Physics Tutorial 2: Exercises in MRI Signal and SNR

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This tutorial will cover MRI pulse sequences and the signals they produce. A MATLAB simulator will be used to get a better understanding of MRI signal and noise characteristics. Questions without any asterisks are those that should be attempted by everyone, whereas more challenging questions are marked with one (*) or two (**) asterisks, and should be considered optional. Take these opportunities to think about these questions, and then discuss your answers with your tutor.

At the end of the tutorial period, you will receive a "take-home tutor", which is an annotated version of this tutorial guide that will help you complete the tutorial at home if you don't manage to make it all the way through with your tutor, or if you missed the tutorial session.

Attendance will be taken by the tutors, and marks will be given by participation. If you would like additional feedback or clarification on the tutorial material, you are welcome to submit your questions or comments to Weblearn, and a tutor will provide written feedback for you. Those unable to attend the tutorial must submit answers to all unstarred questions to receive credit for the tutorial.

Part 0 – Getting Started

Download and unzip the file containing the tutorial resources from Weblearn, or if you have access to the FMRIB internal network, copy them into your current directory from here:

```
~mchiew/GradCourse/2_Signal_and_SNR
```

Start MATLAB, and make sure you're inside the tutorial directory (i.e., the folder containing all the tutorial files).

Part 1 – Pulse Sequences

An MRI pulse sequence is a series of RF (radio-frequency) and gradient magnetic fields, defined with specific timings and amplitudes to produce meaningful MRI signals or images.

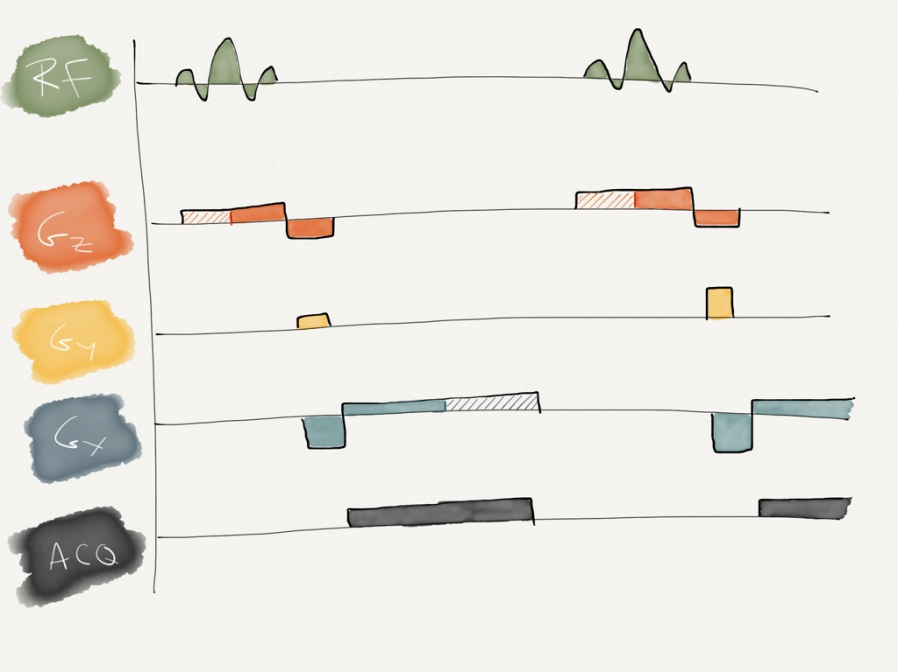

Figure 1

The primary components of a pulse sequence diagram are (see Fig. 1):

1. RF Excitation (typically labelled "RF")
2. z-gradient amplitude (typically labelled G_z or "z-grad")
3. y-gradient amplitude (typically labelled G_y or "y-grad")
4. x-gradient amplitude (typically labelled G_x or "x-grad")
5. signal acquisition (typically labelled ACQ or ADC)

Exercise 1.1
Label the pulse sequence parameters of TR and TE on the diagram in Fig. 1. Indicate which components of the pulse sequence are changed when flip angle and bandwidth are modified.

Question 1.2*
What would happen to the output image from the sequence in Fig. 1 if the G_y and G_x waveforms were completely swapped?

Part 2 – Signal Simulator

(30 minutes)

The MRI signal simulator `simSignal` will be used to demonstrate the relationship between the T1 of the tissue and the sequence parameters TR and flip angle.

Consider the following pulse sequence simulated on a 3T scanner using a TR = 1000 ms and a flip angle = 90°. We assume a nominal value of T1 = 1200 ms to represent grey matter at 3T.

Matlab commands:

```
>> T1=1200;
>> TR=1000;
>> flipAngle=90;
```

```
>> tMax=5000;
>> simSignal;
```

The quantity that M_z represents is the amount of potential or stored magnetization that is available for subsequent measurements. The following analogy may make the role of M_z a little clearer:

- Consider the magnetisation like a battery, which is periodically discharged to produce a "signal", where the signal strength is related to the amount of charge available in the battery. Batteries with a larger T1 value take longer to recharge (recover) than batteries with shorter T1's. Therefore, to get maximum signal, you can wait shorter durations (TRs) for the magnetisation to recharge for batteries with shorter T1's.

Question 2.1
In the bottom sub-plot, how does the M_{xy} signal for the second TR compare to the signal from the first TR? What causes this effect?

Depending on T1, TR and flip angle it can take many measurements before an equilibrium condition ("steady state") is reached. It is because of this effect that the first few images in an fMRI data-series are brighter than the rest, and are discarded (these are sometimes referred to as the dummy or prep scans).

Question 2.2
Consider a flip angle of 65° instead of 90°. What happens to the steady-state signal and why?

The importance of the flip angle becomes noticeable at shorter TR. In fMRI for example, images can be acquired in a single excitation, and time between subsequent excitations can be 1-3 seconds long. In structural imaging however, only single lines of k-space are acquired after each RF excitation pulse. Thus, 192 TRs are required to measure 192 lines of k-space, and these 192 lines would then make up a single slice. The TR therefore needs to be short to keep the total scan times reasonable. A typical structural imaging sequence has a TR around 20ms.

Question 2.3
Using the simulator, find the flip angle that maximises the steady-state magnetisation using a TR = 20ms, assuming T1 = 1200 ms.

Question 2.4**
Find an expression for the steady-state longitudinal magnetisation (Mz) as a function of M_0, T1, TR and flip angle.
Hint: the longitudinal magnetisation Mθ after a flip angle θ is given by:
$M_\theta = M_0 + (M\cos\theta - M_0)e^{-t/T_1}$, *where M is the longitudinal magnetisation just before excitation.*

The "Ernst angle" is a mathematical expression for the optimal flip angle given a TR and T1, which gives the maximum transverse steady-state magnetisation:

$$\cos(\theta) = e^{-TR/T1}$$

The `simSignalvFlip` simulation tool can be used to see the variation of signal with flip angle. The Ernst angle is the flip angle which optimally balances the amount of signal you get (by draining the M_z "battery") and the amount of signal you save for the next

excitation (by leaving some M_z "charge" unused).

Question 2.5*
When TR << T2, then much of the transverse magnetisation will not be decayed by the time the next excitation occurs. What normally happens to this magnetisation, and how can it be useful?

Part 3 – SNR

(30 minutes)

One of the most useful measures of image quality is the signal-to-noise ratio (SNR), which quantifies the signal strength relative to the amount of noise present. SNR is defined as the ratio of the mean signal in a region of interest (ROI) to the standard deviation of the noise in that region:

$$SNR = \frac{\text{mean}(signal)}{\text{st. dev}(noise)}$$

Intuitively, the SNR expresses how many times larger the signal is than the noise:

- **SNR < 1** indicates that the noise variation is larger than the signal *(not good)*
- **SNR = 1** indicates that the signal and noise fluctuation strength are equal (signal indistinguishable from noise)
- **SNR > 1** indicates signal is larger than noise deviations *(higher the better)*

Extra Information*
The source of noise in MRI is a deep and complex (*no pun intended*) topic. In simple terms, noise in MRI images comes from random fluctuations in the received MRI signal that come from the human body and the measurement coil. These intrinsic noise fluctuations have a few properties:

- they are "**white**", which means that they have no preferred frequency (i.e., a flat spectrum)
- they are "**additive**", which means that the noise can be considered as being added on top of the signal
- they have a **Gaussian** distribution, which means that if a histogram of all the noise values is created, they are shaped like a Gaussian or bell-curve
- they have **zero mean**, which means that the noise fluctuations are balanced around zero, and that the average of all noise values is zero.

Ultimately, everything that we record and measure in MRI comes through the receive coil (or coils), and receiver chain electronics (such as pre-amplifiers, filters, etc…). Ignoring noise added or modified after the receive coil, the electrical noise power can be expressed as a function of the temperature and the coupling between the coil and the sample (e.g. body). The Johnson noise formula dictates the differential noise power per unit bandwidth in a system with resistance R and temperature T as:

$$d\sigma^2 = 4 \cdot k_B \cdot T \cdot R \, df$$

where k_B is the Boltzmann constant. After integrating across receiver bandwidth, the total noise power σ^2 is the variance of the additive Gaussian white noise. Note that this integration is the source of the relationship between SNR and bandwidth.
The resistance term is an equivalent resistance of the coupled body-coil system, and is

dependent on specific body and coil geometries and electrical properties (conductivity, capacitance, etc.). This equivalent resistance is dependent on coil resonance frequency, ω_0, which is one of the ways that SNR is related to B_0. It is also dependent on the coil receive field sensitivity, which is why surface coils have higher SNR than volume coils (smaller receive fields mean smaller equivalent resistance, and therefore reduced noise).

Unfortunately, measuring standard deviations within a tissue ROI can also reflect "real" variation of the tissue, making this a poor estimate of the noise. Instead, the noise is often estimated from a background region of the image where there should be no signal. However, because most images only contain the magnitude of the signal (i.e., do not allow negative numbers), this does introduce one mathematical complication.

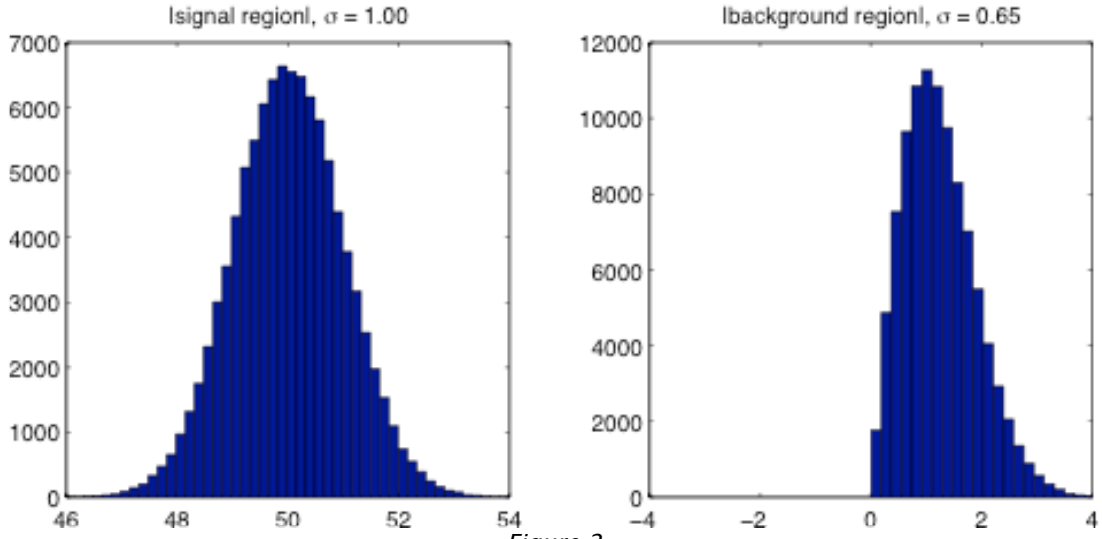

Figure 2

In regions with signal (e.g., tissue), the additive zero-mean noise causes the measured values to fluctuate around the true signal, with equal tendency to add to and subtract from the signal (see Fig. 2, LHS). In background regions with no signal (e.g. outside of the head), the negative component of the noise is forced to be positive by taking the signal magnitude. This causes the standard deviation of the measured values to be smaller in a background region than it is in a signal region; the noise in the background region follows the Rayleigh distribution, of which the standard deviation is 0.65 times that of the standard normal distribution (see Fig. 2, RHS).

Exercise 3.1
Measuring noise from background regions in images can be tricky. Use the `SNR_calc_with_ROIs` function to estimate SNR with normal image scaling. Then, use `SNR_calc_with_ROIs` again with an display windowing factor of 1000 (i.e. `SNR_calc_with_ROIs(structural, 1000);`), and try again. Discuss the difference this makes on the estimated SNR.

Question 3.2
Assuming noise is quantified using a standard deviation measured in a background (no signal) region of a magnitude image, how might the "true" noise and therefore the SNR be estimated from this measure? (Hint: consider the difference between the two plots in Fig. 2)

Question 3.3

Why does increasing the number of averages increase the SNR, and by how much?

Discussion 3.4**

In general, MRI images are composed of complex signals with complex noise. However, it is normally convention to convert the complex image to a real-valued image for display and analysis. What is the SNR difference between an image produced by taking the real part of a complex source image, compared to taking the magnitude? For what reasons aside from SNR might you choose one method over the other? (Hint: assume the complex noise follows a bi-variate normal distribution with equal variance along real and imaginary axes)