An active-selecting neural network method for airfoil self-noise predictions

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PROGRESS UPDATE

Theory

- [Lighthill, 1952] → Conservation equations

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}(\boldsymbol{x}, t) \qquad (1)$$

Where $T_{ij} = \rho u_i u_j + (p - c^2)\ \delta_{ij} - \tau_{ij}$ → Lighthill's stress tensor

- [Bogey et al., 2001] proposed the solution of acoustic pressure from Eq. 1:

$$p'(\boldsymbol{x}, t) = \frac{1}{4\pi c^2}\int_{V_y} \frac{r_i r_j}{r^3}\frac{\partial^2 T_{ij}}{\partial t^2}\left(\boldsymbol{y}, t - \frac{r}{c}\right) d\boldsymbol{y} \qquad (2)$$

The acoustic pressure p' follows the dilatation field

(Large Eddy Simulation & hybrid method on Linearized Euler Equation):

$$\Theta = \nabla \cdot \boldsymbol{u} = -\frac{1}{\rho_0 c^2}\left(\frac{\partial p'}{\partial t} + U_i \frac{\partial p'}{\partial x}\right) \qquad (3)$$

Theory

- With the novel methods [Casalino, 2003] & [Najafi-yazdi et al., 2010] applying control surface function

$f(x,t) = 0$, we can further derive the acoustic pressure at any observer from Eq. 2:

$$p'(\boldsymbol{x},t) = \frac{\partial}{\partial t}\int_{f=0}\left(\frac{Q_i n_i}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\right)_{t_e} dS - \frac{\partial}{\partial x_i}\int_{f=0}\left(\frac{L_{ij} n_j}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\right)_{t_e} dS + \frac{\partial^2}{\partial x_i \partial x_j}\int_{f>0}\left(\frac{T_{ij}}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\right)_{t_e} dV \tag{4}$$

Where

$$\left.\begin{aligned} Q_i &= \rho(u_i - v_i) + \rho_0 v_i \\ L_{ij} &= \rho u_i(u_j - v_j) + P_{ij} \end{aligned}\right\} \text{Source terms} \tag{5}$$

$$T_{ij} = \rho u_i u_j + \left((p - p_0) - c^2(\rho - \rho_0)\right)\delta_{ij} - \tau_{ij} \longrightarrow \text{Lighthill's stress tensor}$$

$$P_{ij} = (p - p_0)\delta_{ij} - \tau_{ij} \longrightarrow \text{Compression tensor}$$

https://cerfacs.fr/antares/doc/src/treatment/fwh/fwh.html

Theory

- Recall the definition of sound pressure level SPL:

$$SPL = -20\log\left(\frac{p'}{p_0}\right), in \text{ dB} \tag{6}$$

- Substitute Eq. 2 into Eq. 6, we have:

$$\begin{aligned} &SPL \\ &= -20\left(\log\left(\frac{\partial}{\partial t}\int_{f=0}\left(\frac{\rho(u_i - v_i)n_i + \rho_0 v_i n_i}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\right)_{t_e} dS \right.\right. \\ &- \frac{\partial}{\partial x_i}\int_{f=0}\left(\frac{\left(\rho u_i\left(u_j - v_j\right) + (p - p_0)\delta_{ij} - \tau_{ij}\right)n_j}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\right)_{t_e} dS \\ &\left.\left. + \frac{\partial^2}{\partial x_i \partial x_j}\int_{f>0}\left(\frac{\rho u_i u_j + \left((p - p_0) - c^2(\rho - \rho_0)\right)\delta_{ij} - \tau_{ij}}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\right)_{t_e} dV\right) - \log p_0\right) \end{aligned} \tag{7}$$

Theory

- From Eq. 7 & Eq. 2, we conclude:

$$SPL = SPL(x, u, p, \rho, t, r, \mu) \tag{8}$$

- Recall the scaled $\mathrm{SPL}_{1/3}$ calculation (1/3 Octave band) [Brooks et al., 1989]:

$$\text{Scaled } SPL_{1/3} = SPL_{1/3} - 10\log\left(M^5 \frac{\delta\Lambda}{r^2}\right)\overline{D} \tag{9}$$

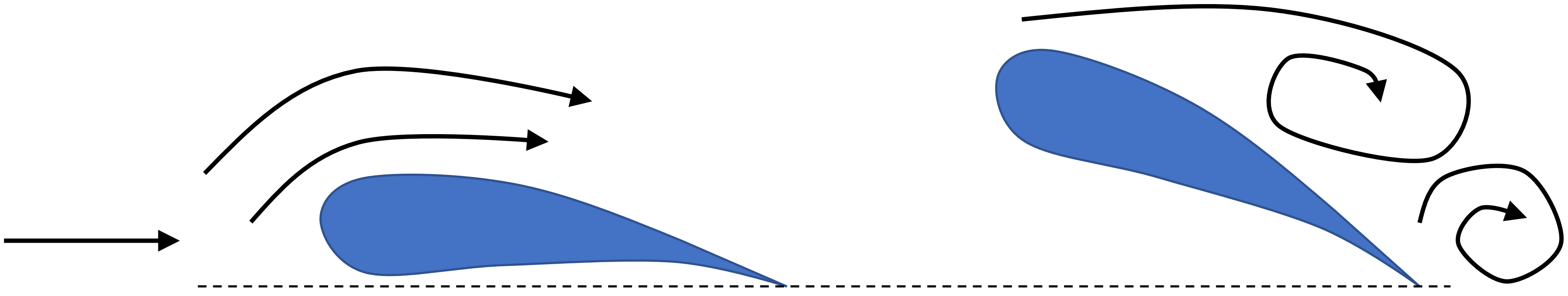
$$\longrightarrow \quad \text{Scaled } SPL_{1/3} = \text{Scaled } SPL_{1/3}(x, u, p, \rho, t, r, \mu, \Lambda) \tag{10}$$

- Recall the NASA database [Brooks et al., 1989], there are five input attributes:

$$\text{Scaled } SPL_{1/3} = \text{Scaled } SPL_{1/3}(\alpha, f, l, u, \delta) \tag{11}$$

Where $\delta = \delta(\mu)$.

Simulation



Flow field: $\{\rho, u, p\} \rightarrow \alpha = \alpha(\rho, u, p)$

- Frequency f corresponds with the $SPL_{1/3}$ from the simulation as output.
- Chord length l is scaled to be 1 in the simulation.
- Hence, the experimental data can be recognized as

$$\text{Scaled } SPL_{1/3} = \text{Scaled } SPL_{1/3}(\alpha(\rho, u, p), f, l, u, \delta(\mu)) \qquad (12)$$

$$\left(\text{Scaled } SPL_{1/3}, f\right) = \left(\text{Scaled } SPL_{1/3}, f\right)(\rho, u, p, \mu) \qquad (13)$$

Simulation

- Compare Eq. 8 with Eq. 12:

$$SPL_{1/3} = SPL_{1/3}(x, u, p, \rho, t, r, \mu)$$

$$\left(\text{Scaled } SPL_{1/3}, f\right) = \left(\text{Scaled } SPL_{1/3}, f\right)(\rho, u, p, \mu)$$

- In the actual simulation, the flow field $\{\rho, u, p\}$ & $\left(\text{Scaled } SPL_{1/3}, f\right)$ is the output data;

  $\{\mu, \text{Turbulence Model}, U, \alpha\}$, where $\text{Turbulence Model} = \text{Turbulence Model}\left(k, \varepsilon, C_\mu, \mu, l\right)$ for k-ε.

- Therefore, the simulation model can be simplified as:

$$\left(\rho, u, p, \left(\text{Scaled } SPL_{1/3}, f\right)\right) = \left(\rho, u, p, \left(\text{Scaled } SPL_{1/3}, f\right)\right)(\mu, \text{Turbulence Model}, U, \alpha) \qquad (14)$$

- Recall the neural network on the experimental data:

$$\text{Scaled } SPL_{1/3} = NN_{\text{exp}}(\alpha, f, l, U, \delta) \qquad (15)$$

Turbulence Model

- **K-ε model:**
- Turbulent kinetic energy k and dissipation ε obeys

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho k u_i)}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)+2\mu_t E_{ij}E_{ij}-\rho\varepsilon \quad (16)$$

$$\frac{\partial(\rho\varepsilon)}{\partial t}+\frac{\partial(\rho\varepsilon u_i)}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right)+C_{1\varepsilon}\frac{\varepsilon}{k}2\mu_t E_{ij}E_{ij}-C_{2\varepsilon}\rho\frac{\varepsilon^2}{k} \quad (17)$$

Where u_i: velocity components; E_{ij}: rate of deformation; μ_t → eddy viscosity $\frac{\mu}{\mu_t}=\frac{\rho C_\mu k^2}{\mu\varepsilon}$

- The terms k and ε is calculated as:

$$k=\frac{3}{2}(Ul)^2\ \&\ \varepsilon=\frac{C_\mu^{\frac{3}{4}}k^{\frac{3}{2}}}{l'}\quad (l'\approx 0.7l) \quad (18)$$

The constants adopts: $C_\mu=0.09, \sigma_k=1.00, \sigma_\varepsilon=1.30, C_{1\varepsilon}=1.44, C_{2\varepsilon}=1.92$

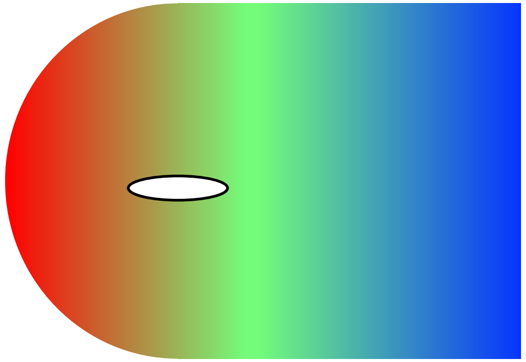
https://en.wikipedia.org/wiki/K-epsilon_turbulence_model

Neural Network

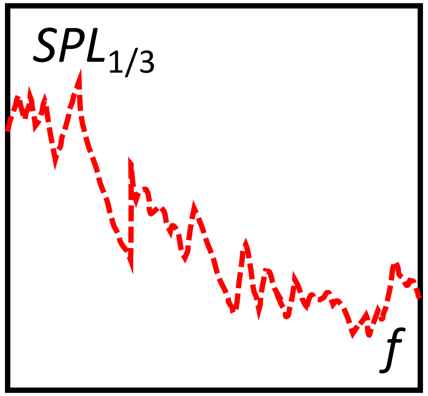
- With the NN_{exp}, we could fit the regression on self-noise experimental model;
- We can also apply a neural network on the simulation data corresponding the theoretical model (Eq. 13);
- The simulation process can be shown as:

Flow field: $\{\rho, u, p\}$

Boundary conditions:
NACA 0012 & $\{\mu, \text{Turbulence Model}, U, \alpha\}$



&



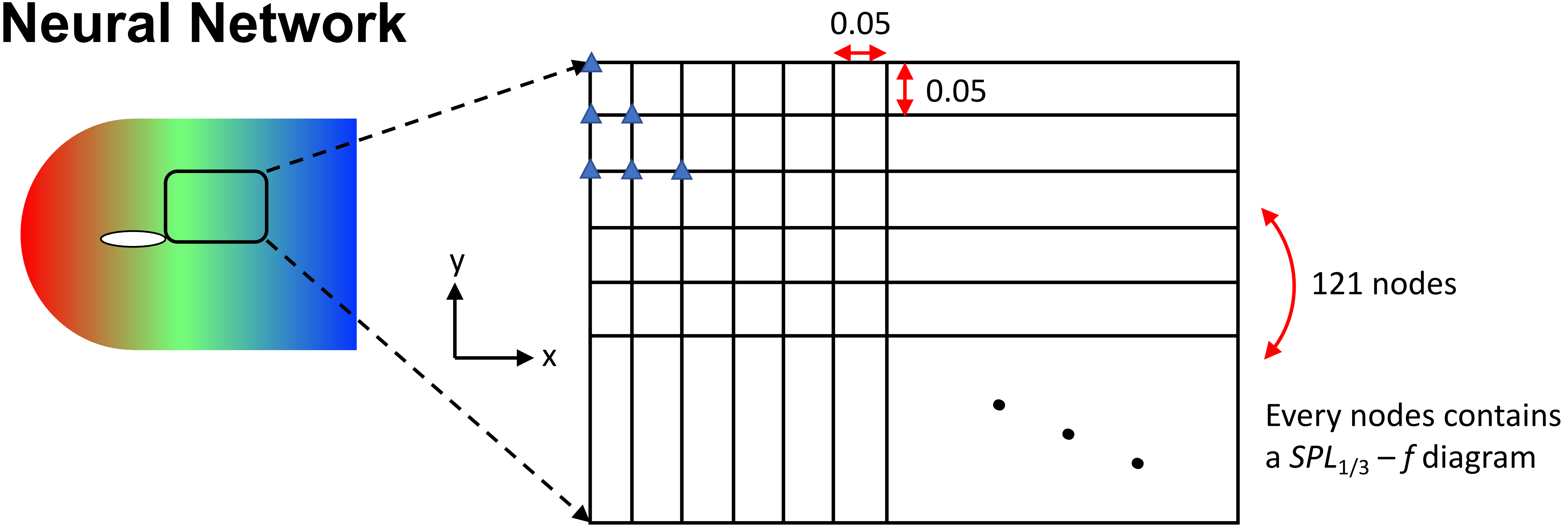
Input data:

Output data:

$\{\mu, \text{Turbulence Model}, U, \alpha, \text{Flow Field}(\rho, u, p)\}$ → Noise field: $\text{SPL}_{1/3}$ distribution;

$$SPL_{1/3}(x, y) = NN_{\text{sim}}\{\mu, \text{Turbulence Model}, U, \alpha, \text{Flow Field}(\rho, u, p)\}$$

Neural Network



Acoustic field: $\{SPL_{1/3}(x, y); SPL(x, y)\}$

- Recall the simulation neural network: $NN_{\mathrm{sim}}\{\mu, \mathrm{Turbulence\ Model}, U, \alpha, \mathrm{Flow\ Field}(\rho, u, p)\}$,

we could further predict the acoustic field considering specific frequency.

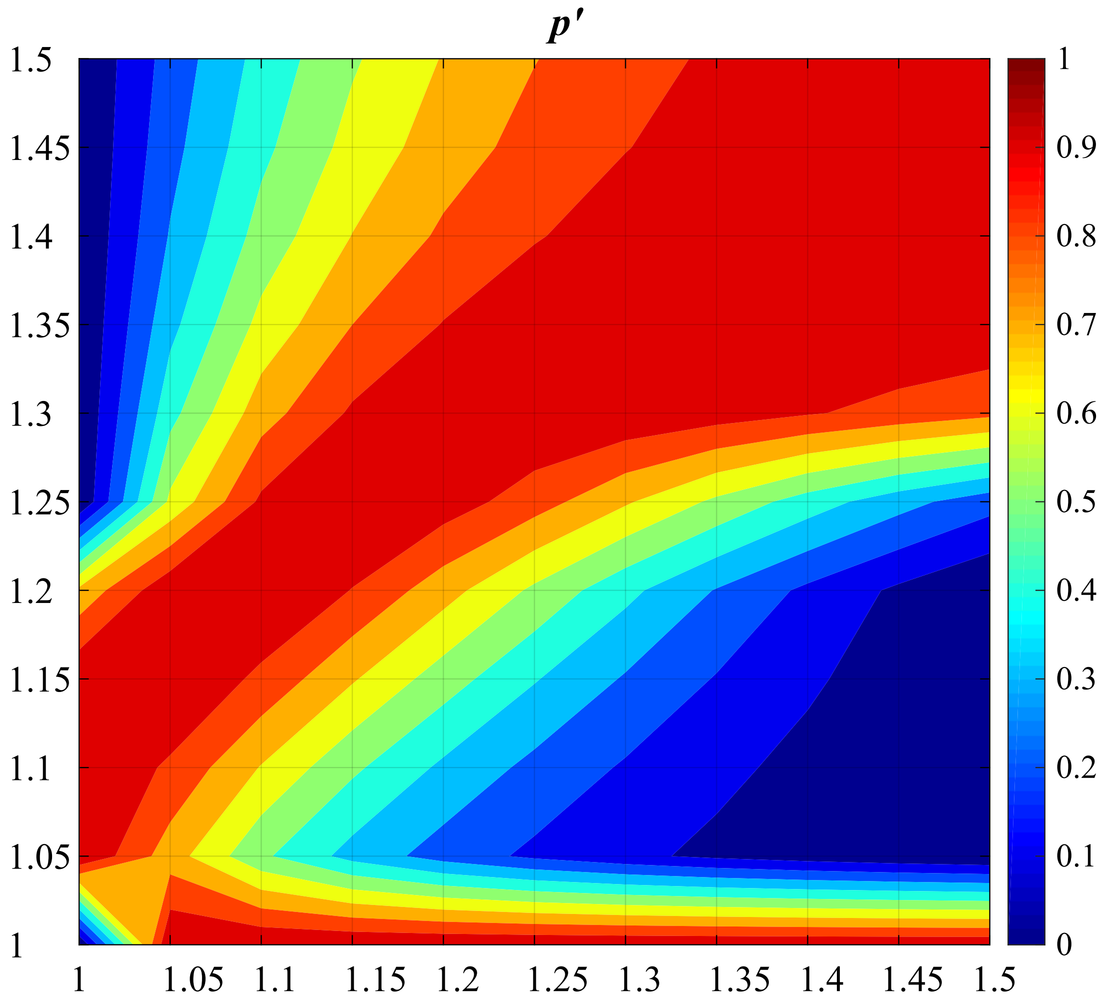
Neural Network

- With the set conditions:
- K-ε, FW-H model parameters:

$$\begin{cases} TKE\ Prandtl\ Number: 1 \\ TDR\ Prandtl\ Number: 1.3 \\ Energy\ Prandtl\ Number: 0.85 \\ Wall\ Prandtl\ Number: 0.85 \\ far\ field: \rho = 1.225kg/m^3 \\ c = 340m/s \end{cases}$$

- Boundary conditions:

$$\begin{cases} far\ field: Mach\ Number = 0.2048 \\ k = 1\ and\ \varepsilon = 1 \\ T = 300K \\ roughness\ height = 0, roughness\ const = 0.5 \end{cases}$$



$\rightarrow p' = p'(u, x, r, \mu, \rho, p, t).$

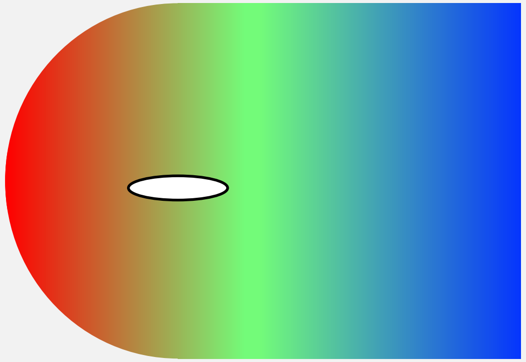
- From Eq. 2, acoustic pressure follows:

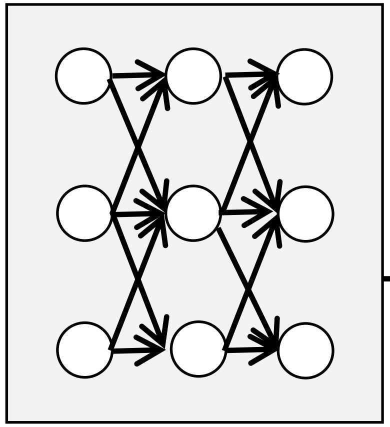
$$p'(\boldsymbol{x}, t) = \frac{1}{4\pi c^2} \int_{V_y} \frac{r_i r_j}{r^3} \frac{\partial^2 \left(\rho u_i u_j + (p - c^2)\, \delta_{ij} - \tau_{ij}\right)}{\partial t^2} \left(\boldsymbol{y}, t - \frac{r}{c}\right) d\boldsymbol{y}$$

Simulation Coupled NN

Field data:

- Pressure field p
- Density field ρ
- Velocity field u
- Energy field E
- Turbulence kinetic energy k
- Turbulence intensity I_T
- Turbulence dissipative rate ε



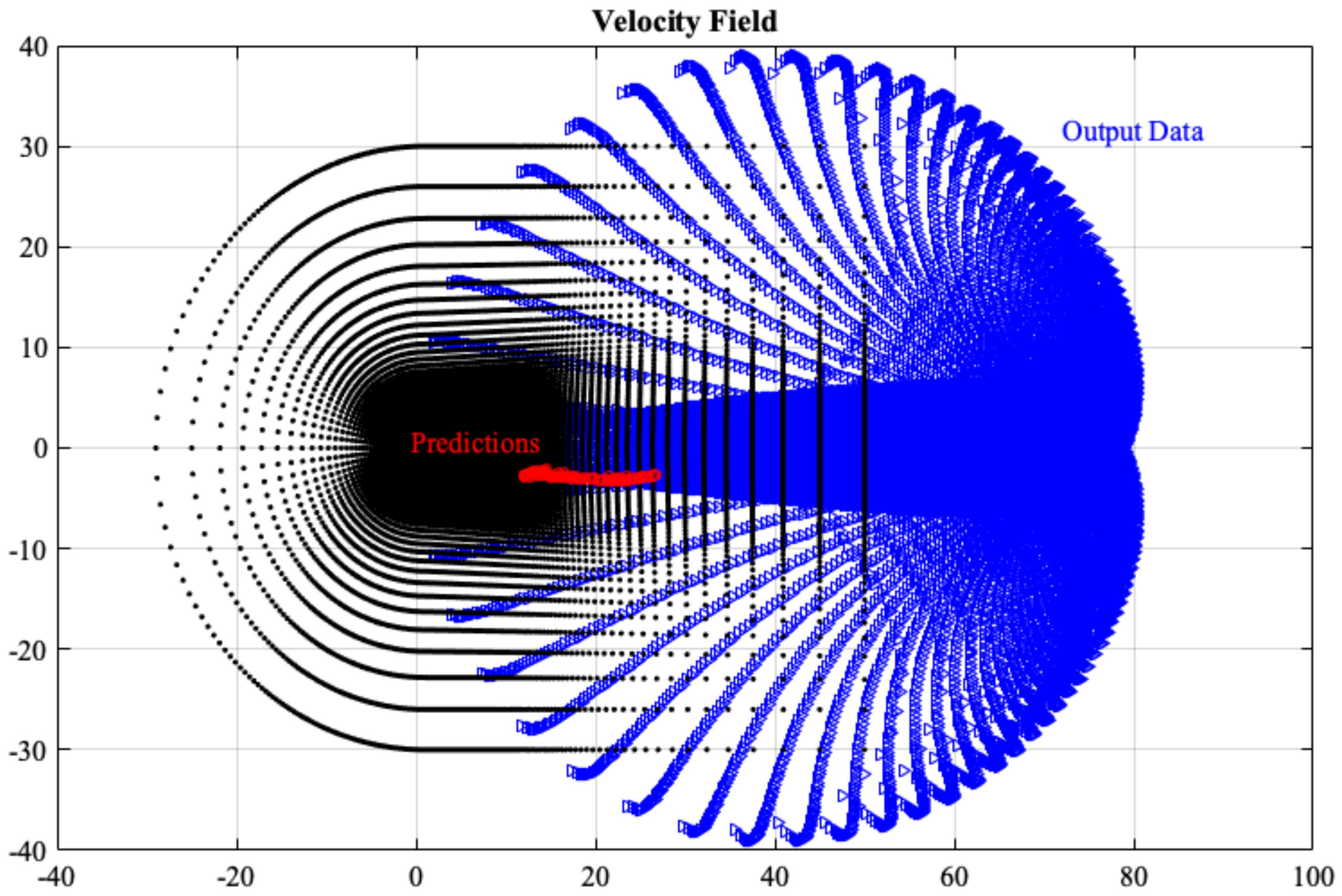


- Velocity field u

Simulation Coupled NN

BAD Results!!!

Because only simple case data is trained



Simulation Coupled NN

Field data:

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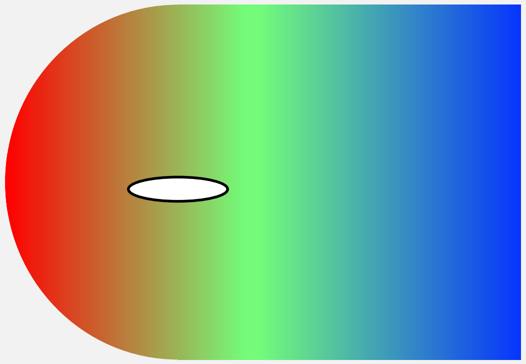
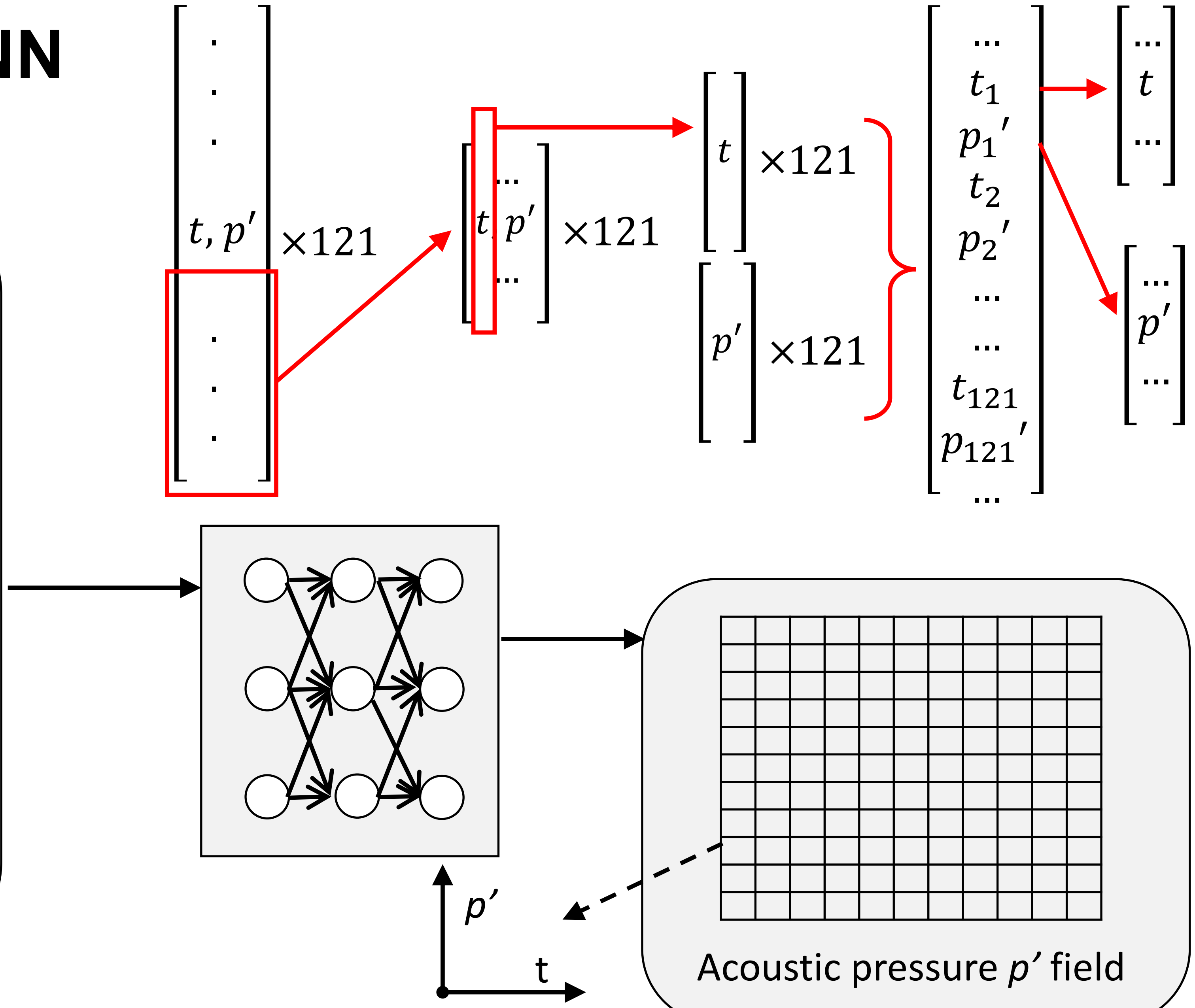
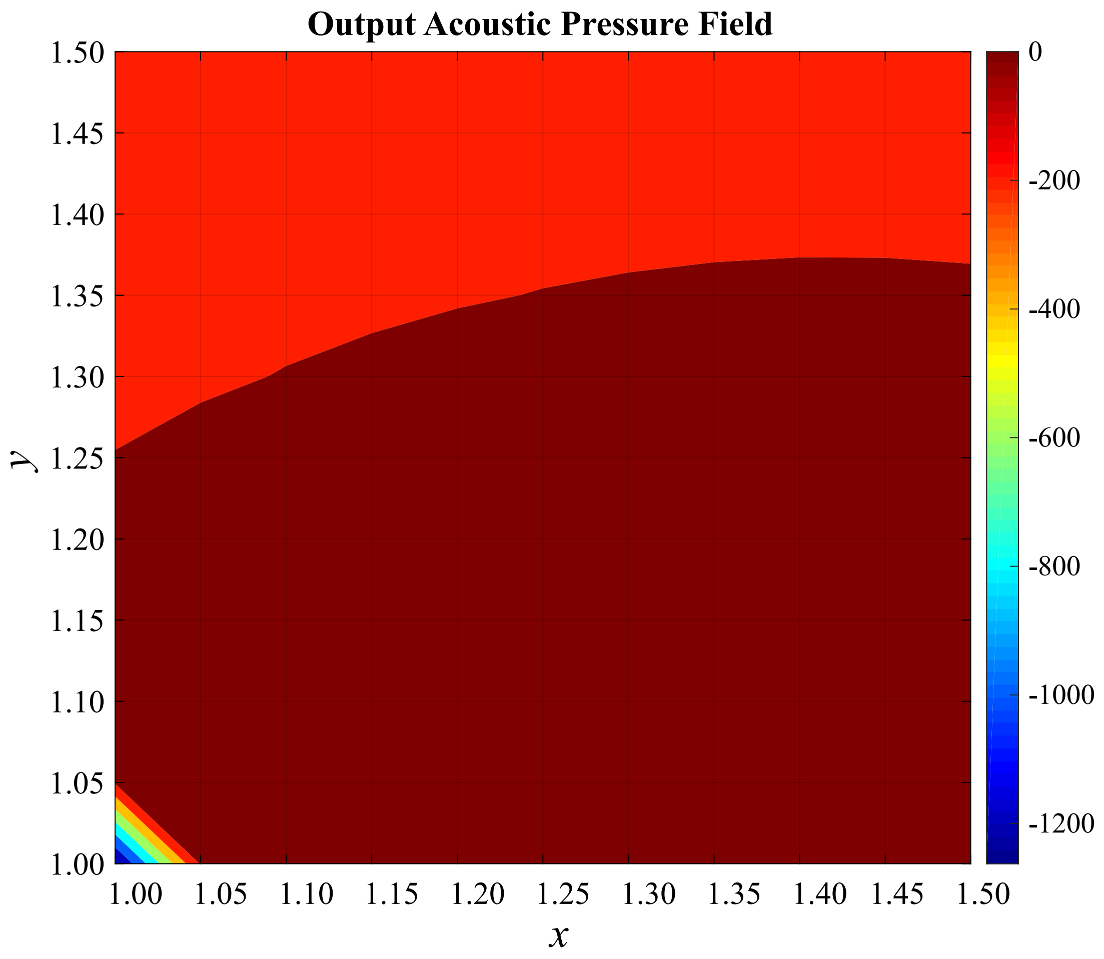
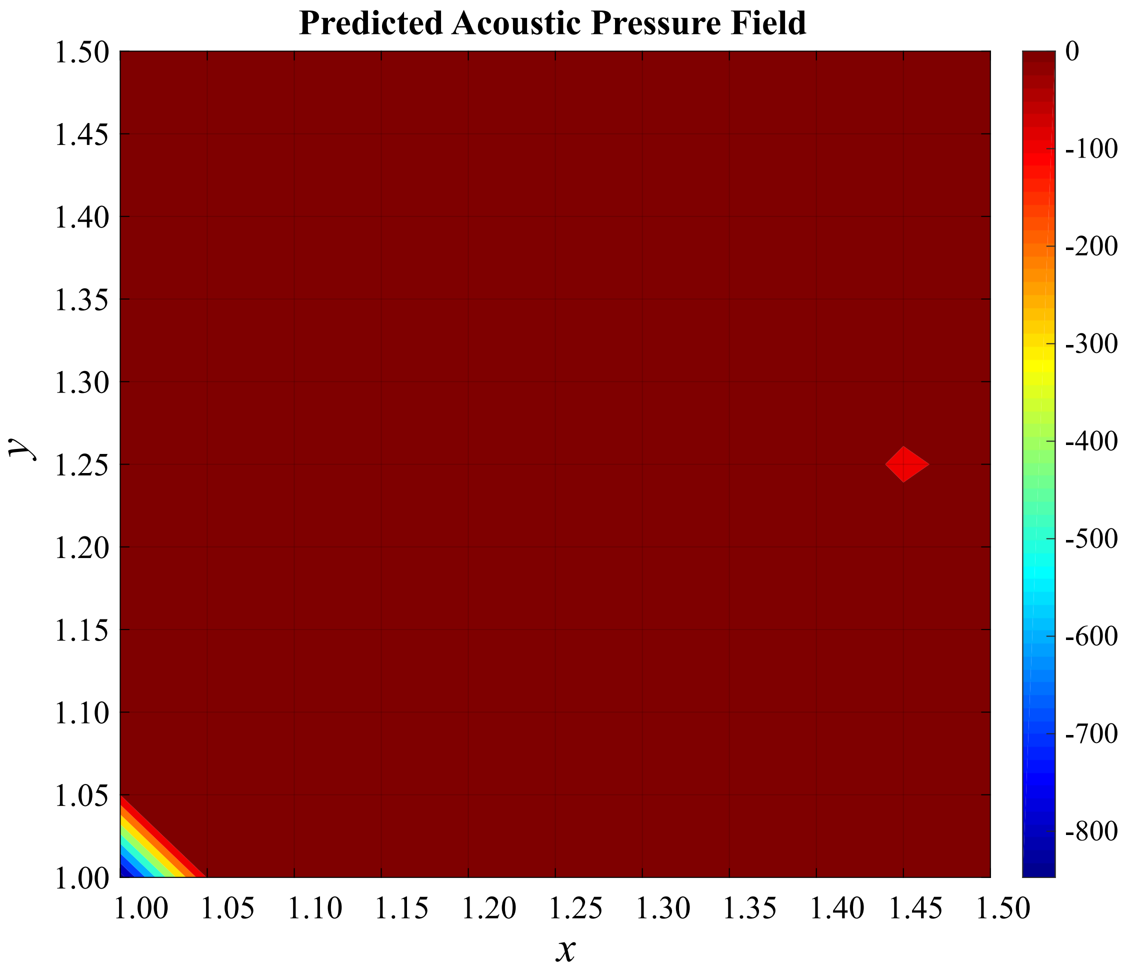




Simulation Coupled NN

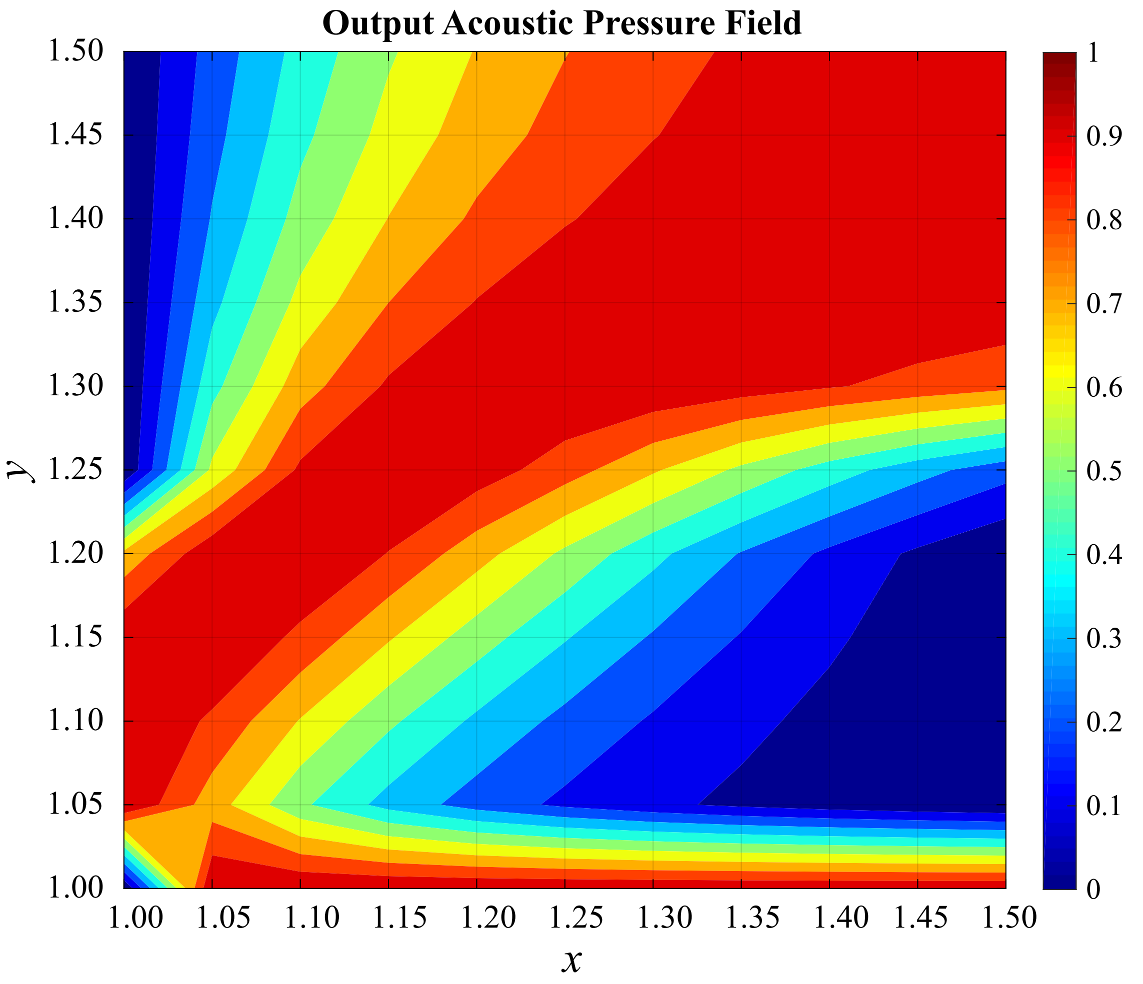
Original Data at Final Time Step

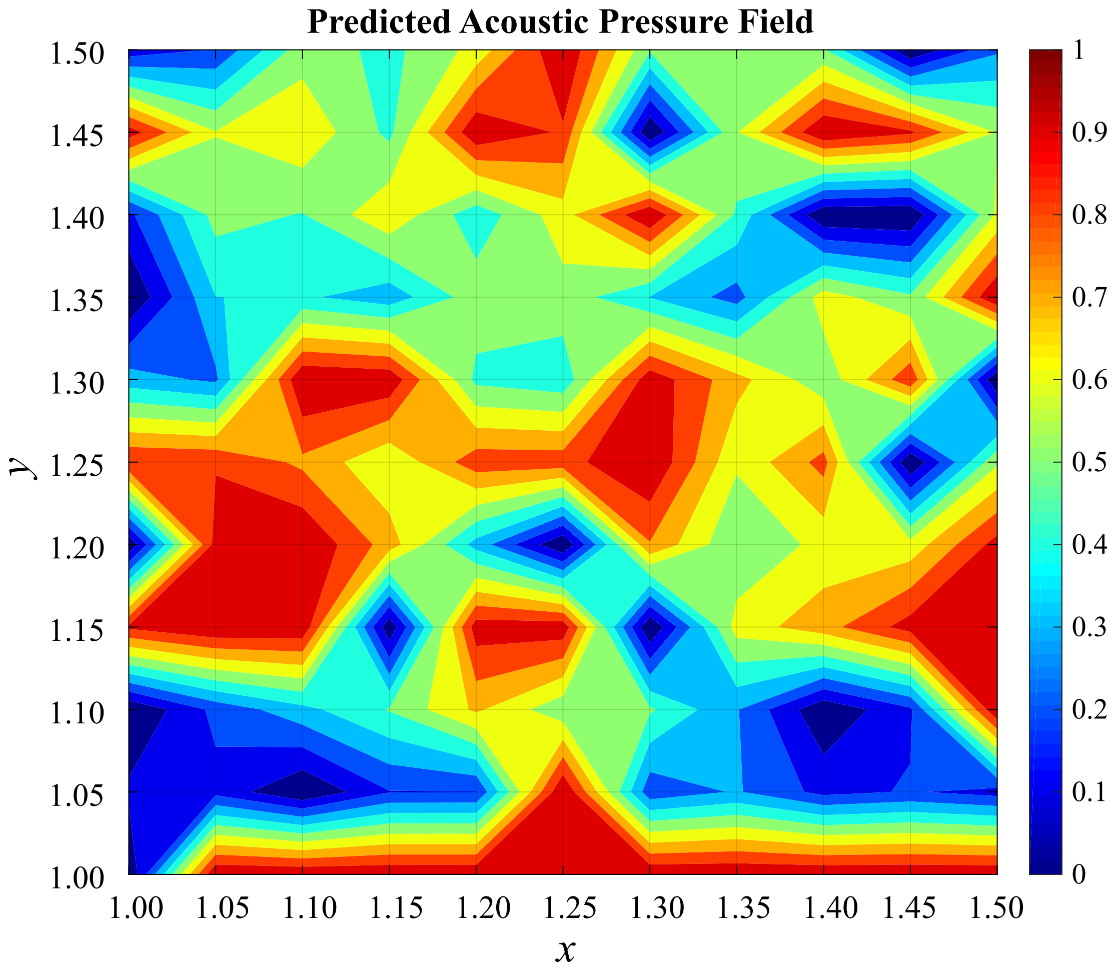




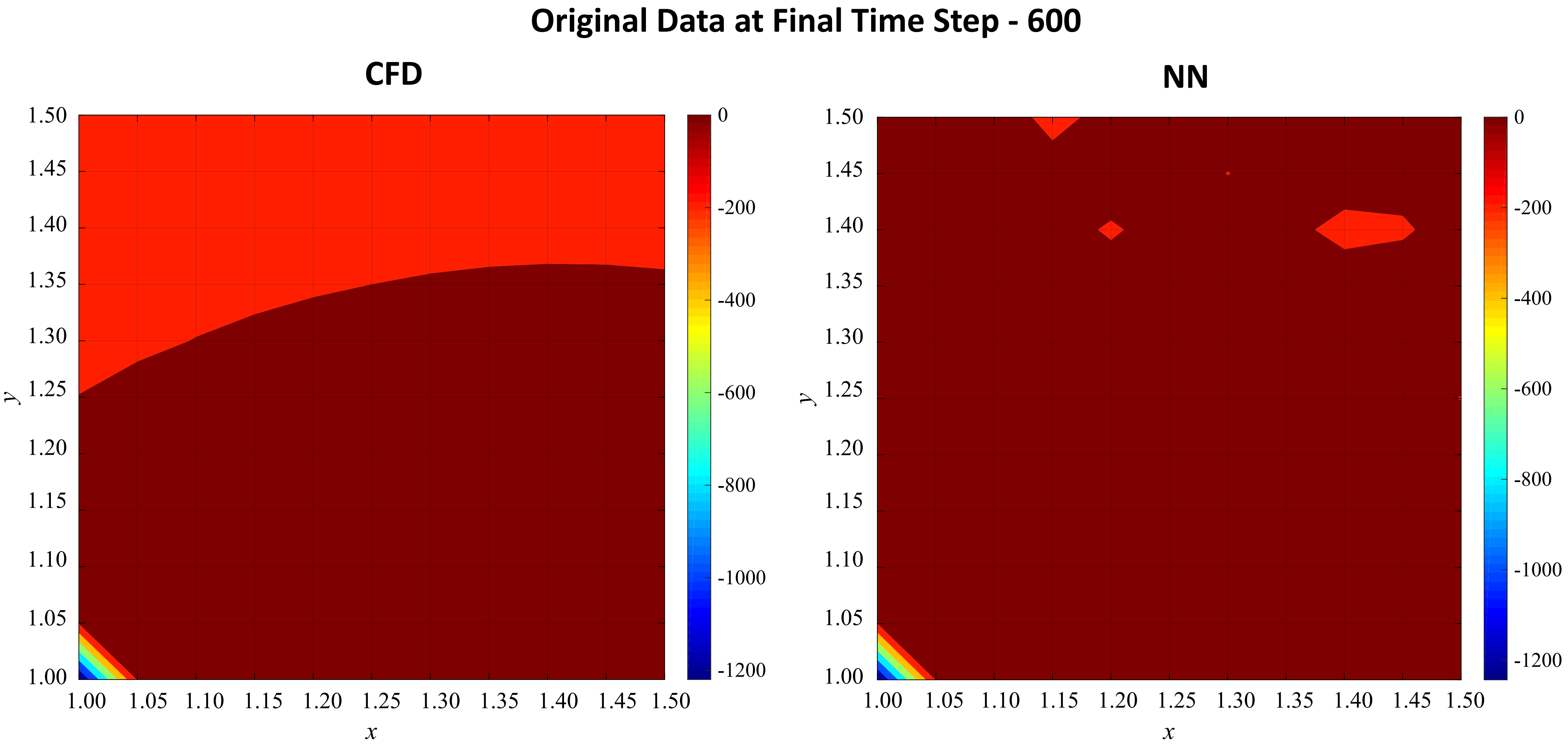
Simulation Coupled NN

Normalized Data at Final Time Step





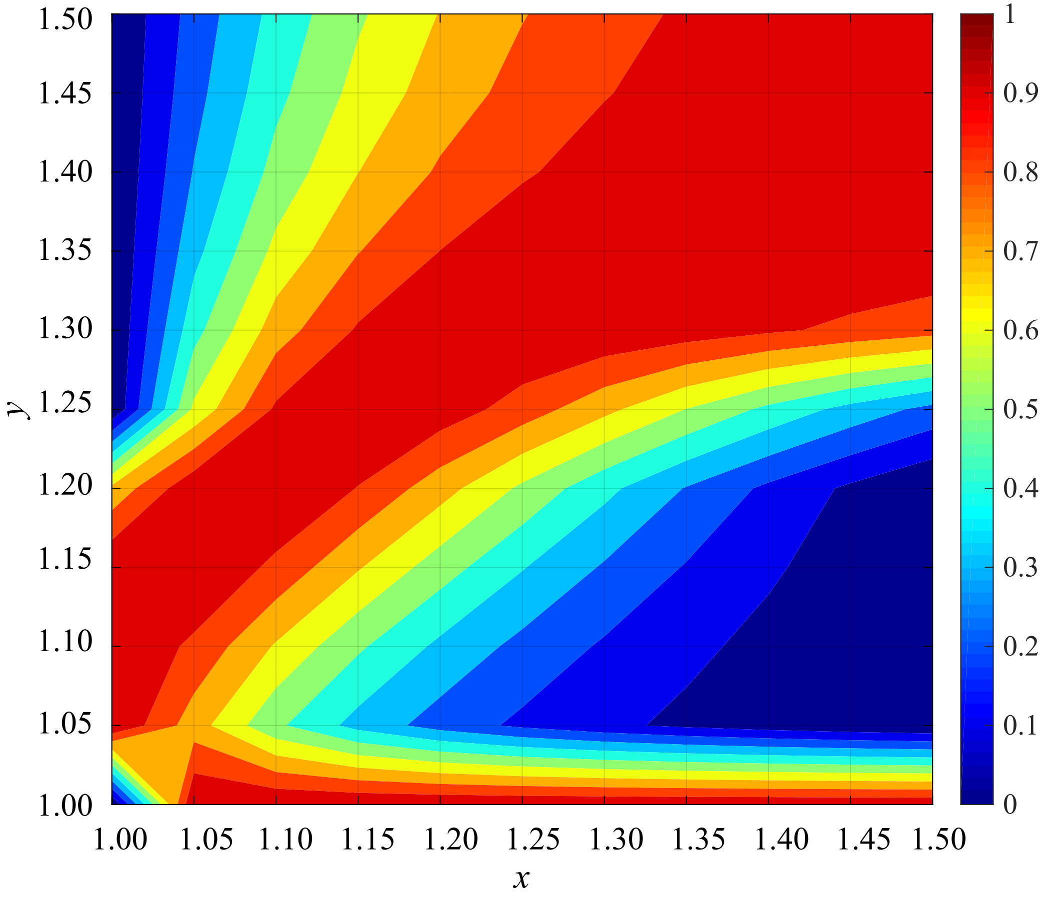
Simulation Coupled NN



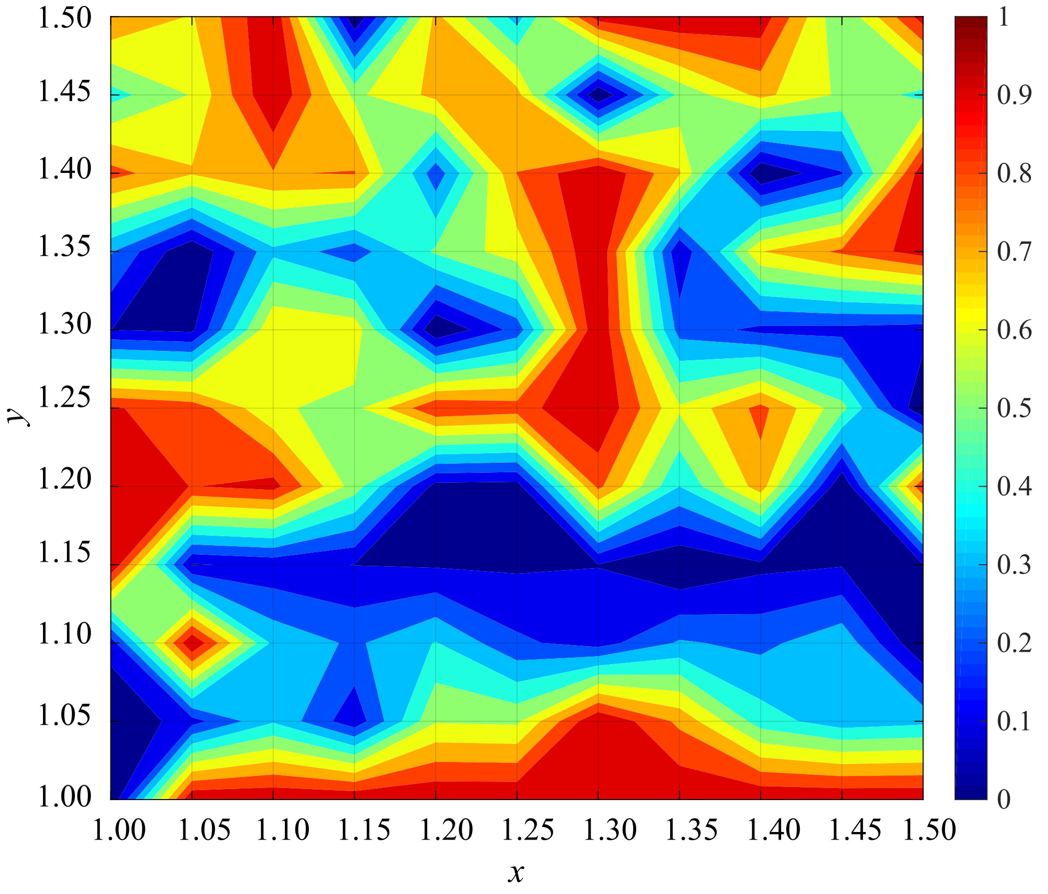
Simulation Coupled NN

Normalized Data at Final Time Step - 600

CFD

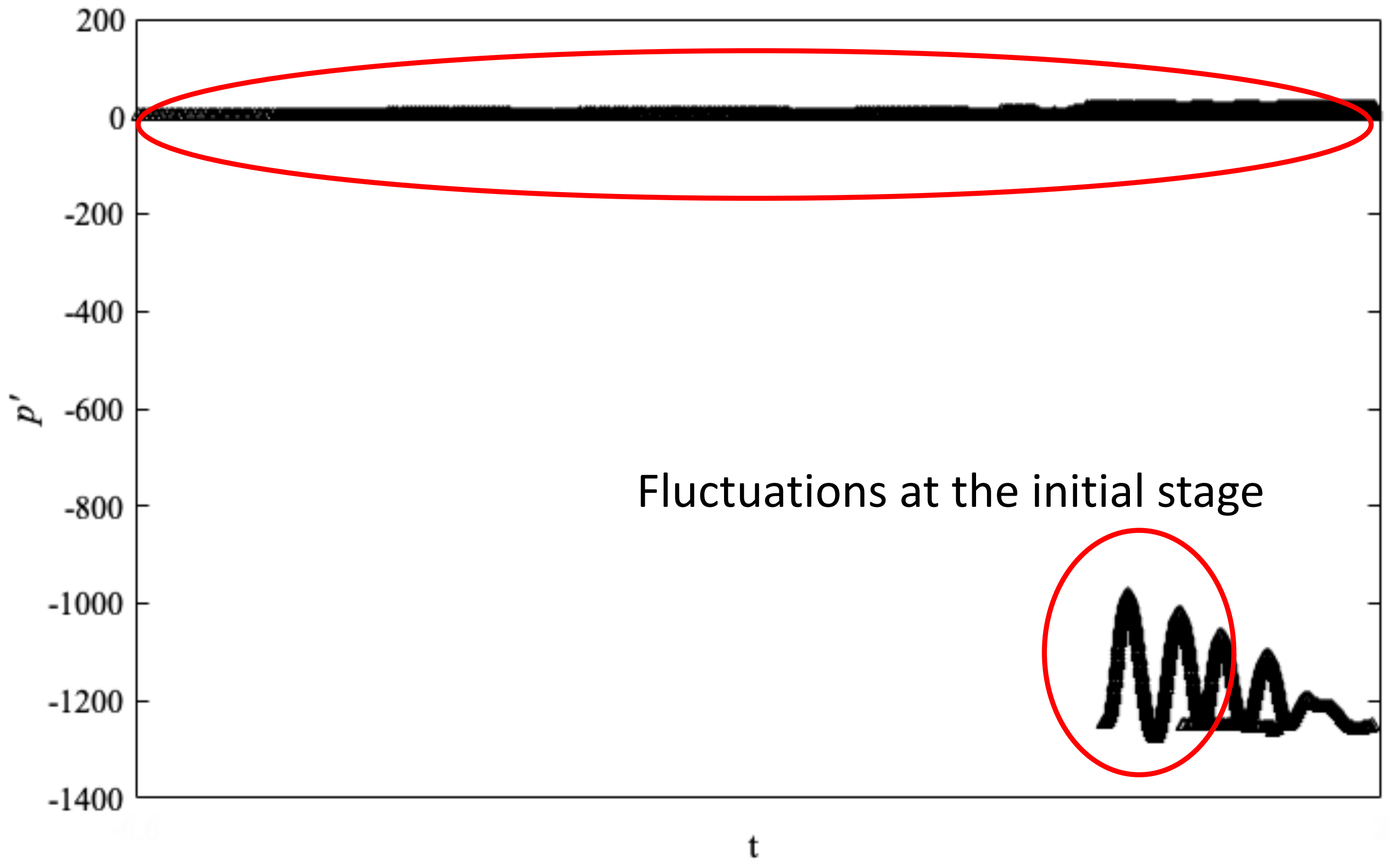


NN



Simulation Coupled NN

Value does not variate significantly



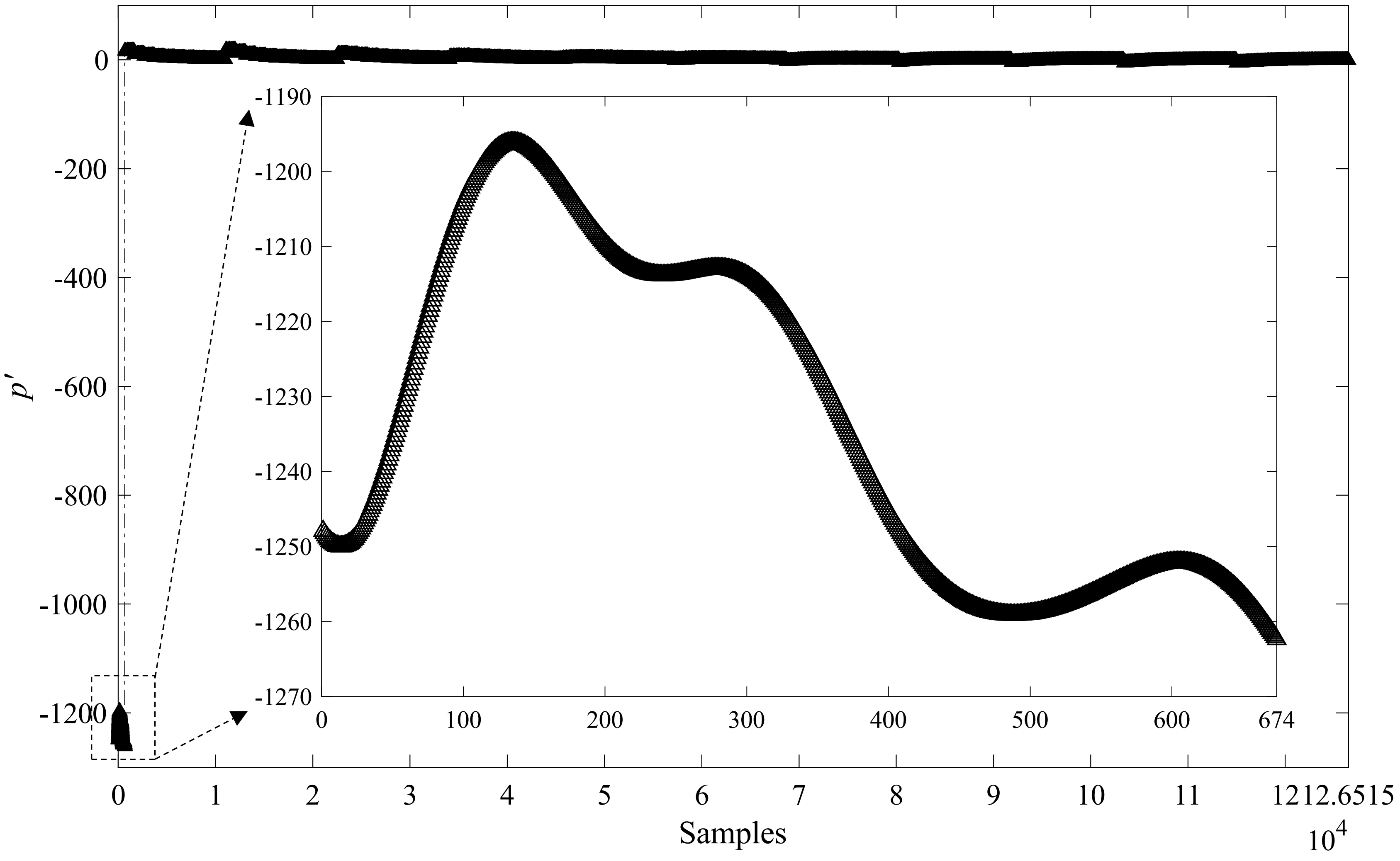
Fluctuations at the initial stage

Contains all information

from 121 receivers

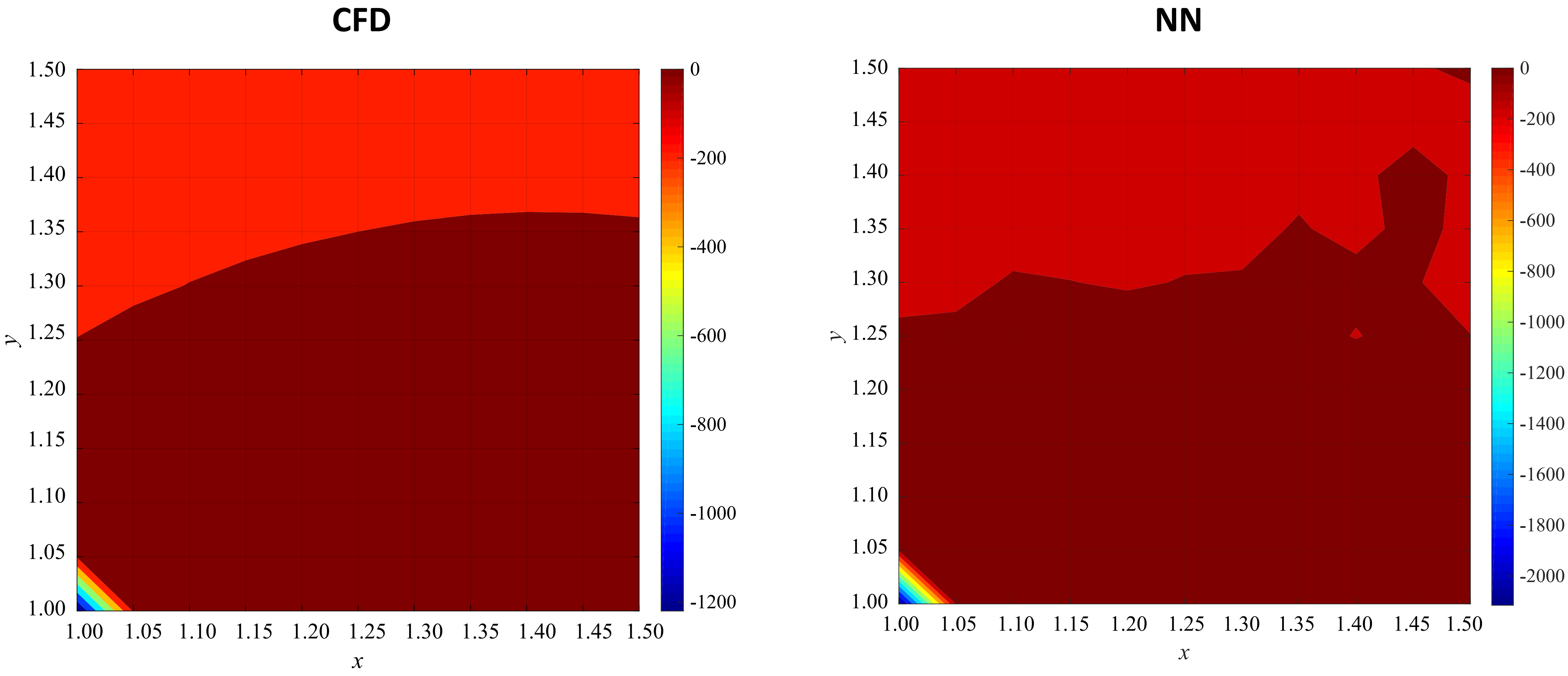
Simulation Coupled NN

- ***How to capture the detailed noise signals***



Simulation Coupled NN

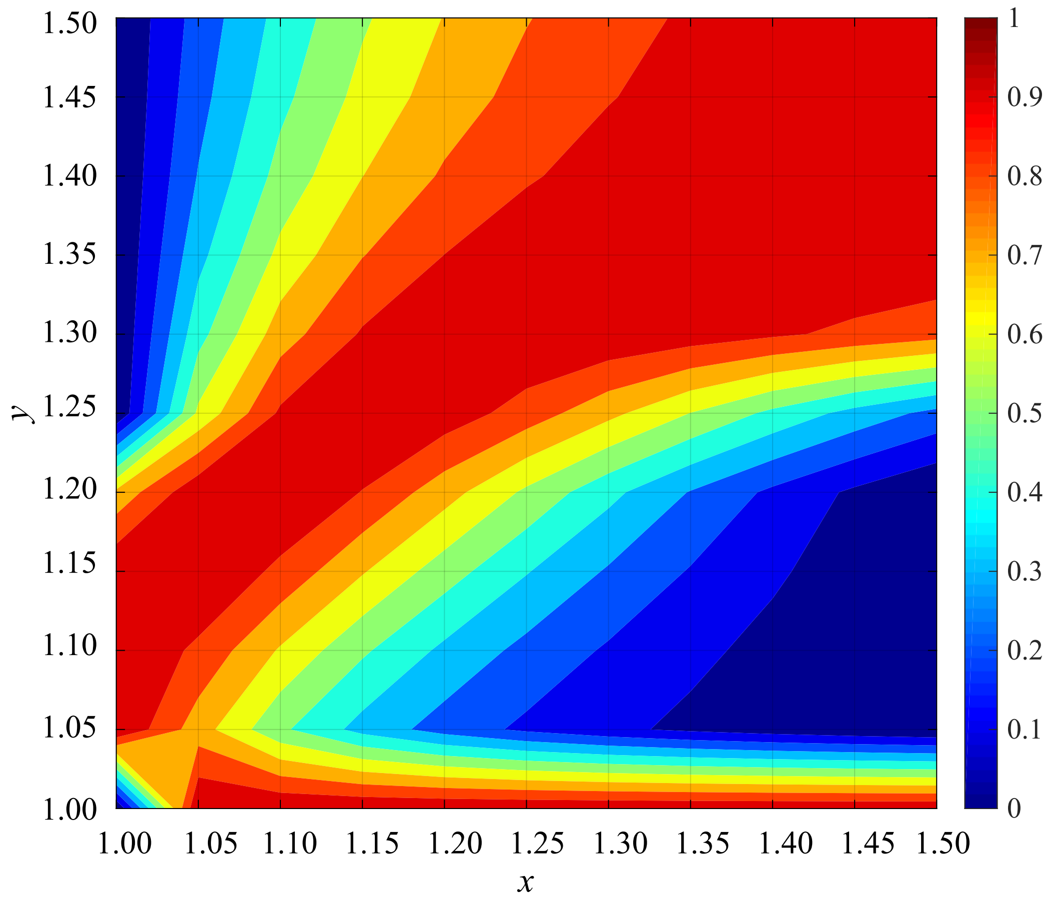
Original Data at Final Time Step



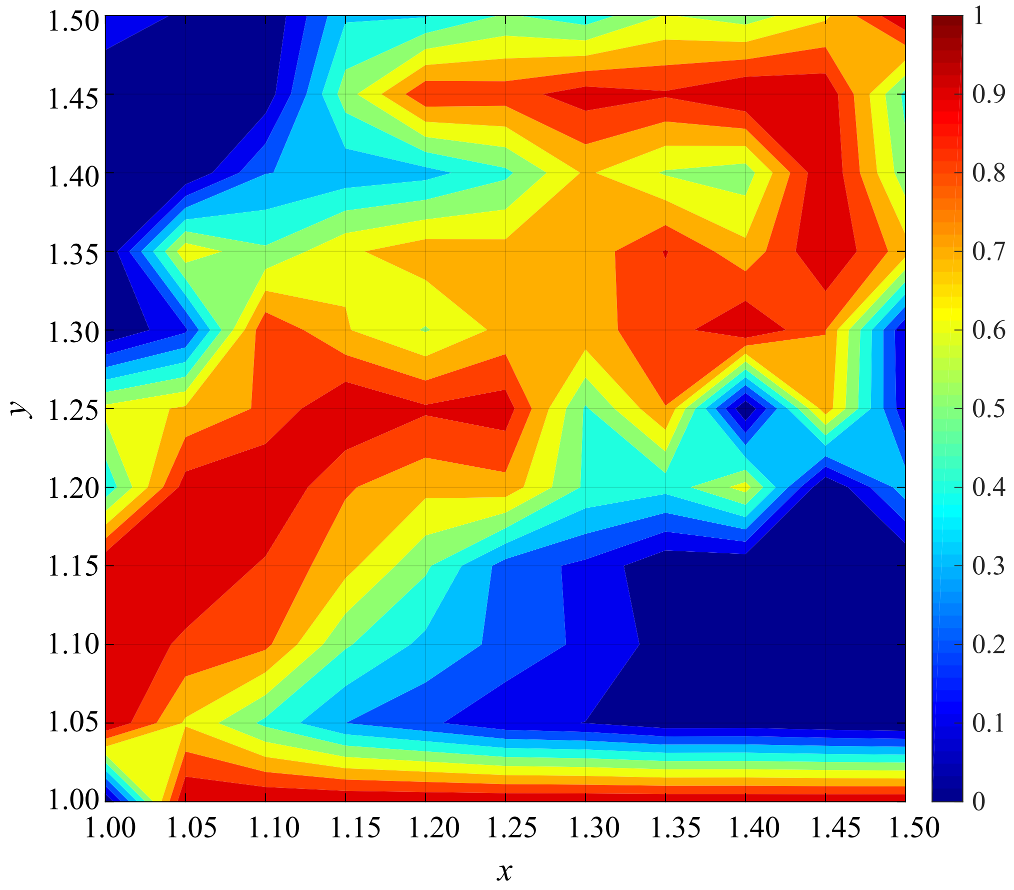
Simulation Coupled NN

Normalized Data at Final Time Step

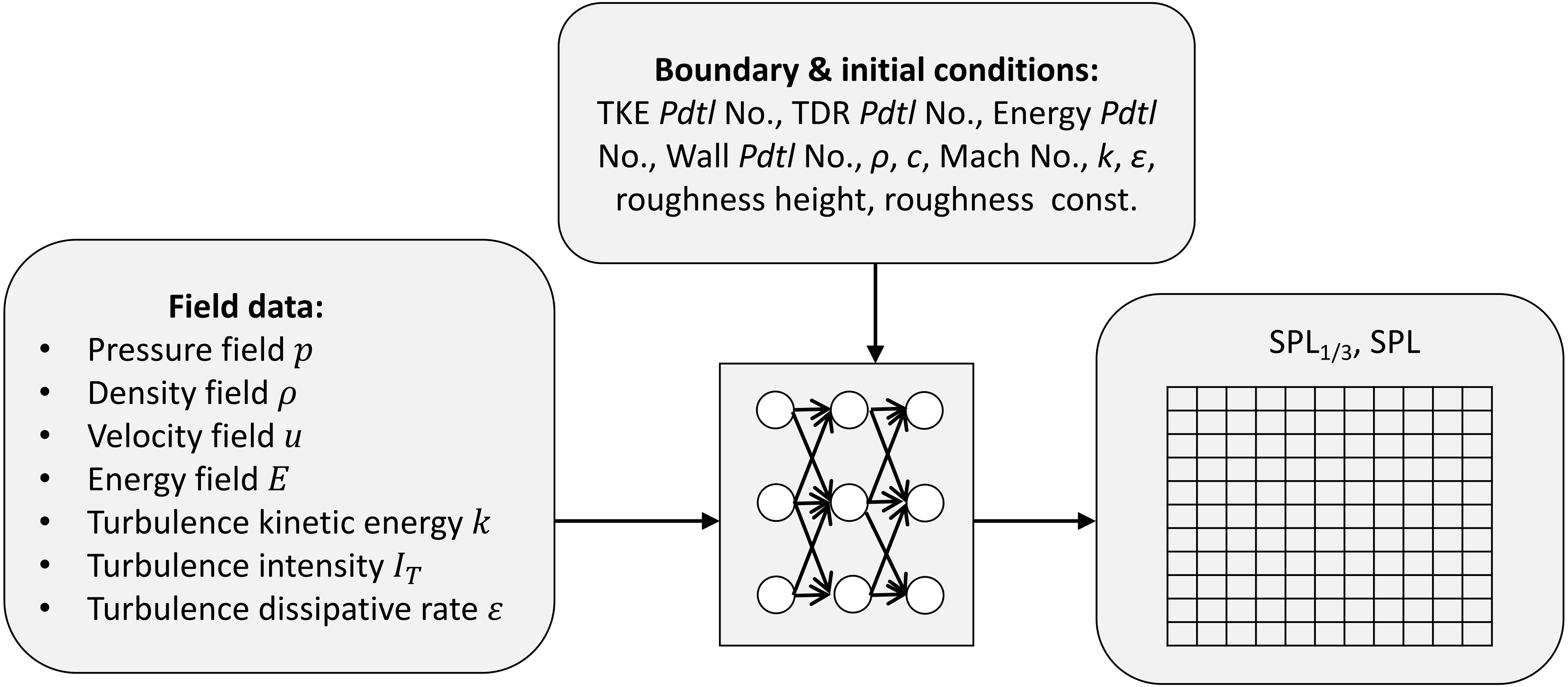
CFD



NN



Simulation Coupled NN



Main Problems

- How to adopt the predictions (NN_{exp}) from the NASA database.
- How to accumulate the physics law into the NN_{sim} predictions.
- The chord length of airfoil.
- Adjust the simulation parameters to the input data.

- The End -