```
> rm(list = ls())
> setwd("E:/Data of R")
> #Question 2
> library(VGAM)
> data2=read.csv("GSS.csv",header = T)
> names(data2)
[1] "gender"
                 "race"
                              "democrat"
                                            "republican" "independent"
> data2
 gender race democrat republican independent
   male white
                   132
                             176
                                        127
1
   male black
                   42
                              6
                                        12
3 female white
                   172
                             129
                                        130
4 female black
                    56
                              4
                                        15
> #(a)
> model21=vglm(cbind(democrat,republican,independent)~gender+race,data =
data2,
             family = multinomial)
> model21
call:
vglm(formula = cbind(democrat, republican, independent) ~ gender +
   race, family = multinomial, data = data2)
Coefficients:
(Intercept):1 (Intercept):2 gendermale:1 gendermale:2
                                                          racewhite:1
                                                        -1.1182884
               -1.1771027 -0.2201865
                                            0.3525732
   1.3882465
 racewhite:2
   1.1598459
Degrees of Freedom: 8 Total; 2 Residual
Residual deviance: 0.1982117
Log-likelihood: -20.17843
This is a multinomial logit model with 3 levels
> summary(model21)
call:
vglm(formula = cbind(democrat, republican, independent) ~ gender +
   race, family = multinomial, data = data2)
Pearson residuals:
 \log(mu[,1]/mu[,3]) \log(mu[,2]/mu[,3])
```

```
1
          -0.07696
                           -0.05743
2
          0.19896
                            0.22498
3
           0.07201
                            0.05961
4
          -0.18480
                           -0.23505
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
               1.3882
                          0.2296 6.045 1.49e-09 ***
(Intercept):1
(Intercept):2 -1.1771
                          0.3807 -3.092 0.00199 **
                          0.1583 -1.391 0.16412
gendermale:1 -0.2202
                         0.1651 2.136 0.03271 *
gendermale:2
              0.3526
racewhite:1
              -1.1183
                         0.2335 -4.789 1.68e-06 ***
racewhite:2
                         0.3801 3.051 0.00228 **
               1.1598
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Number of linear predictors: 2
Names of linear predictors: log(mu[,1]/mu[,3]), log(mu[,2]/mu[,3])
Residual deviance: 0.1982 on 2 degrees of freedom
Log-likelihood: -20.1784 on 2 degrees of freedom
Number of iterations: 3
No Hauck-Donner effect found in any of the estimates
Reference group is level 3 of the response
> #(b)
> deviance(model21)
[1] 0.1982117
> qchisq(0.05,4*2-3*2,lower.tail = FALSE)
[1] 5.991465
> #0.1982117<5.991465
> #We can not reject the null/reduced model in favor of the saturated mode
٦,
> #indicating the model fit with main effects for all the predictors provi
des
> #a reasonable fit.
> #(c)
> coefficients(model21)[3]
gendermale:1
 -0.2201865
> 1/exp(coefficients(model21)[3])
```

gendermale:1

```
1.246309
> #The estimated coefficient for the Gender Male dummy in the Democrat vs.
Independent
> #is -0.2202. This means that men are 1.246309 times less likely choose D
emocrat
> #over Independent.
> coefficients(model21)[4]
gendermale:2
  0.3525732
> exp(coefficients(model21)[4])
gendermale:2
   1.422724
> #The estimated coefficient for the Gender Male dummy in the Republican v
s. Independent
> #is 0.3525732. This means that men are 1.422724 times more likely choos
e Republican
> #over Independent.
> #Gender effect is not significant overall. Gender has a statistically si
gnificant
> #effect when comparing Republican over Independent.
> \#(d)
> #Prob. being Independent for black females
> pi3=1/(1+exp(1.3882465)+exp(-1.1771027))
> pi3
[1] 0.1881118
> #Prob. being Democrat for black females
> pi1=exp(1.3882465)*pi3
> pi1
[1] 0.7539177
> #(e)
> #Intercept1 is >0, which means that pi1/pi3>1, pi1>pi3
> #Intercept2 is <0, which means that pi2/pi3<1, pi2<pi3</pre>
> #So,pi1>pi3>pi2
> #PiD_hat > PiI_hat > PiR_hat
> \#(f)
> coefficients(model21)
(Intercept):1 (Intercept):2 gendermale:1 gendermale:2
                                                           racewhite:1
   1.3882465
                -1.1771027
                             -0.2201865
                                            0.3525732
                                                         -1.1182884
 racewhite:2
   1.1598459
```

>

```
> c1=c(coefficients(model21)[1],coefficients(model21)[3],coefficients(mo
del21)[5])
> c1
(Intercept):1 gendermale:1
                             racewhite:1
   1.3882465
               -0.2201865
                             -1.1182884
> \#log(pi1/pi3)=1.3882465-0.2201865*male-1.1182884*white
> c2=c(coefficients(model21)[2],coefficients(model21)[4],coefficients(mo
del21)[6])
> c2
                             racewhite:2
(Intercept):2 gendermale:2
  -1.1771027
                0.3525732
                              1.1598459
> \#\log(pi2/pi3) = -1.1771027 + 0.3525732 * male + 1.159845 * white
> #log(pi1/pi3)-log(pi2/pi3)=log(pi1/pi2)=c1-c2
> c1-c2
(Intercept):1 gendermale:1
                             racewhite:1
   2.5653492
               -0.5727597
                             -2.2781343
> \#log=(PiD/PiR)=log(pi1/pi2)=2.5653492-0.5727597*male-2.2781343*white
>
> \#(q)
> model22=vglm(cbind(democrat,independent,republican)~gender+race,data =
data2,
            family = multinomial)
> summary(mode122)
call:
vglm(formula = cbind(democrat, independent, republican) ~ gender +
   race, family = multinomial, data = data2)
Pearson residuals:
 \log(mu[,1]/mu[,3]) \log(mu[,2]/mu[,3])
          -0.03865
                            0.08791
1
2
           0.03288
                           -0.29853
3
           0.03077
                           -0.08827
4
          -0.01045
                            0.29881
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
                          0.3437 7.465 8.35e-14 ***
(Intercept):1
               2.5653
(Intercept):2
               1.1771
                          0.3807 3.092 0.001986 **
                          0.1575 -3.636 0.000277 ***
gendermale:1
              -0.5728
gendermale:2
              -0.3526
                          0.1651 -2.136 0.032707 *
                          0.3428 -6.646 3.02e-11 ***
              -2.2781
racewhite:1
racewhite:2
                          0.3801 -3.051 0.002279 **
              -1.1598
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
```

```
Number of linear predictors: 2
Names of linear predictors: log(mu[,1]/mu[,3]), log(mu[,2]/mu[,3])
Residual deviance: 0.1982 on 2 degrees of freedom
Log-likelihood: -20.1784 on 2 degrees of freedom
Number of iterations: 3
No Hauck-Donner effect found in any of the estimates
Reference group is level 3 of the response
> coefficients(mode122)
(Intercept):1 (Intercept):2 gendermale:1 gendermale:2
                                                          racewhite:1
   2.5653492
                 1.1771027
                             -0.5727597
                                           -0.3525732
                                                         -2.2781343
 racewhite:2
  -1.1598459
> \#log=(PiD/PiR)=log(pi1/pi2)=2.5653492-0.5727597*male-2.2781343*white
> #It is the same as the result in part (f)
>
> \#(h)
> # Democrat vs. Independence
> d1=cbind(rep(1,259), rep(1,259), c(rep(1,132), rep(0,127)))
> d2 = cbind(rep(1,54), rep(0,54), c(rep(1,42), rep(0,12)))
> d3=cbind(rep(0,302),rep(1,302),c(rep(1,172),rep(0,130)))
> d4=cbind(rep(0,71),rep(0,71),c(rep(1,56),rep(0,15)))
> data21=rbind(d1,d2,d3,d4)
> colnames(data21)=c("gender","race","democrat")
> data21=data.frame(data21)
> logit1=glm(democrat~gender+race,data=data21,family=binomial(link = log
it))
> # Republican vs. Independence
> d1=cbind(rep(1,303),rep(1,303),c(rep(1,176),rep(0,127)))
> d2 = cbind(rep(1,18), rep(0,18), c(rep(1,6), rep(0,12)))
> d3=cbind(rep(0,259), rep(1,259), c(rep(1,129), rep(0,130)))
> d4=cbind(rep(0,19),rep(0,19),c(rep(1,4),rep(0,15)))
> data22=rbind(d1,d2,d3,d4)
> colnames(data22)=c("gender","race","republican")
> data22=data.frame(data22)
> logit2=glm(republican~gender+race,data=data22,family=binomial(link = 1
ogit))
> summary(logit1)
call:
glm(formula = democrat ~ gender + race, family = binomial(link = logit),
```

```
data = data21
Deviance Residuals:
   Min
            10
                Median
                            3Q
                                   Max
-1.7943 -1.1992 0.7357
                          1.0655
                                   1.1558
Coefficients:
          Estimate Std. Error z value Pr(>|z|)
(Intercept) 1.3866
                               6.040 1.54e-09 ***
                       0.2296
           -0.2181
                       0.1585 - 1.376
gender
                                        0.169
           -1.1175
                      0.2335 -4.785 1.71e-06 ***
race
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 930.60 on 685 degrees of freedom
Residual deviance: 902.32 on 683 degrees of freedom
AIC: 908.32
Number of Fisher Scoring iterations: 4
> summary(logit2)
call:
glm(formula = republican ~ gender + race, family = binomial(link = logit),
   data = data22
Deviance Residuals:
  Min
          10 Median
                         3Q
                               Max
-1.322 -1.171 1.040
                               1.697
                       1.040
Coefficients:
          Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.1698
                       0.3814 -3.067 0.00216 **
            0.3486
                      0.1660 2.099 0.03580 *
gender
            1.1543
                      0.3809
                              3.031 0.00244 **
race
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 828.79 on 598 degrees of freedom
Residual deviance: 813.75 on 596 degrees of freedom
AIC: 819.75
```

Number of Fisher Scoring iterations: 4

> summary(model21)

```
call:
vglm(formula = cbind(democrat, republican, independent) ~ gender +
   race, family = multinomial, data = data2)
Pearson residuals:
 log(mu[,1]/mu[,3]) log(mu[,2]/mu[,3])
1
          -0.07696
                           -0.05743
2
           0.19896
                            0.22498
3
           0.07201
                            0.05961
          -0.18480
                           -0.23505
4
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
                                  6.045 1.49e-09 ***
(Intercept):1
               1.3882
                          0.2296
(Intercept):2 -1.1771
                          0.3807 -3.092 0.00199 **
gendermale:1 -0.2202
                         0.1583 -1.391 0.16412
gendermale:2
              0.3526
                         0.1651 2.136 0.03271 *
racewhite:1
              -1.1183
                         0.2335 -4.789 1.68e-06 ***
                         0.3801 3.051 0.00228 **
racewhite:2
               1.1598
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Number of linear predictors: 2
Names of linear predictors: log(mu[,1]/mu[,3]), log(mu[,2]/mu[,3])
Residual deviance: 0.1982 on 2 degrees of freedom
Log-likelihood: -20.1784 on 2 degrees of freedom
Number of iterations: 3
No Hauck-Donner effect found in any of the estimates
Reference group is level 3 of the response
> #(i)
> #The coefficients and their statistical significance in two separate log
istic
```

- > #models are the same as the corresponding parts of the baseline category logit
- > #model. That is because the submodel of the baseline category logit is e xactly
- > #the logistic model. Taking first logistic model as a example, the sumat
- > #of y=0 is exactly the number of people who choose independent

```
> #Question 3
> library(gee)
> data3=read.csv("attitudes.csv", header = T)
> names(data3)
[1] "gender"
              "response" "question" "case"
> dim(data3)
[1] 5550
> #(a)
> model31=gee(response~gender+as.factor(question),id=case,family=binomia
            corstr="unstructured", scale.fix=T,data = data3)
Beginning Cgee S-function, @(#) geeformula.q 4.13 98/01/27
running glm to get initial regression estimate
       (Intercept)
                               gender as.factor(question)2
       0.023939537
                           0.003582051
                                              -0.097329124
as.factor(question)3
      -0.149347113
> summary(model31)
      GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
Model:
Link:
                        Logit
Variance to Mean Relation: Binomial
Correlation Structure:
                        Unstructured
call:
gee(formula = response ~ gender + as.factor(question), id = case,
   data = data3, family = binomial, corstr = "unstructured",
   scale.fix = T
Summary of Residuals:
     Min
                      Median
                10
                                    3Q
                                             Max
-0.5070710 -0.4827461 -0.4684470 0.5172539 0.5315530
Coefficients:
                     Estimate Naive S.E.
                                            Naive z Robust S.E.
                                                                  Robust z
(Intercept)
                    0.022967184 0.06773175 0.33909035 0.06778176 0.338
8402
                   0.005318694 0.08785139 0.06054194 0.08782143 0.0605
gender
626
as.factor(question)2 -0.097328985 0.02753097 -3.53525447 0.02753163 -3.5
as.factor(question)3 -0.149346974 0.02973943 -5.02184998 0.02973863 -5.0
219851
```

```
Estimated Scale Parameter:
Number of Iterations: 2
Working Correlation
        \lceil,1\rceil
                  [,2]
                           [,3]
[1,] 1.0000000 0.8248498 0.7958825
[2,] 0.8248498 1.0000000 0.8312594
[3,] 0.7958825 0.8312594 1.0000000
> #(b)
> model31$working.correlation
                  [,2]
                           Γ,37
         [,1]
[1,] 1.0000000 0.8248498 0.7958825
[2,] 0.8248498 1.0000000 0.8312594
[3,] 0.7958825 0.8312594 1.0000000
> #The working correlation indicates that in the same case, the correlation
between 1 and 2 is
> #0.8248498, the correlation between 1 and 3 is 0.7958825 and the correla
tion
> #between 2 and 3 is 0.8312594
> #These estimation is large (closing to 1), which indicates that accounti
ng
> #for clustering is necessary
> #(c)
> model32=gee(response~gender+as.factor(question),id=case,family=binomia
1,
            corstr="exchangeable", scale.fix=T,data = data3)
Beginning Cgee S-function, @(#) geeformula.g 4.13 98/01/27
running glm to get initial regression estimate
        (Intercept)
                                gender as.factor(question)2
       0.023939537
                            0.003582051
                                               -0.097329124
as.factor(question)3
       -0.149347113
> model31$working.correlation
                  [,2]
         \lceil,1\rceil
                           Γ,37
[1,] 1.0000000 0.8248498 0.7958825
[2,] 0.8248498 1.0000000 0.8312594
[3,] 0.7958825 0.8312594 1.0000000
> model32$working.correlation
         [,1]
                  [,2]
                           [,3]
[1,] 1.0000000 0.8173308 0.8173308
[2,] 0.8173308 1.0000000 0.8173308
[3,] 0.8173308 0.8173308 1.0000000
> #The working correlation matrix indicates that the correlation between o
bservations
```

```
> #within a subject is estiamted to be 0.8173308
> #Because we spwcified an exchangeable correlation structure, this corre
lation
> #is the same for all pairs in a group
> #0.8173308 is between 0.8248498 and 0.7958825 and it is also closed to e
ither
> #of them, which indicates that is reasonable to use exchangeable correla
> \#(d)
> summary(mode132)
GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
Model:
Link:
                         Logit
Variance to Mean Relation: Binomial
Correlation Structure: Exchangeable
call:
gee(formula = response ~ gender + as.factor(question), id = case,
   data = data3, family = binomial, corstr = "exchangeable",
   scale.fix = T)
Summary of Residuals:
     Min
                       Median
                1Q
                                     3Q
                                              Max
-0.5068644 -0.4825396 -0.4687095 0.5174604 0.5312905
Coefficients:
                     Estimate Naive S.E.
                                            Naive z Robust S.E.
                    0.024021377 0.06774107 0.35460579 0.06779334
(Intercept)
gender
                    0.003437873 0.08787462 0.03912248 0.08784072
as.factor(question)2 -0.097329120 0.02811568 -3.46173752 0.02753161
as.factor(question)3 -0.149347107 0.02813360 -5.30849707 0.02973865
                    Robust z
(Intercept)
                    0.35433237
gender
                   0.03913758
as.factor(question)2 -3.53517666
as.factor(question)3 -5.02198729
Estimated Scale Parameter:
Number of Iterations: 2
Working Correlation
                 [,2]
        \lceil,1\rceil
[1,] 1.0000000 0.8173308 0.8173308
```

```
[2,] 0.8173308 1.0000000 0.8173308
[3,] 0.8173308 0.8173308 1.0000000
> exp(coefficients(model32)[2])
 gender
1.003444
> #The estimated coefficient for the gender is 0.003437873
> #This means that females are 1.003444 times more likley to support legal
ized abortion
> qnorm(0.975)
[1] 1.959964
> #Naive z 0.03912248 < 1.959964</pre>
> #Can not reject HO
> #(e)
> 1b=0.003437873-qnorm(0.975)*0.08787462
> ub=0.003437873+qnorm(0.975)*0.08787462
> #the 95% CI for
                     is
> c(1b,ub)
[1] -0.1687932 0.1756690
> #the 95% CI for
> c(exp(lb),exp(ub))
[1] 0.8446836 1.1920434
> \#(f)
> #response=0.024021377+0.003437873*female-0.097329120*Q2-0.149347107*Q3
> #estimated odds of support for legalized abortion in scenario 2 for a ma
> odds=0.024021377-0.097329120
> odds
[1] -0.07330774
> \#(g)
> model33=gee(response~gender+question,id=case,family=binomial,
             corstr="exchangeable", scale.fix=T,data = data3)
Beginning Cgee S-function, @(#) geeformula.q 4.13 98/01/27
running glm to get initial regression estimate
(Intercept)
                  gender
                             question
0.091085560 0.003581948 -0.074688363
> summary(mode133)
GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
```

Model:

```
Link:
                        Logit
Variance to Mean Relation: Binomial
Correlation Structure:
                           Exchangeable |
call:
gee(formula = response ~ gender + question, id = case, data = data3,
   family = binomial, corstr = "exchangeable", scale.fix = T)
Summary of Residuals:
                      Median
     Min
                10
                                    3Q
                                             Max
-0.5049607 -0.4862922 -0.4668213 0.5137078 0.5331787
Coefficients:
             Estimate Naive S.E.
                                    Naive z Robust S.E. Robust z
(Intercept) 0.091155117 0.07149404 1.27500299 0.07221536 1.26226763
gender
           0.003376898 0.08787193 0.03842977 0.08783897 0.03844419
          -0.074688407 0.01406974 -5.30844210 0.01487266 -5.02186055
question
Estimated Scale Parameter:
Number of Iterations: 2
Working Correlation
        [,1]
                [,2]
                          [,3]
[1,] 1.0000000 0.8172912 0.8172912
[2,] 0.8172912 1.0000000 0.8172912
[3,] 0.8172912 0.8172912 1.0000000
> #Comparing to part(c)
> summary(mode132)
GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
Model:
Link:
                        Logit
Variance to Mean Relation: Binomial
Correlation Structure:
                           Exchangeable
Call:
gee(formula = response ~ gender + as.factor(question), id = case,
   data = data3, family = binomial, corstr = "exchangeable",
   scale.fix = T
Summary of Residuals:
     Min
                      Median
                10
                                    3Q
                                             Max
-0.5068644 -0.4825396 -0.4687095 0.5174604 0.5312905
```

```
Coefficients:
                     Estimate Naive S.E.
                                            Naive z Robust S.E.
                    0.024021377 0.06774107 0.35460579 0.06779334
(Intercept)
                   0.003437873 0.08787462 0.03912248 0.08784072
gender
as.factor(question)2 -0.097329120 0.02811568 -3.46173752 0.02753161
as.factor(question)3 -0.149347107 0.02813360 -5.30849707 0.02973865
                    Robust z
(Intercept)
                    0.35433237
gender
                   0.03913758
as.factor(question)2 -3.53517666
as.factor(question)3 -5.02198729
Estimated Scale Parameter:
Number of Iterations: 2
Working Correlation
        \lceil,1\rceil
                 Γ,2]
                          [,37
[1,] 1.0000000 0.8173308 0.8173308
[2,] 0.8173308 1.0000000 0.8173308
[3,] 0.8173308 0.8173308 1.0000000
> #(h)
> model34=glm(response~gender+as.factor(question),family=binomial,data=d
ata3)
> summary(model34)
call:
glm(formula = response ~ gender + as.factor(question), family = binomial,
   data = data3
Deviance Residuals:
          10 Median
                         3Q
  Min
                               Max
-1.189 -1.148 -1.125
                       1.207
                               1.231
Coefficients:
                  Estimate Std. Error z value Pr(>|z|)
(Intercept)
                    0.023940
                             0.055528
                                         0.431
                                                 0.6664
                   0.003582
                              0.054138
                                         0.066
                                                0.9472
as.factor(question)2 -0.097329  0.065783 -1.480
                                                   0.1390
as.factor(question)3 -0.149347  0.065825 -2.269  0.0233 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 7689.5 on 5549 degrees of freedom
Residual deviance: 7684.2 on 5546 degrees of freedom
AIC: 7692.2
```

```
Number of Fisher Scoring iterations: 3
> #Comparing to part(c)
> summary(model32)
GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
Model:
Link:
                        Logit
Variance to Mean Relation: Binomial
Correlation Structure: Exchangeable
call:
gee(formula = response ~ gender + as.factor(question), id = case,
   data = data3, family = binomial, corstr = "exchangeable",
   scale.fix = T
Summary of Residuals:
     Min
                      Median
                10
                                     3Q
                                             Max
-0.5068644 -0.4825396 -0.4687095 0.5174604 0.5312905
Coefficients:
                     Estimate Naive S.E.
                                            Naive z Robust S.E.
                    0.024021377 0.06774107 0.35460579 0.06779334
(Intercept)
gender
                   0.003437873 0.08787462 0.03912248 0.08784072
as.factor(question)2 -0.097329120 0.02811568 -3.46173752 0.02753161
as.factor(question)3 -0.149347107 0.02813360 -5.30849707 0.02973865
                    Robust z
                    0.35433237
(Intercept)
gender
                   0.03913758
as.factor(question)2 -3.53517666
as.factor(question)3 -5.02198729
Estimated Scale Parameter:
Number of Iterations: 2
Working Correlation
        [,1]
                 [,2]
[1,] 1.0000000 0.8173308 0.8173308
[2,] 0.8173308 1.0000000 0.8173308
[3,] 0.8173308 0.8173308 1.0000000
> #The coefficents estimates of these two models are the same
> #The standard error of exchangeable model is smaller than the standard e
> #of independence model
```

```
> #(i)
> #It is because that as for GEE, there is a large correlation within subject, so
> #standard error is smaller than GLM. However, using the same data, the total
> #standard errors are always the same. Therefore, GEE has larger between-subject
> #standard error.
> #standard error.
```