full range can either go from [0,5] or from [-1,1]

1 Algebra

$$\text{current: } y = x^{\frac{h_{mean}}{1-h_{mean}}} \tag{1}$$
$$x - h \tag{2}$$
$$x \tag{3}$$
$$x/h \tag{4}$$
$$xx - xh \tag{5}$$
$$\tag{6}$$

2 Wave mechanics and rotation

$$h(x) + h(-x) \tag{7}$$

3 Calc of Variations, least time principle

$$\frac{d}{dx}\frac{\partial}{\partial y'}\left(\frac{\sqrt{1+y'^2}}{v(x,y)}\right)=\frac{\partial}{\partial y}\left(\frac{\sqrt{1+y'^2}}{v(x,y)}\right) \tag{8}$$

$$v(x,y)=\frac{c}{ay+b} \tag{9}$$

$$\frac{d}{dx}\left(\frac{ay+b}{c}\frac{\partial}{\partial y'}\left(\sqrt{1+y'^2}\right)\right)=\sqrt{1+y'^2}\frac{\partial}{\partial y}\left(\frac{ay+b}{c}\right) \tag{10}$$

$$\frac{d}{dx}\left((ay+b)\frac{\partial}{\partial y'}\left(\sqrt{1+y'^2}\right)\right)=\sqrt{1+y'^2}\frac{\partial}{\partial y}(ay+b) \tag{11}$$

$$\frac{d}{dx}\left((ay+b)\frac{\partial}{\partial y'}\left(\sqrt{1+y'^2}\right)\right)=a\sqrt{1+y'^2} \tag{12}$$

$$\frac{d}{dx}\left((ay+b)\left(y'\left(1+y'^2\right)^{-\frac{1}{2}}\right)\right)=a\sqrt{1+y'^2} \tag{13}$$

$$\frac{d}{dx}\left((ay+b)(y')\left(1+y'^2\right)^{-\frac{1}{2}}\right)=a\sqrt{1+y'^2} \tag{14}$$

$$\begin{aligned}&\left(ay'(y')\left(1+y'^2\right)^{-\frac{1}{2}}\right)+\left((ay+b)(y'')\left(1+y'^2\right)^{-\frac{1}{2}}\right)+\\&\left((ay+b)(y')\left(1+y'^2\right)^{-\frac{3}{2}}\times-\frac{1}{2}\times 2y'\right)=a\sqrt{1+y'^2}\end{aligned} \tag{15}$$

$$\frac{ay'^2}{\sqrt{1+y'^2}}+\frac{(ay+b)(y'')}{\sqrt{(1+y'^2)}}-\frac{(ay+b)\left(y'^2\right)}{\sqrt{(1+y'^2)^3}}=a\sqrt{1+y'^2} \tag{16}$$

$$ay'^2+(ay+b)(y'')-\frac{(ay+b)\left(y'^2\right)}{(1+y'^2)}=a\left(1+y'^2\right) \tag{17}$$

$$ayy''-\frac{ayy'^2}{(1+y'^2)}=a\left(1+y'^2\right)-ay'^2-by''+\frac{by'^2}{(1+y'^2)} \tag{18}$$

$$y\left(ay''-\frac{ay'^2}{(1+y'^2)}\right)=a\left(1+y'^2\right)-ay'^2-by''+\frac{by'^2}{(1+y'^2)} \tag{19}$$

$$y=\frac{a-by''+\frac{by'^2}{(1+y'^2)}}{\left(ay''-\frac{ay'^2}{(1+y'^2)}\right)} \tag{20}$$

$$b=0\rightarrow y=\frac{a}{\left(ay''-\frac{ay'^2}{(1+y'^2)}\right)} \tag{21}$$

$$u=y'\rightarrow y=\frac{1}{\left(u'-\frac{u^2}{(1+u^2)}\right)} \tag{22}$$

$$y=\frac{1+u^2}{(u'(1+u^2)-u^2)} \tag{23}$$

$$\tag{24}$$

4 simple Snells

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \rightarrow \theta_2 = \arcsin(\frac{n_1}{n_2} \sin\theta_1) \tag{25}$$
$$(x_1, y_1) \text{ and } (x_2, y_2) \tag{26}$$
$$y = mx + b \rightarrow y - y_1 = m_1(x - x_1) \text{ and } y - y_2 = m_2(x - x_2) \tag{27}$$
$$m = \tan(\theta) = \tan() \tag{28}$$
$$y - y_1 = \tan\theta_1(x - x_1) \text{ and } y - y_2 = \tan\theta_2(x - x_2) \tag{29}$$
$$x_{interface} \text{ defined by } \tan\theta_1(x_{interface} - x_1) + y_1 = \tan\theta_2(x_{interface} - x_2) + y_2 \tag{30}$$
$$\tan\theta_1(x_{interface} - x_1) - \tan\theta_2(x_{interface} - x_2) = y_2 - y_1 \tag{31}$$
$$\tan\theta_1 x_{interface} - \tan\theta_1 x_1 - \tan\theta_2 x_{interface} + \tan\theta_2 x_2 = y_2 - y_1 \tag{32}$$
$$\tan\theta_1 x_{interface} - \tan\theta_2 x_{interface} = y_2 - y_1 + \tan\theta_1 x_1 - \tan\theta_2 x_2 \tag{33}$$
$$x_{interface} = \frac{y_2 - y_1 + \tan\theta_1 x_1 - \tan\theta_2 x_2}{\tan\theta_1 - \tan\theta_2} \tag{34}$$
$$\tag{35}$$
$$y - y_1 = \tan\theta_1(x - x_1) \text{ and } y - y_2 = \tan\theta_2(x - x_2) \tag{36}$$
$$\tag{37}$$

5 snell bizarre idea

$$n \sin\theta = s_0 \tag{38}$$
$$\tag{39}$$

6 Iterated snell's law

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) = ... = n_i \sin(\theta_i) \tag{40}$$
$$\theta_i = \arcsin(\frac{n_j}{n_i} \sin(\theta_j))\forall j \tag{41}$$
$$\text{option 1:} \qquad \theta_{start} = 90^o - \theta_{end} \tag{42}$$
$$sin(\theta) = dx/dy \;\&\; \theta = \theta(y) \text{ related to the density} \tag{43}$$
$$n_1 \frac{dx}{dy}\bigg|_{y_1} = n_2 \frac{dx}{dy}\bigg|_{y_2} = ... = n_i \frac{dx}{dy}\bigg|_{y_i} \tag{44}$$
$$\tag{45}$$

7 Parametric

$$x(t) = \qquad\qquad y(t) = \tag{46}$$