CHAPTER 6. Artificial Dissipative Terms

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Abstract

The artificial dissipative term was raised by A. Jameson in applying Runge-Kutta method solving Euler equations by finite volume method [Jameson et al., 1981]. Here we show how the artificial dissipative term is derived from the flux H.

We first start with the Euler equation:

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}=0 \tag{1}$$

We apply the finite volume method as given in Chap. 3 on the equation and obtains

$$\int\left(\frac{\partial U}{\partial t}\right)dS+\int\left(\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}\right)dS=0 \tag{2}$$

The second term can be transformed through the Green-Gauss transformation:

$$\int\left(\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}\right)dS=\oint_{\partial\Omega}\left(En_x+Fn_y\right)dS \tag{3}$$

Referred from Chap. 3, the term in Eq. 3 is considered as RHS, taking the form:

$$RHS=H_{i+\frac{1}{2},j}-H_{i-\frac{1}{2},j}+H_{i,j+\frac{1}{2}}-H_{i,j-\frac{1}{2}} \tag{4}$$

Referred from Eq. 18 which is given in Chap. 3, the form of Euler equation that discretized by the finite volume method can be written as:

$$\frac{dU}{dt}\Omega+RHS=0 \tag{5}$$

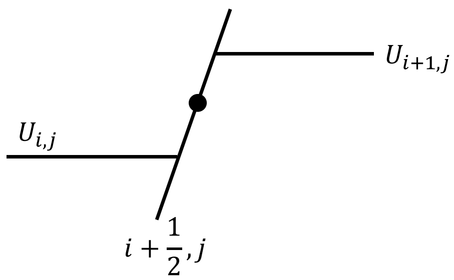

Fig. 1 Discretization on the boundary.

The timing step can be discretized as:

$$U_{i+\frac{1}{2},j}=\frac{1}{2}\left(U_{i+1,j}+U_{i,j}\right) \tag{6}$$

Based on the discretization on the boundary surface, and as referred from Eq. 4, the discretization of the flux H on the boundary surface can be written as:

$$H_{i+\frac{1}{2},j} = \frac{1}{2}\left(H_{i+1,j} + H_{i,j}\right) - D \tag{7}$$

Where D is the artificial dissipative term.

The artificial dissipative term $D = D_{i,j}$ is derived from the discretization of the boundary surface flux $H_{i+\frac{1}{2},j}$. Here, the term $D_{i,j}$ takes the form:

$$D_{i,j} = D_{i+\frac{1}{2},j} - D_{i-\frac{1}{2},j} + D_{i,j+\frac{1}{2}} - D_{i,j-\frac{1}{2}} \tag{8}$$

Where the artificial dissipative term on the boundary surface can be discretized as:

$$D_{i+\frac{1}{2},j} = \frac{H_{i+\frac{1}{2},j}}{\Delta t}\left(\varepsilon^{(2)}_{i+\frac{1}{2},j}\left(U_{i+1,j} - U_{i,j}\right) - \varepsilon^{(4)}_{i+\frac{1}{2},j}\left(U_{i+2,j} - 3U_{i+1,j} + 3U_{i,j} - U_{i-1,j}\right)\right) \tag{9}$$

Where

$$\varepsilon^{(2)}_{i+\frac{1}{2},j} = \kappa^{(2)} max\left(v^{(2)}_{i+1}, v^{(2)}_{j}\right) \tag{10}$$

$$\varepsilon^{(4)}_{i+\frac{1}{2},j} = max\left(0, \left(\kappa^{(4)} - \varepsilon^{(2)}_{i+\frac{1}{2},j}\right)\right) \tag{11}$$

Hence, based on Eq. 7, and the discretization of the artificial dissipative term on the boundary surface shown from Eq. 8 and 9, we deduce that

$$H_{i+\frac{1}{2},j} = \frac{1}{2}\left(H_{i,j} - H_{i+1,j}\right) - \frac{1}{2} D_{i+\frac{1}{2},j} \tag{9}$$

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