Physics Tutorial 2: Exercises in MRI Signal and SNR

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This tutorial will cover MRI pulse sequences and the signals they produce. A MATLAB simulator will be used to get a better understanding of MRI signal and noise characteristics. Questions without any asterisks are those that should be attempted by everyone, whereas more challenging questions are marked with one (*) or two (**) asterisks, and should be considered optional. Take these opportunities to think about these questions, and then discuss your answers with your tutor.

At the end of the tutorial period, you will receive a "take-home tutor", which is an annotated version of this tutorial guide that will help you complete the tutorial at home if you don't manage to make it all the way through with your tutor, or if you missed the tutorial session.

Attendance will be taken by the tutors, and marks will be given by participation. If you would like additional feedback or clarification on the tutorial material, you are welcome to submit your questions or comments to Weblearn, and a tutor will provide written feedback for you. Those unable to attend the tutorial must submit answers to all unstarred questions to receive credit for the tutorial.

Part 0 – Getting Started

Download and unzip the file containing the tutorial resources from Weblearn, or if you have access to the FMRIB internal network, copy them into your current directory from here:

```
~mchiew/GradCourse/2_Signal_and_SNR
```

Start MATLAB, and make sure you're inside the tutorial directory (i.e., the folder containing all the tutorial files).

NB: we need the jvm for this practical, so DO NOT use the -nojvm option when starting MATLAB

Part 1 – Pulse Sequences *[10-15 minutes]*

An MRI pulse sequence is a series of RF (radio-frequency) and gradient magnetic fields, defined with specific timings and amplitudes to produce meaningful MRI signals or images.

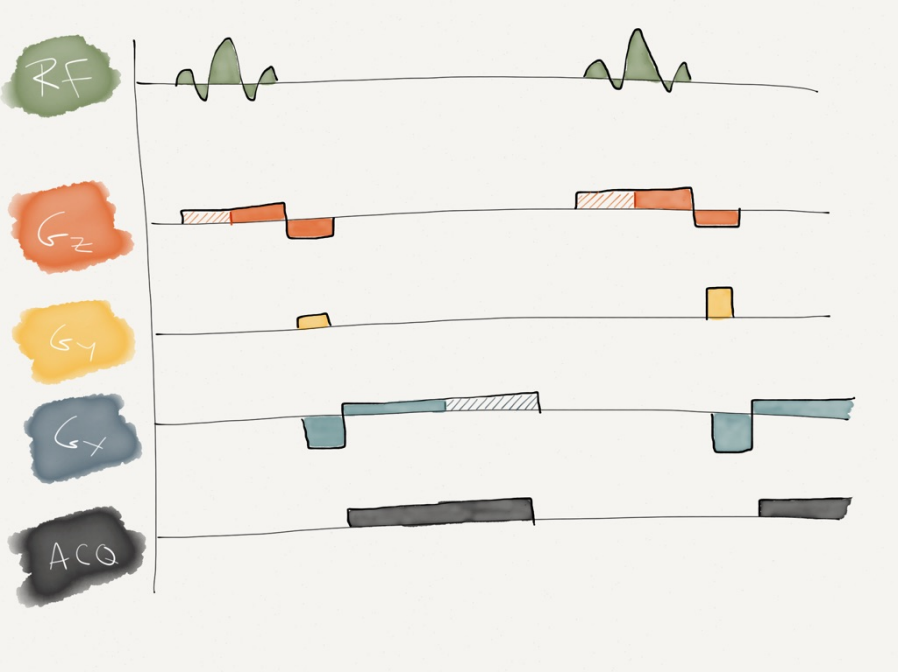

Figure 1

The primary components of a pulse sequence diagram are (see Fig. 1):

1. RF Excitation (typically labelled "RF")
2. z-gradient amplitude (typically labelled G_z or "z-grad")
3. y-gradient amplitude (typically labelled G_y or "y-grad")
4. x-gradient amplitude (typically labelled G_x or "x-grad")
5. signal acquisition (typically labelled ACQ or ADC)

Advanced students may be interested in the distinction between the "physical" (as described above), the "imaging" (slice select, phase-encode, readout) and the "anatomical" co-ordinate system (left-right, anterior-posterior, superior-inferior). This is one of the reasons neuroimaging researchers have to take some care in ensuring what orientation output images are in (i.e., is left on the image the left side of the brain, etc…)

Exercise 1.1

Label the pulse sequence parameters of TR and TE on the diagram in Fig. 1. Indicate which components of the pulse sequence are changed when flip angle and bandwidth are modified.

- ***TR*** *is the "repetition time", normally the time between successive RF excitation pulses. In Figure 1, TR spans between the two RF pulses. Note that this refers to successive RF excitations of the same slice.*
- ***TE*** *is the "echo time", which is the duration between the RF excitation pulse and the centre of the k-space readout. In Figure 1, TE is from the RF to the centre of the ACQ window.*
- ***Flip angle*** *is the amount by which the RF excitation pulse rotates or "flips" the magnetisation from the z-axis onto the transverse x-y plane. This is controlled by increasing the strength or duration of the RF excitation pulse.*
- ***Bandwidth*** *refers to the spread of resonance frequencies across the image (when referred to as bandwidth-per-pixel, it is the spread of frequencies across a voxel) that is produced by varying the amplitude of the readout gradient.*

Question 1.2*

What would happen to the output image from the sequence in Fig. 1 if the G_y and G_x waveforms were completely swapped?

There are multiple answers to this question with varying levels of detail.

- *Assuming square voxels and a square FOV, and the entire sample fits inside the FOV, and disregarding T2 decay, and any field inhomogeneity/off-resonance, nothing happens. The image would look identical. There are no fundamental differences between phase and frequency encoding.*
- *If the sample was larger in the former readout direction than the FOV, then aliasing would occur because the anti-aliasing filter is now applied along a different dimension. Conversely, if there was aliasing previously along the former phase-encode direction, there would be none following the axis change.*
- *With non-square voxels or FOV, the voxel dimensions or FOV would change with the axes.*
- *Any artefacts due to T2 decay along the readout, as well as off-resonance induced distortion or chemical shift would change with the axes*

Part 2 – Signal Simulator

(30 minutes)

NB: we need the jvm for this practical, so DO NOT use the -nojvm option when starting MATLAB

The MRI signal simulator `simSignal` will be used to demonstrate the relationship between the T1 of the tissue and the sequence parameters TR and flip angle.

This function requires the definition of the parameters `T1, TR, flipAngle and tMax`. (T1 and TR are in milliseconds, the flip angle in degrees; tMax is the total time to be simulated).

Consider the following pulse sequence simulated on a 3T scanner using a TR = 1000 ms and a flip angle = 90°. We assume a nominal value of T1 = 1200 ms to represent grey matter at 3T.

Matlab commands:

```
>> T1=1200;
>> TR=1000;
>> flipAngle=90;
>> tMax=5000;
>> simSignal;
```

Walk through the components of the resulting figure. Pay particular attention to the relationship between the available M_z magnetisation (upper subplot) and the measureable M_{xy} magnetisation (bottom subplot). Additionally, note that the simulation begins from equilibrium and shows the approach to steady state M_z.

The quantity that M_z represents is the amount of potential or stored magnetization that is available for subsequent measurements. The following analogy may make the role of M_z a little clearer:

- Consider the magnetisation like a battery, which is periodically discharged to produce a "signal", where the signal strength is related to the amount of charge available in the battery. Batteries with a larger T1 value take longer to recharge (recover) than batteries with shorter T1's. Therefore, to get maximum signal, you

can wait shorter durations (TRs) for the magnetisation to recharge for batteries with shorter T1's.

Question 2.1
In the bottom sub-plot, how does the M_{xy} signal for the second TR compare to the signal from the first TR? What causes this effect?
The measured M_{xy} signal in the second TR is lower than the first because the available magnetisation is reduced due to incomplete T1 recovery. That is, if not enough time is given for the magnetisation to "recharge" in between excitations, the measured signal will be reduced in all measurements after the first.

Depending on T1, TR and flip angle it can take many measurements before an equilibrium condition ("steady state") is reached. It is because of this effect that the first few images in an fMRI data-series are brighter than the rest, and are discarded (these are sometimes referred to as the dummy or prep scans).

Question 2.2
Consider a flip angle of 65° instead of 90°. What happens to the steady-state signal and why?
Two changes are evident from the flip angle change:

1. *The steady state M_z (and M_{xy}) signal levels are higher in the 65° case. This is because with a flip angle < 90°, not all of the M_z magnetisation is "used up". That is, given the same amount of time for T1 recovery, starting at a higher value means ending up at a higher value. Using a lower flip angle means less of the longitudinal magnetisation ends up in the transverse plane, but in this case the higher M_z just before each RF pulse leads to a larger M_{xy} after the RF pulse.*
2. *It takes longer to establish the "steady-state". This is a more subtle point, given that steady-state is reached when the difference in M_z from immediately prior to one excitation to immediately prior to the next is negligible. The reason 90° flip angle pulses reach equilibrium immediately is due to the fact that after a 90° excitation, Mz always starts recovering from 0. Therefore the steady state condition is immediately satisfied. The battery analogy may be useful in helping students visualize this effect. Another way of looking at this is that the Mz lost due to the RF excitation is exactly cancelled by the T1 recovery during the TR.*

The importance of the flip angle becomes noticeable at shorter TR. In fMRI for example, images can be acquired in a single excitation, and time between subsequent excitations can be 1-3 seconds long. In structural imaging however, only single lines of k-space are acquired after each RF excitation pulse. Thus, 192 TRs are required to measure 192 lines of k-space, and these 192 lines would then make up a single slice. The TR therefore needs to be short to keep the total scan times reasonable. A typical structural imaging sequence has a TR around 20ms.

Question 2.3
Using the simulator, find the flip angle that maximises the steady-state magnetisation using a TR = 20ms, assuming T1 = 1200 ms.
Use a range of flip angles to demonstrate the effects of flip angle on the steady-state magnetisation in a low-TR sequence. A flip angle of ~10-11° should maximise the steady-state signal at around 0.109.

Question 2.4**
Find an expression for the steady-state longitudinal magnetisation (Mz) as a function of M_0, T1, TR and flip angle.
Hint: the longitudinal magnetisation $M\theta$ after a flip angle θ is given by:

$M_\theta = M_0 + (M\cos\theta - M_0)e^{-t/T_1}$, *where M is the longitudinal magnetisation just before excitation.*

Solve this by setting $M_z^{n+1} = M_z^n$, *where* n *is the measurement/repetition index.*

Answer: $M_z = \frac{M_0\left(1-e^{-TR/T1}\right)}{1-\cos\theta e^{-TR/T1}}$

The optimal flip angle ("Ernst angle") is actually found by setting the derivative of the transverse magnetisation ($M_{xy} = M_z\sin\theta$*), with respect to θ, to zero and solving for θ.*

The "Ernst angle" is a mathematical expression for the optimal flip angle given a TR and T1, which gives the maximum transverse steady-state magnetisation:

$$\cos(\theta) = e^{-TR/T1}$$

The `simSignalvFlip` simulation tool can be used to see the variation of signal with flip angle. The Ernst angle is the flip angle which optimally balances the amount of signal you get (by draining the M_z "battery") and the amount of signal you save for the next excitation (by leaving some M_z "charge" unused).
Explore this plot with students, noting that apart from the maximum, many steady-state signal values can occur by under- or over-flipping the magnetisation. Note the large range in steady-state signal values that occur between 0 – 90°, and that consequences of choosing the wrong flip angle can cause significant changes to output images.

Question 2.5**
When TR << T2, then much of the transverse magnetisation will not be decayed by the time the next excitation occurs. What normally happens to this magnetisation, and how can it be useful?
Typically in non-steady-state sequences (this is a different use of the term "steady state"), transverse magnetisation is spoiled away using either or both of RF spoiling and gradient spoiling, to effectively mimic a long TR acquisition without having to wait. This prevents the transverse magnetisation from being flipped and manipulated along with the M_z *magnetisation that we typically consider. This is how FLASH (or SPGR) sequences operate.*
Steady-state sequences (such as SSFP, FISP, GRASS and derivatives) operate by explicitly not spoiling away the residual transverse magnetisation, instead carefully allowing it to evolve along with the normal recovered M_z *magnetisation to form a complicated steady state. While these sequences require more complicated signal models, they have the advantage of retaining much more magnetisation that ultimately produces signal, resulting in higher SNR efficiency compared to their spoiled counterparts.*

Part 3 – SNR

(30 minutes)

One of the most useful measures of image quality is the signal-to-noise ratio (SNR), which quantifies the signal strength relative to the amount of noise present. SNR is defined as the ratio of the mean signal in a region of interest (ROI) to the standard deviation of the noise in that region:

$$SNR = \frac{\text{mean}(signal)}{\text{st. dev}(noise)}$$

Intuitively, the SNR expresses how many times larger the signal is than the noise:

- **SNR < 1** indicates that the noise variation is larger than the signal *(not good)*
- **SNR = 1** indicates that the signal and noise fluctuation strength are equal (signal indistinguishable from noise)
- **SNR > 1** indicates signal is larger than noise deviations *(higher the better)*

It may be useful to go over the definition of standard deviation and variance (where std.dev = sqrt(variance)), in case there is a lack of understanding on how you would quantify the size or strength of noise, which by definition has a non-fixed value.

Extra Information**

The source of noise in MRI is a deep and complex (*no pun intended*) topic. In simple terms, noise in MRI images comes from random fluctuations in the received MRI signal that come from the human body and the measurement coil. These intrinsic noise fluctuations have a few properties:

- they are "**white**", which means that they have no preferred frequency (i.e., a flat spectrum)
- they are "**additive**", which means that the noise can be considered as being added on top of the signal
- they have a **Gaussian** distribution, which means that if a histogram of all the noise values is created, they are shaped like a Gaussian or bell-curve
- they have **zero mean**, which means that the noise fluctuations are balanced around zero, and that the average of all noise values is zero.

Some of these properties may benefit from a diagram or additional elaboration. Referring to Figure 2 (LHS) may be instructive as well.

Ultimately, everything that we record and measure in MRI comes through the receive coil (or coils), and receiver chain electronics (such as pre-amplifiers, filters, etc…). Ignoring noise added or modified after the receive coil, the electrical noise power can be expressed as a function of the temperature and the coupling between the coil and the sample (e.g. body). The Johnson noise formula dictates the differential noise power per unit bandwidth in a system with resistance R and temperature T as:

$$d\sigma^2 = 4 \cdot k_B \cdot T \cdot R \, df$$

where k_B is the Boltzmann constant. After integrating across receiver bandwidth, the total noise power σ^2 is the variance of the additive Gaussian white noise. Note that this integration is the source of the relationship between SNR and bandwidth.

The resistance term is an equivalent resistance of the coupled body-coil system, and is dependent on specific body and coil geometries and electrical properties (conductivity, capacitance, etc.). This equivalent resistance is dependent on coil resonance frequency, ω_0, which is one of the ways that SNR is related to B_0. It is also dependent on the coil receive field sensitivity, which is why surface coils have higher SNR than volume coils (smaller receive fields mean smaller equivalent resistance, and therefore reduced noise).

Demonstrate the measurement of SNR from a tissue ROI, for both signal and noise, using the interactive ROI drawing tools

```
>> load structural;
>> SNR_calc_with_ROIs(structural)
```

Unfortunately, measuring standard deviations within a tissue ROI can also reflect "real" variation of the tissue, making this a poor estimate of the noise. Instead, the noise is often estimated from a background region of the image where there should be no signal. However, because most images only contain the magnitude of the signal (i.e., do not

allow negative numbers), this does introduce one mathematical complication.
Discussion of the difference between intrinsic noise vs. artefacts may also be useful

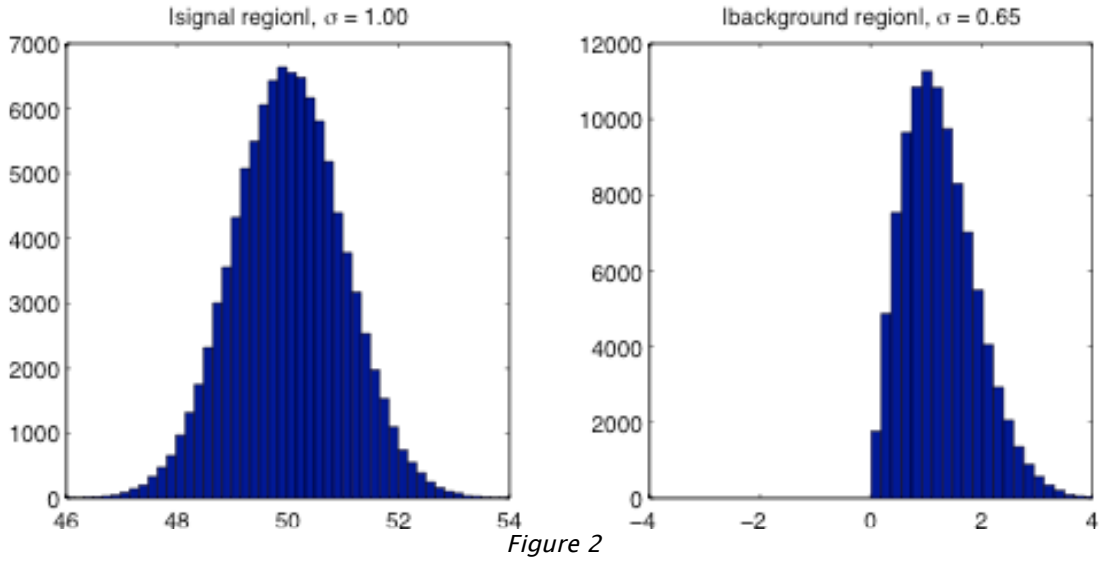

Figure 2

In regions with signal (e.g., tissue), the additive zero-mean noise causes the measured values to fluctuate around the true signal, with equal tendency to add to and subtract from the signal (see Fig. 2, LHS). In background regions with no signal (e.g. outside of the head), the negative component of the noise is forced to be positive by taking the signal magnitude. This causes the standard deviation of the measured values to be smaller in a background region than it is in a signal region; the noise in the background region follows the Rayleigh distribution, of which the standard deviation is 0.65 times that of the standard normal distribution (see Fig. 2, RHS).
Consider showing students histograms of the signal values in tissue_mask and background_mask on the same horizontal bin width, demonstrating the difference in variance (essentially live replicating Fig. 2).

```
>> tissue_roi = find(tissue_mask);
>> background_roi = find(background_mask);
>> tissue_vals = structural(tissue_roi);
>> background_vals = structural(background_roi);
>> S = mean(tissue_vals);
>> N = std(tissue_vals);
>> tissue_bins = linspace(S-3*N, S+3*N, 50);
>> background_bins = linspace(-3*N, 3*N, 50);
>> figure();
>> subplot(1,2,1);
>> hist(tissue_vals, tissue_bins);
>> xlim([S-2*N, S+2*N]);
>> subplot(1,2,2);
>> hist(background_vals, background_bins);
>> xlim([-2*N, 2*N]);
```

Also show that the 'background' might also contain signal—in the form of ghosting, and that selection of the background region should therefore be informed by phase-encode direction, amongst others.

Exercise 3.1

Measuring noise from background regions in images can be tricky. Use the `SNR_calc_with_ROIs` function to estimate SNR with normal image scaling. Then, use `SNR_calc_with_ROIs` again with an display windowing factor of 1000 (i.e. `SNR_calc_with_ROIs(structural, 1000);`), and try again. Discuss the difference this makes on the estimated SNR.

Question 3.2

Assuming noise is quantified using a standard deviation measured in a background (no signal) region of a magnitude image, how might the "true" noise and therefore the SNR be estimated from this measure? (Hint: consider the difference between the two plots in Fig. 2)

If the noise standard deviation was measured in a no-signal background region, we expect the standard deviation to be 0.65 smaller than it should have been. Therefore, the standard deviation term should be scaled by 1/0.65 to correct for the under-estimation:

$$SNR = \frac{S}{\sigma_{BG}/0.65} = 0.65 \cdot \frac{S}{\sigma_{BG}}$$

Use the provided datasets `epi_ave1` and `epi_ave9` to demonstrate the SNR differences resulting from a different number of averages.

Question 3.3

Why does increasing the number of averages increase the SNR, and by how much?

The SNR of an image can be modified by averaging multiple image acquisitions. This is because adding together images containing random noise causes the signal to increase (or add coherently), while the noise decreases due to cancellation (adding incoherently). Generally, SNR is increased by $\sqrt{N}$, where N is the number of separate images included in the average.

Discussion 3.4**

In general, MRI images are composed of complex signals with complex noise. However, it is normally convention to convert the complex image to a real-valued image for display and analysis. What is the SNR difference between an image produced by taking the real part of a complex source image, compared to taking the magnitude? For what reasons aside from SNR might you choose one method over the other? (Hint: assume the complex noise follows a bi-variate normal distribution with equal variance along real and imaginary axes)