Lecture notes

Statistical Methods

Typeset by John Möller

Contents

1 Page 1

Minimal Variance Unbiased Estimator (MVUE):

Theorem 1.1 (Lehman-Schally). Let $T(X_1, \dots, X_n)$ be an unbiased estimator of $g(\theta)$.

$$E(T(X_1, \dots, X_n)) = g(\theta)$$

with finite variance. T is MVUE for θ iff $E(T(X_1, \dots, X_n) = S(X_1, \dots, X_n)) = 0$ for every S s.t. $E(S(X_1, \dots, X_n)) = 0$, $\mathrm{Var}(S) < \infty$, we can use "special estimators" ex BLUE (Best linear unbiased estimator) in lin reg MSE is usually BLUE. CDF:

$$F(x) = P(X \leq x)$$

Definition 1.1 (Empirical cumulative distribution function).

$$F_n(u, (X_1, \dots, X_n)) = n^{-1} \sum_{i=0}^{n} I_{(-u,u)}(X_i) = \begin{cases} 0, & u \leq X_{(1)} \\ \frac{k}{n} X_{(k)} \leq u \leq X_{k+1} & \\ 1 & u > X_{(n)}. \end{cases}$$

1.1 Order statistics

$$X_1, \dots, X_n \Rightarrow X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$$

[!!image 1]

Theorem 1.2 (Glivenko-Conteli).

$$F_n \rightarrow_{n \to \infty} F$$

[!add type of convergence]

$$X_i \sim \mathrm{N}(\mu, \sigma^2)$$
$$\overline{X} \sim N(\mu \frac{\sigma^2}{n}$$
$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Show:

$$\overline{X} \pm \epsilon$$

$$P(-1.96 \le \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \le 1.96) = 95\% / - \frac{\sigma}{\sqrt{n}}$$
$$P(-1.96\frac{\sigma}{\sqrt{n}} \le \overline{X} - \mu \le 1.96\frac{\sigma}{\sqrt{n}}) = 95\% / - \overline{X}$$
$$P(-1.96\frac{\sigma}{\sqrt{n}} - \overline{X} \le -\mu \le 1.96\frac{\sigma}{\sqrt{n}} - \overline{X}) = 95\% / - (-1)$$
$$P(\overline{X} - 1.96\frac{\sigma}{\sqrt{n}} \le \mu \le \overline{X} + 1.96\frac{\sigma}{\sqrt{n}}) = 95\%.$$

Definition 1.2 (Confidence Interval). A RANDOM interval (L, U)
$[L, U]$ [?] is a confidence interval for parameter θ with confidence level α if

$$P(L \le \theta \le U) = 1 - \alpha$$

Confidence interval (L, U) covers θ with $1 - \alpha$ probability.

$$\overline{X} \pm z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

is a α-level confidence interval for μ.

Show: $(-\infty, \overline{X}+$