One-step logics, parity automata and μ-calculi sec:parityaut

This section introduces and studies the type of parity automata that will be used in the characterisation of and on tree models. In order to define these automata in a uniform way, we introduce, at a slightly higher level of abstraction, the notion of a one-step logic, a concept from coalgebraic modal logic cirs:modu04 which provides a nice framework for a general approach towards the theory of automata operating on infinite objects. As salient specimens of such one-step logics we will discuss monadic first-order logic with equality () and its extension with the infinity quantifier (). We then define, parametric in the language of such a one-step logic, the notions of an -automaton and of a mu-calculus μ, and we show how various classes of -automata effectively correspond to fragments of μ.

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definitiondef:one-step Given a finite set A of monadic predicates, a one-step model is a pair (D, V) consisting of a domain set D and a valuation or interpretation $V : A \to D$. Where $B \subseteq A$, we say that $V' : A \to D$ is a B-extension of $V : A \to D$, notation $V \leq_B V'$, if $V(b) \subseteq V'(b)$ for every $b \in B$ and $V(a) = V'(a)$ for every $a \in A \setminus B$.

A one-step language is a map assigning blueto any set A a collection (A) of objects bluethat we will refer to as one-step formulas over A. We assume that one-step languages come with a truth relation $\models$ between one-step formulas and models, writing $(D, V) \models \phi$ to denote that (D, V) satisfies ϕ.