

# Métodos Numéricos

## Modalidad virtual por pandemia COVID-19



DEPARTAMENTO  
DE COMPUTACION

Facultad de Ciencias Exactas y Naturales - UBA

# Factorización LU

Resolver varios sistemas de ecuaciones con Eliminación Gaussiana tiene un costo de  $\mathcal{O}(n^3)$  por cada uno. ¿Cómo evitarlo?

$A \in R^{n \times n}$ ,  $L \in R^{n \times n}$  triang. inferior,  $U \in R^{n \times n}$  triang. superior

$$A = LU$$

¿Para qué resulta útil?

$$Ax = b$$

$$LUx = b$$

Resolvemos en dos etapas:

$$Ly = b$$

$$Ux = y$$

Dos sistemas triangulares con costo  $\mathcal{O}(n^2)$ !

Sea  $A \in R^{n \times n}$ . Supongamos que aplicamos eliminación Gaussiana y se verifica que  $a_{ii}^{i-1} \neq 0$  para todo  $i = 1, \dots, n-1$

Sea la matriz elemental (tipo 2)

$$E = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ -m_{21} & 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -m_{21} & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -m_{21} & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix} = \begin{bmatrix} a_{11}^0 & & & a_{1j}^0 & \cdots & a_{1n}^0 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -m_{21} & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -m_{21} & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -m_{21} & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix}$$



# Factorización LU

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -m_{21} & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\ a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\ a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0 \end{bmatrix}$$

$$F_2 - m_{21}F_1!$$

$$M^1 = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & \dots & 0 \\ -m_{12} & 1 & 0 & \dots & 0 & \dots & 0 \\ -m_{13} & 0 & 1 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \dots & \vdots \\ -m_{1i} & 0 & 0 & \dots & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \ddots & \vdots \\ -m_{1n} & 0 & 0 & \dots & 0 & \dots & 1 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix}
 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{12} & 1 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{13} & 0 & 1 & \cdots & 0 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
 -m_{1i} & 0 & 0 & \cdots & 1 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\
 -m_{1n} & 0 & 0 & \cdots & 0 & \cdots & 1
 \end{bmatrix}
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\
 a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 & a_{i2}^0 & \cdots & a_{ij}^0 & \cdots & a_{in}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0
 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix}
 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{12} & 1 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{13} & 0 & 1 & \cdots & 0 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
 -m_{1i} & 0 & 0 & \cdots & 1 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\
 -m_{1n} & 0 & 0 & \cdots & 0 & \cdots & 1
 \end{bmatrix}
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\
 a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 & a_{i2}^0 & \cdots & a_{ij}^0 & \cdots & a_{in}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0
 \end{bmatrix}
 =
 \begin{bmatrix}
 a_{11}^0 & & & & & \\
 & a_{12}^0 & & & & \\
 & & \cdots & & & \\
 & & & a_{1j}^0 & & \\
 & & & & \cdots & \\
 & & & & & a_{1n}^0
 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix}
 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{12} & 1 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{13} & 0 & 1 & \cdots & 0 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
 -m_{1i} & 0 & 0 & \cdots & 1 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\
 -m_{1n} & 0 & 0 & \cdots & 0 & \cdots & 1
 \end{bmatrix}
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\
 a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 & a_{i2}^0 & \cdots & a_{ij}^0 & \cdots & a_{in}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0
 \end{bmatrix}
 =
 \begin{bmatrix}
 a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\
 \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
 a_{n1}^0 - m_{n1}a_{11}^0 & a_{n2}^0 - m_{n1}a_{12}^0 & \cdots & a_{nj}^0 - m_{n1}a_{1j}^0 & \cdots & a_{nn}^0 - m_{n1}a_{1n}^0
 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix}
 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{12} & 1 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{13} & 0 & 1 & \cdots & 0 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
 -m_{1i} & 0 & 0 & \cdots & 1 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\
 -m_{1n} & 0 & 0 & \cdots & 0 & \cdots & 1
 \end{bmatrix}
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\
 a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 & a_{i2}^0 & \cdots & a_{ij}^0 & \cdots & a_{in}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0
 \end{bmatrix}
 =
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\
 a_{31}^0 - m_{31}a_{11}^0 & a_{32}^0 - m_{31}a_{12}^0 & \cdots & a_{3j}^0 - m_{31}a_{1j}^0 & \cdots & a_{3n}^0 - m_{31}a_{1n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots
 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix}
 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{12} & 1 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{13} & 0 & 1 & \cdots & 0 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
 -m_{1i} & 0 & 0 & \cdots & 1 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\
 -m_{1n} & 0 & 0 & \cdots & 0 & \cdots & 1
 \end{bmatrix}
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\
 a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 & a_{i2}^0 & \cdots & a_{ij}^0 & \cdots & a_{in}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0
 \end{bmatrix}
 =
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\
 a_{31}^0 - m_{31}a_{11}^0 & a_{32}^0 - m_{31}a_{12}^0 & \cdots & a_{3j}^0 - m_{31}a_{1j}^0 & \cdots & a_{3n}^0 - m_{31}a_{1n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 - m_{i1}a_{11}^0 & a_{i2}^0 - m_{i1}a_{12}^0 & \cdots & a_{ij}^0 - m_{i1}a_{1j}^0 & \cdots & a_{in}^0 - m_{i1}a_{1n}^0
 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix}
 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{12} & 1 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{13} & 0 & 1 & \cdots & 0 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
 -m_{1i} & 0 & 0 & \cdots & 1 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\
 -m_{1n} & 0 & 0 & \cdots & 0 & \cdots & 1
 \end{bmatrix}
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\
 a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 & a_{i2}^0 & \cdots & a_{ij}^0 & \cdots & a_{in}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0
 \end{bmatrix}
 =
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\
 a_{31}^0 - m_{31}a_{11}^0 & a_{32}^0 - m_{31}a_{12}^0 & \cdots & a_{3j}^0 - m_{31}a_{1j}^0 & \cdots & a_{3n}^0 - m_{31}a_{1n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 - m_{i1}a_{11}^0 & a_{i2}^0 - m_{i1}a_{12}^0 & \cdots & a_{ij}^0 - m_{i1}a_{1j}^0 & \cdots & a_{in}^0 - m_{i1}a_{1n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 - m_{n1}a_{11}^0 & a_{n2}^0 - m_{n1}a_{12}^0 & \cdots & a_{nj}^0 - m_{n1}a_{1j}^0 & \cdots & a_{nn}^0 - m_{n1}a_{1n}^0
 \end{bmatrix}$$



# Factorización LU

$$\begin{bmatrix}
 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{12} & 1 & 0 & \cdots & 0 & \cdots & 0 \\
 -m_{13} & 0 & 1 & \cdots & 0 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
 -m_{1i} & 0 & 0 & \cdots & 1 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\
 -m_{1n} & 0 & 0 & \cdots & 0 & \cdots & 1
 \end{bmatrix}
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 & a_{22}^0 & \cdots & a_{2j}^0 & \cdots & a_{2n}^0 \\
 a_{31}^0 & a_{32}^0 & \cdots & a_{3j}^0 & \cdots & a_{3n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 & a_{i2}^0 & \cdots & a_{ij}^0 & \cdots & a_{in}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 & a_{n2}^0 & \cdots & a_{nj}^0 & \cdots & a_{nn}^0
 \end{bmatrix}
 =
 \begin{bmatrix}
 a_{11}^0 & a_{12}^0 & \cdots & a_{1j}^0 & \cdots & a_{1n}^0 \\
 a_{21}^0 - m_{21}a_{11}^0 & a_{22}^0 - m_{21}a_{12}^0 & \cdots & a_{2j}^0 - m_{21}a_{1j}^0 & \cdots & a_{2n}^0 - m_{21}a_{1n}^0 \\
 a_{31}^0 - m_{31}a_{11}^0 & a_{32}^0 - m_{31}a_{12}^0 & \cdots & a_{3j}^0 - m_{31}a_{1j}^0 & \cdots & a_{3n}^0 - m_{31}a_{1n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{i1}^0 - m_{i1}a_{11}^0 & a_{i2}^0 - m_{i1}a_{12}^0 & \cdots & a_{ij}^0 - m_{i1}a_{1j}^0 & \cdots & a_{in}^0 - m_{i1}a_{1n}^0 \\
 \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
 a_{n1}^0 - m_{n1}a_{11}^0 & a_{n2}^0 - m_{n1}a_{12}^0 & \cdots & a_{nj}^0 - m_{n1}a_{1j}^0 & \cdots & a_{nn}^0 - m_{n1}a_{1n}^0
 \end{bmatrix}$$

Primer paso de Eliminación Gaussiana

$$M^i = \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & -m_{i+1i} & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -m_{ni} & 0 & \cdots & 1 \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & & \vdots & \dots & \vdots \\ 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & -m_{i+1i} & 1 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -m_{ni} & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & \dots & a_{i+1i}^{i-1} & \dots & a_{i+1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ni}^{i-1} & \dots & a_{nn}^{i-1} \end{bmatrix}$$

# Factorización LU

$$= \begin{bmatrix} \begin{bmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & & \vdots & \dots & \vdots \\ 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & -m_{i+1i} & 1 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -m_{ni} & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & \dots & a_{i+1i}^{i-1} & \dots & a_{i+1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ni}^{i-1} & \dots & a_{nn}^{i-1} \end{bmatrix} \\ a_{11}^{i-1} & a_{12}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \end{bmatrix}$$

# Factorización LU

$$\begin{bmatrix}
 1 & \dots & 0 & 0 & \dots & 0 \\
 \vdots & \ddots & & \vdots & \dots & \vdots \\
 0 & \dots & 1 & 0 & \dots & 0 \\
 0 & \dots & -m_{i+1i} & 1 & \dots & 0 \\
 \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\
 0 & \dots & -m_{ni} & 0 & \dots & 1
 \end{bmatrix}
 \begin{bmatrix}
 a_{11}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\
 \vdots & \dots & \vdots & \dots & \vdots \\
 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\
 0 & \dots & a_{i+1i}^{i-1} & \dots & a_{i+1n}^{i-1} \\
 \vdots & \dots & \vdots & \dots & \vdots \\
 0 & \dots & a_{ni}^{i-1} & \dots & a_{nn}^{i-1}
 \end{bmatrix}
 =
 \begin{bmatrix}
 a_{11}^{i-1} & a_{12}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\
 0 & a_{22}^{i-1} & \dots & a_{2i}^{i-1} & \dots & a_{2n}^{i-1} \\
 & & \ddots & & & \\
 & & & \ddots & & \\
 & & & & \ddots & \\
 & & & & & \ddots
 \end{bmatrix}$$

# Factorización LU

$$= \begin{bmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & & \vdots & \dots & \vdots \\ 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & -m_{i+1i} & 1 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -m_{ni} & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & \dots & a_{i+1i}^{i-1} & \dots & a_{i+1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ni}^{i-1} & \dots & a_{nn}^{i-1} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}^{i-1} & a_{12}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ 0 & a_{22}^{i-1} & \dots & a_{2i}^{i-1} & \dots & a_{2n}^{i-1} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \end{bmatrix}$$

# Factorización LU

$$\begin{aligned}
 & \begin{bmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & -m_{i+1i} & 1 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -m_{ni} & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & \dots & a_{i+1i}^{i-1} & \dots & a_{i+1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ni}^{i-1} & \dots & a_{nn}^{i-1} \end{bmatrix} \\
 = & \begin{bmatrix} a_{11}^{i-1} & a_{12}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ 0 & a_{22}^{i-1} & \dots & a_{2i}^{i-1} & \dots & a_{2n}^{i-1} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & 0 & \dots & a_{i+1i}^{i-1} - m_{i+1i}a_{ii}^{i-1} & \dots & a_{i+1n}^{i-1} - m_{i+1i}a_{in}^{i-1} \end{bmatrix}
 \end{aligned}$$

# Factorización LU

$$\begin{aligned}
 & \begin{bmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & -m_{i+1i} & 1 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -m_{ni} & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & \dots & a_{i+1i}^{i-1} & \dots & a_{i+1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ni}^{i-1} & \dots & a_{nn}^{i-1} \end{bmatrix} \\
 = & \begin{bmatrix} a_{11}^{i-1} & a_{12}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ 0 & a_{22}^{i-1} & \dots & a_{2i}^{i-1} & \dots & a_{2n}^{i-1} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & 0 & \dots & a_{i+1i}^{i-1} - m_{i+1i}a_{ii}^{i-1} & \dots & a_{i+1n}^{i-1} - m_{i+1i}a_{in}^{i-1} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & a_{ni}^{i-1} - m_{ni}a_{ii}^{i-1} & \dots & a_{nn}^{i-1} - m_{ni}a_{in}^{i-1} \end{bmatrix}
 \end{aligned}$$



$$\begin{aligned}
 & \begin{bmatrix} 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & -m_{i+1i} & 1 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -m_{ni} & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} a_{11}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & \dots & a_{i+1i}^{i-1} & \dots & a_{i+1n}^{i-1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & a_{ni}^{i-1} & \dots & a_{nn}^{i-1} \end{bmatrix} \\
 = & \begin{bmatrix} a_{11}^{i-1} & a_{12}^{i-1} & \dots & a_{1i}^{i-1} & \dots & a_{1n}^{i-1} \\ 0 & a_{22}^{i-1} & \dots & a_{2i}^{i-1} & \dots & a_{2n}^{i-1} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & a_{ii}^{i-1} & \dots & a_{in}^{i-1} \\ 0 & 0 & \dots & a_{i+1i}^{i-1} - m_{i+1i}a_{ii}^{i-1} & \dots & a_{i+1n}^{i-1} - m_{i+1i}a_{in}^{i-1} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & a_{ni}^{i-1} - m_{ni}a_{ii}^{i-1} & \dots & a_{nn}^{i-1} - m_{ni}a_{in}^{i-1} \end{bmatrix}
 \end{aligned}$$

i-ésimo paso de Eliminación Gaussiana!

## Método de Eliminación Gaussiana

$M^{n-1}M^{n-2}\dots M^1A = U$  con  $U$  triangular superior

$$\text{con } M^i = \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & -m_{i+1i} & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -m_{ni} & 0 & \cdots & 1 \end{bmatrix}$$

IMPORTANTE: asumimos  $a_{ii}^{i-1} \neq 0 \forall i = 1, \dots, n$

## Propiedades de $M^i$

$$M^i = \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & -m_{i+1i} & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -m_{ni} & 0 & \cdots & 1 \end{bmatrix}$$

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$$M^i = \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & -m_{i+1i} & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -m_{ni} & 0 & \cdots & 1 \end{bmatrix} = I - \begin{bmatrix} 0 \\ \vdots \\ m_{i+1i} \\ \vdots \\ m_{ni} \end{bmatrix} \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \end{bmatrix}$$

## Propiedades de $M^i$

$$M^i = \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & -m_{i+1i} & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -m_{ni} & 0 & \cdots & 1 \end{bmatrix} = I - \begin{bmatrix} 0 \\ \vdots \\ m_{i+1i} \\ \vdots \\ m_{ni} \end{bmatrix} \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \end{bmatrix}$$

$$M^i = I - m_i^t e_i$$

con  $m_i = (0, \dots, m_{i+1i}, \dots, m_{ni})$  y  $e_i$  el  $i$ -ésimo vector canónico.

## Propiedades de $M^i$

- $M^i$  es triangular inferior.

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$$\text{pero } e_i m_i^t = \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ m_{i+1i} \\ \vdots \\ m_{ni} \end{bmatrix} = 0$$

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$$\text{pero } e_i m_i^t = [0 \quad \cdots \quad 1 \quad \cdots \quad 0] \begin{bmatrix} 0 \\ \vdots \\ m_{i+1i} \\ \vdots \\ m_{ni} \end{bmatrix} = 0$$

$$\text{Entonces } (I - m_i^t e_i)(I + m_i^t e_i) = I \implies (I - m_i^t e_i)^{-1} = I + m_i^t e_i$$

$$M^{n-1}M^{n-2}\dots M^1A = U$$

$$M^{n-1}M^{n-2} \dots M^1 A = U$$

$$A = (M^1)^{-1}(M^2)^{-1} \dots (M^{n-1})^{-1} U$$

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$$A = (I + m_1^t e_1)(I + m_2^t e_2)\dots(I + m_{n-1}^t e_{n-1})U$$

$$M^{n-1}M^{n-2} \dots M^1 A = U$$

$$A = (M^1)^{-1}(M^2)^{-1} \dots (M^{n-1})^{-1} U$$

$$A = (I + m_1^t e_1)(I + m_2^t e_2) \dots (I + m_{n-1}^t e_{n-1}) U$$

Observando que  $m_i^t e_i m_j^t e_j = 0$  si  $i < j$



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Observando que  $m_i^t e_i m_j^t e_j = 0$  si  $i < j$

$$A = (I + m_1^t e_1 + m_2^t e_2 + \dots + m_{n-1}^t e_{n-1}) U$$

# Factorización LU

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1i} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2i} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ii} & \cdots & a_{in} \\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{ni} & \cdots & a_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ m_{21} & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ m_{i1} & m_{i2} & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{ni} & \cdots & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1i} & \cdots & u_{1n} \\ 0 & u_{22} & \cdots & u_{2i} & \cdots & u_{2n} \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & u_{ii} & \cdots & u_{in} \\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & u_{nn} \end{bmatrix}$$

## Ejemplo

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 4 \\ 6 & -1 & 2 & -3 \end{bmatrix}$$

## Ejemplo

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 4 \\ 6 & -1 & 2 & -3 \end{bmatrix} \rightarrow \begin{array}{l} F_2 - (-1)F_1 \\ F_3 - (2)F_1 \\ F_4 - (-3)F_1 \end{array}$$

## Ejemplo

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 4 \\ 6 & -1 & 2 & -3 \end{bmatrix} \xrightarrow{\begin{array}{l} F_2 - (-1)F_1 \\ F_3 - (2)F_1 \\ F_4 - (-3)F_1 \end{array}} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & 0 & -2 \\ -3 & 2 & -1 & 6 \end{bmatrix}$$

## Ejemplo

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 4 \\ 6 & -1 & 2 & -3 \end{bmatrix} \xrightarrow{\substack{F_2 - (-1)F_1 \\ F_3 - (2)F_1 \\ F_4 - (-3)F_1}} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & 0 & -2 \\ -3 & 2 & -1 & 6 \end{bmatrix}$$

$$\xrightarrow{\substack{F_3 - (-1)F_2 \\ F_4 - (2)F_2}}$$

## Ejemplo

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 4 \\ 6 & -1 & 2 & -3 \end{bmatrix} \xrightarrow{\substack{F_2 - (-1)F_1 \\ F_3 - (2)F_1 \\ F_4 - (-3)F_1}} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & 0 & -2 \\ -3 & 2 & -1 & 6 \end{bmatrix}$$

$$\xrightarrow{F_3 - (-1)F_2} \xrightarrow{F_4 - (2)F_2} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & -1 & 1 \\ -3 & 2 & 1 & 0 \end{bmatrix}$$

## Ejemplo

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 4 \\ 6 & -1 & 2 & -3 \end{bmatrix} \xrightarrow{\substack{F_2 - (-1)F_1 \\ F_3 - (2)F_1 \\ F_4 - (-3)F_1}} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & 0 & -2 \\ -3 & 2 & -1 & 6 \end{bmatrix}$$

$$\xrightarrow{\substack{F_3 - (-1)F_2 \\ F_4 - (2)F_2}} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & -1 & 1 \\ -3 & 2 & 1 & 0 \end{bmatrix} \xrightarrow{F_4 - (-1)F_3}$$



## Ejemplo

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 4 \\ 6 & -1 & 2 & -3 \end{bmatrix} \xrightarrow{\substack{F_2 - (-1)F_1 \\ F_3 - (2)F_1 \\ F_4 - (-3)F_1}} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & 0 & -2 \\ -3 & 2 & -1 & 6 \end{bmatrix}$$

$$\xrightarrow{\substack{F_3 - (-1)F_2 \\ F_4 - (2)F_2}} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & -1 & 1 \\ -3 & 2 & 1 & 0 \end{bmatrix} \xrightarrow{F_4 - (-1)F_3} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & -1 & 1 \\ -3 & 2 & -1 & 1 \end{bmatrix}$$

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$$\xrightarrow{\substack{F_3 - (-1)F_2 \\ F_4 - (2)F_2}} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & -1 & 1 \\ -3 & 2 & 1 & 0 \end{bmatrix} \xrightarrow{F_4 - (-1)F_3} \begin{bmatrix} 2 & 1 & -1 & 3 \\ -1 & 1 & -1 & 3 \\ 2 & -1 & -1 & 1 \\ -3 & 2 & -1 & 1 \end{bmatrix}$$

$$A = LU$$

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 4 \\ 6 & -1 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 \\ -3 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 & 3 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- LU asociada a Eliminación Gaussiana sin necesidad de intercambio de filas. Se verifica que  $l_{ii} = 1$  para todo  $i = 1, \dots, n$ .

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- Si  $A \in R^{n \times n}$  es estrictamente diagonal dominante, entonces tiene factorización  $LU$ .

Existe algún  $a_{ii}^{i-1} = 0$

- La Eliminación Gaussiana puede continuar mediante permutaciones de filas.
- Se obtiene factorización LU de la matriz original *permutada*.



## Ejemplo

$$\begin{bmatrix} 0 & 2 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 2 & 3 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \quad \text{Inicialamos } P = I = [1, 2, 3, 4]$$

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$a_{11}^0 = 0$ . Permutamos  $P = [3, 2, 1, 4]$

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$$\begin{bmatrix} 0 & 2 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 2 & 3 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \quad \text{Inicialamos } P = I = [1, 2, 3, 4]$$

$a_{11}^0 = 0$ . Permutamos  $P = [3, 2, 1, 4]$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 0 & 2 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \begin{array}{l} F_2 - (2)F_1 \\ F_3 - (0)F_1 \\ F_4 - (3)F_1 \end{array} \rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix}$$

$a_{22}^1 = 0$ . Permutamos  $P = [3, 1, 2, 4]$

## Ejemplo

$$\begin{bmatrix} 0 & 2 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 2 & 3 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \quad \text{Inicialamos } P = I = [1, 2, 3, 4]$$

$a_{11}^0 = 0$ . Permutamos  $P = [3, 2, 1, 4]$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 0 & 2 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \xrightarrow{\begin{array}{l} F_2 - (2)F_1 \\ F_3 - (0)F_1 \\ F_4 - (3)F_1 \end{array}} \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix}$$

$a_{22}^1 = 0$ . Permutamos  $P = [3, 1, 2, 4]$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix}$$

## Ejemplo

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$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 0 & 2 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \xrightarrow{\begin{array}{l} F_2 - (2)F_1 \\ F_3 - (0)F_1 \\ F_4 - (3)F_1 \end{array}} \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix}$$

$a_{22}^1 = 0$ . Permutamos  $P = [3, 1, 2, 4]$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix} \xrightarrow{\begin{array}{l} F_3 - (0)F_2 \\ F_4 - (-\frac{1}{2})F_1 \end{array}}$$



## Ejemplo

$$\begin{bmatrix} 0 & 2 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 2 & 3 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \quad \text{Inicialamos } P = I = [1, 2, 3, 4]$$

$a_{11}^0 = 0$ . Permutamos  $P = [3, 2, 1, 4]$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 0 & 2 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \xrightarrow{\substack{F_2 - (2)F_1 \\ F_3 - (0)F_1 \\ F_4 - (3)F_1}} \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix}$$

$a_{22}^1 = 0$ . Permutamos  $P = [3, 1, 2, 4]$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix} \xrightarrow{F_3 - (0)F_2} \xrightarrow{F_4 - (-\frac{1}{2})F_1} \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & \color{red}{0} & -11 & -3 \\ \color{red}{3} & \color{red}{-\frac{1}{2}} & -22 & -8 \end{bmatrix}$$

## Ejemplo

$$\begin{bmatrix} 0 & 2 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 2 & 3 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \quad \text{Inicialamos } P = I = [1, 2, 3, 4]$$

$$a_{11}^0 = 0. \text{ Permutamos } P = [3, 2, 1, 4]$$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 0 & 2 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \xrightarrow{\substack{F_2 - (2)F_1 \\ F_3 - (0)F_1 \\ F_4 - (3)F_1}} \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix}$$

$$a_{22}^1 = 0. \text{ Permutamos } P = [3, 1, 2, 4]$$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix} \xrightarrow{\substack{F_3 - (0)F_2 \\ F_4 - (-\frac{1}{2})F_1}} \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & \color{red}{0} & -11 & -3 \\ \color{red}{3} & \color{red}{-\frac{1}{2}} & -22 & -8 \end{bmatrix}$$

$$\rightarrow F_4 - (-2)F_3$$

## Ejemplo

$$\begin{bmatrix} 0 & 2 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 2 & 3 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \quad \text{Inicialamos } P = I = [1, 2, 3, 4]$$

$$a_{11}^0 = 0. \text{ Permutamos } P = [3, 2, 1, 4]$$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 0 & 2 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} \xrightarrow{\substack{F_2 - (2)F_1 \\ F_3 - (0)F_1 \\ F_4 - (3)F_1}} \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix}$$

$$a_{22}^1 = 0. \text{ Permutamos } P = [3, 1, 2, 4]$$

$$\rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & 0 & -11 & -3 \\ \color{red}{3} & -1 & -25 & -10 \end{bmatrix} \xrightarrow{\substack{F_3 - (0)F_2 \\ F_4 - (-\frac{1}{2})F_1}} \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & \color{red}{0} & -11 & -3 \\ \color{red}{3} & \color{red}{-\frac{1}{2}} & -22 & -8 \end{bmatrix}$$

$$\rightarrow F_4 - (-2)F_3 \rightarrow \begin{bmatrix} 2 & 3 & 6 & 4 \\ \color{red}{0} & 2 & 6 & 4 \\ \color{red}{2} & \color{red}{0} & -11 & -3 \\ \color{red}{3} & \color{red}{-\frac{1}{2}} & \color{red}{-2} & -2 \end{bmatrix}$$

# Factorización PLU-Ejemplo

$$P = [3, 1, 2, 4] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & -\frac{1}{2} & -2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 3 & 6 & 4 \\ 0 & 2 & 6 & 4 \\ 0 & 0 & -11 & -3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$PA = LU$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 6 & 4 \\ 4 & 6 & 1 & 5 \\ 2 & 3 & 6 & 4 \\ 6 & 8 & -7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & -\frac{1}{2} & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 6 & 4 \\ 0 & 2 & 6 & 4 \\ 0 & 0 & -11 & -3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$