Homework # 3

Due Wednesday 3/7

Turn in R code where applicable.

1. *Transforming vs. Linking:* Consider two GLMs both assuming normality: one with a log-transformed dependent variable Y and the other with a log link. State the three GLM components (random, systematic, linking function) of these two models. Explain what the regression model parameters β describe in the two models.

2. *Program your own Newton-Raphson Algorithm!*

   (a) State the likelihood, score function and Hessian matrix for the binomial distribution.

   (b) Program (in R) the Newton-Raphson algorithm to maximize the likelihood for a binomial random variable. Turn in your code.

   (c) For $\hat{\pi} = .3$ based on $n = 10$, show results of the first 6 iterations for 9 different starting values: .1, .2, .3, ..., .9.

   (d) Summarize the effects of the starting value on the speed of convergence.

   (e) Repeat part (b) for $\hat{\pi} = 0$ and $\hat{\pi} = 1$. What happens? You do not need to submit results, just describe in words.

3. *Fisher Scoring vs. Newton-Raphson Algorithms*

   (a) For n independent observations from a Poisson distribution, show that Fisher Scoring gives $\mu^{(t+1)} = \bar{y}$ for all $t > 0$. By contrast, what happens with the Newton-Raphson algorithm?

   (b) For noncanonical links in a GLM, show that the observed information matrix depends on the data and hence differs from the expected information. Relate this result to the Fisher Scoring vs. Newton-Raphson algorithms.

4. *Negative Binomial GLM*: The negative binomial probability mass function is often written as:

$$f(Y = y) = \frac{\Gamma(y + 1/\gamma)}{\Gamma(y+1)\Gamma(1/\gamma)} \left(\frac{1}{1+\gamma\mu}\right)^{1/\gamma} \left(\frac{\gamma\mu}{1+\gamma\mu}\right)^{y}, y = 0, 1, 2, \ldots$$

   where γ is a dispersion parameter.

   (a) Arrange this distribution in an exponential family form (Equation 4.17 in Agresti) and identify all the relevant components. Assume the dispersion parameter γ is known.

   (b) What is the canonical link and variance function for the negative binomial GLM with known γ?

   (c) Using the components of the exponential family form, find the expectation and variance of Y.

   (d) Using the components of the exponential family form, find the likelihood equations.

   (e) Instead of the canonical link, now assume a *log* link. Find the likelihood equations.

   (f) Why do you think the *log* link is commonly used instead of the canonical link for the negative binomial GLM?

5. An experiment analyzes number of seizures for 20 patients. For ten patients who received an anti-seizure drug (Treatment A), the number of seizures were $8, 7, 6, 6, 3, 4, 7, 2, 3, 4$. For ten patients who did not receive an anti-seizure drug (Treatment B), the number of seizures were $9, 9, 8, 14, 8, 13, 11, 5, 7, 6$. .

   (a) Treat the counts as independent Poisson random variables with means μ_A and μ_B. Using the `glm` function in R, fit the Poisson loglinear model $\log \mu = \alpha + \beta x$, where $x = 1$ for Treatment B and $x = 0$ for Treatment A. Show that $exp(\beta) = \mu_B/\mu_A$ and interpret the estimate.

   (b) Test $H_0 : \mu_A = \mu_B$ using the Wald method. State the null and alternative hypothesis, test statistic and conclusion.

   (c) Is there evidence of overdispersion for this Poisson model? Explain.

   (d) Show that for any link function g (where $g(\mu_i) = \alpha + \beta x_i$) used with the Poisson distribution, the GLM likelihood equations imply that the fitted means $\hat{\mu}_A$ and $\hat{\mu}_B$ equal the sample means.

   (e) Now fit the negative binomial GLM with a *log* link using the `glm.nb` function in R. What is the estimated dispersion parameter? Interpret. *We did not go through an R example of the negative binomial regression in class, however there are many examples online such as: stats.idre.ucla.edu/r/dae/negative-binomial-regression/*

   (f) Compare the coefficient estimate $\hat{\beta}$ and its standard error for the Poisson and negative binomial GLM.

   (g) Now fit the Poisson loglinear and negative binomial loglinear model without the covariate x (i.e. the intercept only model). Compare results for the overall mean response (i.e. the intercept). Why do they differ?

   (h) Is there evidence of overdispersion for the intercept-only Poisson model? Explain.

Group Project Topic Form

Please list the topics your group is interested in presenting in rank order. You may only choose from the list of topics posted on Blackboard, unless you have consulted with me before turning in this form.

Only turn in one form per group.

Group Number:

Group Member Names:

Topics (Book Section and Title):

1.

2.

3.