Confluence of Graph Rewriting with Interfaces

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EUROPEAN JOINT CONFERENCES ON THEORY & PRACTICE OF SOFTWARE

Joint work with

Filippo
Bonchi

Fabio
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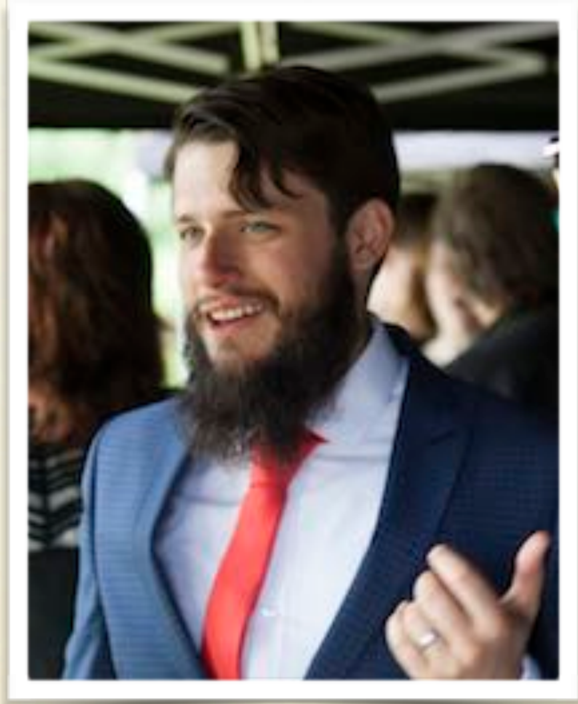
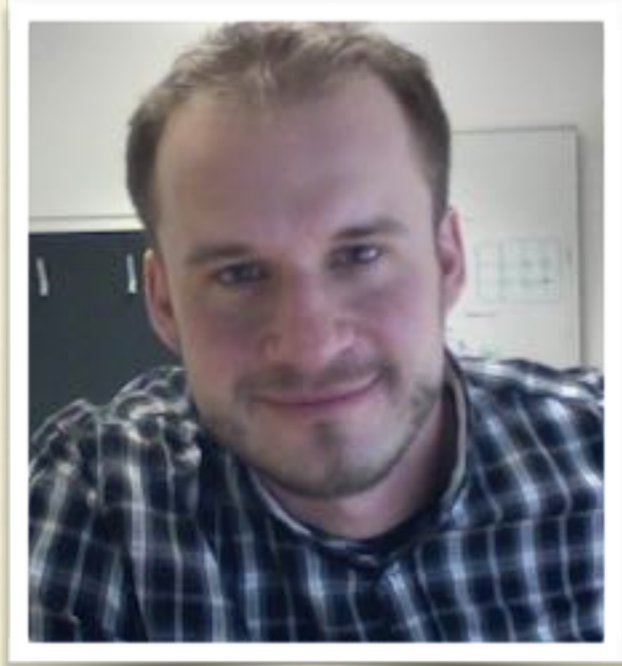


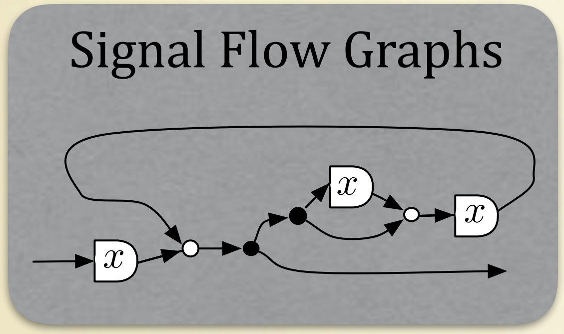
Aleks
Kissinger

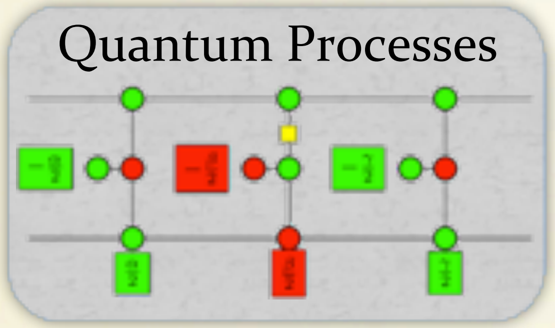


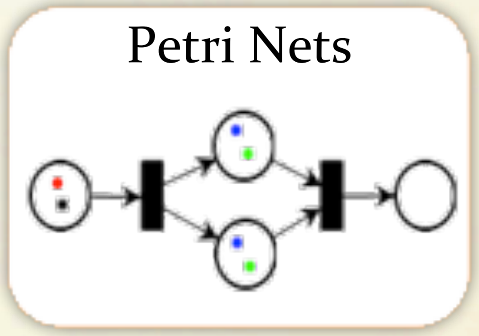
Pawel
Sobocinski

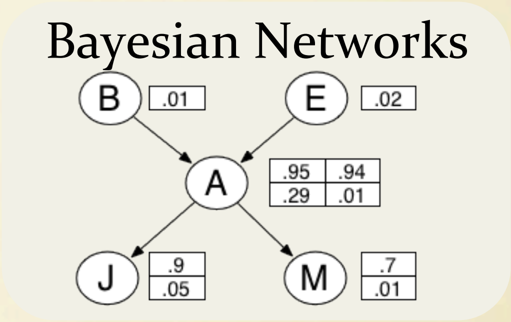
Context and Motivation

The algebraic approach to network diagrams





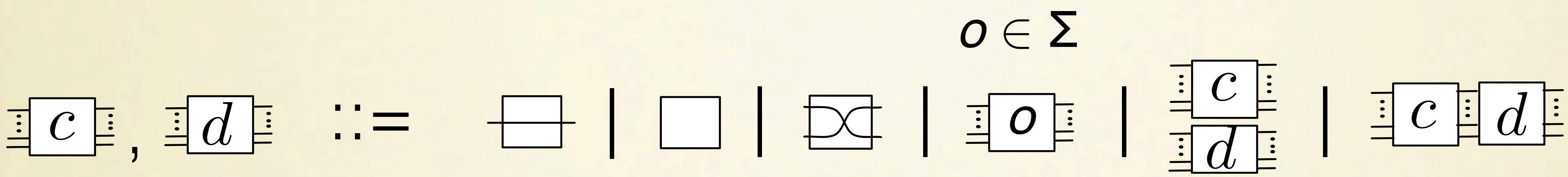




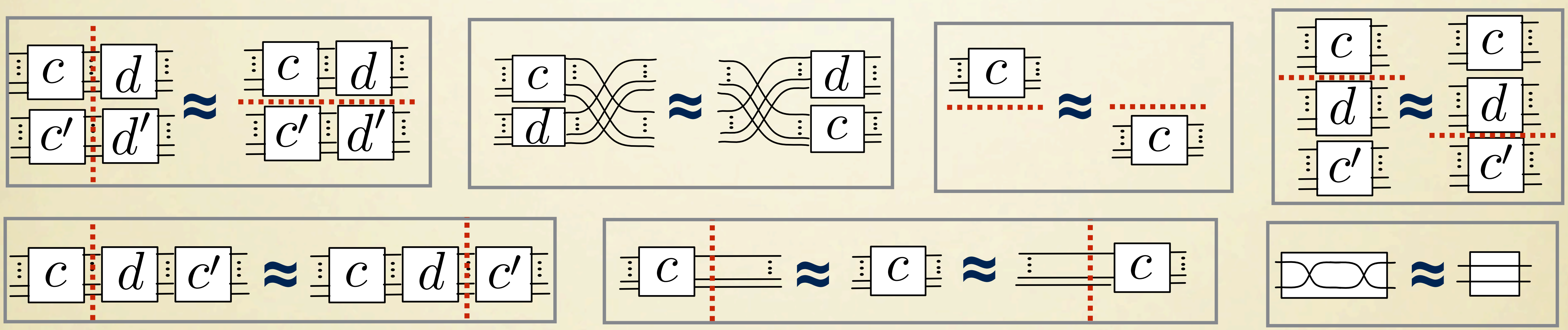
Diagrammatic languages studied using the compositional methods of programming language theory.

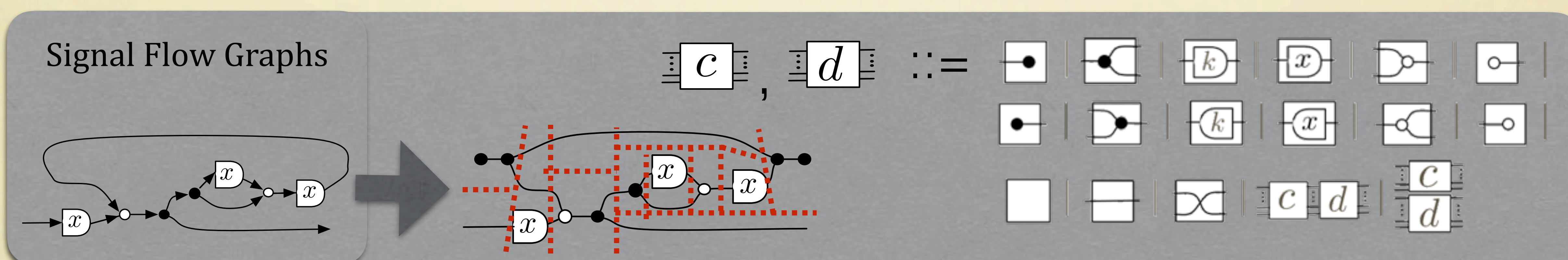
Syntax + **Equations**

Graphical syntax



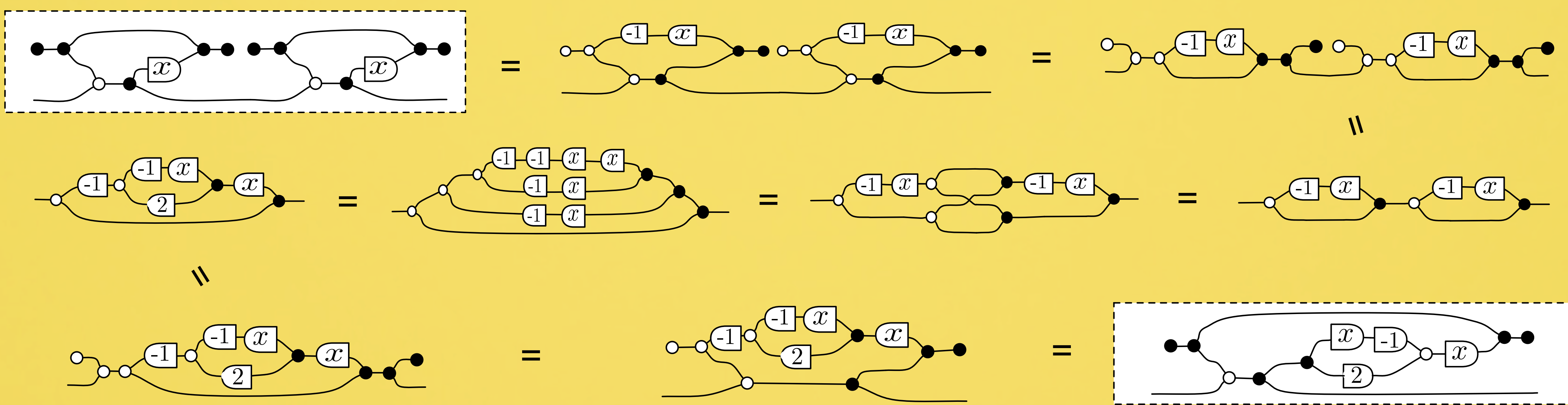
Quotiented by the *laws of symmetric monoidal categories*:



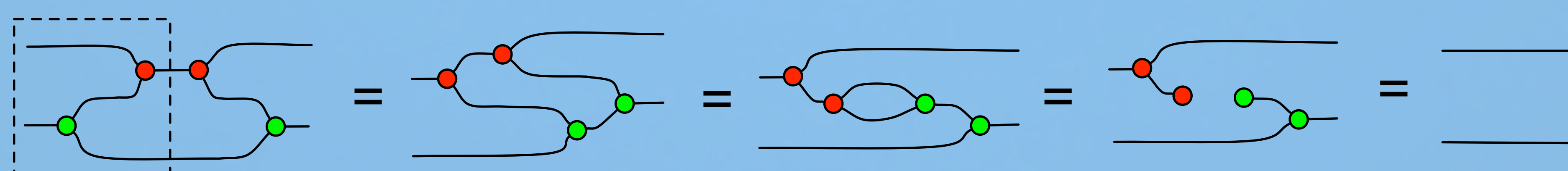


Equational theories of diagrams

Signal flow graphs: circuit equivalence



Quantum processes: the CNOT gate is unitary

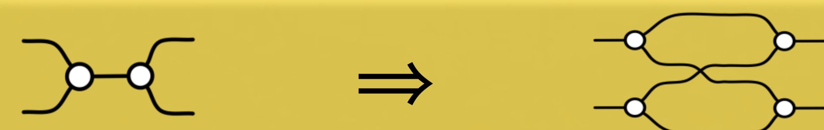


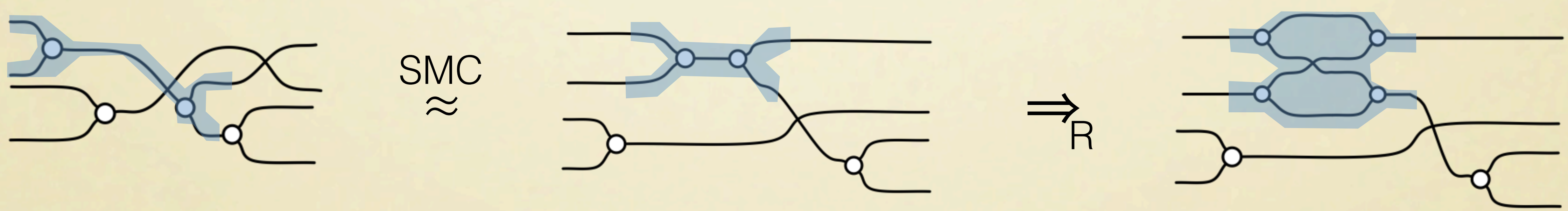
Bayesian reasoning: disintegration

Perspective of this work

Syntax + **Equations**

Modulo the laws of SMCs — **Diagram rewriting system**

(R) 

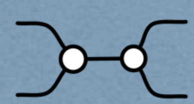


Solution: diagram rewriting is interpreted as graph rewriting with interfaces

Implementing Diagram Rewriting

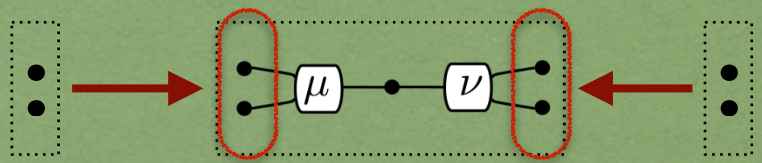
The graph interpretation (LICS'16)

Diagram



Faithful

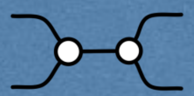
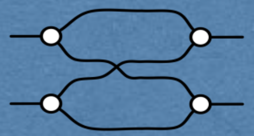
Hypergraph with interfaces

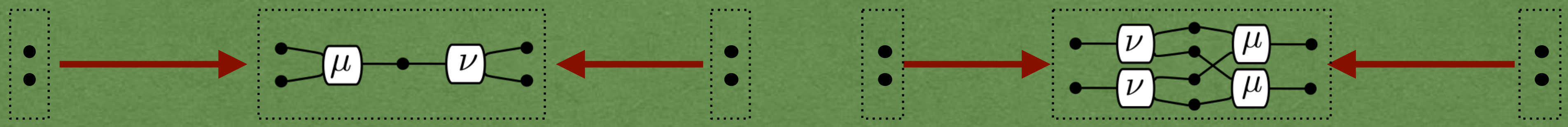


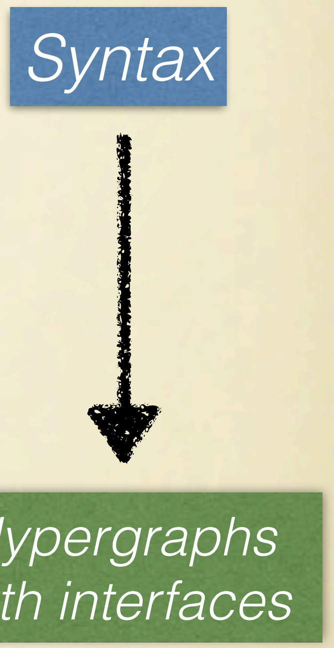
Rewriting modulo symmetric monoidal structure

Double pushout (DPO) rewriting of hypergraphs with interfaces

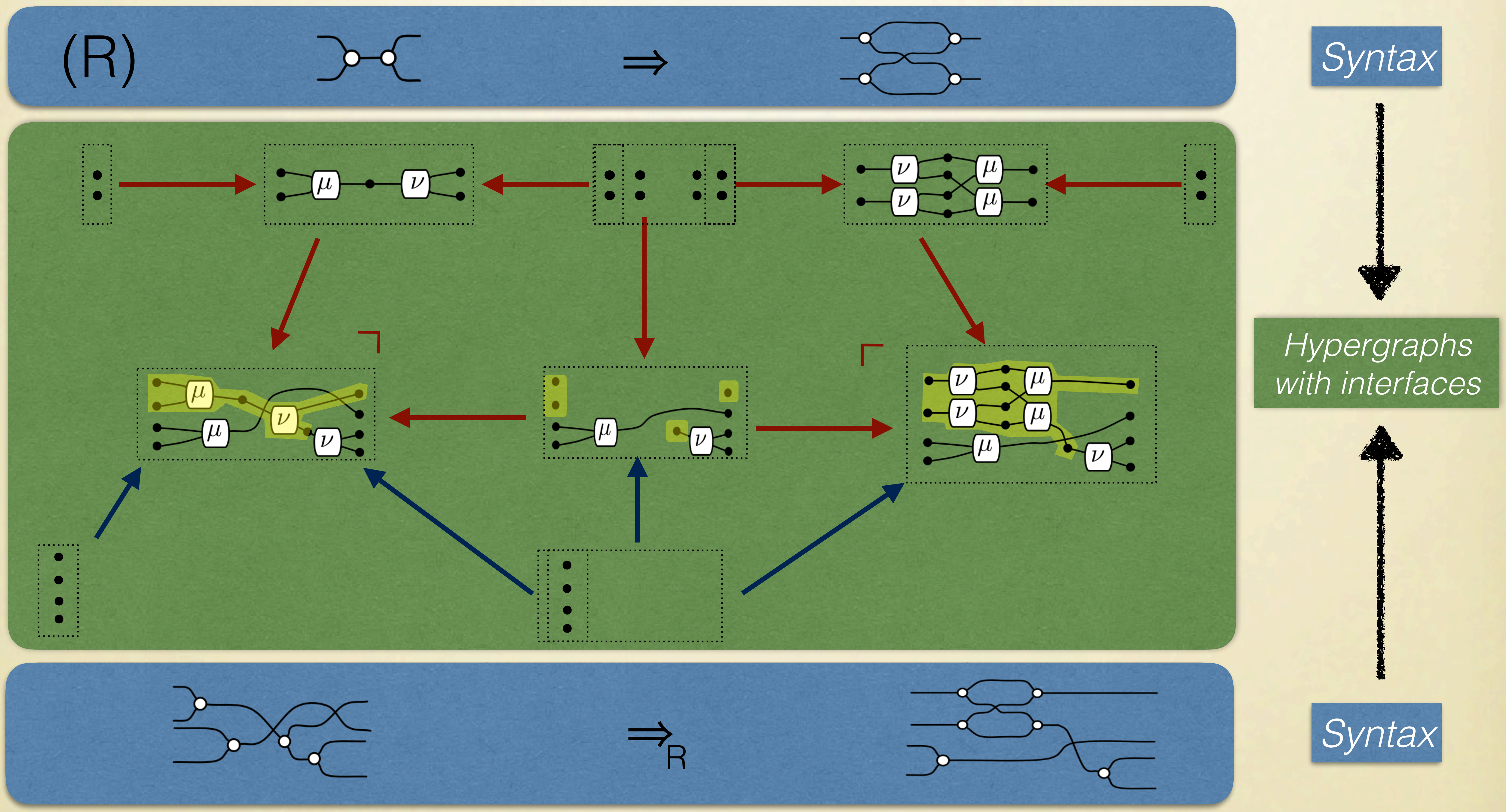
DPO rewriting with interfaces

(R)  $\Rightarrow$ 



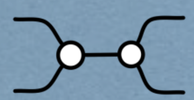


DPO rewriting with interfaces



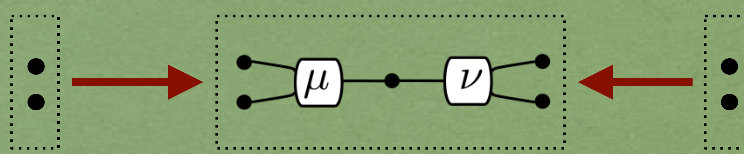
The graph interpretation (LICS’16)

Diagram



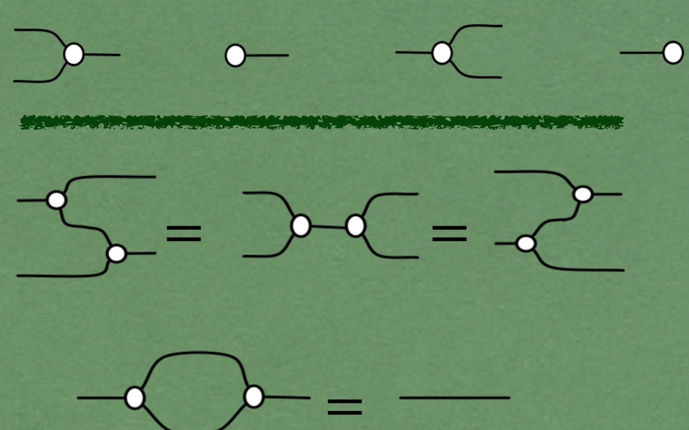
Faithful

Hypergraph with interfaces



Rewriting modulo symmetric monoidal structure

Rewriting modulo SM + Frobenius structure



Complete but unsound

Sound & Complete

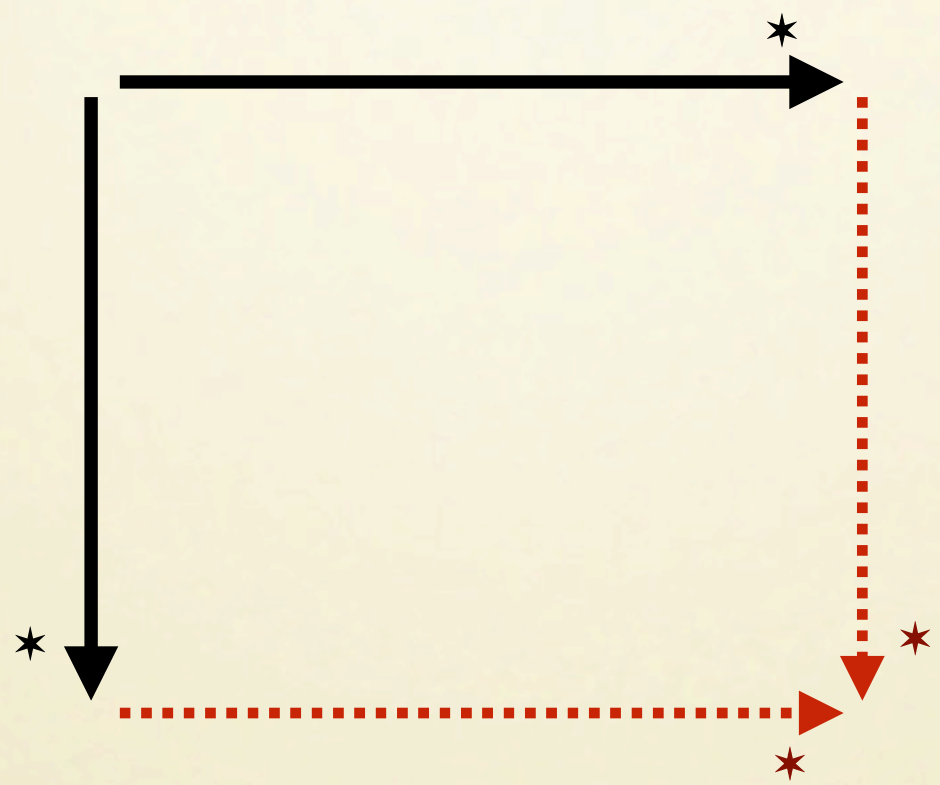
Sound & Complete

Double pushout (DPO) rewriting of hypergraphs with interfaces

<u>Convex</u> DPO rewriting

Confluence

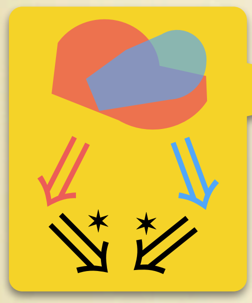
Confluence, abstractly



If E is confluent & terminating
then $x \overset{E}{=} y$ becomes decidable.

Decidability of Confluence

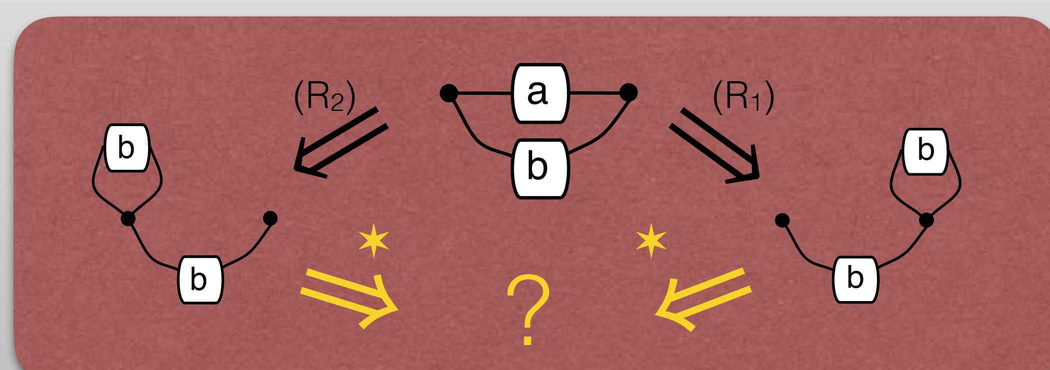
In term rewriting, confluence is **decidable** for terminating systems



All the critical pairs are joinable ⟺ The system is confluent (Knuth-Bendix)

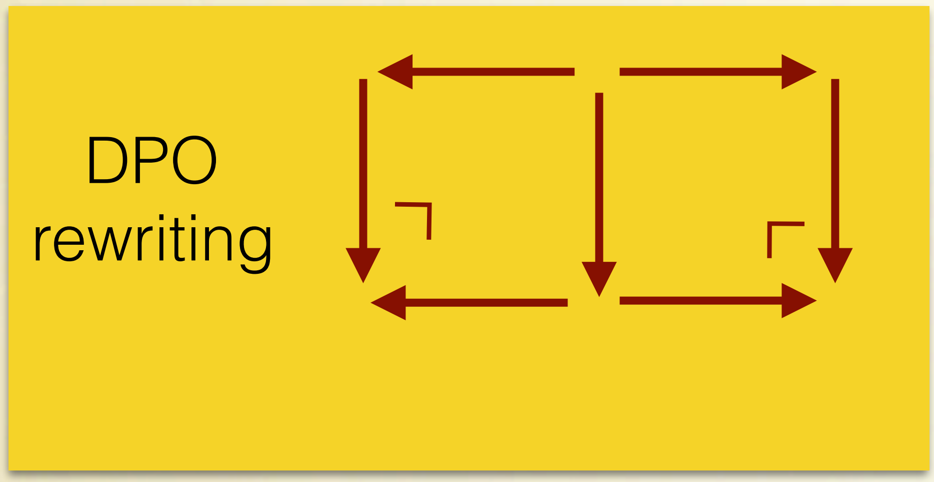
In DPO (hyper)graph rewriting, confluence is **undecidable** (Plump)

All the critical pairs are joinable…

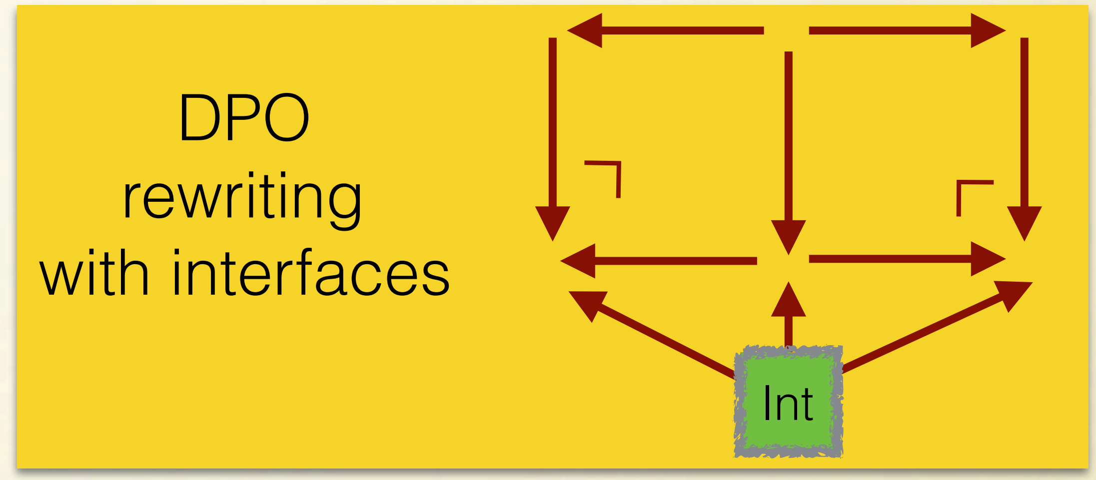


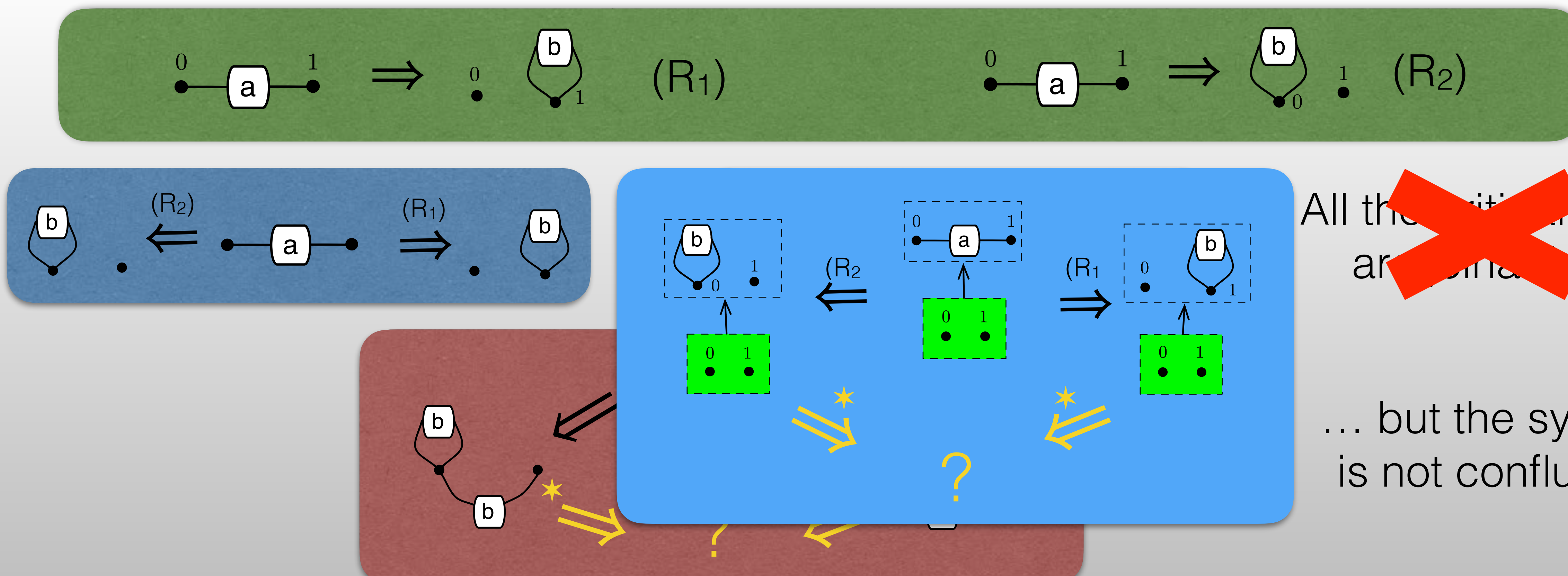
… but the system is not confluent.

Interfaces to the Rescue



VS



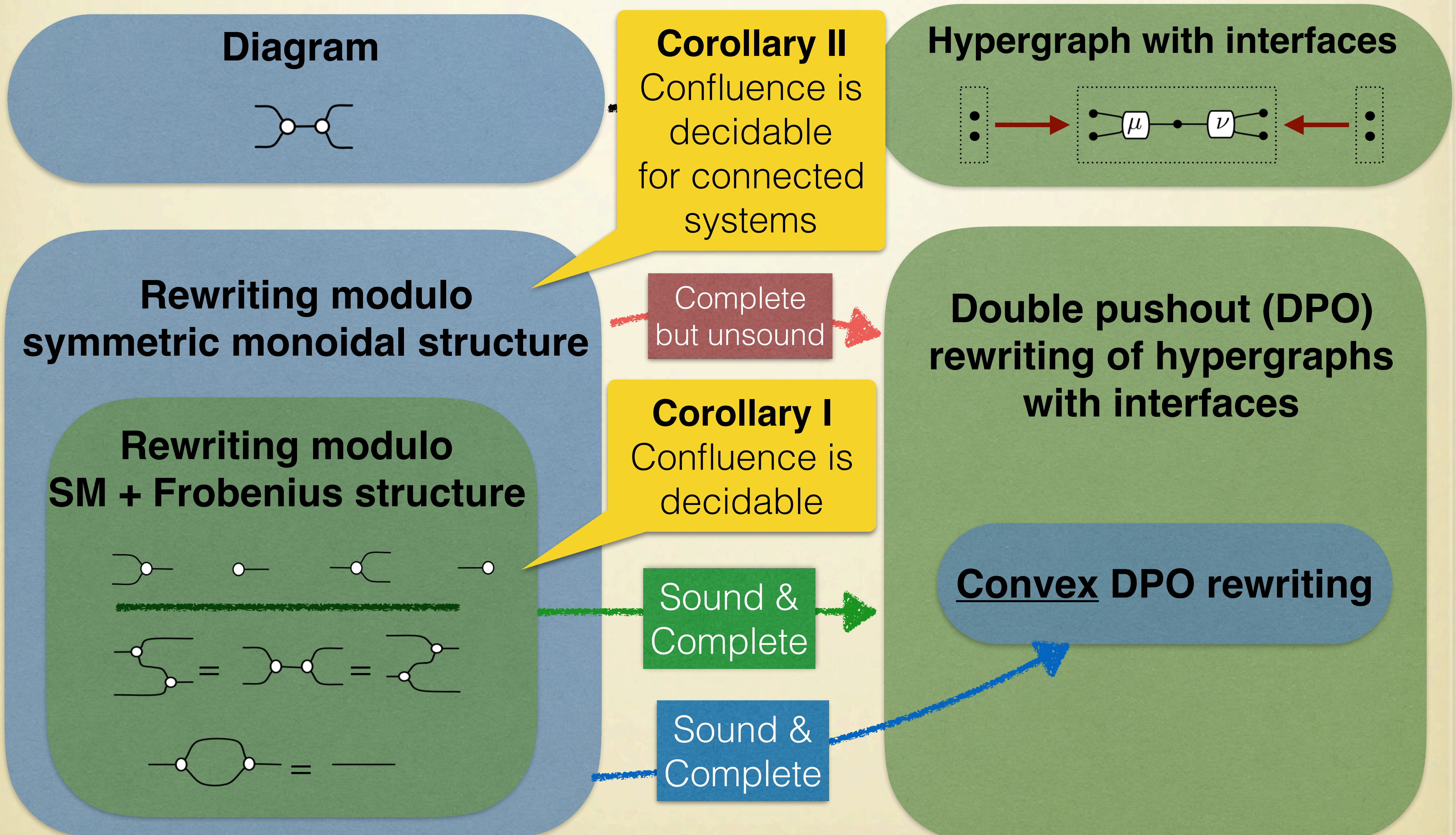


~~All th… pairs ar…e…~~

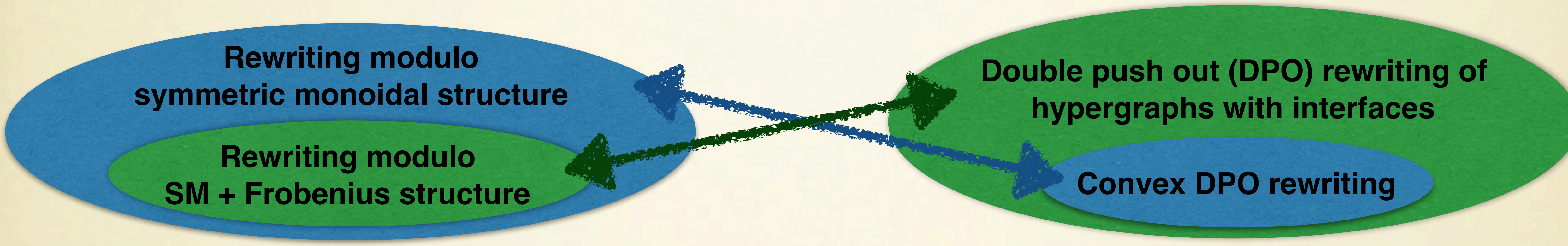
… but the system is not confluent.

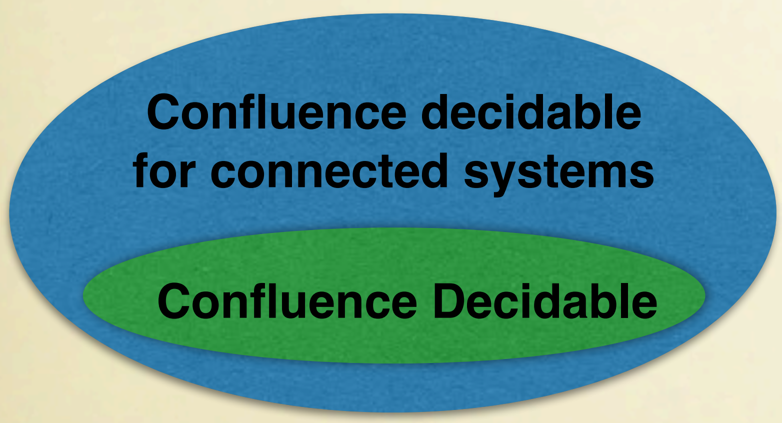
Theorem In DPO rewriting *with interfaces*, confluence is decidable.

The graph interpretation



Conclusions



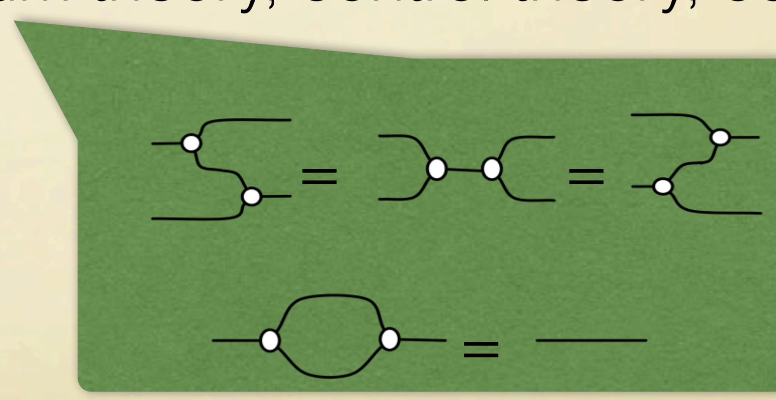


	Terminating term rewriting systems	Terminating DPO-with-interface systems
Confluence for ground objects	***undecidable (Kapur et al.)***	***undecidable (Plump)***
Confluence	***decidable (Knuth-Bendix)***	***decidable***

Applications:

Algebras of network diagrams in quantum theory, control theory, concurrency, ….

Any adhesive category



What's next: termination