The three card puzzle

Suppose we have three cards in a hat:

- **R****B** - One card is painted blue on one side and red on the other.
- **BB** - One card is painted blue on both sides.
- **RR** - One card is painted red on both sides.

The setup

- I pick one of the three cards at random, flip it to a random side, and place it on the table.

- U be the color of the side of the card facing up. (**B** or **R**)

Do you want to bet?

- If the other side of the card has a different I pay you $1,
- If the other side has the same color you pay me $1.

Why is this a fair bet ?

- Suppose U is **R**.
- Then the card is either **RR** or **R****B**.
- Therefor the other side can be either **R** or **B**
- Therefor in this case the odds are equal.
- A similar argument holds for the case where U is **B**

Lets use a monte-carlo simulation

The code below selects one of the three cards at random and selects a random side to be "up".

It then prints the card and indicates if the two sides have the same or different colors.

In [3]:
```python
red_bck="\x1b[41m%s\x1b[0m"
blue_bck="\x1b[44m%s\x1b[0m"
red=red_bck%'R'
black=blue_bck%'B'
Cards=[(red,black),(red,red),(black,black)]
counts={'same':0,'different':0}
from random import random
for j in range(50):
    i=int(random()*3.)  # Select a random card
    side=int(random()*2.)
    C=Cards[i]
    if(side==1):        # select which side to be "up"
        C=(C[1],C[0])
    same= 'same' if C[0]==C[1] else 'different'  # count the number of times the
 two sides are the same or different.
    counts[same]+=1
    print(''.join(C)+' %-9s'%same, end='')
    if (j+1)%5==0:
        print()
print()
print(counts)
```

```
BR differentRR same     BR differentBB same     RR same
BB same     BR differentRR same     RR same     RB different
BB same     RR same     BB same     RR same     BB same
BB same     BB same     RB differentBB same     RR same
RR same     RR same     BR differentRR same     RB different
RR same     BB same     BR differentRR same     BB same
BB same     RB differentRB differentRR same     BB same
RR same     BB same     BB same     BB same     RR same
BB same     BB same     RR same     BB same     BR different
BR differentBB same     RR same     RR same     RR same

{'same': 38, 'different': 12}
```

The simulation does not agree with the argument

- In Simulation: the two sides have the same color about **twice** the number of times that they have different color.

- you are twice as likely to lose as you are to win.

- On average you lose 33 cents per iteration:
  $\$1 \times (2/3) - \$1 \times (1/3)$

Alternative argument

If we pick a card at random 2/3 of the time we pick a card where the two sides have the same color, and only 1/3 where the color is different.

How can we be sure?

- The original argument also sounds convincing, but is wrong.
- To be sure that our argument is correct, we need to define some concepts, including **outcome** and **event**. Which we will do next week.