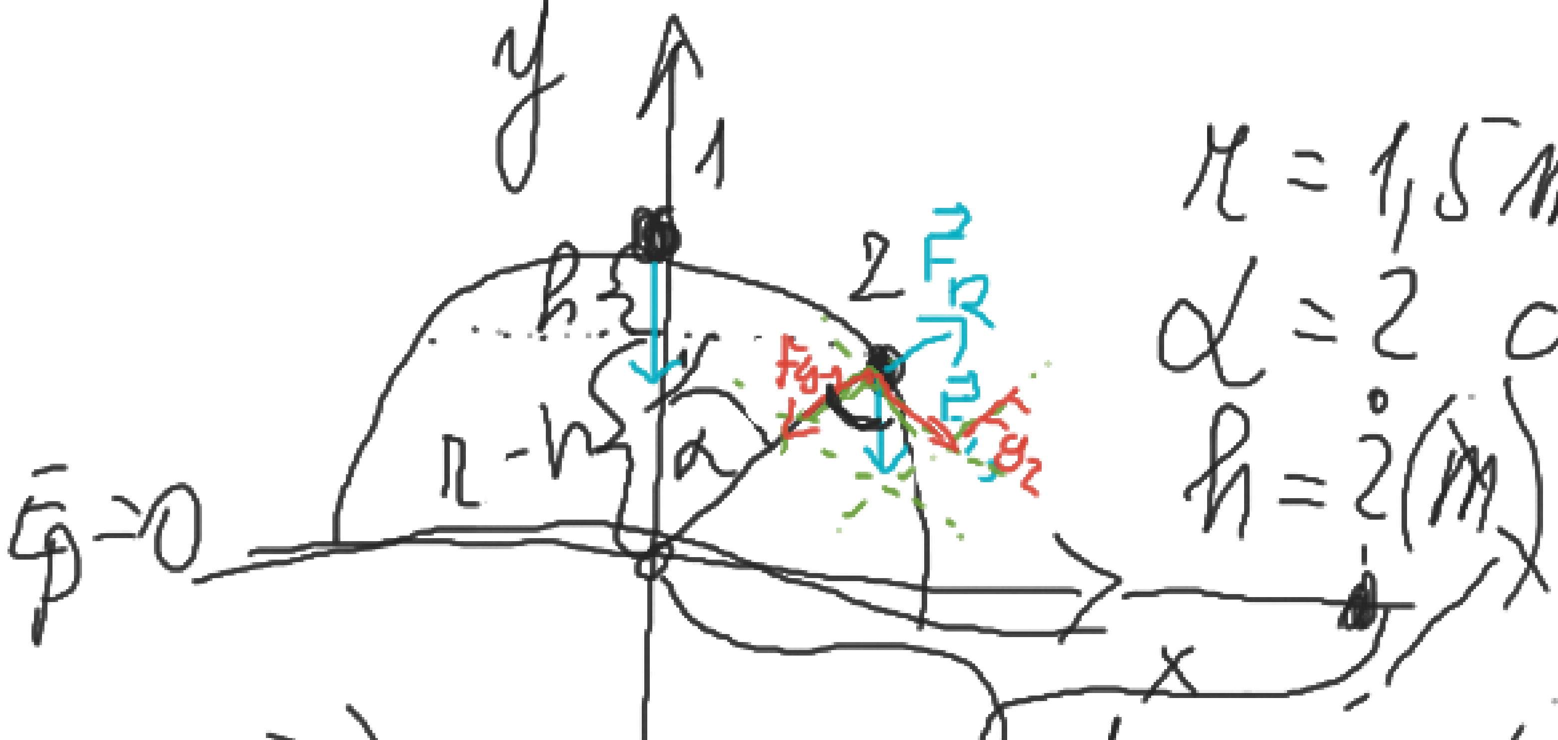


$r = 1,5\,m$

$\alpha = 2 \quad d = ?$

$h = ?\,(m)$

$\vec{F} = \vec{F}_g + \vec{F}_R$

$\vec{F}_{G1} + \vec{F}_R = \vec{F}_n$

$\vec{F}_{G2} = \vec{F}_t$

$g\,r = \frac{1}{2} g\,(r-h) + g\,(r-h)$

$h = \ldots$

$v = \ldots$

$\vec{F}_d = \vec{F}_{G1} + \vec{F}_R$

$F_d = F_{G1} - F_R$

$F_d = F_{G1}$

$F_d = mg\cos\alpha - ,$

$m\,a_d = m\,g\cos\alpha$

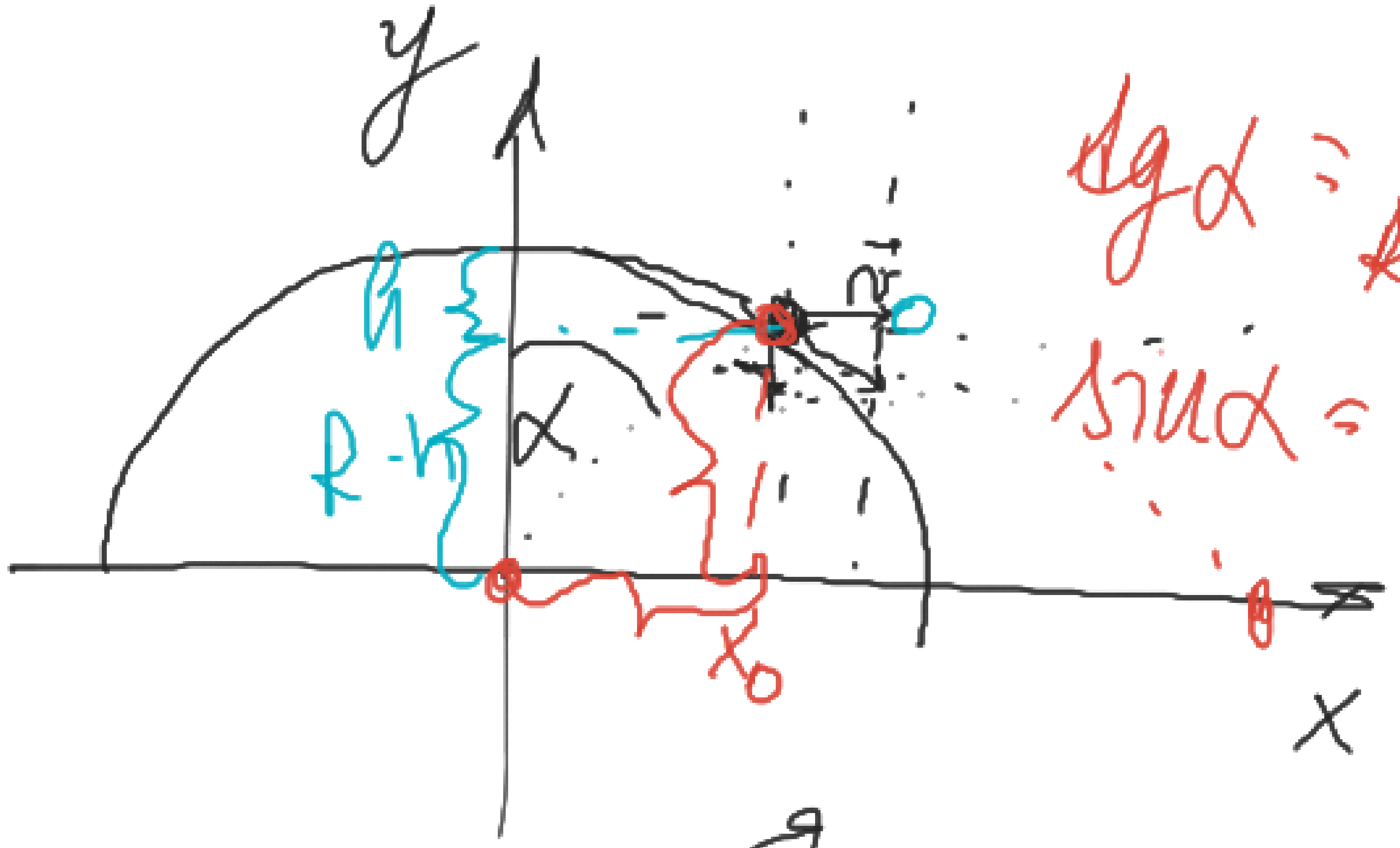
$m\frac{v^2}{r} = mg\cos\alpha \qquad \cos\alpha = \frac{r-h}{r}$

$E_1 = E_2$

$\boxed{\begin{aligned} \cancel{m}gr &= \tfrac{1}{2}\cancel{m}v^2 + \cancel{m}g(r-h) \\ \frac{\cancel{m}v^2}{\cancel{r}} &= \cancel{m}g\cdot\frac{r-h}{\cancel{r}} \end{aligned}}$

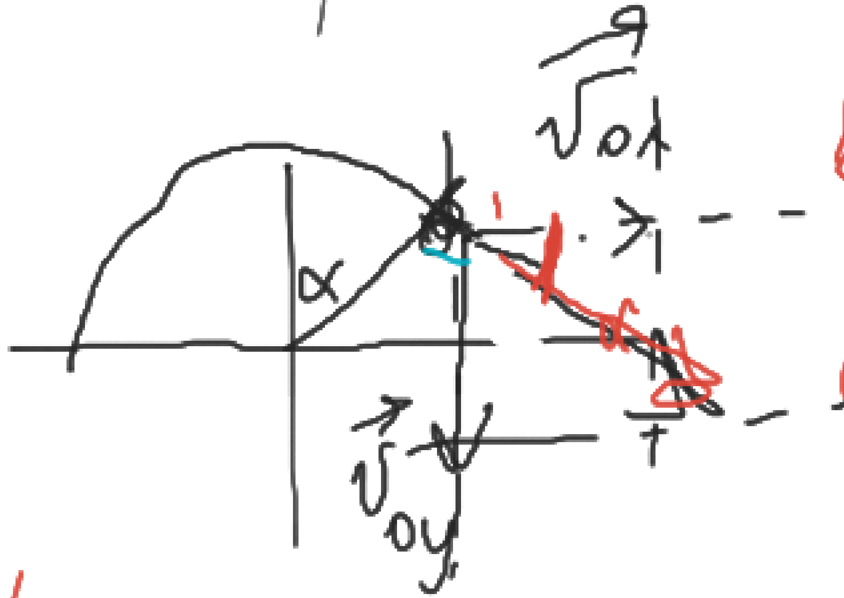
$v^2 = g\,(r-h)$

$h = 2$



$tg\alpha = \frac{x_0}{R-h}$

$\sin\alpha =$



$\sin\alpha = \frac{v_y}{v}$

$\cos\alpha = \frac{v_x}{v}$

HB Lona 59/84

$x: \quad v_x = v_{0x} = \underline{\underline{v_0 \cos\alpha}}$ RPP

$y: \quad v_y = v_{0y} + gt$ RZP

$v_y = v_0 \cdot \sin\alpha + gt$

(velkosť)

$x = v_0 \cos\alpha \cdot t + x_0$

$y = R - h - v_0 \sin\alpha \, t - \frac{1}{2} g t^2$

V okamihu dopadu:

$y = 0 \Rightarrow t_{dopadu}$

$x = d =$ dĺžka vrhu

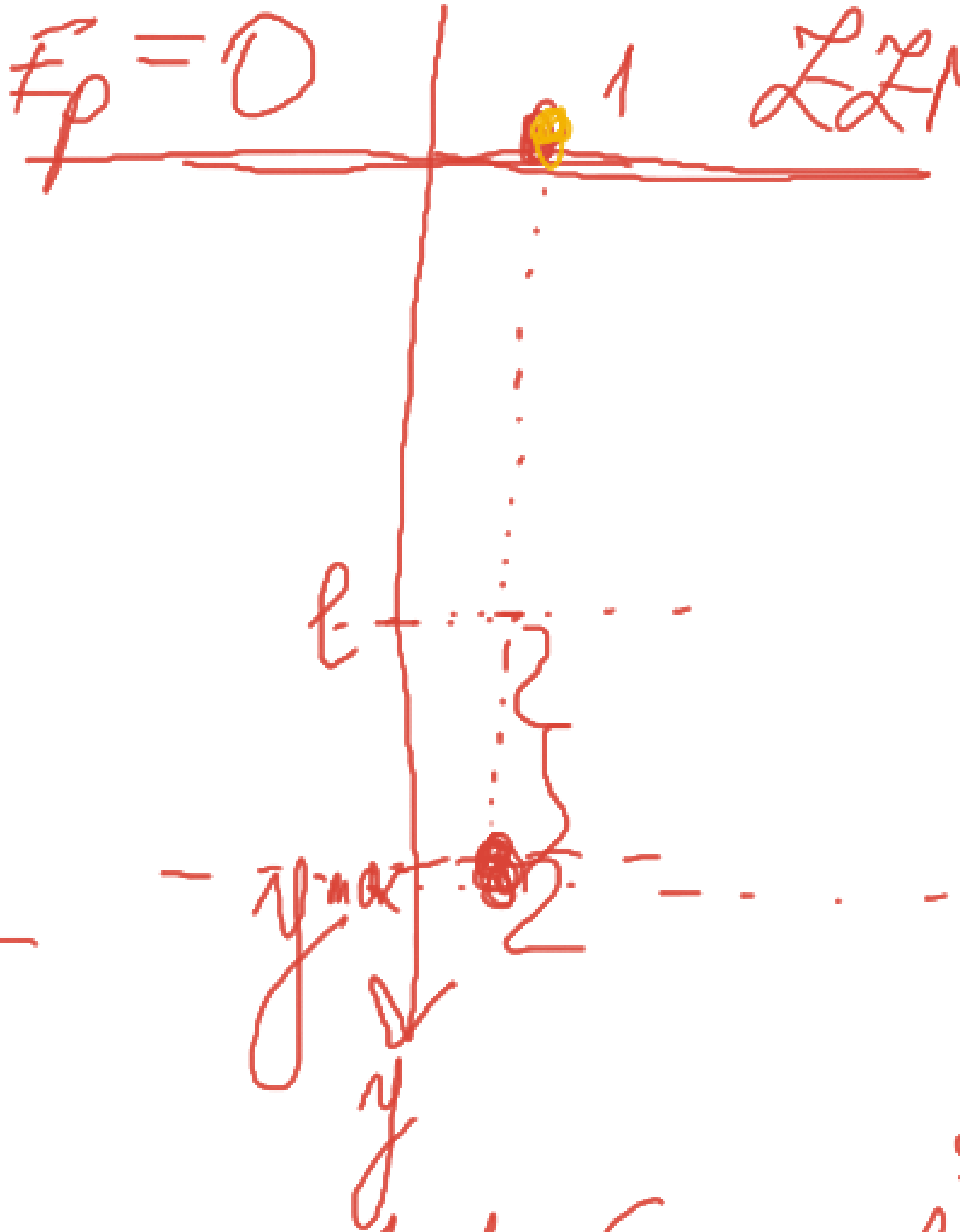
$m = 61\,kg$

$l = 25\,m$

$k = 160\,N/m$

$h = 45\,m$

$y_{max} = ?\ (m)$



ZZME – kin. en.
– pot. en. težnosti
– pot. en. prožnosti

$E_{pr} = \frac{1}{2} k \Delta y^2$

podaljšek

$E_1 = E_2$

$$0 = 0 - mg\,y_{max} + \frac{1}{2} k \left(y_{max} - l\right)^2 \Rightarrow y_{max} = ?$$

TAŽISKO

HB $\rightarrow$ teleso

1d - 1

$$\vec{r}_T = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$$x_T = \frac{\sum m_i x_i}{\sum m_i}$$

$$y_T = \dots$$

$$z_T = \dots$$

m_1, m_2, m_3, m_4

$A(-2, -3, 4)$ B

C D

$$x_T = \frac{m_1 x_1 + m_2 x_2}{}$$

$$y_T =$$

$$z_T =$$

Sústava 2 HB m_1, m_2

$$x_T = \frac{m_2 \overset{=0}{x_2} + m_1 \overset{=\frac{a}{2}}{x_1}}{m_2 + m_1}$$

Въпрос 12

$$m_1 = \rho \cdot V_1 = \frac{m}{V} \quad V_1 = \frac{m}{S \cdot \cancel{h}} \cdot S_1 \cdot \cancel{h} = \frac{m}{S} \cdot S_1$$

$$m_2 = \rho \cdot V_2 =$$

Bruyac 12

59, 84, 119

151, 152, 153, 154 } odovzdať

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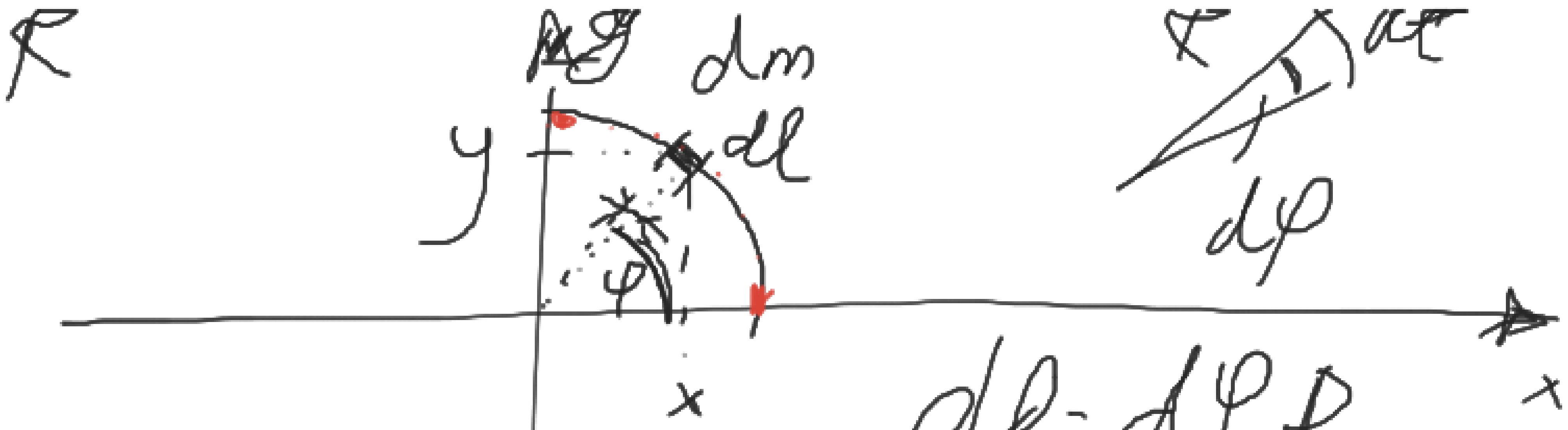
práca
energia } otázky?

ťažisko
1 veta imp.
moment. zotrvač.



$$x = R\cos\varphi$$

$$y = R\sin\varphi$$

$$dl = d\varphi \cdot R$$

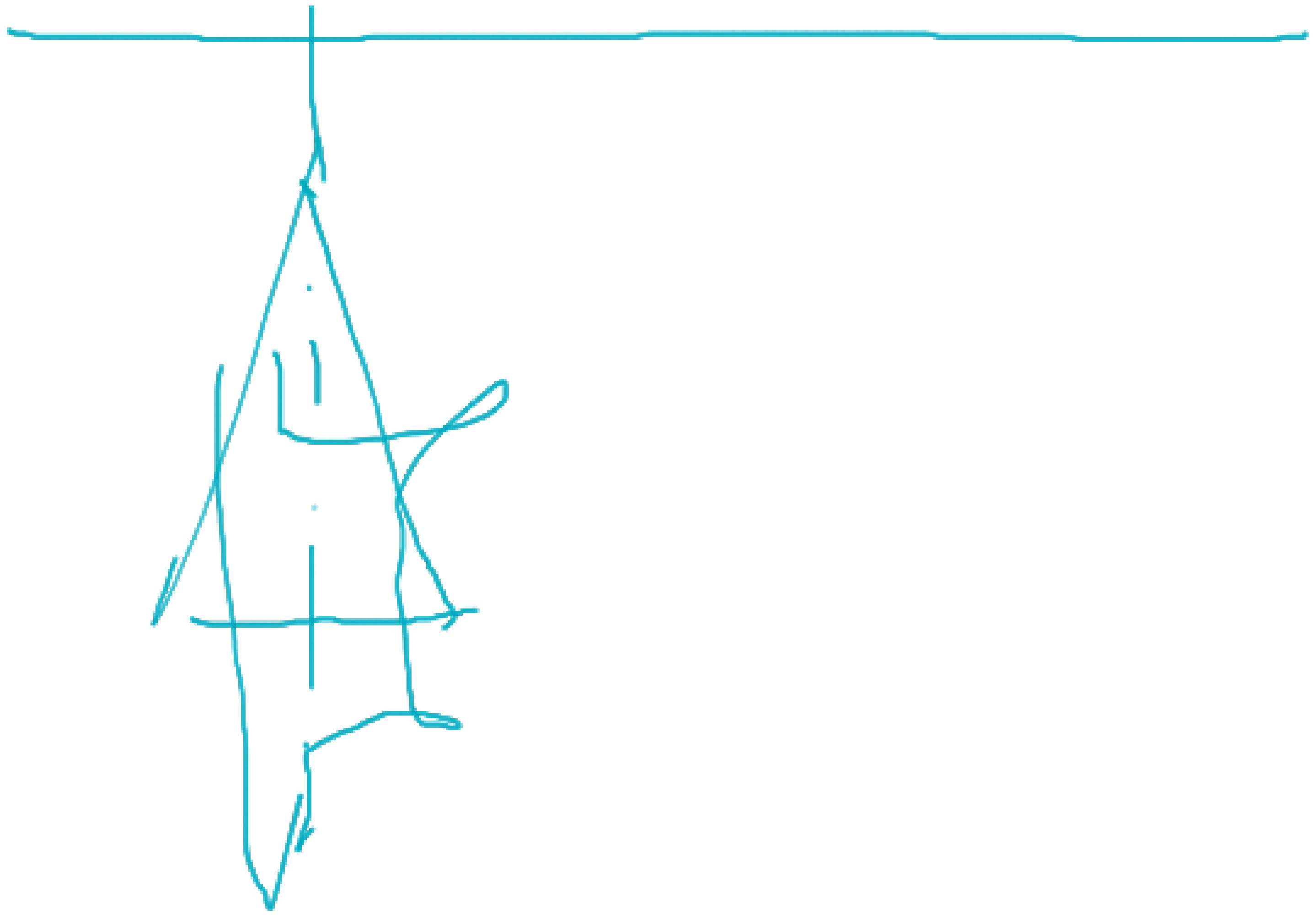
$$dm = \rho \cdot dV = \frac{m}{\cancel{\rho} \cdot \frac{2\pi R}{4}} \cdot \cancel{\rho} \cdot dl = \frac{2m}{\pi R}\, dl = \frac{2m}{\pi R} \cdot R\, d\varphi$$

$$x_T = \frac{\int x\, dm}{m} = \frac{\int R\cos\varphi \cdot \frac{2\cancel{m}}{\pi \cancel{R}} R\, d\varphi}{\cancel{m}}$$

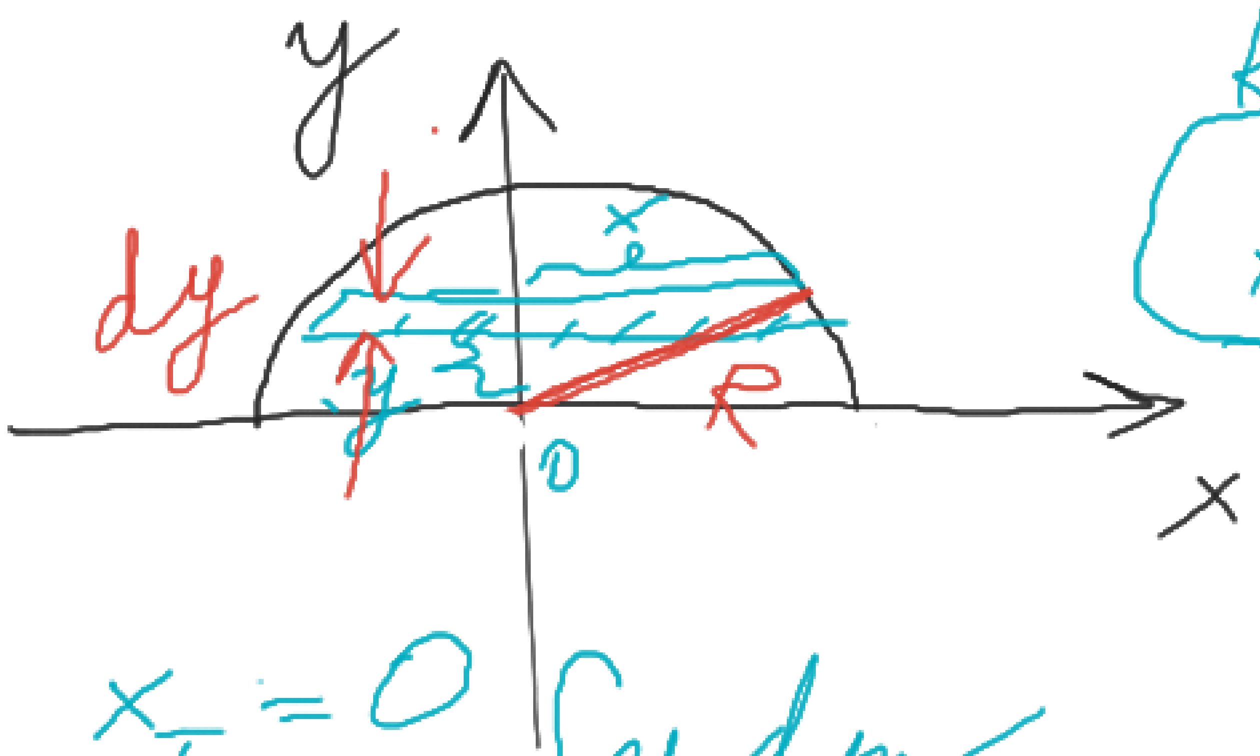
$$x_T = \frac{2R}{\pi} \int_0^{\pi/2} \cos\varphi\, d\varphi = \frac{2R}{\pi}\Big[\sin\varphi\Big]_0^{\pi/2} =$$

$$= \frac{2R}{\pi}\Big[\underbrace{\sin\frac{\pi}{2}}_{1} - \underbrace{\sin 0}_{0}\Big] = \frac{2R}{\pi}$$

ŤAŽNKO KAŽELA



ŤAŽISKO POLOGULE, R – súčasť kruh doštek



Kruh dosta / polomer x

$$\boxed{R^2 = x^2 + y^2} \Rightarrow x^2 = R^2 - y^2$$

$$dm = \rho \cdot dV = \frac{m}{\frac{2}{3}\pi R^3} \cdot \pi x^2 \, dy$$

$x_T = 0$

$$y_T = \frac{\int y \, dm}{m} = \frac{\int y \frac{3\cancel{m}}{2\cancel{\pi} R^3} \cancel{\pi} x^2 \, dy}{\cancel{m}} = \frac{3}{2R^3} \int y \, x^2 \, \cancel{dy} =$$

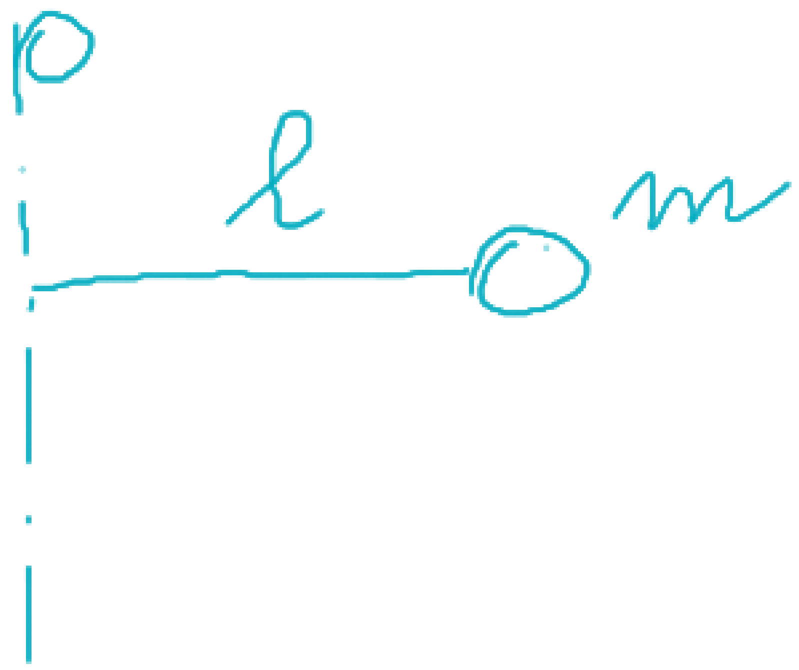
$$= \frac{3}{2R^3} \int_0^R y \left(R^2 - y^2\right) dy = \dots\dots$$

$$\int y \, dy = \frac{y^2}{2} \qquad \int y^3 \, dy = \left[\frac{y^4}{4}\right]_0^R$$

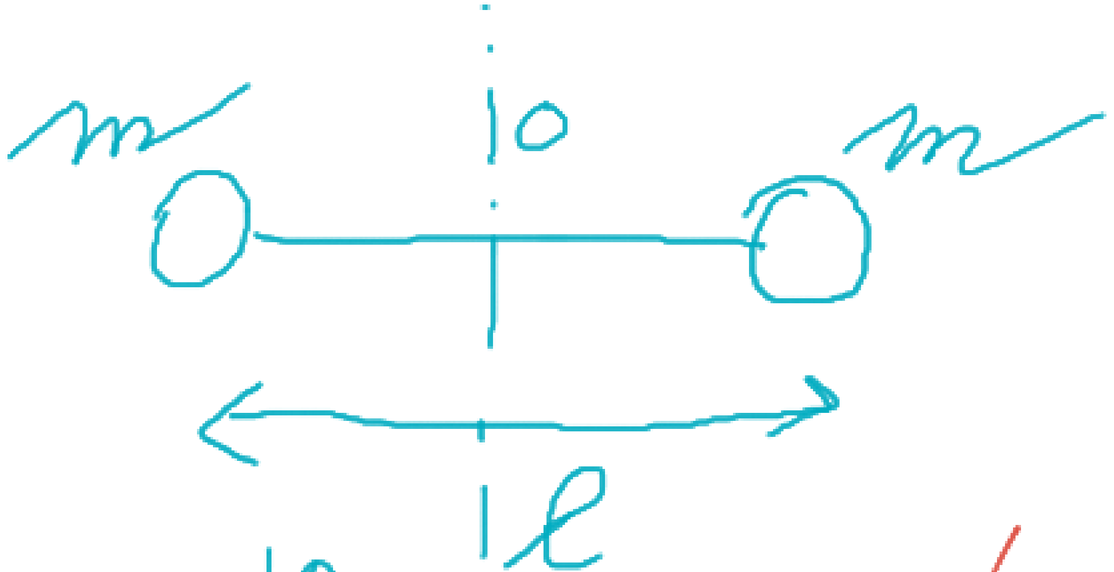
Moment zotrvačnosti – otáčavý pohyb

$$I_i = m_i r_i^2$$

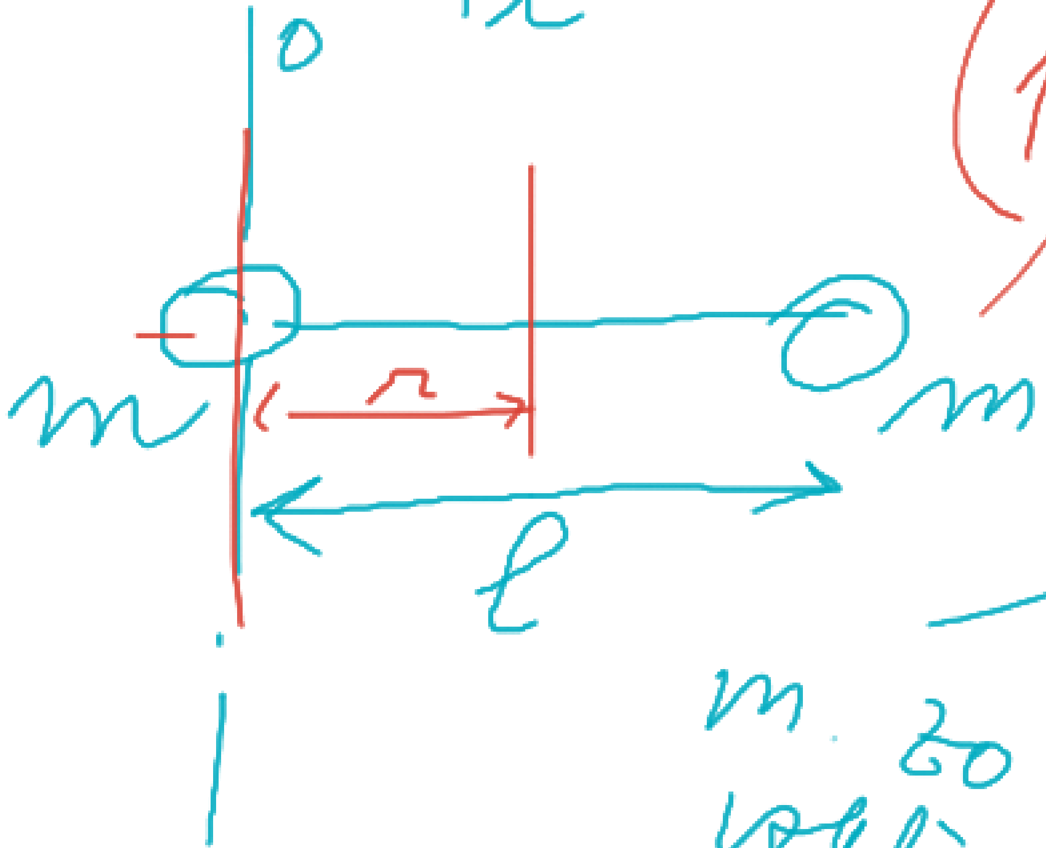
$$I = \sum_i m_i r_i^2$$



$$I = m \cdot l^2$$



$$I_T = m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2 = 2m\left(\frac{l}{2}\right)^2$$



(1) $I = m.0^2 + m.l^2$ (1)

Steinerova veta

$$\boxed{I = I_T + m r_0^2}$$

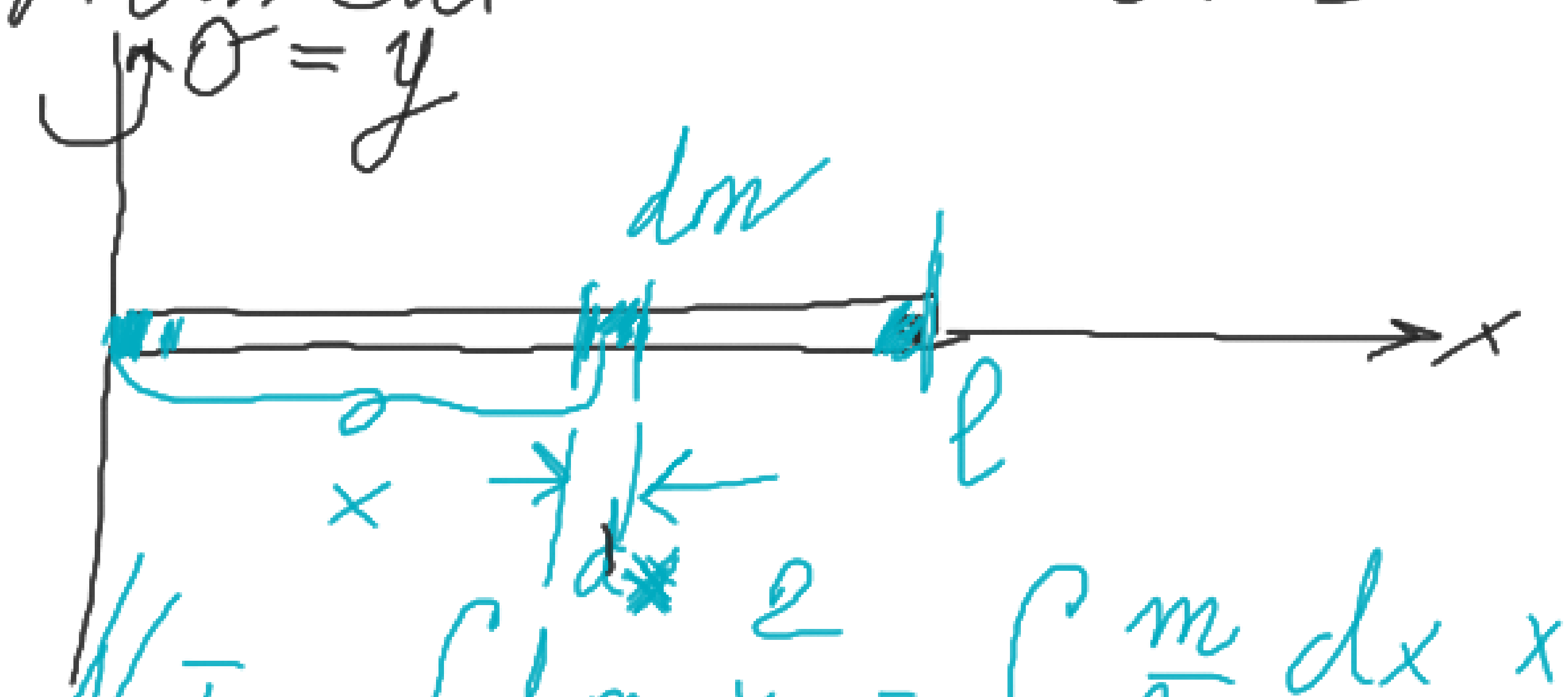
vzd. 2 osí

m. zotrv. vzhľ. na os neprech. ťažiskom

m. zotrv vzhľ. na os prech. ťažiskom

(2) $I = I_T + 2m\left(\frac{l}{2}\right)^2 = 2m\left(\frac{l}{2}\right)^2 + 2m\left(\frac{l}{2}\right)^2 = \cancel{4}m\frac{l^2}{\cancel{4}} = ml^2$ (2)

Moment zotrvačnosti tyče, m, ℓ $\quad I = \sum m_i r_i^2$



$$I = \int dm\, r^2$$

vzd. dm od osi otáč.

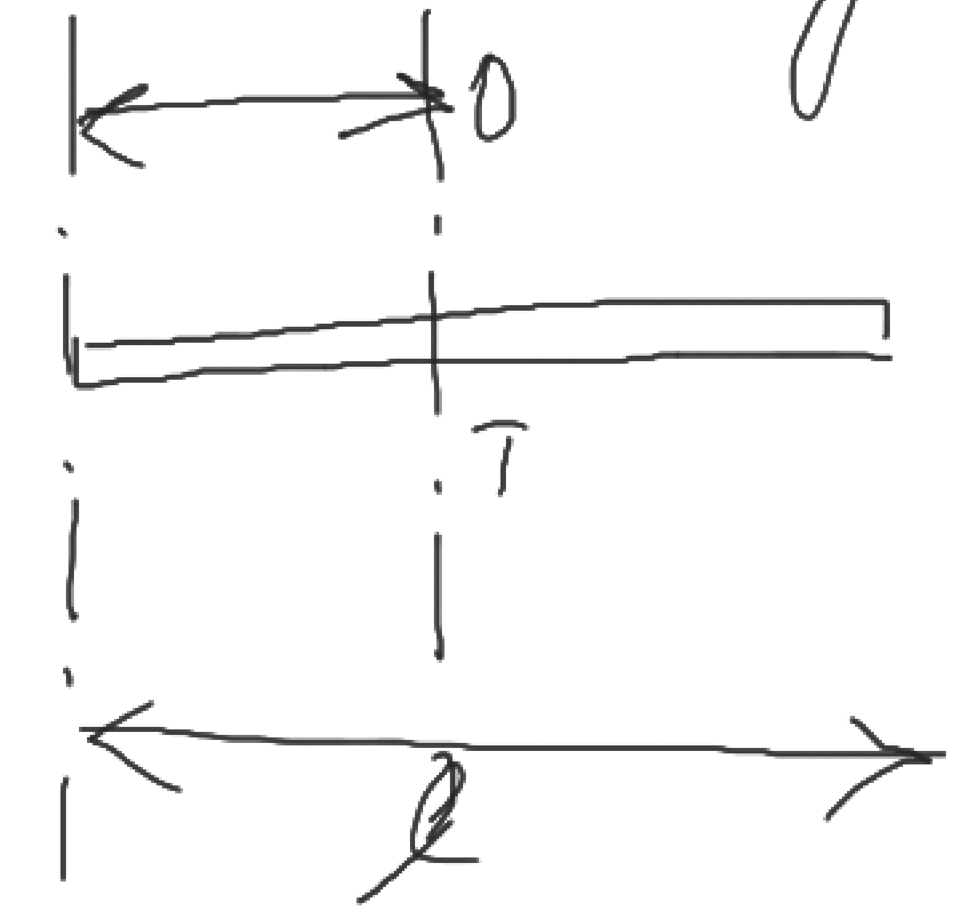
$$\boxed{I = \int dm\, x^2} = \int \frac{m}{\ell}\, dx\, x^2 = \frac{m}{\ell}\int_0^{\ell} x^2\, dx = \frac{m}{\ell}\cdot\left[\frac{x^3}{3}\right]_0^{\ell} = (*)$$

$$\boxed{dm = \rho\, dV = \frac{m}{V}\, dV = \frac{m}{\cancel{S}\ell}\, \cancel{S}\, dx = \frac{m}{\ell}\, dx}$$

dĺžk. hustota

$$\left(\frac{x^3}{3}\right)' = x^2 \qquad (*) = \frac{m}{\ell}\left(\frac{\ell^3}{3} - \frac{0^3}{3}\right) = \frac{m\ell^2}{3}$$

M. zotrv. tyče m, l – os prech T



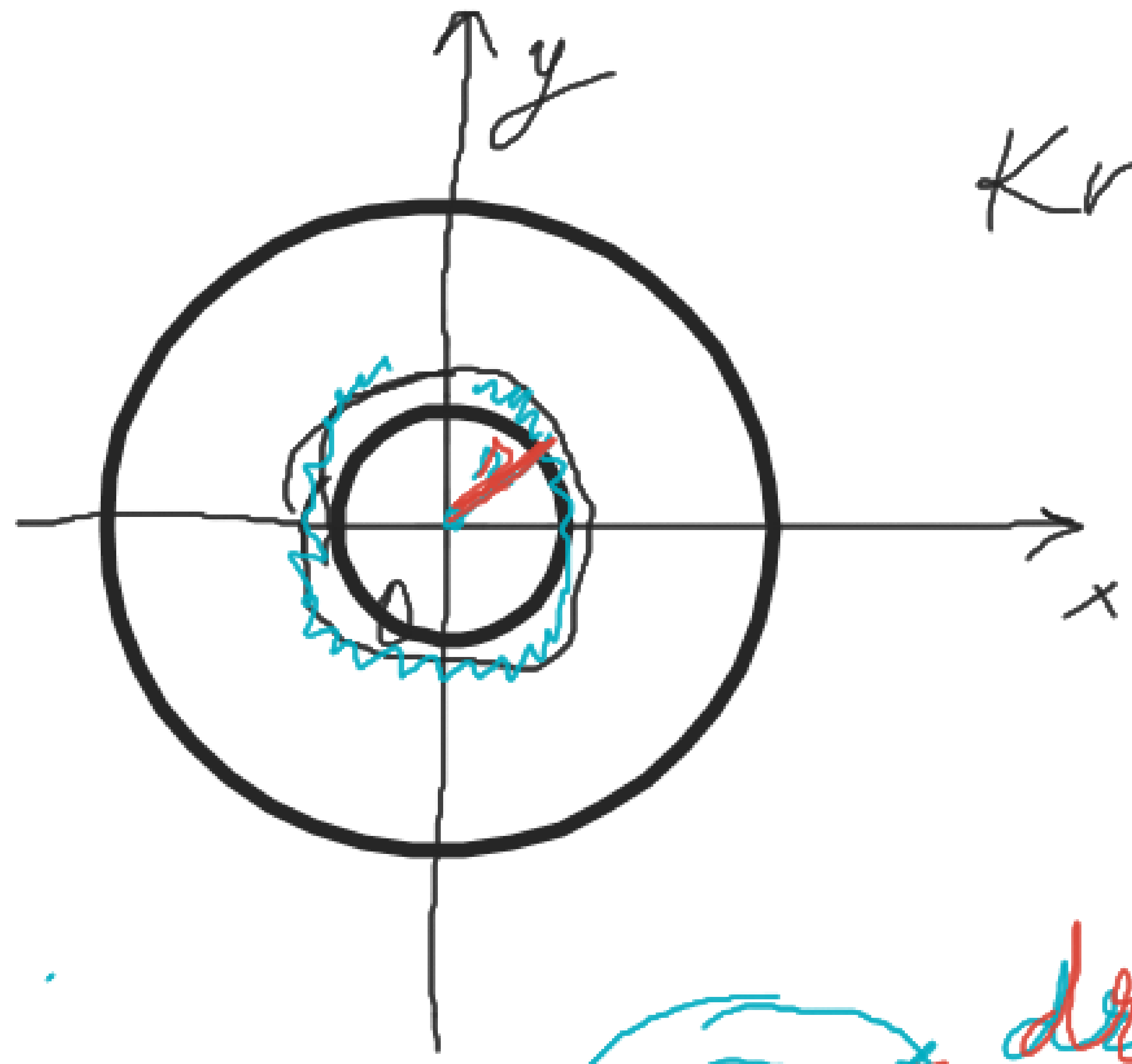
Stein veta:

$$I = I_T + mr^2 = I_T + m\left(\frac{l}{2}\right)^2$$

vzdj vzd. 2 osí

$$I_T = I - \frac{ml^2}{4} = \frac{ml^2}{3} - \frac{ml^2}{4}$$

$$= \frac{1}{12} m l^2$$



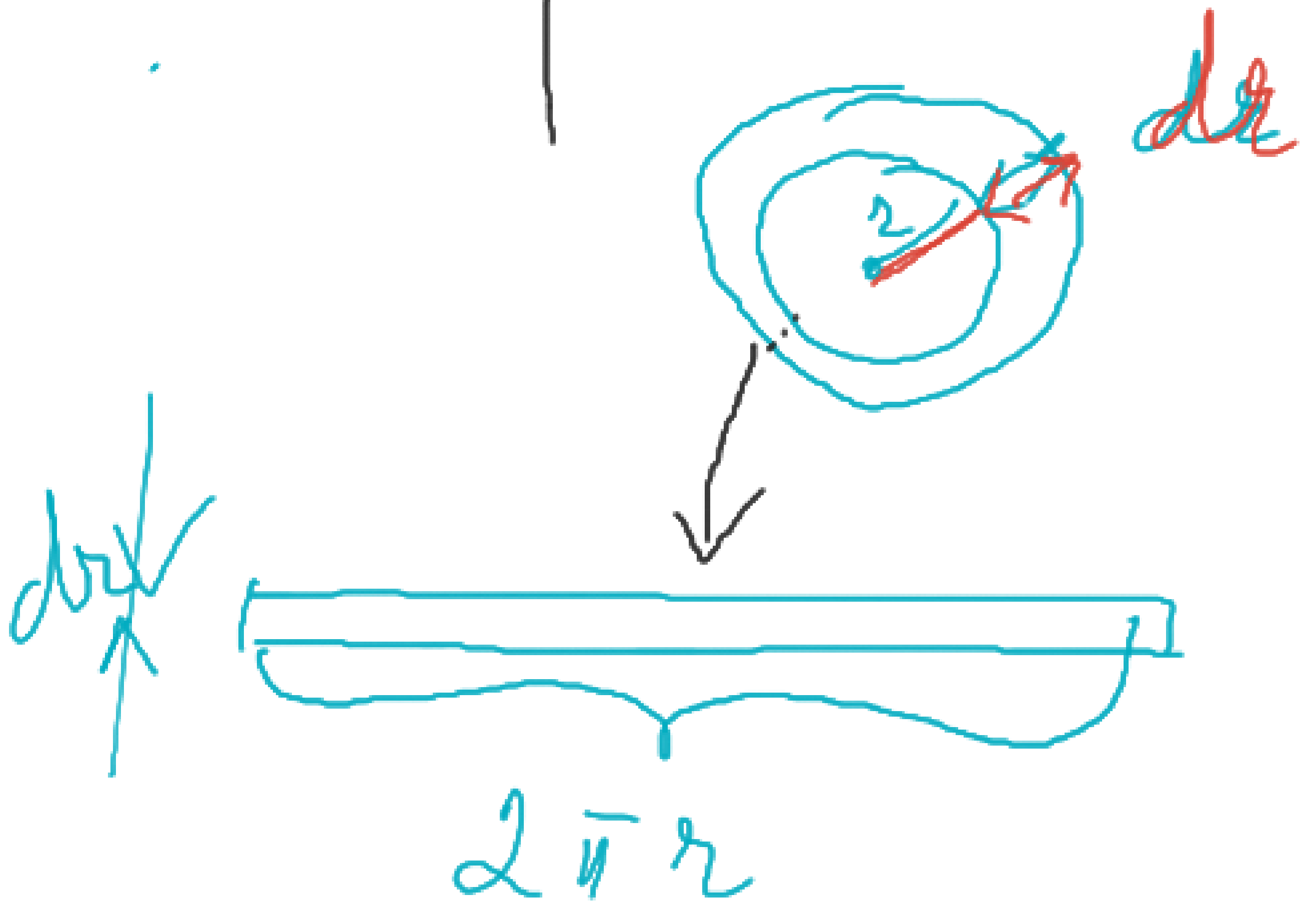
Kruh. doska m, R

$$I = \int dm\, r^2$$

$$dm = \rho\, dV = \frac{m}{V}\, dV = \frac{m}{S\cdot \cancel{h}}\, dS \cdot \cancel{h} =$$

$$= \frac{m}{S}\, dS = \frac{m}{\pi R^2} \cdot 2\pi r\, dr$$

plošná hustota



$$I = \int \frac{m}{\cancel{\pi} R^2} \cdot 2\cancel{\pi} r\, dr \cdot r^2 = \frac{2m}{R^2}\int_0^R r^3\, dr =$$

$$= \frac{2m}{R^2}\,\frac{R^4}{4} = \frac{mR^3}{2}$$

172 $E_k = 0{,}1\,J$

$m =$

$r =$

$\omega = 2\,(s^{-1})$

$E_k = \frac{1}{2} I \omega^2 \Rightarrow \omega = \sqrt{\frac{2E_k}{I}}$

$I = \int \frac{2}{5} m r^2$

$I = \int dI = \int \frac{dm\, r^2}{2} =$

$I = \int \frac{1}{2} \frac{m}{\frac{4}{3}\pi R^3} \cdot \pi x^2 dy \cdot x^2$

$dm = \rho\, dV = \frac{m}{\frac{4}{3}\pi R^3} \cdot \pi x^2\, dy$

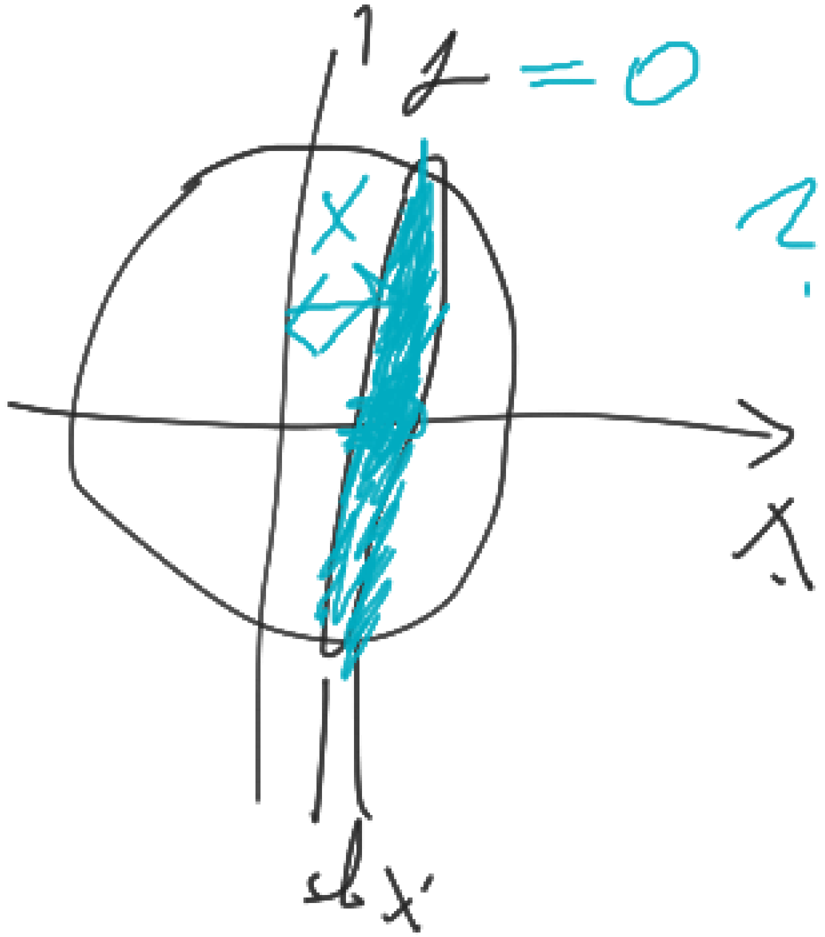
$x^2 = R^2 - y^2$

$= \int x^4 dy$

$= k \cdot 2 \int_0^R$

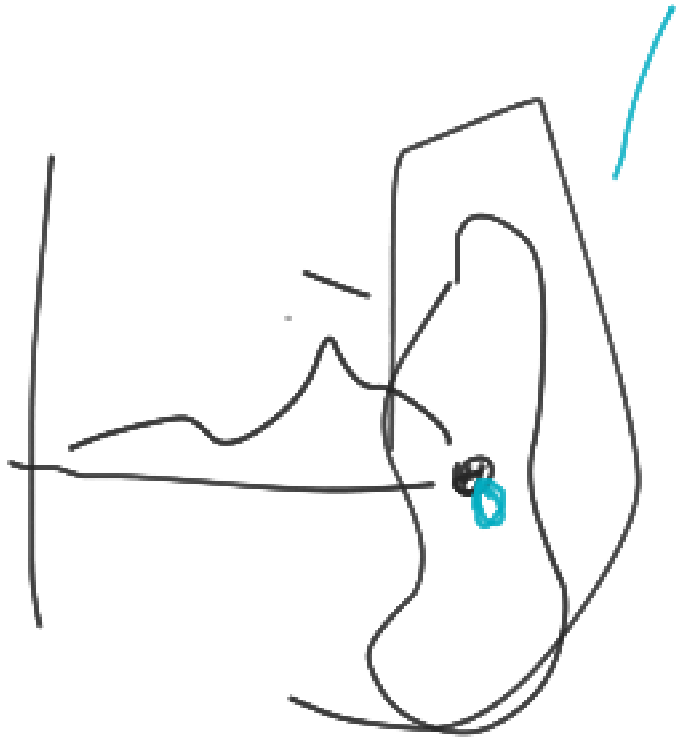
$= \int_{-R}^{R} \frac{1}{2} \frac{3m}{4R^3} (R^2 - x^2)\, dx$

$(R^2 - x^2)$

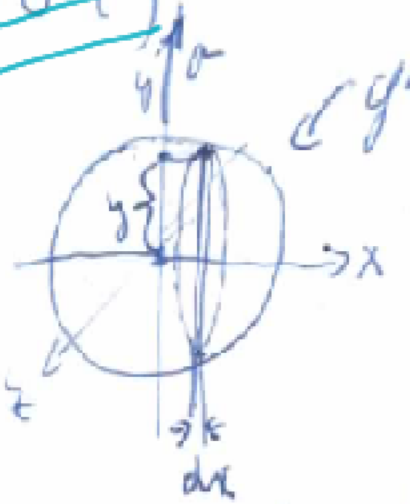


$dm = \rho\, dV =$

$\int dm\, \frac{R^2}{3}$ $\quad \frac{1}{2} dm\, x^2$



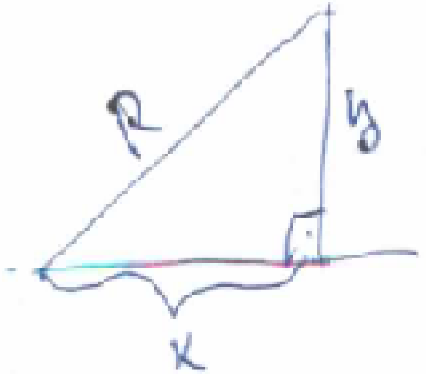
7(64)



cirgala $dm = \rho\, dV = \frac{m}{V} \cdot d(S \cdot x) = \frac{m}{\frac{4}{3}\pi R^3} \cdot y^2 \pi \cdot dx = \frac{3}{4}\frac{m}{R^3} \cdot y^2 \cdot dx$

$dx \cdot S = dx \cdot \pi \cdot y^2$

$dm = \frac{3}{4}\frac{m}{R^3}(R^2 - x^2)\, dx$ $\quad dm$



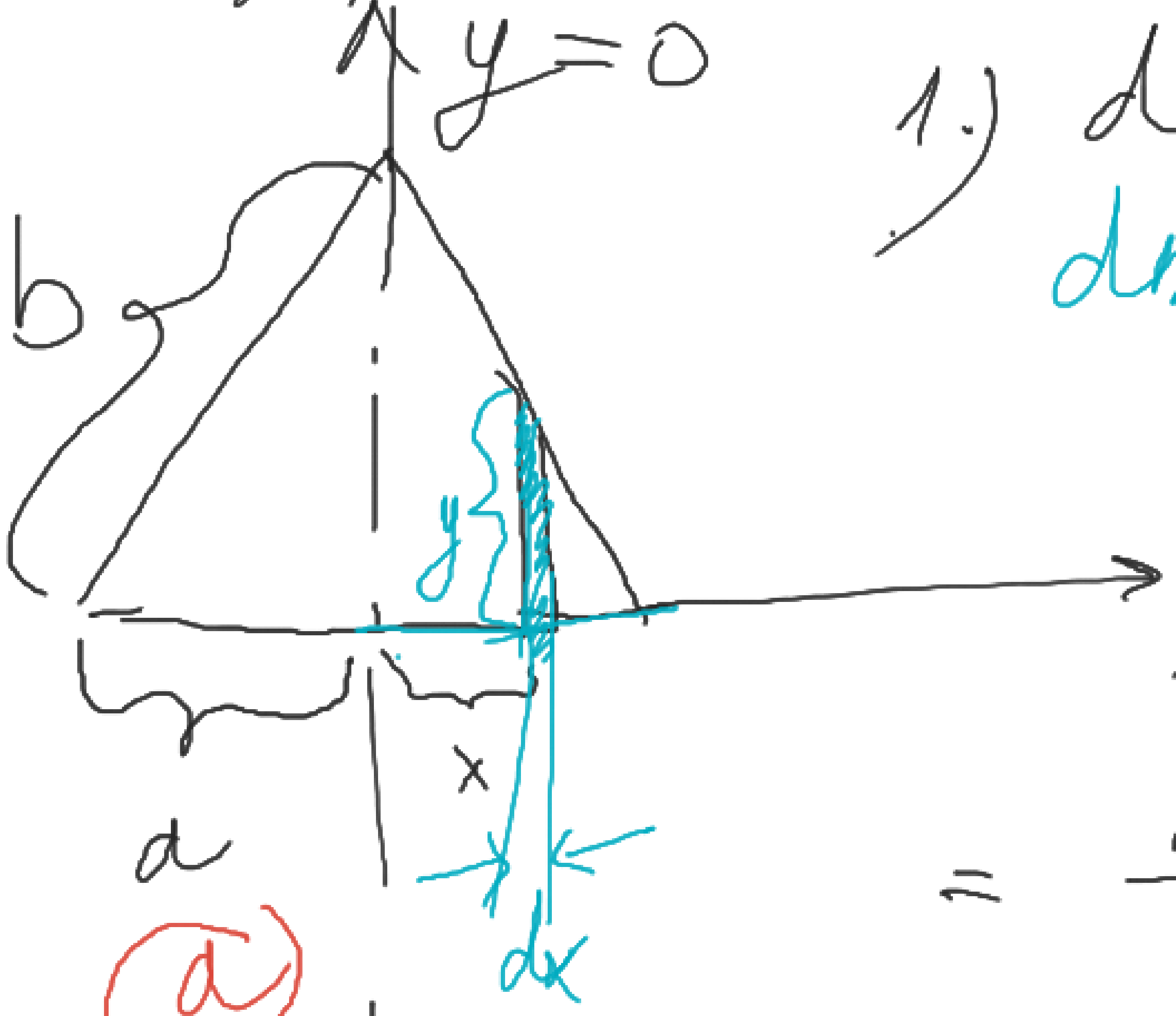
$R^2 = y^2 + x^2$

$y^2 = R^2 - x^2$

$I = \int_{-R}^{R} x^2 \cdot dm = \int_{-R}^{R} x^2 \cdot \frac{3}{4}\frac{m}{R^3}(R^2 - x^2)\, dx =$

$= \int_{-R}^{R} \frac{3}{4}\frac{m}{R^3}(R^2 x^2 - x^4)\, dx = \frac{3}{4}\frac{m}{R^3}\int_{-R}^{R}(R^2 x^2 - x^4)\, dx =$

m, zotrvač trojuholník dosky m, a, b



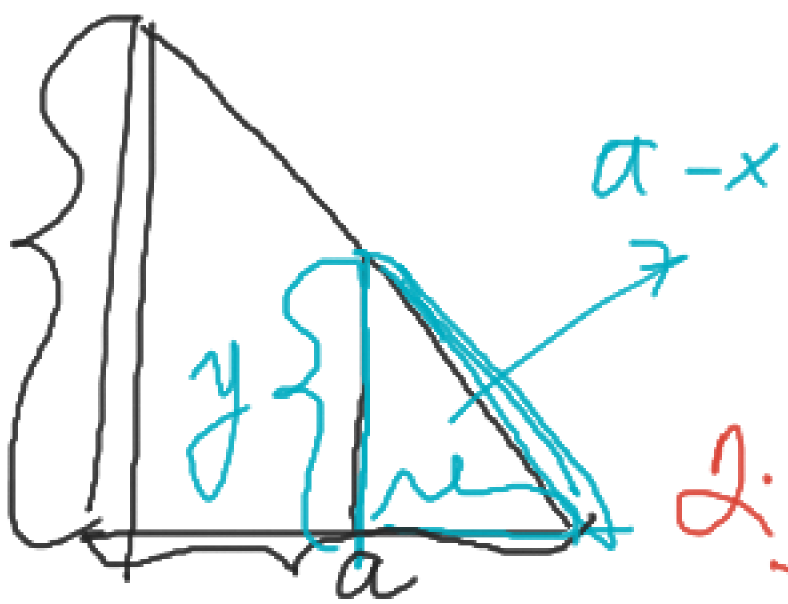
1.) dm ?

$dm = \rho \, dV = \sigma \cdot dS = \frac{m}{S} dS =$

plošná hustota

$= \frac{m}{S} dx \, y$

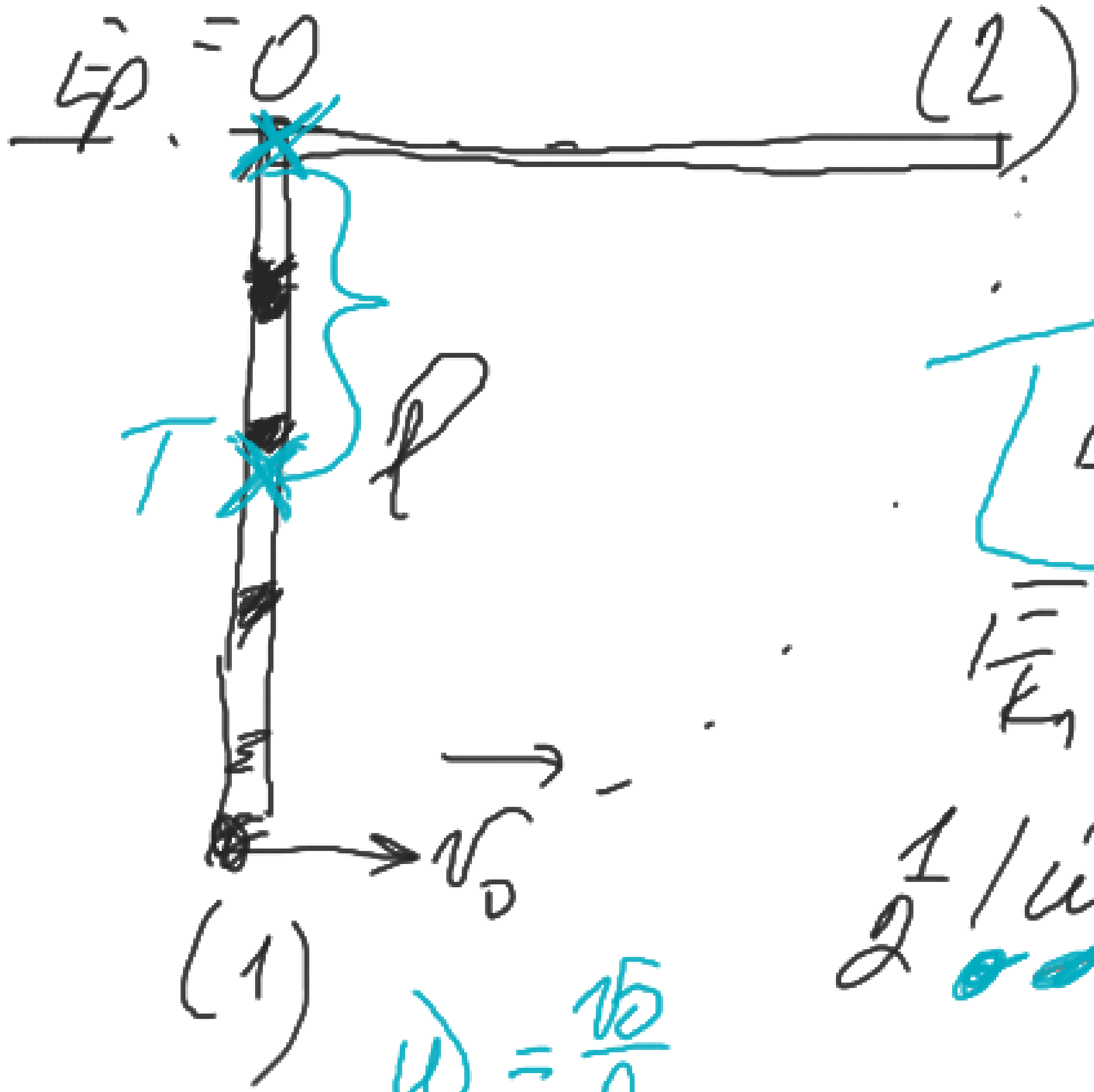
podobné △



$I = \int_{-a}^{a} \frac{m}{S} dx \, y$

$\frac{v}{a} = \frac{y}{a-x} \Rightarrow y = f(x)$

$2 \cdot \int_0^a \ldots =$



$v_0 = ?$ ZZME izolovaná sústava

tyč + Zem

$\boxed{E_k + E_p = konst}$ Ep tiažová

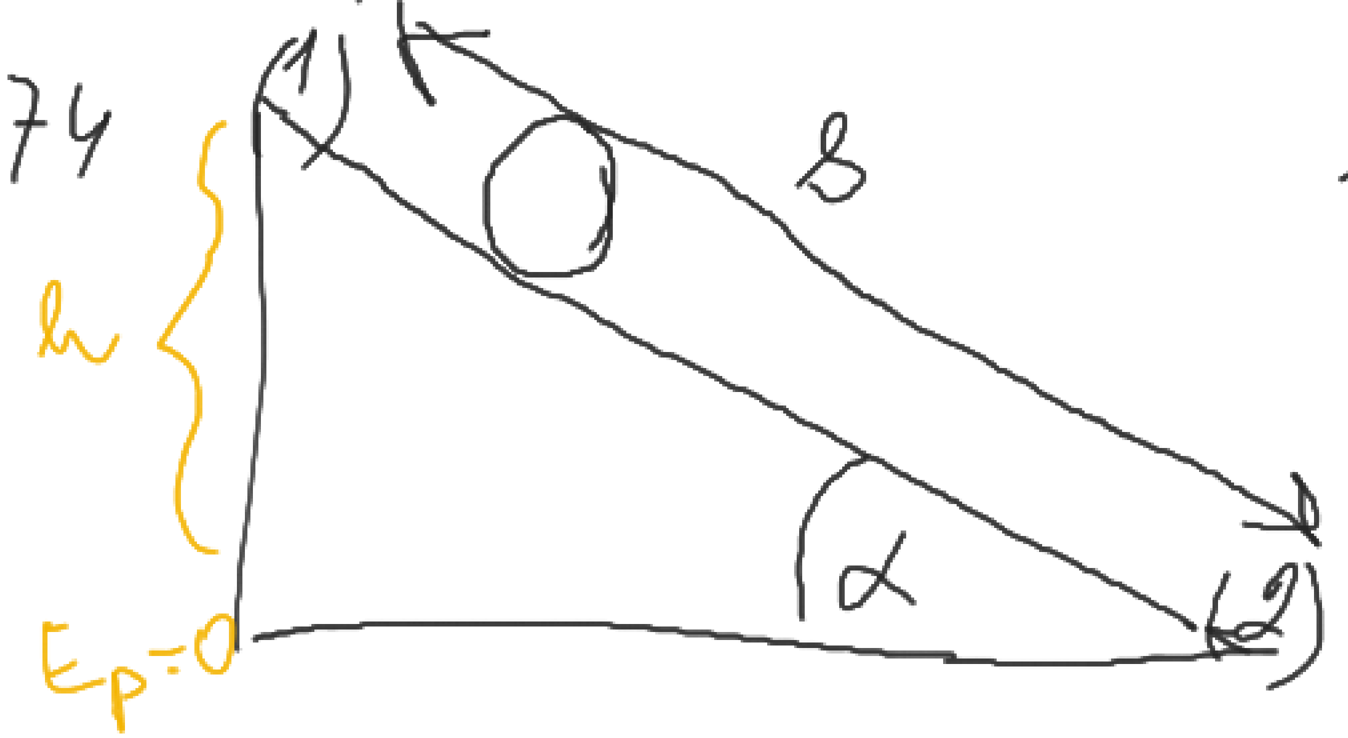
$E_{k_1} + E_{p_1} = E_{k_2} + E_{p_2}$

$\frac{1}{2} I \omega^2 - mg\frac{l}{2} = 0 + 0 \Rightarrow v_0 = \dots$

$\omega = \frac{v_0}{l}$

$I = \frac{ml^2}{3}$

174



m
r
α
s
v = ?

ZZME

$$E_{k_1} + E_{p_1} = E_{k_2} + E_{p_2}$$

$$0 + E_{p_1} = E_{k_2} + 0$$

$$\omega = \frac{v_T}{r}$$

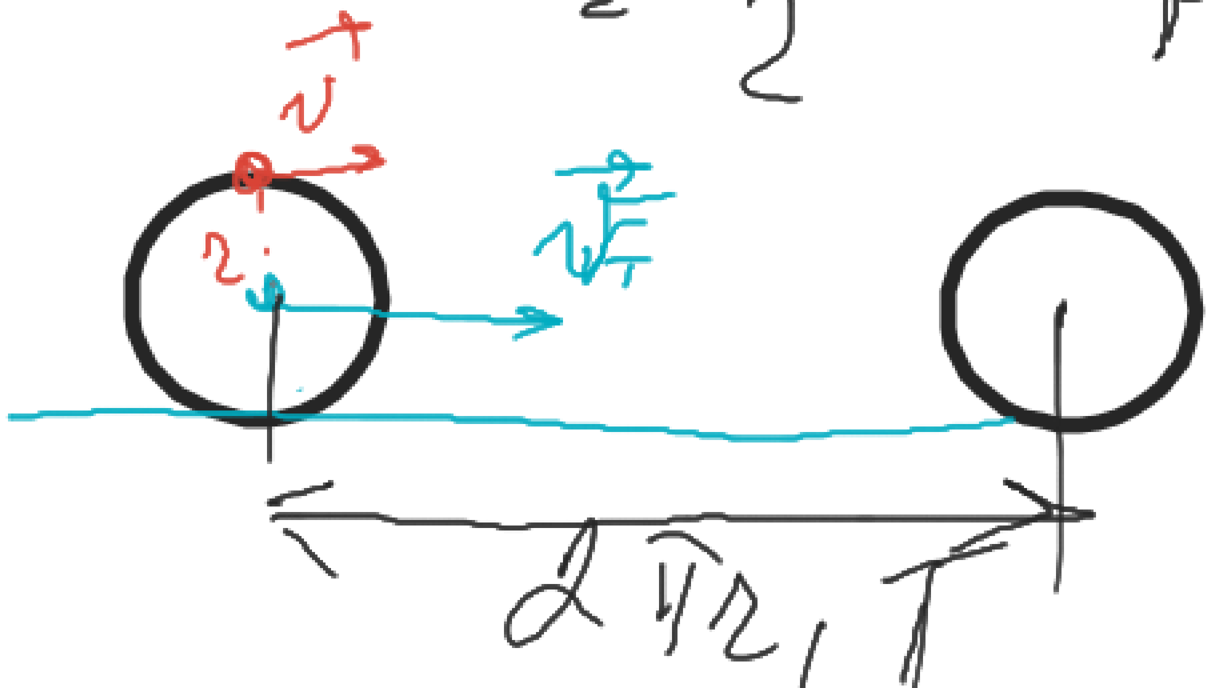
$$E_{p_1} = mg\cdot(\sin\alpha \cdot s)$$

$$E_{k_2} = \frac{1}{2} m v_T^2 + \frac{1}{2} I \omega^2$$

$$mg\sin\alpha \cdot s = \frac{1}{2}\left(m v_T^2 + I\omega^2\right)$$

$$mg\sin\alpha \, s = \frac{1}{2} m v_T^2 + \frac{1}{2} I \frac{v_T^2}{r^2}$$

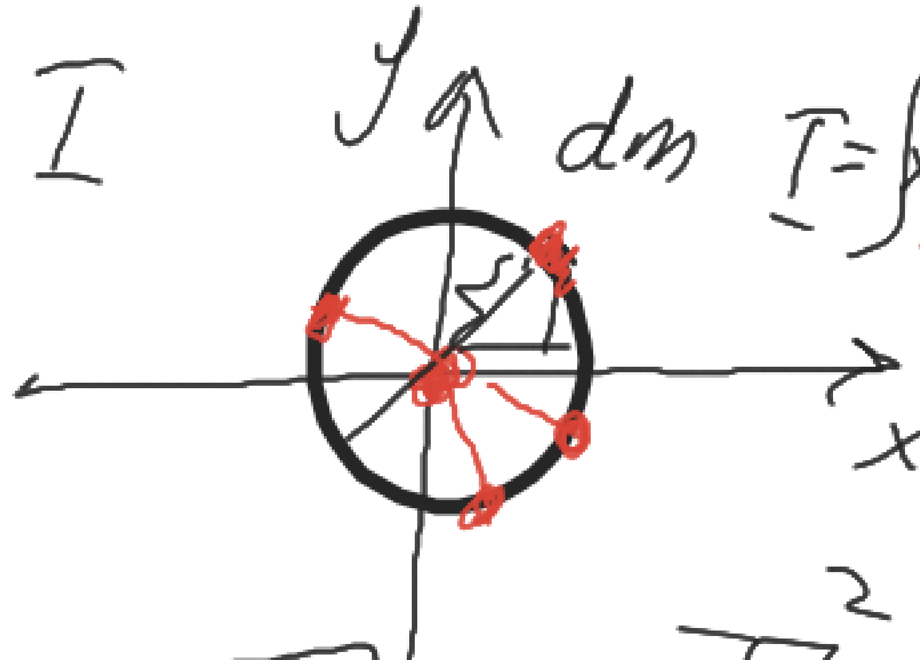
I



$$v = \omega \cdot r$$

$$v_T = v$$

$$v_T = \sqrt{sg \cdot \sin\alpha}$$



dm

$$I = \int x^2 \, dm = \int r^2 \, dm$$

$$= r^2 \int dm = r^2 m$$

Pohyb. rov pod pohybu: $\vec{F} = m\vec{a}_T$

$$\vec{F} = \vec{F}_g + \vec{F}_R + \vec{F}_T = \vec{F}_1 + \underbrace{\vec{F}_2 + \vec{F}_R + \vec{F}_T}_{0} = \vec{F}_1 + \vec{F}_T$$

$$\boxed{mg\sin\alpha - F_T = m\cdot a_T} \quad (1)$$

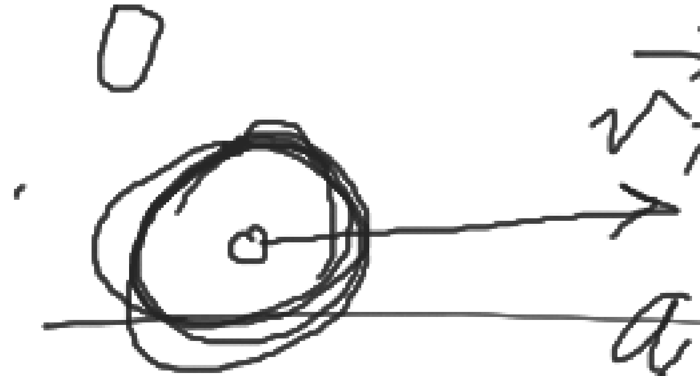
$0 = m a_y$

$0 = m \cdot a_z$

$F_x = m a_x$

$F_y = m a_y$

$F_z = m a_z$



$v_T = \omega \cdot R$

$a_t = \frac{dv_T}{dt} = \frac{d\omega}{dt} R$

$a_t = \varepsilon R$

$\varepsilon = \frac{a_t}{R}$

Pohyb. rovnica rotač. pohybu: $M = I \cdot \varepsilon$

$M = F_t \cdot r$

$F_t \cdot r = I \cdot \varepsilon$

$$\boxed{F_t\, r = I \frac{a_t}{r}} \quad (2)$$

$\Rightarrow a_t =$ konšt RZP

$v_T = ?$

171, 172, 173, 174, 146, 145