Bayesian Inference for two-way contingency table

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* Recall 2×2 contingency table.

		Y: 1	2	
X	1	n_{11}	n_{12}	n_{1+}
	2	n_{21}	n_{22}	n_{2+}

$$\hat{\pi}_1 = \frac{n_{11}}{n_{1+}} \qquad \hat{\pi}_2 = \frac{n_{21}}{n_{2+}}$$

"Freqnencist"

* Bayesian inference for contingency table. (After Adjustment)

	B	
A	y_1	n_1-y_1
	y_2	n_2-y_2

$y_1 \sim bin(n_1, \pi_1)$

$y_2 \sim bin(n_2, \pi_2)$

— Prior distribution of π_i, $i=1,2$ ☆ : π_i is random variable. not fixed.

$\pi_1 \sim beta(a,b)$. $\pi_2 \sim beta(c,d)$,

— deviation : posterior distribution of $\pi_i | Y_i$

① bayes formula: $f(\pi|Y) = \dfrac{f(Y|\pi)\cdot f(\pi)}{f(Y)}$

$\Rightarrow f(\pi|Y)\cdot f(Y) = f(Y|\pi)\cdot f(\pi)$

② e.g. $f(\pi_1|Y_1)$

$$f(Y_1|\pi_1)\cdot f(\pi_1) = \binom{n_1}{Y_1}\pi_1^{Y_1}(1-\pi_1)^{n_1-Y_1}\,\frac{1}{B(a,b)}\,\pi_1^{a-1}(1-\pi_1)^{b-1}$$

$$= \binom{n}{Y_1}\pi_1^{a+Y_1-1}(1-\pi_1)^{n_1-Y_1+b-1}\cdot\frac{1}{B(a,b)}$$

$$= \underbrace{\binom{n}{Y_1}\frac{B(a+Y_1, n_1-Y_1+b)}{B(a,b)}}_{f(Y_1)}\cdot\underbrace{\frac{\pi_1^{a+Y_1-1}(1-\pi_1)^{n_1-Y_1+b-1}}{B(a+Y_1, n_1-Y_1+b)}}_{f(\pi_1|Y_1)}$$

Thus, $\pi_1|Y_1 \sim beta(a+Y_1, n_1-Y_1+b)$

Similarly, $\pi_2|Y_2 \sim beta(c+Y_2, n_2-Y_2+d)$

* Hypothesis test

- generally $H_0: \pi_1 \le \pi_2$. $H_a: \pi_1 > \pi_2$

- Bayesian p-value: $p(\pi_1 \le \pi_2 \mid y_1, y_2)$

- calculate p-value:

  - Howard (1998)

* Interval of association parameters

— Recall: association parameters

① difference of proportion: $\pi_1 - \pi_2$

② Relative Risk: $\frac{\pi_1}{\pi_2}$

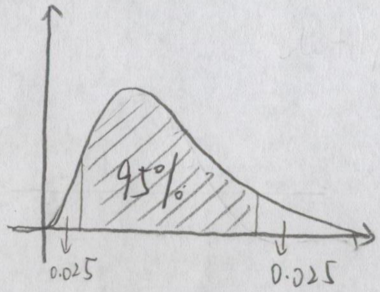
③ odds Ratio: $\frac{\pi_1/(1-\pi_1)}{\pi_2/(1-\pi_2)}$

— Definition

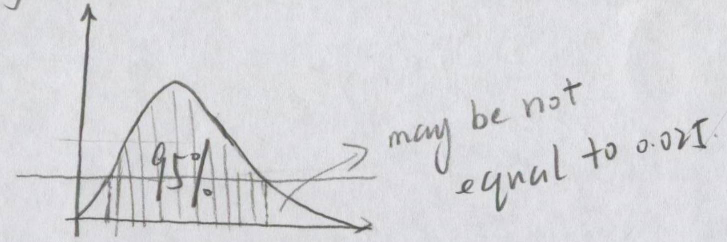
① credible interval: a range of values within which an unobserved parameter value falls with a particular subjective probability. It is an interval in the domain of a posterior probability distribution.

② equal-tail interval: choosing the interval where the probability of being below the interval is as likely as being above it.

③ highest posterior desity (HPD) interval: it's the narrowest interval, which for a unimodal distribution will involve choosing those values of highest probability density including the mode.



95% equal-tail interval



95% HPD interval

— Equal-tail interval

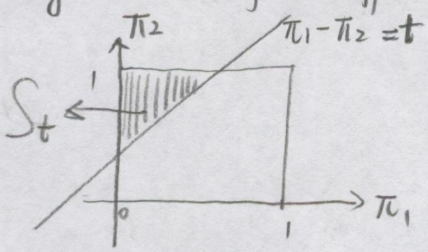
- w: association parameter
- $F_w(t)$: cdf of w
- $f(\pi_i|y_i)$: posterior density of π_i
- $F_w(t) = \iint_{S_t} f(\pi_1|y_1) \cdot f(\pi_2|y_2)\, d\pi_1 d\pi_2$.

  where $S_t = \{(\pi_1, \pi_2): w \le t,\ 0 < \pi_1, \pi_2 < 1\}$

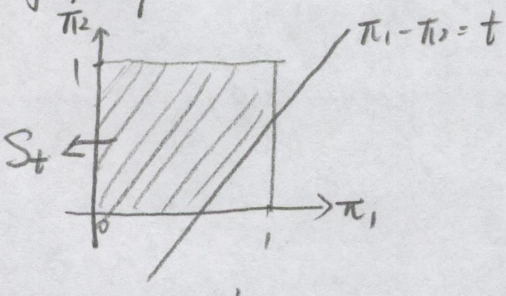
$\Rightarrow$ 95% equal-tail interval (U, L) satisfies.

$F_w(U) = 0.025$, $F_w(L) = 0.975$

e.g. S_t of different of proportion



$t<0$



$t>0$

— Highest posterior density interval

- application: difference of proportion
- disadvantage: can't not apply to relative risk, odds ration

  (not invariant under nonlinear transformation)

* Approach to estimate the prior distribution of π_i

- Empirical Bayes
- Hierarchical Bayesian Approach

* Frequentist VS Bayesian inference

content	frequentist	Bayesian
probability is	limiting relative frequency	degree of belief
parameter π is a	fixed constant	random variable.