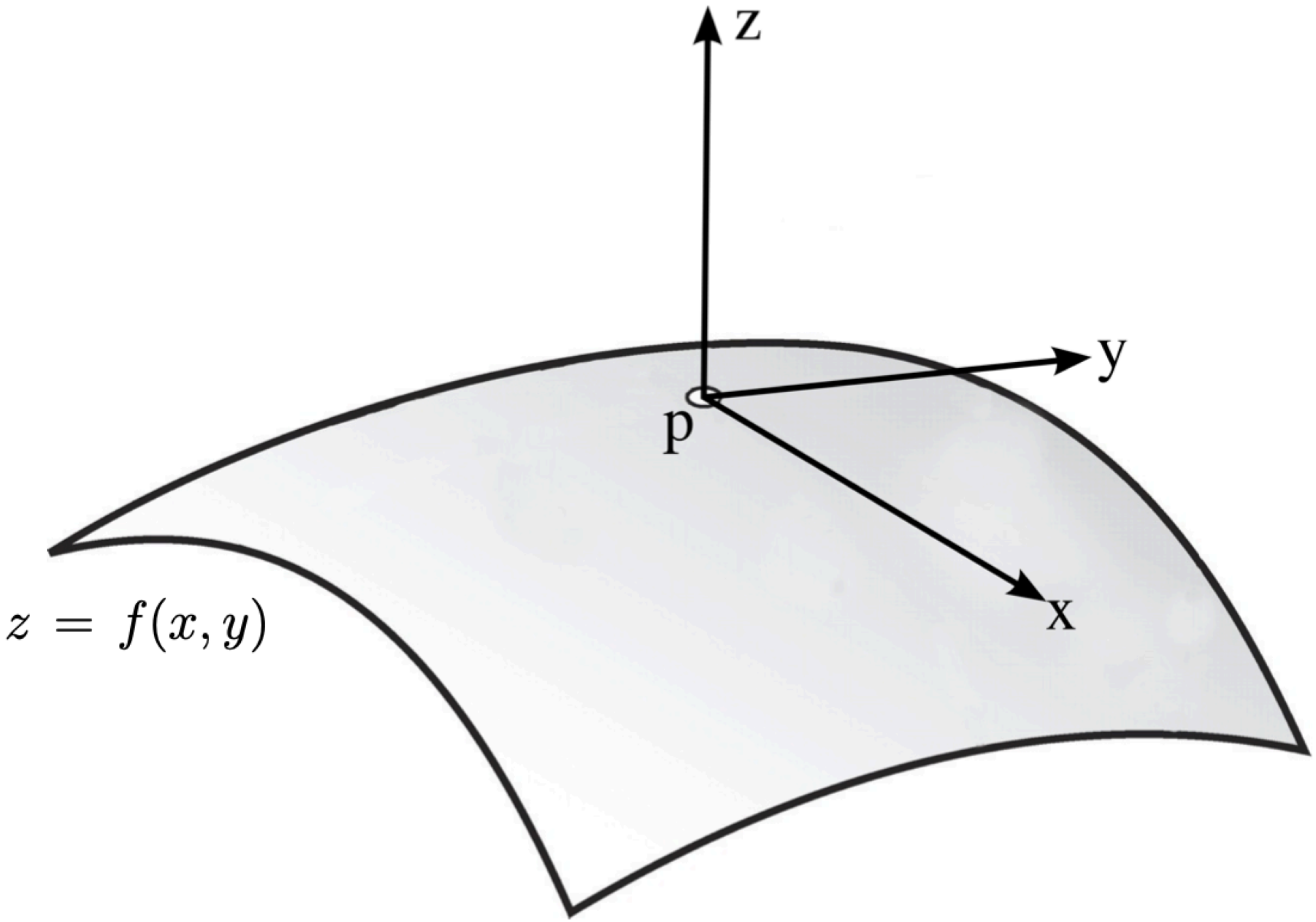
$$\frac{1}{\left(1+y'^2\right)^2}\left((0,y'')\cdot(1+y'^2)-(1,y')\cdot y'y''\right)$$

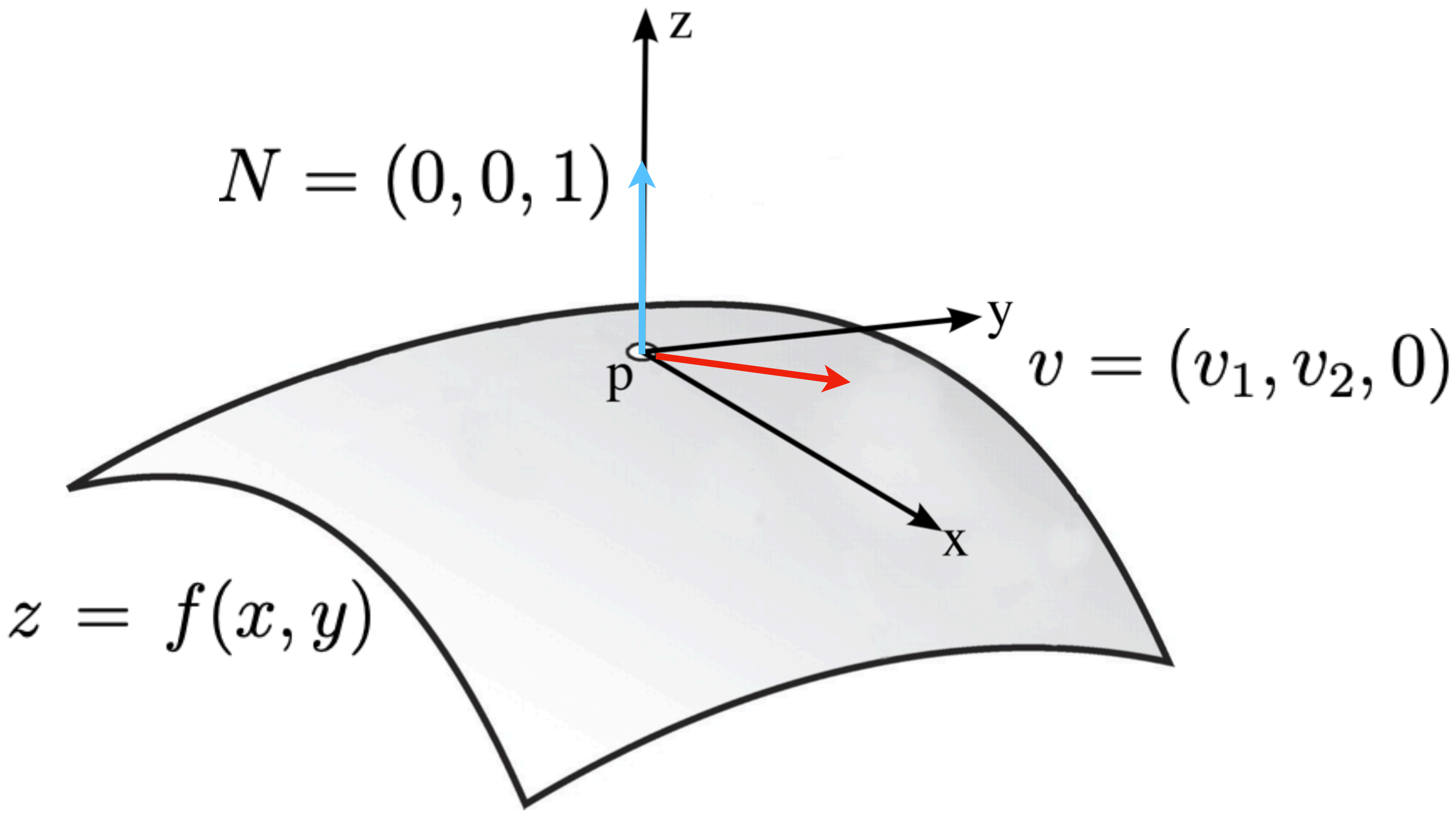
$$=\frac{1}{\left(1+y'^2\right)^2}(-y'y''，\ \mathrm{y}'')$$

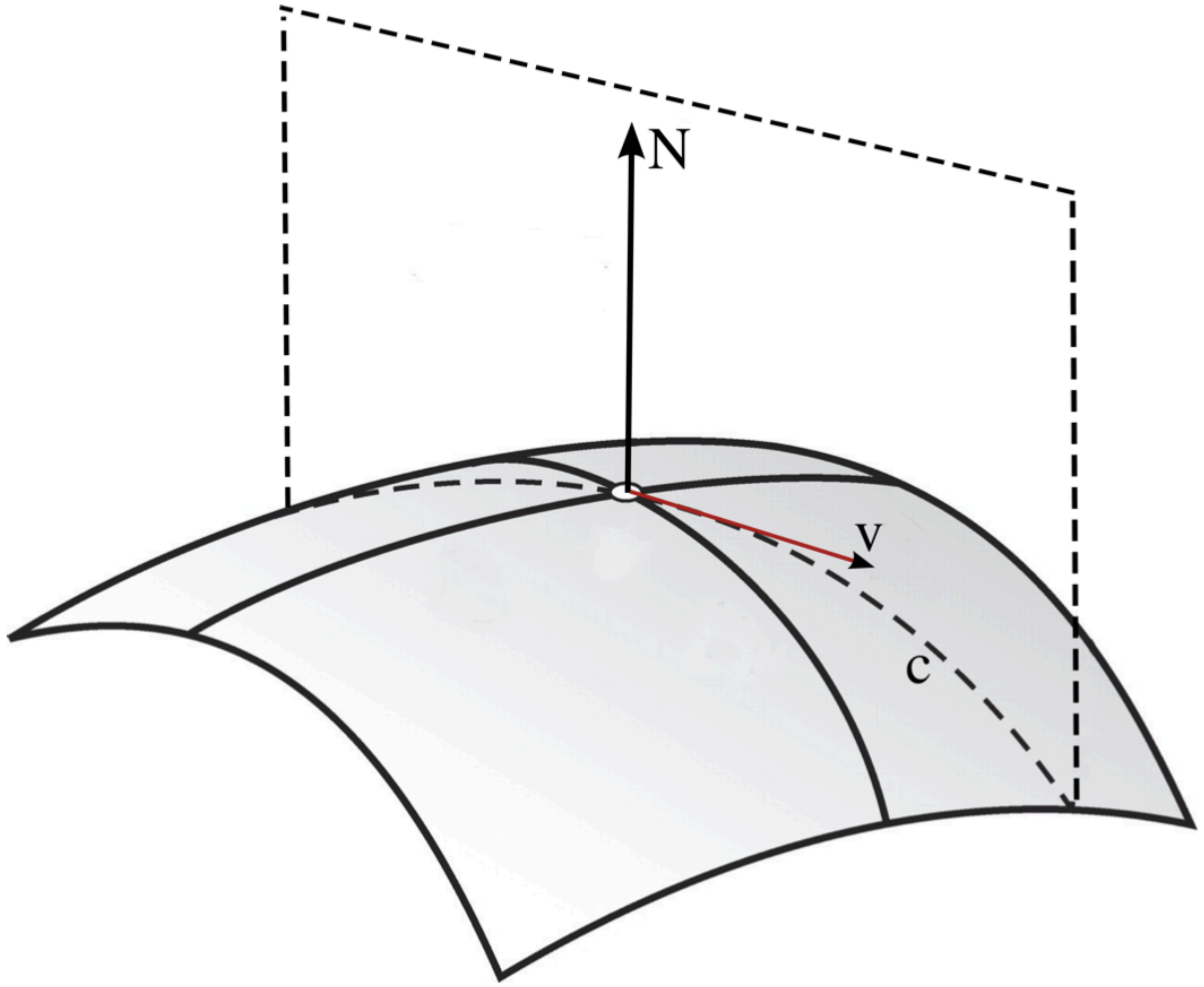
取模得：

$$\left|\frac{1}{\left(1+y'^2\right)^2}(-y'y''，\ \mathrm{y}'')\right|=\frac{\sqrt{y'^2y''^2+y''^2}}{(1+y'^2)^2}=\frac{|y''|}{(1+y'^2)^{\frac{3}{2}}}$$

证明:



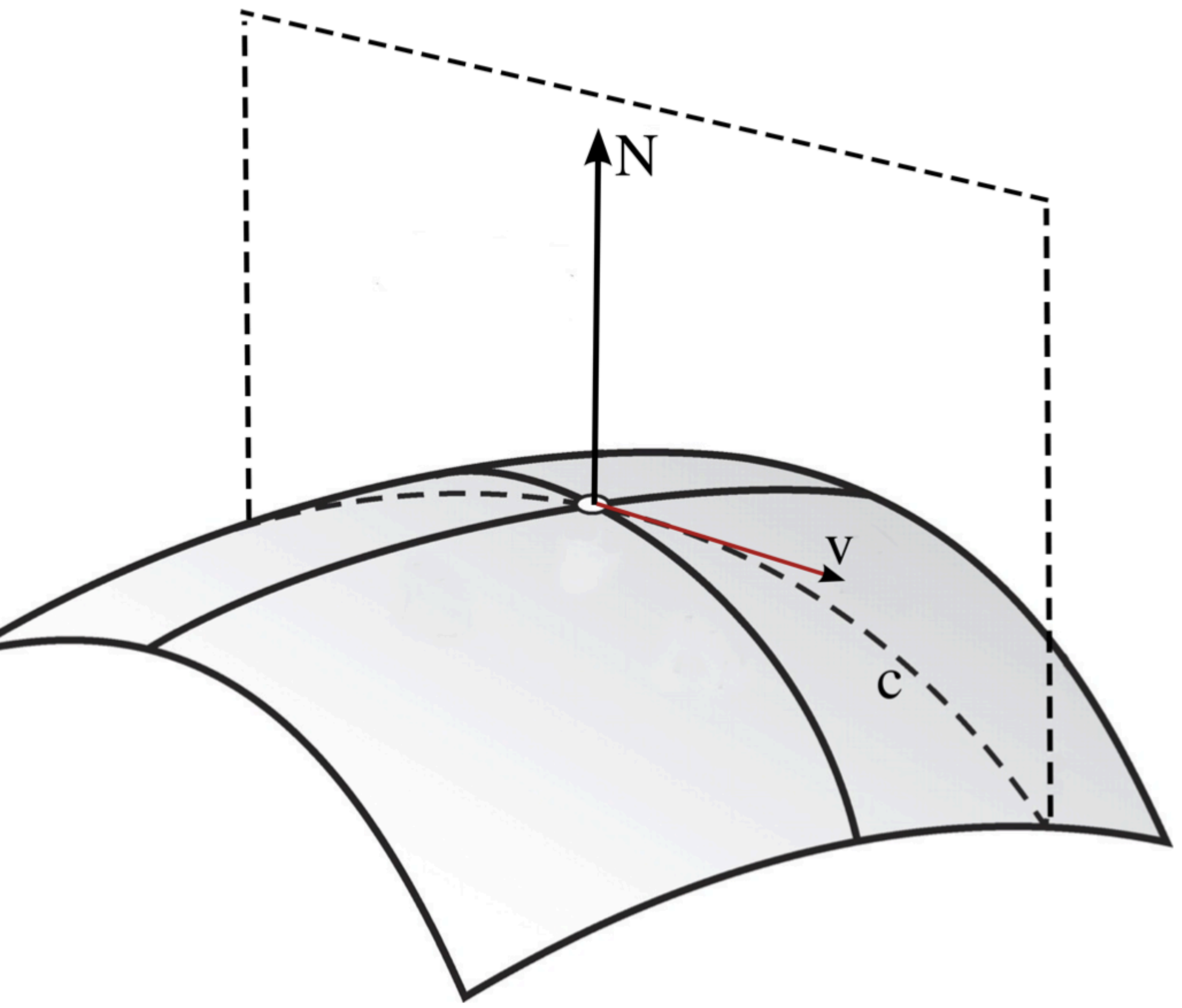




$$c(t) = (v_1 t, v_2 t, f(v_1 t, v_2 t))$$

根据之前的讨论，t0处曲率= $\vec{C}''(s)|_{t=t_0} = \frac{\partial}{\partial s}\frac{\partial \vec{C}}{\partial s}|_{t=t_0}$

故点p处曲率= $\vec{C}''(s)|_{t=0} = \frac{\partial}{\partial s}\frac{\partial \vec{C}}{\partial s}|_{t=0}$



$$c(t) = (v_1 t, v_2 t, f(v_1 t, v_2 t))$$

而在p(0,0,0)处,

单位切向量 $\frac{\partial \vec{C}}{\partial s}|_{t=0} = \frac{\partial \vec{C}}{\partial t}\frac{\partial t}{\partial s}|_{t=0}$

而 $\frac{\partial \vec{C}}{\partial t}|_{t=0} = (v_1, v_2, 0)$ 模长为一

故 $\frac{\partial t}{\partial s}|_{t=0} = 1$

故曲率:

$$\vec{C}''(s)|_{t=0} = \frac{\partial}{\partial s}\frac{\partial \vec{C}}{\partial s}|_{t=0} = \frac{\partial(\frac{\partial \vec{C}}{\partial t})}{\partial s}|_{t=0} = \frac{\partial(\frac{\partial \vec{C}}{\partial t})}{\partial t}\frac{\partial t}{\partial s}|_{t=0} = \vec{C}''(t)|_{t=0}$$

曲率: $\vec{C}''(t)|_{t=0} = (0,0,{v_1}^2 f_{xx}(v_1 t, v_2 t) + 2v_1 v_2 f_{xy}(v_1 t, v_2 t) + {v_2}^2 f_{yy}(v_1 t, v_2 t))|_{t=0}$

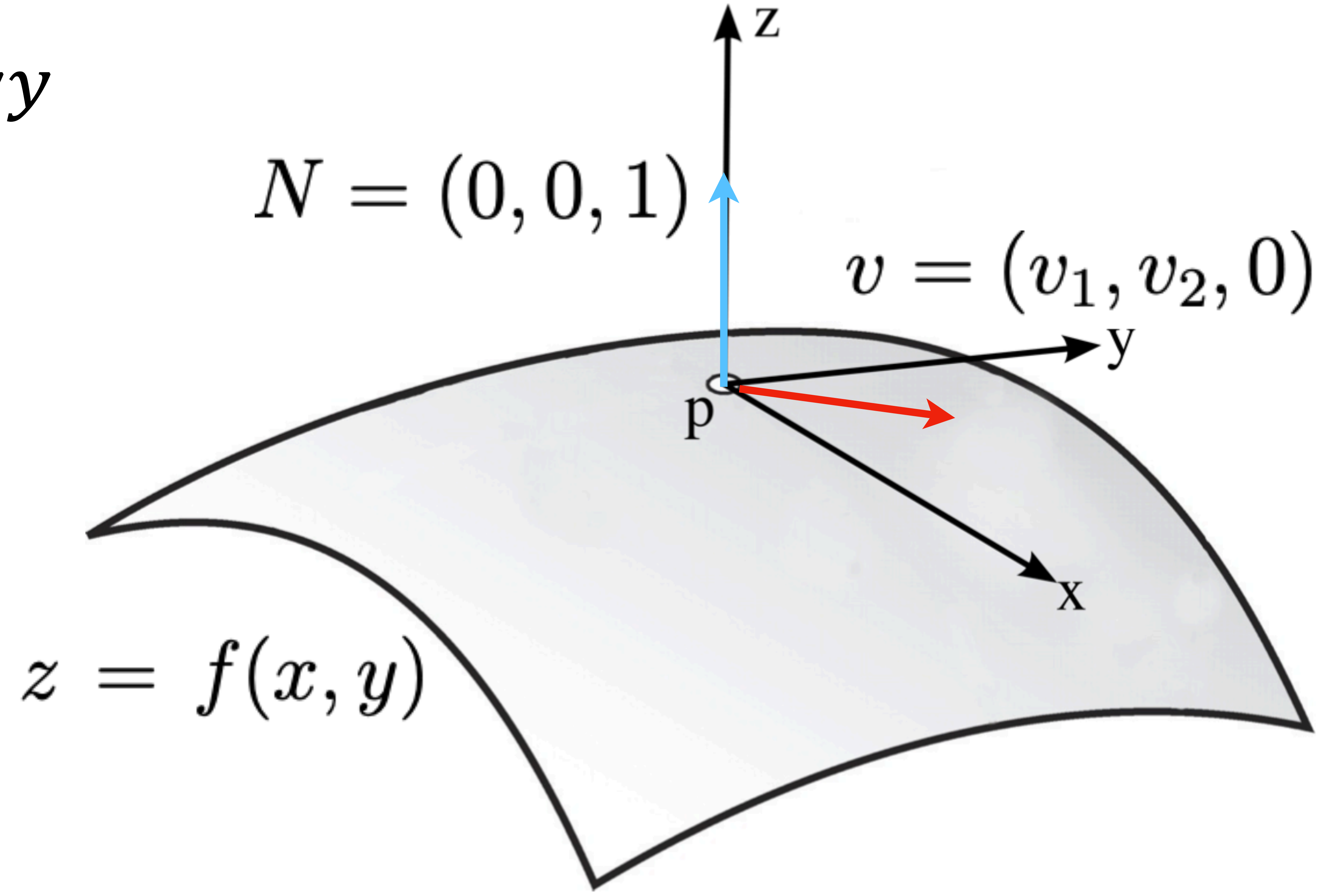
由于曲率= $(0,0,{v_1}^2 f_{xx}(v_1 t, v_2 t) + 2v_1 v_2 f_{xy}(v_1 t, v_2 t) + {v_2}^2 f_{yy}(v_1 t, v_2 t))|_{t=0}$

故曲率大小取决于 $k = {v_1}^2 f_{xx} + 2v_1 v_2 f_{xy} + {v_2}^2 f_{yy}$

$$= \begin{pmatrix} v_1 & v_2 \end{pmatrix} \begin{pmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$= \begin{pmatrix} {v_1}' & {v_2}' \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} {v_1}' \\ {v_2}' \end{pmatrix}$$

$$= \lambda_1 ({v_1}')^2 + \lambda_2 ({v_2}')^2$$



其中 $({v_1}')^2 + ({v_2}')^2 = v_1^2 + v_2^2 = 1$

故 $\begin{cases} k_{max} = \lambda_{max} \\ k_{min} = \lambda_{min} \end{cases}$