CHAPTER 7. Meshing Transformation & Viscosity Discretization

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Abstract

In meshing objects with structural meshes such as meshing of airfoils, we need to transform the meshing to the coordinate system. The transformation process of the meshing coordinates is provided. The methods for discretization of the viscosity is hence given.

We first recall the viscous term as gives in the Navier-Stokes equation in Chap. 1:

$$\sigma_{x_i x_i} = \mu\left(\frac{\partial u_i}{\partial x_i}\right) \tag{1}$$

Here, the derivative term takes the form based on the Green-Gauss integration:

$$\frac{\partial u}{\partial x} = \frac{\int \frac{\partial u}{\partial x} d\Omega}{\int d\Omega} = \frac{1}{\Omega}\oint \vec{u}\cdot\vec{n}dS \tag{2}$$

$$= \frac{1}{\Omega} u_{i+\frac{1}{2},j} n_{i+\frac{1}{2},j} \Delta S \tag{3}$$

Hence, the partial derivatives of u can be discretized taking the forms:

$$\left.\frac{\partial u}{\partial x}\right|_{i,j} = \frac{1}{\Omega_{i,j}}\left(u_{s_{i+\frac{1}{2},j}} - u_{s_{i-\frac{1}{2},j}} + u_{s_{i,j+\frac{1}{2}}} - u_{s_{i,j-\frac{1}{2}}}\right) \tag{4}$$

Substituting the relation as given in Eq. 3, we deduce

$$\left.\frac{\partial u}{\partial x}\right|_{i,j} = \frac{1}{2\Omega_{i,j}}\left(u_{i+1,j} + u_{i,j}\right) n_{i+\frac{1}{2},j} S_{i+\frac{1}{2},j} + \left(u_{i,j+1} + u_{i,j}\right) n_{i,j+\frac{1}{2}} S_{i,j+\frac{1}{2}}$$
$$- \left(u_{i-1,j} + u_{i,j}\right) n_{i-\frac{1}{2},j} S_{i-\frac{1}{2},j} - \left(u_{i,j} + u_{i,j-1}\right) n_{i,j-\frac{1}{2}} S_{i,j-\frac{1}{2}} \tag{5}$$

Here, the meshing elements in a row can be visualized as shown in Fig. 1. The structural mesh are shown in the *x*-*y* coordinates (adapted mesh). The transformed mesh in the coordinate system is the *ζ*-*η* system as shown in the down view.

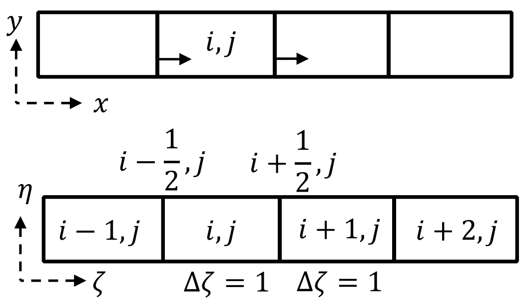

Fig. 1 Schematic for meshing elements in a row.

The derivatives on the top side can be transformed to the coordinate system as written:

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \zeta}\zeta_x + \frac{\partial u}{\partial \eta}\eta_x \tag{6}$$

In which the term $\frac{\partial u}{\partial \zeta}$ can be written as

$$\frac{\partial u}{\partial \zeta} = \left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}\right) \tag{7}$$

Where the two terms on the right side can be derived from the down view in Fig. 1:

$$u_{i+\frac{1}{2},j} = \frac{u_{i+1,j} - u_{i,j}}{2} \tag{8}$$

$$u_{i-\frac{1}{2},j} = \frac{u_{i,j} - u_{i-1,j}}{2}$$

Similarly, the component in the η can also be decomposed as:

$$\frac{\partial u}{\partial \eta} = \left(u_{i,j+\frac{1}{2}} - u_{i,j-\frac{1}{2}}\right) \tag{9}$$

Where the two terms follows

$$u_{i,j+\frac{1}{2}} = \frac{u_{i,j+1} - u_{i,j}}{2} \tag{10}$$

$$u_{i,j-\frac{1}{2}} = \frac{u_{i,j} - u_{i,j-1}}{2}$$

Thence, the term $\frac{\partial u}{\partial x}$ can be discretized as:

$$\left.\frac{\partial u}{\partial x}\right|_{i,j} = \left.\frac{\partial u}{\partial \zeta}\zeta_x\right|_{i+\frac{1}{2},j} - \left.\frac{\partial u}{\partial \eta}\eta_x\right|_{i+\frac{1}{2},j} \tag{11}$$

$$= \frac{1}{4}\left(\left.\frac{\partial u}{\partial x}\right|_{i+\frac{1}{2},j} + \left.\frac{\partial u}{\partial x}\right|_{i-\frac{1}{2},j} + \left.\frac{\partial u}{\partial x}\right|_{i,j+\frac{1}{2}} + \left.\frac{\partial u}{\partial x}\right|_{i,j-\frac{1}{2}}\right) \tag{12}$$

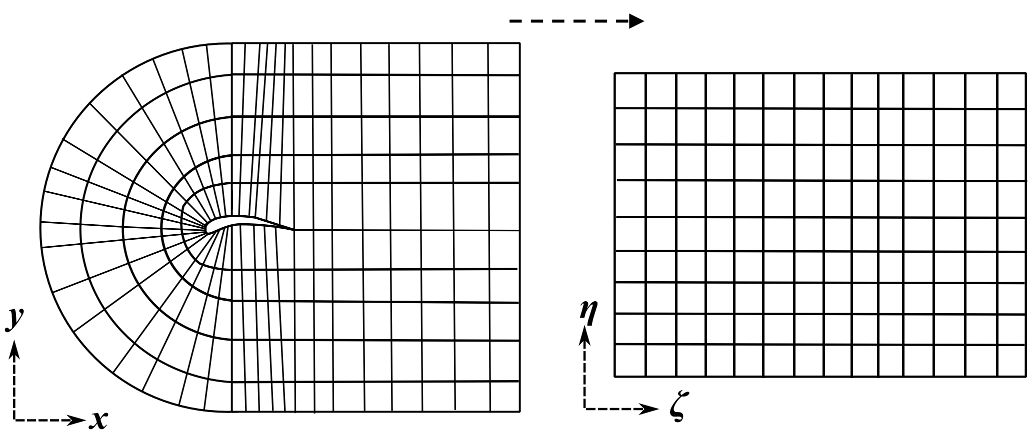

Fig. 2 Schematic for the meshing transformation of the two coordinates with an airfoil.

Here in Fig. 2 we present an example of the meshing transformation based on an airfoil. The

x-y coordinate indicate the original structural meshing corresponds to the coordinate system ζ-η.

With the presented transformation, we can discretize the terms $\vec{H}\cdot\vec{n}$ of the viscosity term as given in Chap. 3 considering a 2D situation:

$$\vec{H}\cdot\vec{n}=\begin{pmatrix} 0 \\ \sigma_{11}n_x+\sigma_{12}n_y \\ \sigma_{12}n_x+\sigma_{22}n_y \\ (u_1\sigma_{11}+u_2\sigma_{12})n_x+(u_1\sigma_{12}+u_2\sigma_{22})n_x+\kappa\left(\frac{\partial T}{\partial x}n_x+\frac{\partial T}{\partial y}n_y\right) \end{pmatrix} \tag{13}$$

Where the derivatives can be transformed through the Green-Gauss integration:

$$\int \frac{\partial T}{\partial x}d\Omega=\oint T\cdot n_x dS \tag{14}$$

$$\int \frac{\partial T}{\partial y}d\Omega=\oint T\cdot n_y dS \tag{15}$$

Referring from Eq. 7 to 8, we can write derivatives

$$\begin{cases} \dfrac{\partial u}{\partial x}=\dfrac{\partial u}{\partial \zeta}\zeta_x+\dfrac{\partial u}{\partial \eta}\eta_x \\ \dfrac{\partial u}{\partial y}=\dfrac{\partial u}{\partial \zeta}\zeta_y+\dfrac{\partial u}{\partial \eta}\eta_y \end{cases} \tag{16}$$

Here, based on Eq. 13, let us consider the derivative terms on the boundary as shown in Fig. 3:

$$\left.\frac{\partial u}{\partial x}\right|_{i+\frac{1}{2}}=\frac{u_{i+1}-u_i}{\Delta\zeta} \tag{17}$$

$$=(u_{i+1}-u_i)\zeta_{x_{i+\frac{1}{2}}}+(u_{j+1}-u_j)\eta_{x_{j+\frac{1}{2}}} \tag{18}$$

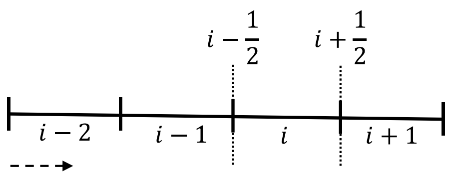

Fig. 3 The meshing points in the row in i-direction.

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