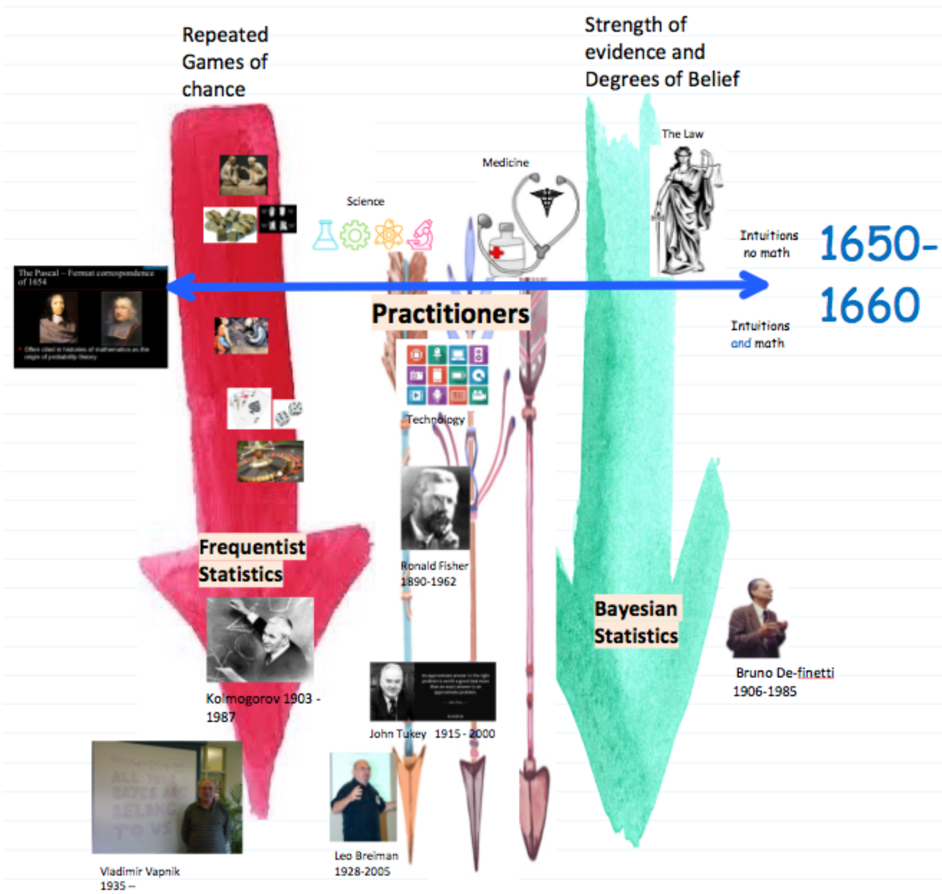
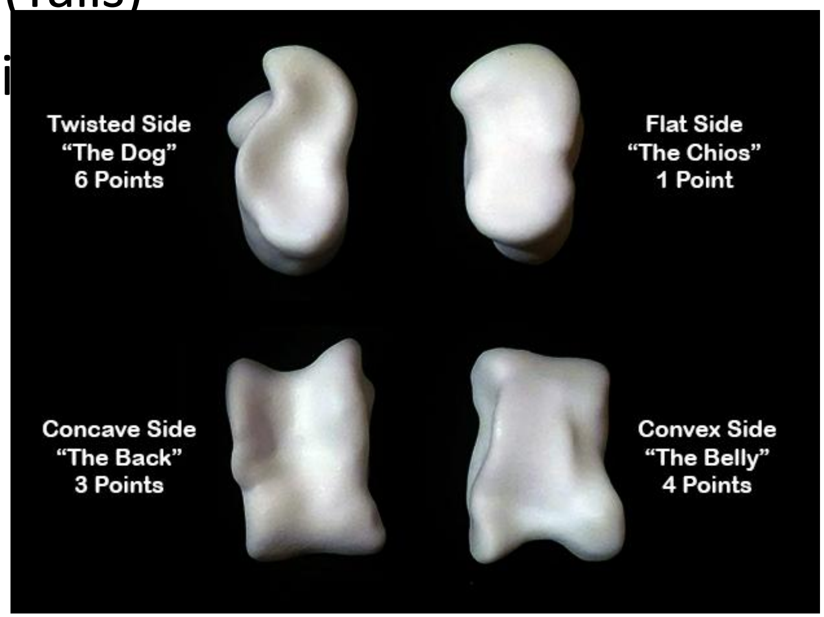
A short history of probability And Statistics

Games of chance VS. Strength of evidence



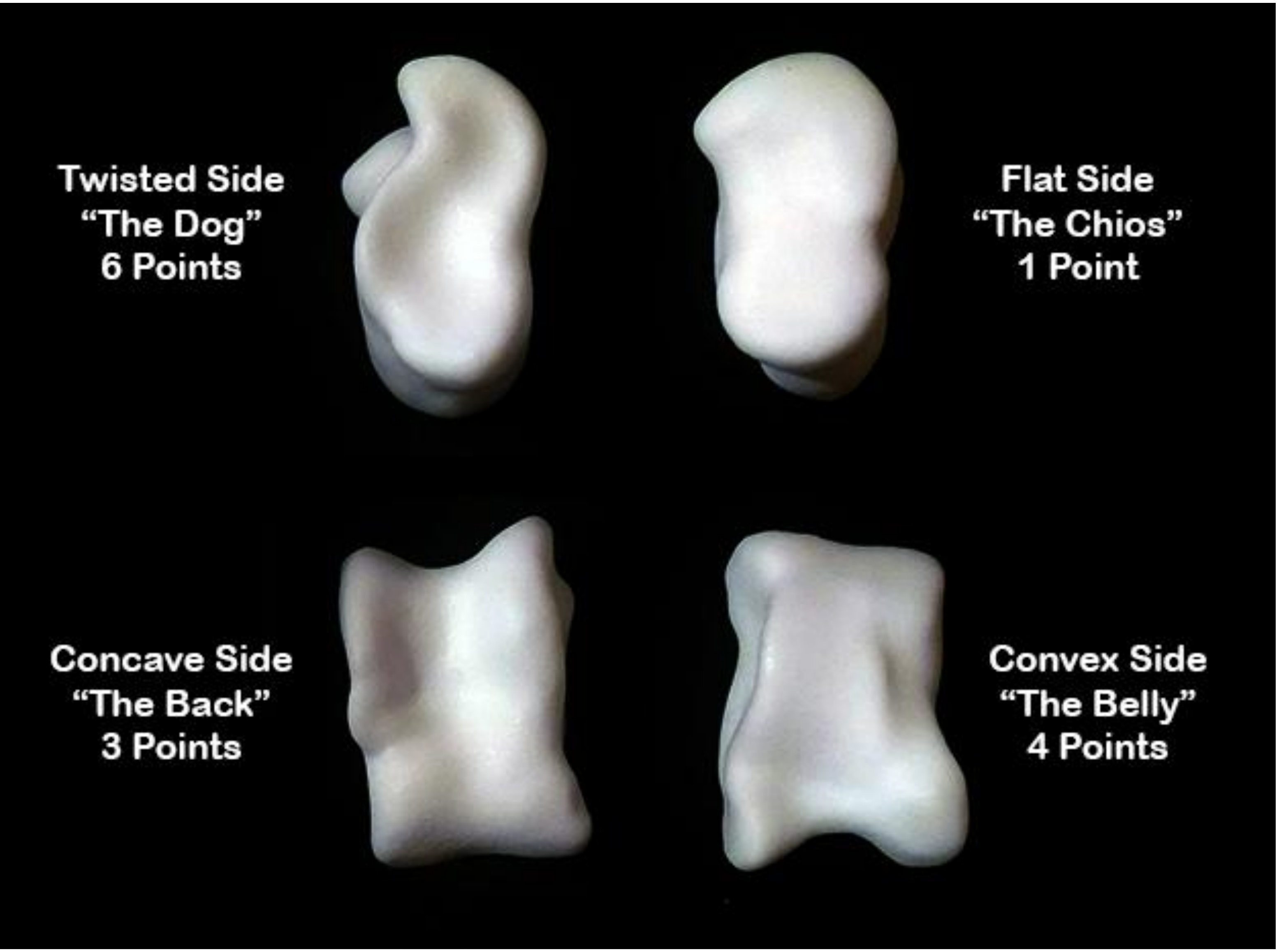
Games of chance

- Sumeria, Assyria, ancient Greece, ancient Rome
- Knuckle Bones (Talis)
- Repeat the basi



From knuckle bones to dice and cards

- Winning or losing is up to chance, luck, or god.
- **Equal probability Assumption**: all outcomes have the same probability.
- True for dice and roulette
- Not true for knuckle bones.


Twisted Side
“The Dog”
6 Points
Flat Side
“The Chios”
1 Point
Concave Side
“The Back”
3 Points
Convex Side
“The Belly”
4 Points

Long Term Frequencies

- The probability that a knucklebone lands on a narrow face is smaller than it lands on a wide face.
- Each knucklebone is different, the probabilities are different.
- Suppose we have P(**6**)=0.1, P(**1**)=0.2,p(**3**)=0.3, p(**4**)=0.4
- Flip 1000 times:

```
43334333434411141464343613434611333344643133314644364463433314141343434333313466131136346443346313464363
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41611136443344333434434143114314314334134333
```

```
probability=0.10 frequency= 105/1000 = 0.10
probability=0.20 frequency= 197/1000 = 0.20
probability=0.30 frequency= 291/1000 = 0.29
probability=0.40 frequency= 407/1000 = 0.41
```

Long Term Frequencies

- The probability of landing on a narrow face is smaller than that of landing on a wide face.
- Each knucklebone is different, the probabilities are different.
- Suppose we have P(6)=0.1, P(1)=0.2,p(3)=0.3, p(4)=0.4
- Flip 100 times:

```
631134463164444341434114143643136444113434434463134334333434311464634614143343643463631114141311
```

```
6 probability=0.10 frequency= 12/100 = 0.12
1 probability=0.20 frequency= 21/100 = 0.21
3 probability=0.30 frequency= 29/100 = 0.29
4 probability=0.40 frequency= 38/100 = 0.38
```

Long Term Frequencies

- The probability of landing on a narrow face is smaller than that of landing on a wide face.
- Each knucklebone is different, the probabilities are different.
- Suppose we have P(6)=0.1, P(1)=0.2,p(3)=0.3, p(4)=0.4
- Flip 10 times:

```
6414114444

6 probability=0.10 frequency= 1/10 = 0.10
1 probability=0.20 frequency= 3/10 = 0.30
3 probability=0.30 frequency= 0/10 = 0.00
4 probability=0.40 frequency= 6/10 = 0.60
```

Stopping a game in the middle

- Simplified version of problem in famous letter from Pascal to Fermat in 1654
- Suppose a card game of pure chance is played until one side wins.
- Both players put in 1$.
- The winner takes the 2$
- Suppose the game is **stopped** before either side wins.
- How should the 2$ be split?
- What is the probability that player 1 will win given the cards currently held?

The frequentist point of view

- To assign a probabilities to the outcomes of a game/experiment is the same as saying that if we repeat the game many times, the long term frequencies of the outcomes converge to the probabilities.
- Provides a solid foundation on which probability theory is built.
- Makes sense in games and other situations where one can repeat the same random choice many times.
- Not always possible ….

Situations where repetition is hard

1. A meteorologist says that the probability of rain tomorrow is 10%.
   - What does that mean?
   - It will either rain or not rain.
   - Tomorrow happens only once.
2. Suppose a surgeon says that there is a 2% chance of complications with a particular surgery.
   - It might mean that 2% of the patients that underwent the surgery had complications.
   - What does it mean for <u>you</u> ?
   - Maybe most of the complications where with patients older than 90 (and you are 35) …

The colloquial meaning of probability

- The word “probable” was in use before 1650. But it’s meaning was not quantitative
- Even today the words “probable” and “probably” have common use meanings that is qualitative, not quantitative.

Definition of PROBABLY Merriam Webster Dictionary

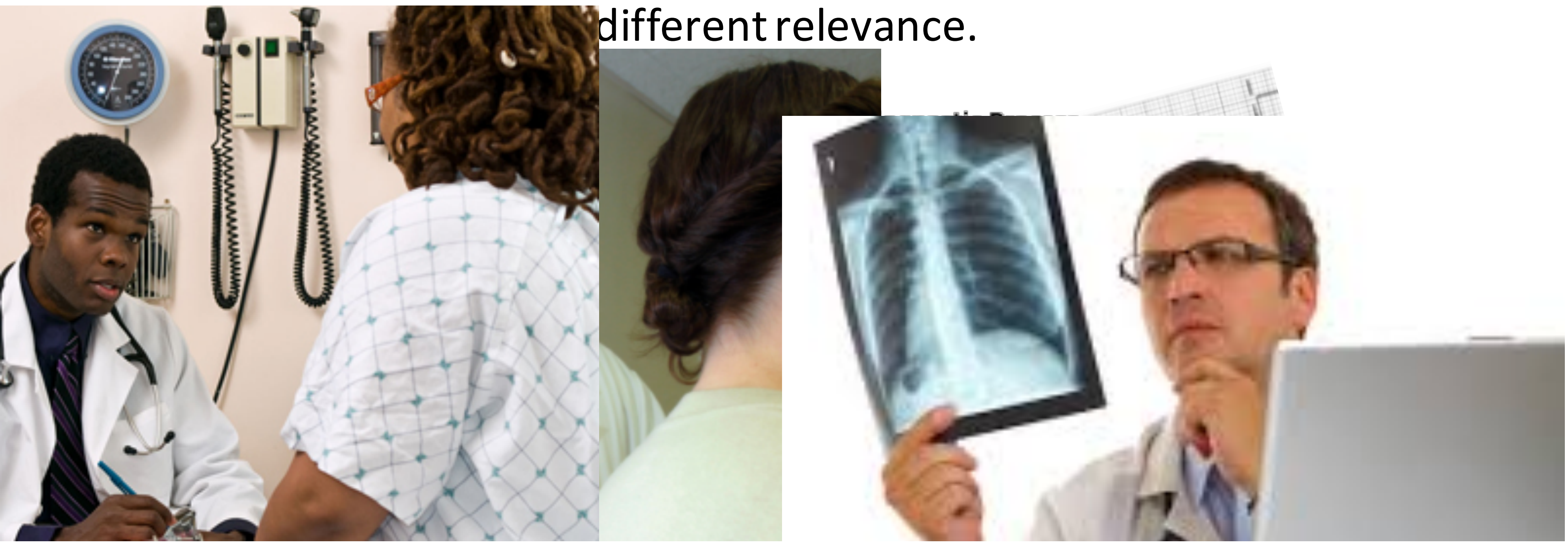
: insofar as seems reasonably true, factual, or to be expected : without much doubt • is *probably* happy • it will *probably* rain

A probable doctor

- Before 1660 it was common to say that someone is a “probable doctor”.
- It meant that the doctor was **approved** by some authority.
- At the time, in Europe, the authority was usually the church.
- Today MDs are approved by a board, after passing the board exams.

Combining evidence for Diagnosis

- Diagnosing a patient requires combining pieces of information.
- Most information is uncertain (measurement error)
- different relevance.



Combining evidence

- Central to many fields: Medicine, economics, investment, Law, Science, Technology …..
- Typically, you don't repeat an experiment many times.
- The math used is probability theory, but much of the discussion is not mathematical.
- Closely related concepts: Fairness, pricing.
- A popular approach: Bayesian Statistics.

Next video: an exploration of duality