Motor operation inspired optimization for Hierarchical Influence governed Differential Evolution

ABSTRACT

Operational maturity of biological control systems have fuelled the inspiration for a large number of mathematical and logical models for control, automation and optimisation. The human brain represents the most sophisticated control architecture known to us and is a central motivation for several research attempts across various domains. In the present work, we introduce an algorithm for mathematical optimisation that derives its intuition from the hierarchical and distributed operations of the human motor system. The system comprises global leaders, local leaders and an effector population that adapt dynamically to attain global optimisation via a feedback mechanism coupled with the structural hierarchy. The hierarchical system operation is distributed into local control for movement and global controllers that facilitate gross motion and decision making. We present our algorithm as a variant of the classical Differential Evolution algorithm, introducing a hierarchical crossover operation. The discussed approach is tested exhaustively on standard test functions from the CEC 2017 benchmark. Our algorithm significantly outperforms various standard algorithms as well as their popular variants as discussed in the results.

CCS CONCEPTS

•Computer systems organization \rightarrow Embedded systems; Redundancy; Robotics; •Networks \rightarrow Network reliability;

KEYWORDS

Hierarchical Optimization

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1 INTRODUCTION

Evolutionary algorithms are classified as meta-heuristic search algorithms, where possible solution elements span the n-dimensional search space to find the global optimum solution. Over the years, natural phenomena and biological processes have laid the foundation for several algorithms for control and optimization that have highlighted their applicability in solving intricate optimization problems in various fields. At the cellular level in the E.Coli Bacterium, there is sensing and locomotion involved in seeking

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nourishment and avoid harmful chemicals. These behavioral characterisitics have fueled the inspiration for the bacterial foraging optimization algorithm. Ant Colony Optimization (ACO) deals with behaviour of ants and has been a successful model for solving complex problems. Particle Swarm Optimization (PSO) is a swarm intelligence algorithm based on behaviour of birds and fishes that models these particles as they traverse an n-dimensional search space and share information in order to obtain global optimum. From a biological control point, the human brain represents one of the most sophesticated architectures and several research attempts seek to mimic its accuracy, precision and efficiency. The brain function activities can be classified into 2 categories: sensory and motor operations. Sensory cortical functions inspired the concept of neural networks and they are being scaled successfully in deep learning to solve vast amount of problems.

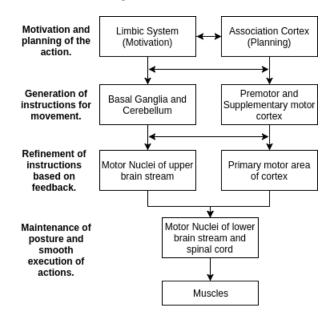


Figure 1: Description of Heirarchical Motor Operations in Humans

The human motor function represents a distributed neural and hierarchical control system. It can be classified as having local control functions for movement as well as higher level controllers for gross motion and decision making for planning actions. The optimal execution of motor operation involves distributed brain structures at different levels of hierarchy. These include the pre-frontal cortex, motor cortex, spinal cord, anterior horn cells etc. For generating an actions sequence, a sequence of actions is implemented by a string of subsequences of actions each implemented in a different part of the body. The operational structure has been depicted in Figure 1.[]

the motor cortex act like global leaders and send inhibitory or facilitatory influence over the anterior horn cells, local leaders, located in the spinal cord. These local leaders, are connected to the muscle fibres, effectors through a peripheral nerve and neuromuscular junction. Efficient execution of task requires feedback based facilitation and inhibition of the effectors over the anterior horn cells. These sequence of operations consitute the optimal convergence of the system leading to smooth motor execution.

The present paper focusses on introducing an optimisation algorithm modelled intuitively on the distributed and hierarchical operation of the brain motor function.

The Classical Differential Algorithm, proposed by Storn and Price [] has been hailed as one of the best available evolutionary algorithms, owing to it's simple yet effective structure but it has been criticised for it's slow convergence rate. Through this paper, we seek to improve the algorithm's performance by proposing a hierarchical structure in the pipeline of the algorithm. Introduction of a hierarchical architecture in the algorithm enables the algorithm to control the flow of agents based on the influences of local leaders and a global leader, as depicted by the brain motor function in Fig. 1. The algorithmic performance has been exhaustively compared with some popular variants of DE, namely joint Adaptive Differential Evolution (JADE), Particle Swarm Optimization-Differential Evolution (PSODE) and the original Classical Differential Evolution (DE) algorithms. Through this exhaustive comparision, we show even with fixed parameters and the introduction of a hierarchical crossover operator, HIDE is able to outperform adaptive architectures such as JADE by a high margin, as discussed in the result sections. This effective behaviour results from the co-ordinated hierarchy between the global and local control, owing to the selected hierarchical function as discussed in the algorithmic approach. The result section () comprises a thorough analysis and comparision on algorithmic performance on the CEC 2017 objective function set, where we report the objective function optimal values and display the convergence plots for each algorithm, while discussing the efficiency of HIDE.

2 CLASSICAL DIFFERENTIAL EVOLUTION

The classical Differential Evolution (DE) algorithm is a population-based global optimization algorithm, utilizing a crossover and mmutation approach to generate new individuals in the population for achieving optimum solutions. For each individual x_i that belongs to the population, DE randomly samples three other individuals from the population a_i , b_i and c_i . Then using the randomly chosen points, a new individual vector is generated using equation (1):

$$u_i = a_i + F(b_i - c_i) \tag{1}$$

Where, F is called the differential weight (Usually lies between [0, 1]).

And to obtain the new position of the individual, a crossover operation is implemented between x_i and u_i , controlled by the parameter CR called the crossover probability. The value for CR always lies between [0,1].

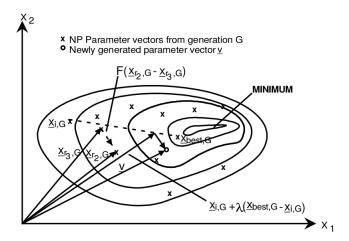


Figure 2: Two dimensional example of an objective function showing its contour lines and the process for generating v in scheme DE2

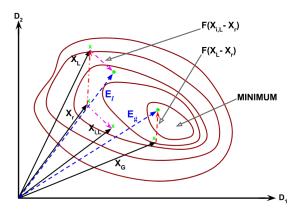


Figure 3: The process for generating generation of E_g and E_l in a 2 dimensional optimization.

3 DISTRIBUTED LEADER OPTIMIZATION

Taking inspiration from the human motor system, we model the hierarchical motor operations in our optimization agents, where we define a global leader which influences the action of several distributed local leaders and the particle agents which act as the effectors. The global leader is analogous to the decision making and planning section in the motor system hierarchy whilst, the local leaders correspond to motion generators acting under the influence of the global leader.

The position of each particle in the population is affected by the influence of global leaders and local leaders, while also being affected by a randomly chosen particle from the population to induce some stochasticity in the optimization pipeline. We first model the influence of the global leader on the local leaders and the influences of the local leaders on each population element using equation (2) and (3). We introduce a hierarchical crossover between the two influencing equations governed by a hierarchical crossover parameter HC.

Algorithm 1 Distributed Leader Optimization

```
1: procedure Start
       Initialize parameters (HC, P, N_1, N).
 2:
       Generate initial global leader q_L as a random point.
 3:
       Generate N_l local leader points around g_L.
 4:
       Using a Normal distribution, generate N points for popula-
    tion P around the local leaders.
       while termination criteria is not met do
            for each individual x_i in P do
 7:
               compute the corresponding local leader l based on
 8:
    nearest position.
               Let u = 0 be an empty vector.
 9:
               Let i_c and i_N be the current generation and total
10:
    generations of the procedure.
               if i_c < (HC * i_N) then
11:
                   u = E_g from (2).
12:
               else
13:
                   u = E_l from (3).
14:
               end if
15:
               for each dimension i do
16:
17:
                   Generate r_i = U(0, 1), a random number be-
    tween 0 and 1.
                   if r_i < HC then
18:
                       Set x_i = x_i.
19:
20:
                       Set x_{i}' = u_{i}
21:
                   end if
22:
               end for
23:
               if f(x') < f(x) then
24:
                   Replace x with x^{'} in the population.
25:
               end if
26:
            end for
27:
            Alter local leaders in each population cluster based on
28:
    objective function value.
            Compute updated global leader q_L.
29:
       end while
30:
31: end procedure
```

Analogously to [step 3] in the brain motor operation, the updation of particle positions requires generating feedback for the leaders as a part of the optimization procedure, and hence the local leaders and the global leader are updated based onn their objective function value generated from the perturbations in population particles. This series of events comprise of one optimization pass (one loop step). On execution of several optimization passes as described, the system is able to converge to an optimal configuration, analogous to the successful execution of the required task as shown in [step 4].

The updated position of the particle x is governed by the hierarchical crossover operation and a mutation operation. The hierarchical operation is affected by the global leader g_L and the local leader l through the parametric equations (2) and (3). Switching between

the two is governed by the hierarchical crossover parameter HC. The given equations are discussed as follows:

$$E_q = g_L + F(l - c) \tag{2}$$

$$E_l = l + F(x - c) \tag{3}$$

In the algorithm 1, The Hierarchical crossover is controlled by the conditional equation $i_c < (HC*i_N)$. According to this equation, during the initial phases (HC fraction of total generations) of the optimization procedure, only the local leader is responsible for the motion of the agents, and after a certain amount of time has passed, the global leader also takes part in the motion generation process, signifying the motor control operation. Additionally, The hierarchical crossover parameter HC also influences the mutation process wherein the degree of final mutation is decided based on the probability HC.

Table 1: Test Functions

| F_{id} | Problem Function | F* |
|----------|--|------|
| 1 | Shifted and Rotated Bent Cigar Function | 100 |
| 2 | Shifted and Rotated Sum of Different Power | 200 |
| | Function | |
| 3 | Shifted and Rotated Zakharov Function | 300 |
| 4 | Shifted and Rotated Rosenbrockfis Func- | 400 |
| | tion | |
| 5 | Shifted and Rotated Rastriginfis Function | 500 |
| 6 | Shifted and Rotated Expanded Scafferfis F6 | 600 |
| | Function | |
| 7 | Shifted and Rotated Lunacek Bi_Rastrigin | 700 |
| | Function | |
| 8 | Shifted and Rotated Non-Continuous Rast- | 800 |
| | riginfis Function | |
| 9 | Shifted and Rotated Levy Function | 900 |
| 10 | Shifted and Rotated Schwefelfis Function | 1000 |
| 11 | Hybrid Function 1 (N=3) | 1100 |
| 12 | Hybrid Function 2 (N=3) | 1200 |
| 13 | Hybrid Function 3 (N=3) | 1300 |
| 14 | Hybrid Function 4 (N=4) | 1400 |
| 15 | Hybrid Function 5 (N=4) | 1500 |
| 16 | Hybrid Function 6 (N=4) | 1600 |
| 17 | Hybrid Function 7 (N=5) | 1700 |
| 18 | Hybrid Function 8 (N=5) | 1800 |
| 19 | Hybrid Function 9 (N=5) | 1900 |
| 20 | Hybrid Function 10 (N=6) | 2000 |
| 21 | Composition Function 1 (N=3) | 2100 |
| 22 | Composition Function 2 (N=3) | 2200 |
| 23 | Composition Function 3 (N=4) | 2300 |
| 24 | Composition Function 4 (N=4) | 2400 |
| 25 | Composition Function 5 (N=5) | 2500 |
| 26 | Composition Function 6 (N=5) | 2600 |
| 27 | Composition Function 7 (N=6) | 2700 |
| 28 | Composition Function 8 (N=6) | 2800 |
| 29 | Composition Function 9 (N=3) | 2900 |
| | Composition Function 10 (N=3) | |

4 RESULTS AND DISCUSSIONS

All evaluations were performed using Python 2.7.12 with Scipy and Numpy for numerical computations and Matplotlib package for graphical representation of the result data. This section is divided into two sub-sections: Section A provides description about the problem set used for analysis of algorithmic efficiency and accuracy, and section B comprises of tabular and graphical data to support the claim of eminence of the proposed approach.

4.1 Problem Set Description and Parameter settings

The set of objective functions considered for testing the proposed algorithm and compare it's performance against classsical DE and it's variants PSODE and Joint Adaptive Differential Evolution (JADE) have been taken from the CEC 2017 [] set of scalable benchmark functions. Exhaustive comparisions and analysis have been depicted on dimensions D = 10, 30, 50 and 100 for a clear understanding of the strengths of the proposed algorithm. For fair comparisions, the parameters for all algorithms are fixed to the following values: [values of params]. Summarized in Table [] are the 30 objective functions from the CEC 2017 dataset and the optimal value for each fuction denoted by F*. In all simulation runs, we set the population size NP to a fixed value of 40 and the number of runs for each dimenstions are: [Number of runs], and the results are shown in a tabular structure depicting the best and average values of the population individuals for the simulations. For clarity, we employ a scoring method for determining the reliability of algorithms where the most suitable values for each test function have been highlighted in [bold], and allocated an incremental score of 1. At the end of each table, we summarize the performance of each algorithm based on the final score allocated for all 30 objective functions, and ultimately discuss the overall performance of the algorithms based on these scores.

Additionally, several graphical results have been discussed to observe the convergence rate and efficiency of the algorithms used in the simulation. These graphs were plotted based on the numerical results obtained from the simulation runs used to build the tables.

4.2 Numerical and Graphical Results

In Tables 2-5, the best and mean values obtained for the population agents in the simulation runs have been reported, and the optimum results for each objective function have been highlighted in [bold] and allocated an incremental score as discussed in the Parameter section.

In addition to the tabular information, the plots in Fig. 4 aptly depict the performance of all algorithms across various dimensions and objective functions. On overall compilation of the plots, it is clear that HIDE shows a better convergence rate as compared to rest of the algorithms. Moreover, on higher dimensions, the proposed algorithm evidently outperforms all other algorithms both in terms of convergence rate and optimality. On lower dimensions (D=10, 30), HIDE shows a performance trend similar to PSODE as shown in Fig. 4 (a, b, e, f, i, j, m, n) and can been seen to produce better end-results as compared to PSODE, DE and JADE in almost all these cases, as was evident from the tabular information as well. For problems in higher dimensions (D=50, 100), it is

very clear that HIDE optperforms rest of the algorithms by a high margin, especially in Fig. 4 (s, t), where the algorithm displays an impressive convergence rate and a far better optimality than DE, PSODE and JADE. Similarly, in Fig. 4 (k, l, o, p) HIDE shows a faster convergence than any other algorithms considered for comparisions, hence validating the claim that the proposed approach is able to overcome the issue of slow convergence with Differential Evolution algorithm. This remarkable trait in HIDE enhances it's utility for high dimensional problems where fast convergence to global optimum value is required, hence making it superior to the other considered algorithms and several variants of the Differential Algorithm

4.3 Results

5 CONCLUSION

Differential Evolution has been one of the most successful optimization algorithms and over the years, several variants have been proposed to enhance its convergence time and performance. In the present work, we introduced a hierarchical influence governed variant of the classical Differential Evolution. Our algorithm derives its motivation from the distributed and hierarchical operations of the human motor system, one of the most sophesticated biological control architecture known. Our proposed approach involved introduction of a hierarchical crossover parameter, HC, that has enabled scaling the classical algorithm to more distributed settings. The collaborative influence of the global leader-local leader and local leader-effector interactions resulted in faster and more robust optimization. We exhaustively tested the algorithm performance on transformed hybrid and composite functions introduced in CEC 2017 Benchmark. The results and graphs discussed in Section 4 highlight the particular effectivess of the proposed approach on higher dimensional settings on more involved test functions.

The results observed provide sufficient motivation to extend the work to solving challenging real life problems in future.

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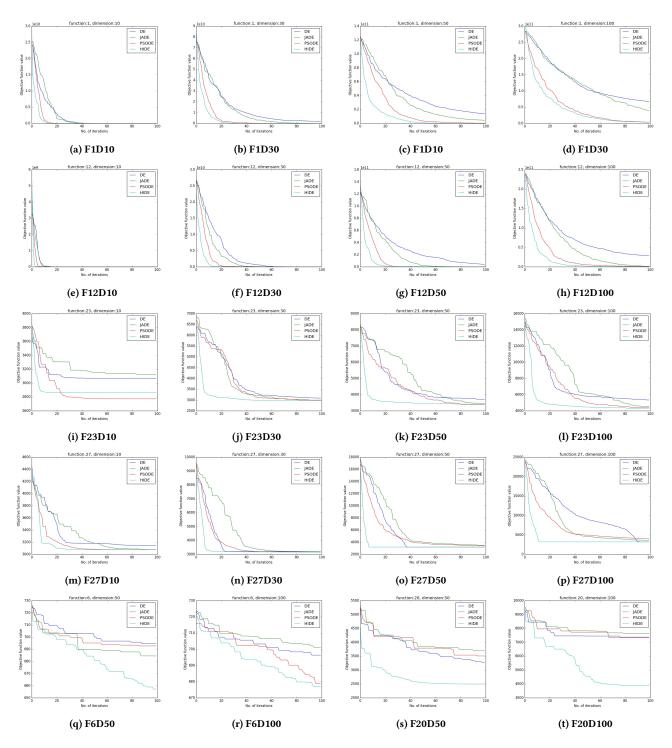


Figure 4: Comparision analysis over various functions and dimensions

Table 2: Objective Function Value for Dimension: 10

| ID | DE | | JADE | | PSO-DE | | Ours | |
|----|-------------|--------------|-------------|-------------|-------------|--------------|-------------|-------------|
| | best | mean | best | mean | best | mean | best | mean |
| 1 | 100.000051 | 100.011085 | 100.0 | 100.0 | 100.000712 | 185.975885 | 100.0 | 100.0 |
| 2 | 200.0 | 200.1 | 200.0 | 200.0 | 200.0 | 200.0 | 200.0 | 200.0 |
| 3 | 300.00134 | 300.214502 | 300.0 | 300.0 | 300.000006 | 300.000985 | 300.0 | 300.0 |
| 4 | 400.042617 | 403.674837 | 400.0 | 400.409399 | 400.064644 | 404.307763 | 400.0 | 400.000003 |
| 5 | 566.661791 | 604.867489 | 523.908977 | 541.521084 | 525.868824 | 575.61616 | 533.803201 | 579.483815 |
| 6 | 621.914237 | 634.807962 | 620.878276 | 636.034759 | 603.187964 | 635.865001 | 613.730565 | 629.293758 |
| 7 | 724.831278 | 739.129935 | 717.016542 | 723.983312 | 725.44788 | 733.15638 | 720.345706 | 725.233785 |
| 8 | 818.904202 | 829.749207 | 821.914433 | 826.321588 | 820.8941 | 830.246691 | 821.064763 | 828.160987 |
| 9 | 900.0 | 908.104383 | 900.0 | 1084.478253 | 900.0 | 1124.102561 | 900.0 | 903.454324 |
| 10 | 1911.510092 | 2447.443751 | 1760.956867 | 2162.648588 | 2049.644727 | 2518.241095 | 1694.437597 | 2049.074266 |
| 11 | 1102.985708 | 1113.423105 | 1105.661676 | 1117.509748 | 1105.97013 | 1120.192974 | 1101.769749 | 1108.863598 |
| 12 | 2531.746305 | 6509.743078 | 1438.605713 | 5430.674683 | 4089.006352 | 10810.387667 | 1308.438341 | 1327.405881 |
| 13 | 1313.130226 | 1404.903601 | 1304.681558 | 1328.755262 | 1319.839199 | 1453.340785 | 1306.682039 | 1344.282241 |
| 14 | 1409.949612 | 1426.571937 | 1412.934432 | 1428.169439 | 1420.91065 | 1434.112884 | 1404.928993 | 1410.000769 |
| 15 | 1504.131392 | 1521.446614 | 1502.496189 | 1508.31154 | 1501.389515 | 1518.310358 | 1500.08137 | 1503.169264 |
| 16 | 1958.42062 | 2104.555728 | 1958.857997 | 2094.630816 | 1958.411527 | 2048.156879 | 1958.433511 | 2062.385949 |
| 17 | 1728.194973 | 1743.155244 | 1730.715318 | 1748.129878 | 1727.80039 | 1791.607742 | 1723.853972 | 1747.589077 |
| 18 | 1801.586012 | 1838.840555 | 1804.298538 | 1825.091639 | 1817.154641 | 1840.546923 | 1800.235516 | 1804.014301 |
| 19 | 1901.195482 | 1903.604767 | 1900.399786 | 1902.152965 | 1902.71174 | 1906.252333 | 1900.005632 | 1901.014116 |
| 20 | 2204.55412 | 2289.226577 | 2148.538938 | 2178.313173 | 2140.561308 | 2261.038768 | 2139.915527 | 2172.816519 |
| 21 | 2337.772994 | 2387.230357 | 2314.421135 | 2338.688719 | 2337.207339 | 2351.898856 | 2320.496212 | 2344.61612 |
| 22 | 2300.805852 | 2304.132879 | 2300.0 | 2300.093485 | 2300.684181 | 2301.710478 | 2300.000015 | 2301.095975 |
| 23 | 3070.177083 | 3145.772296 | 3003.678563 | 3091.22041 | 2773.372859 | 3060.022519 | 2867.020036 | 3047.982305 |
| 24 | 2500.0 | 2500.0 | 2500.0 | 2500.0 | 2500.0 | 2500.0 | 2500.0 | 2500.0 |
| 25 | 2899.584968 | 2933.249812 | 2899.584968 | 2930.266506 | 2897.742869 | 2921.27479 | 2897.833388 | 2927.976511 |
| 26 | 2800.0 | 4117.597033 | 2800.0 | 2956.064173 | 2800.0 | 3367.60765 | 2800.0 | 3161.548079 |
| 27 | 3113.157656 | 3358.806434 | 3072.439023 | 3178.509645 | 3078.873134 | 3240.501812 | 3071.203569 | 3107.268539 |
| 28 | 3184.75565 | 3230.921422 | 3184.75565 | 3195.113042 | 3184.755652 | 3198.370691 | 3100.0 | 3195.411961 |
| 29 | 3148.587115 | 3266.979786 | 3172.400194 | 3233.707677 | 3191.348193 | 3244.892638 | 3189.211417 | 3292.420474 |
| 30 | 3442.555095 | 11927.404685 | 3207.766942 | 4615.591316 | 4573.358512 | 16415.162901 | 3205.740954 | 3249.710975 |
| # | 6/30 | 2/30 | 12/30 | 13/30 | 8/30 | 4/30 | 19/30 | 18/30 |

Table 3: Objective Function Value for Dimension: 30

| ID | DE | | JADE | | PSO-DE | | Ours | |
|----|--------------|--------------|--------------|---------------|--------------|--------------|--------------|-------------|
| | best | mean | best | mean | best | mean | best | mean |
| 1 | 100.001508 | 4334.438478 | 100.001338 | 100.056201 | 364.295574 | 4236.363207 | 100.0 | 100.0 |
| 2 | 40412441.0 | 5.129601e+19 | 200.0 | 1535352368.6 | 332899.0 | 9.590679e+11 | 200.0 | 159855.5 |
| 3 | 17926.872873 | 22131.542719 | 69304.926091 | 74080.700372 | 15792.547575 | 21683.209092 | 3679.811599 | 8999.947269 |
| 4 | 481.255055 | 519.422652 | 403.633939 | 442.206911 | 468.341175 | 479.341966 | 400.004163 | 443.016156 |
| 5 | 689.041352 | 737.79326 | 667.50756 | 735.204027 | 715.904429 | 746.548906 | 685.40454 | 738.842184 |
| 6 | 643.626307 | 652.582714 | 651.39169 | 655.142819 | 642.724237 | 655.106996 | 644.701241 | 652.002395 |
| 7 | 883.347367 | 962.591129 | 779.907693 | 818.344111 | 790.014281 | 854.285524 | 812.923573 | 856.90477 |
| 8 | 923.37426 | 967.251501 | 931.500175 | 957.362003 | 915.414882 | 960.486239 | 930.288539 | 964.11663 |
| 9 | 5652.483961 | 7878.781444 | 4953.05469 | 5146.600953 | 6018.417197 | 9042.410178 | 4003.118072 | 4734.984364 |
| 10 | 3596.63104 | 4536.989761 | 4012.723292 | 4204.18969 | 3934.606704 | 4863.741107 | 3793.781776 | 4346.741344 |
| 11 | 1162.405965 | 1184.634006 | 1152.748529 | 1174.58813 | 1165.144993 | 1189.171787 | 1149.748499 | 1171.130409 |
| 12 | 56679.435092 | 317650.61349 | 24821.171765 | 58930.090242 | 10221.077465 | 161046.05540 | 9208.289246 | 41947.22269 |
| 13 | 3002.029489 | 18794.835991 | 4276.907742 | 13775.816239 | 3871.279833 | 10612.26359 | 1664.06241 | 2453.606969 |
| 14 | 1773.180798 | 5502.160382 | 1496.219858 | 42868.9158 | 1555.452763 | 4029.808535 | 1462.926848 | 1504.191515 |
| 15 | 1860.435669 | 2484.689969 | 1688.05046 | 2222.674323 | 1651.747476 | 2223.060542 | 1611.074402 | 1852.66177 |
| 16 | 2517.439623 | 2827.004968 | 2344.19818 | 2621.618684 | 2239.242719 | 2664.114667 | 2298.041965 | 2691.674809 |
| 17 | 2321.175936 | 2604.529778 | 2062.898023 | 2546.995596 | 2107.43677 | 2457.34021 | 1820.806639 | 2418.723829 |
| 18 | 38987.282456 | 94156.328505 | 11841.60813 | 184888.162181 | 62294.853257 | 118430.28912 | 12578.003784 | 23024.11193 |
| 19 | 2043.469888 | 3010.235379 | 1959.71819 | 2156.957875 | 3049.52231 | 6840.408394 | 1949.271714 | 1987.866761 |
| 20 | 2625.539158 | 2864.832611 | 2706.314441 | 2805.600064 | 2619.996493 | 2895.107238 | 2753.806213 | 2966.035793 |
| 21 | 2412.081757 | 2504.777775 | 2414.52134 | 2456.718982 | 2431.740293 | 2478.841357 | 2200.0 | 2442.734316 |
| 22 | 2300.481796 | 5655.569322 | 2300.0 | 4157.698784 | 2307.721358 | 6811.069162 | 2300.009985 | 6795.24842 |
| 23 | 3050.654508 | 3572.965066 | 2772.002023 | 2946.749322 | 2764.922461 | 3199.874364 | 2883.276891 | 3543.839343 |
| 24 | 3104.623692 | 3290.698756 | 2891.557648 | 2965.225566 | 2911.63347 | 2983.772932 | 2500.0 | 2940.75997 |
| 25 | 2916.180657 | 2946.711753 | 2875.106846 | 2881.091389 | 2875.498843 | 2889.943671 | 2874.171109 | 2877.484904 |
| 26 | 4043.691403 | 6756.3724 | 2900.0 | 3266.510982 | 2800.007809 | 3273.128769 | 2900.0 | 3298.490539 |
| 27 | 3200.005857 | 3998.876498 | 3145.810354 | 3189.82261 | 3145.425231 | 3639.634132 | 3132.816283 | 3284.28897 |
| 28 | 3290.744025 | 3326.263983 | 3100.0 | 3131.027315 | 3195.486838 | 3225.594053 | 3100.0 | 3115.505829 |
| 29 | 3720.314598 | 4115.185803 | 3305.310139 | 3626.887552 | 3535.952295 | 3867.593068 | 3352.845055 | 3709.102375 |
| 30 | 3359.030768 | 3900.826662 | 3263.496536 | 3749.610722 | 3312.635025 | 3524.714477 | 3298.704645 | 3421.715322 |
| # | 2/30 | 0/30 | 9/30 | 11/30 | 3/30 | 1/30 | 17/30 | 16/30 |

Table 4: Objective Function Value for Dimension: 50

| ID | DE | | JADE | | PSO-DE | | Ours | |
|----|---------------|---------------|---------------|---------------|--------------|---------------|---------------------|-------------|
| | best | mean | best | mean | best | mean | best | mean |
| 1 | 5884574.87314 | 367294248.521 | 136.072384 | 3708.75086 | 5811.218992 | 154233.646744 | 106.072862 | 3665.419272 |
| 2 | 4.718137e+24 | 3.364977e+44 | 2635725.0 | 5.02374e+26 | 2.212101e+19 | 2.544543e+23 | 2.279950e+17 | 1.00729e+31 |
| 3 | 45520.966376 | 62237.296819 | 143481.793147 | 156166.762356 | 52308.42743 | 64435.24063 | 44613.29993 | 58182.83733 |
| 4 | 574.400328 | 801.384952 | 418.580378 | 470.113207 | 477.080964 | 574.528479 | 400.005049 | 447.775413 |
| 5 | 816.394775 | 843.258843 | 809.899483 | 834.131266 | 778.59312 | 831.066954 | 791.405194 | 830.218472 |
| 6 | 652.541914 | 655.794152 | 633.217881 | 654.893828 | 653.291336 | 658.183613 | 645.25633 | 656.060597 |
| 7 | 1109.02123 | 1263.038487 | 889.036574 | 944.90319 | 915.153525 | 1047.43879 | 989.957862 | 1186.248741 |
| 8 | 1139.278925 | 1175.893113 | 1118.339103 | 1144.604745 | 1092.62639 | 1159.032351 | 1100.476077 | 1168.529946 |
| 9 | 22196.387817 | 29218.775982 | 11958.280061 | 13174.66236 | 24753.040541 | 32233.95451 | 10251.4763 1 | 14752.7168 |
| 10 | 6228.49289 | 7289.183679 | 6054.707691 | 6833.306317 | 6207.795302 | 7055.595231 | 6050.434374 | 6609.804567 |
| 11 | 1170.858603 | 1258.517635 | 1202.694857 | 1232.204268 | 1206.154564 | 1252.939541 | 1156.439606 | 1205.254497 |
| 12 | 677263.0799 | 16987989.981 | 74784.6159 | 530814.6481 | 584300.6983 | 3448448.7906 | 126908.2157 | 494471.0756 |
| 13 | 6005.535308 | 16893.949921 | 2041.488125 | 4332.5945 | 1572.252973 | 4301.829606 | 1484.761799 | 7760.056137 |
| 14 | 38490.532315 | 174367.45065 | 2466.047056 | 238838.470051 | 16327.42317 | 67939.000264 | 2967.818485 | 26290.31618 |
| 15 | 2278.141229 | 26989.255509 | 13553.041864 | 25636.769611 | 3443.587343 | 9167.267098 | 1938.200405 | 14976.72189 |
| 16 | 2722.026011 | 3176.916902 | 2345.400708 | 2916.561016 | 2521.93881 | 3146.04527 | 2436.449338 | 2978.37746 |
| 17 | 2799.949776 | 3289.61565 | 2568.383575 | 2907.869272 | 2887.281107 | 3236.957928 | 2561.370306 | 2874.965038 |
| 18 | 264037.12570 | 872072.47739 | 36176.58677 | 113941.3176 | 26965.28512 | 114846.121366 | 260540.781819 | |
| 19 | 10051.912407 | 20380.25713 | 2089.172253 | 7763.17234 | 9905.850822 | 16555.756926 | 2013.126904 | 3609.258962 |
| 20 | 2950.923195 | 3274.334015 | 3041.81309 | 3113.289461 | 2991.589293 | 3361.823946 | 2495.031774 | 3080.137478 |
| 21 | 2596.725663 | 2689.688363 | 2526.190898 | 2597.677199 | 2555.8788 | 2642.381597 | 2447.758274 | 2570.911014 |
| 22 | 9713.993241 | 10803.653732 | 10759.59674 | 11032.880953 | 8918.436264 | 10465.022457 | 8181.446081 | 9755.070369 |
| 23 | 3451.104943 | 4200.174424 | 2971.160647 | 3237.778662 | 2977.554961 | 3490.639751 | 2851.650254 | 3162.313622 |
| 24 | 3434.465028 | 3682.846708 | 3103.955173 | 3185.382676 | 3036.799607 | 3158.330504 | 3136.927747 | 3284.656095 |
| 25 | 3141.144886 | 3292.303449 | 2931.162959 | 2962.471758 | 2931.926959 | 3008.895353 | 2931.142314 | 2954.767839 |
| 26 | 4906.132848 | 7989.490966 | 2900.0 | 3346.874039 | 2900.441895 | 3653.757741 | 2900.0 | 3262.668498 |
| 27 | 3200.010703 | 3792.645588 | 3143.038057 | 3184.646353 | 3158.178238 | 3397.130323 | 3141.010872 | 3176.011524 |
| 28 | 3300.010827 | 3431.570911 | 3240.725865 | 3288.253039 | 3263.207144 | 3300.257609 | 3243.631996 | 3294.373237 |
| 29 | 3812.475517 | 4605.349537 | 3533.945743 | 3956.835243 | 3955.324537 | 4364.18129 | 3653.675553 | 3966.471956 |
| 30 | 3673.711968 | 5813.173755 | 3916.725719 | 4869.089335 | 3730.309354 | 5143.078706 | 3346.483679 | 4747.88675 |
| # | 0/30 | 0/30 | 9/30 | 8/30 | 4/30 | 4/30 | 17/30 | 18/30 |

Table 5: Objective Function Value for Dimension: 100

| ID | DE | | JADE | | PSO-DE | | Ours | |
|----|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | best | mean | best | mean | best | mean | best | mean |
| 1 | 3427212811.79 | 13807281895.7 | 141.263356 | 13516.698933 | 6067123.52108 | 29751976.5091 | 122.398748 | 11708.82360 |
| 2 | 4.19617e+84 | 1.54741e+112 | 8.73752e+74 | 2.54362e+87 | 6.1536e+66 | 3.2118e+73 | 3.8835e+80 | 8.8914e+114 |
| 3 | 208808.969094 | 242699.687639 | 312244.360944 | 332179.290693 | 241427.723667 | 257462.977885 | 220765.0838 | 251901.1093 |
| 4 | 1975.651157 | 2752.246068 | 539.386275 | 677.054657 | 777.314462 | 836.965399 | 531.169819 | 621.219143 |
| 5 | 1223.536503 | 1286.153332 | 1249.195036 | 1307.110127 | 1248.410134 | 1310.887657 | 1068.11742 | 1272.47682 |
| 6 | 651.650133 | 657.84974 | 654.709342 | 659.421427 | 656.877048 | 662.318417 | 642.33355 | 654.132758 |
| 7 | 1614.003864 | 1920.797726 | 1367.066537 | 1536.357878 | 1311.849757 | 1534.207764 | 1562.379772 | 2076.702502 |
| 8 | 1595.418732 | 1736.367379 | 1672.567849 | 1768.082435 | 1678.127263 | 1761.94051 | 1293.552115 | 1592.162983 |
| 9 | 59726.514621 | 71986.043905 | 28906.90908 | 30336.745335 | 63640.331351 | 74961.220998 | 23466.57501 | 27067.02959 |
| 10 | 12005.889721 | 14725.348334 | 14227.801909 | 15355.621891 | 12937.027857 | 14972.950738 | 11153.58683 | 13298.09210 |
| 11 | 7540.617987 | 11481.260145 | 40447.548688 | 57228.683666 | 3521.901521 | 4544.804011 | 5380.432052 | 9916.347692 |
| 12 | 529993877.325 | 1881773956.29 | 2893556.27222 | 6415173.6097 | 26105108.937 | 41876679.0862 | 3680108.181 | 10059039.63 |
| 13 | 7943.9249 | 508209.562668 | 4622.698553 | 8892.775994 | 8246.515295 | 12675.845535 | 2976.841354 | 11376.986338 |
| 14 | 728122.833253 | 1329183.17224 | 132194.7952 | 365560.8816 | 548410.338286 | 941547.524763 | 234045.940166 | 867160.306892 |
| 15 | 2660.465784 | 181957.060133 | 1799.506503 | 3362.509604 | 1899.073444 | 2914.44348 | 1976.789124 | 4485.415275 |
| 16 | 4749.254663 | 5847.826738 | 4817.483738 | 5632.3022 | 3852.700054 | 5228.663526 | 3519.494945 | 4796.802728 |
| 17 | 4397.496352 | 4958.418182 | 3842.206015 | 4450.177422 | 3790.72056 | 4730.994585 | 3582.785882 | 5463.216947 |
| 18 | 1357845.39305 | 1938893.27972 | 146426.2736 | 763318.8226 | 1004224.20385 | 2315010.29868 | 631040.14635 | 1335739.59138 |
| 19 | 2482.170159 | 26455.706954 | 2098.9496 | 4767.529535 | 2263.725158 | 3927.459947 | 2071.077067 | 3664.159878 |
| 20 | 4968.497438 | 5436.604051 | 5231.026486 | 5690.748998 | 5109.460563 | 5781.300835 | 3627.777893 | 5228.430669 |
| 21 | 3180.746656 | 3355.4783 | 2921.900122 | 3085.692252 | 2885.574085 | 3127.356835 | 2926.350399 | 3199.986183 |
| 22 | 17808.897744 | 19562.986646 | 19213.375668 | 20278.929093 | 18695.522312 | 20167.413741 | 17548.33905 | 19547.15124 |
| 23 | 4907.519646 | 5819.207866 | 3352.556985 | 4222.436894 | 3582.043556 | 4779.921248 | 3418.983204 | 3609.098575 |
| 24 | 5173.249408 | 5946.12042 | 4060.951302 | 4095.429519 | 3801.368588 | 4042.426859 | 3998.054028 | 4216.824895 |
| 25 | 4089.118918 | 4548.285768 | 3153.485413 | 3236.61784 | 3348.382262 | 3407.526581 | 3176.3038 | 3264.318532 |
| 26 | 8557.498566 | 20159.11458 | 2900.077371 | 11924.799473 | 3021.136025 | 4682.035439 | 2900.000382 | 9867.5518 |
| 27 | 3200.023355 | 3772.409153 | 3194.809213 | 3201.670732 | 3200.024171 | 3494.618132 | 3200.023542 | 3200.023953 |
| 28 | 4947.745152 | 5948.213156 | 3295.122914 | 3340.280383 | 3456.828432 | 3542.571307 | 3300.807691 | 3354.717338 |
| 29 | 6004.774424 | 7090.642544 | 5208.711727 | 5970.628689 | 5462.328635 | 6178.559061 | 4541.195471 | 5739.291549 |
| 30 | 7798.106217 | 202435555.594 | 3584.974771 | 10674.217331 | 3920.327039 | 7139.460728 | 3850.317099 | 15318.554601 |
| # | 0/30 | 0/30 | 7/30 | 8/30 | 6/30 | 4/30 | 17/30 | 18/30 |