Chapter 1

Algorithm Description

The algorithm described in Chapter!!!Unresolved reference!!!3 was implemented in Python programming language. An open source machine vision library OpenCV was utilized to perform feature extraction and tracking. The feature extraction method used was the Shi-Tomasi corner detector. Feature tracking was accomplished through the pyramid implementation of Lucas-Kanade optical flow method .

1.1 Camera Centric Inverse Depth Parameterization

The standard way of describing a feature's position is to use the Euclidean XYZ parameterization. In practical outdoor range estimation problem, the algorithm must deal with features located at near infinity. Inverse depth parameterization overcomes this problem by representing range d in its inverse form $\rho = 1/d$. In addition, features at infinity contribute in estimating the camera rotational motion even though they offer little information on camera translational motion. Furthermore, inverse depth parameterization allows us to initialize features into the EKF framework before it is safely triangulated.

The inverse depth parameterization used in this work was first introduced in . All features and camera positions are referred to a world reference frame. When used in an Extended Kalman Filter framework, the system suffers decreasing linearity when camera is moving away from the world origin. A modified method which uses the camera center as the origin was proposed in . Our work has adopted the camera centered approach with minor modification to integrate the inertial measurements.

A scene point p_i^C can be defined by 6 parameters, with the superscript C representing a camera reference frame. ():

$$p_i^C = \begin{bmatrix} x_i^C & y_i^C & z_i^C & \rho_i & \varphi_i^C & \theta_i^C \end{bmatrix} \tag{1}$$

The first three parameters $[x_i^C, y_i^C, z_i^C]$ represent the initial position where the feature is first observed. ρ_i is the inverse distance from the initialization position to the feature. The elevation-azimuth pair $[\phi_i^C, \theta_i^C]$ encodes a unit vector pointing from the initialization point to the feature. The vector is given by

$$m(\phi_i^C, \theta_i^C) = \begin{bmatrix} \cos\varphi_i^C \cos\theta_i^C \\ \cos\varphi_i^C \sin\theta_i^C \\ \sin\varphi_i^C \end{bmatrix} \tag{2}$$

1.2 Modeling the System with Extended Kalman Filter

1.2.1 Full State Vector

The EKF state vector is defined as

$$x = \begin{bmatrix} OX_W^C & c^C & r^C & p_1^C & p_2^C & \dots \end{bmatrix} \tag{3}$$

where $OX_W^C = \begin{bmatrix} O_x^C & O_y^C & O_z^C & W_x^C & W_y^C & W_z^C \end{bmatrix}^T$ contains translation parameters $O_{x,y,z}^C$ and rotation parameters $W_{x,y,z}^C$ to transform the camera reference frame to the world reference frame. $\left(c^C, r^C\right)^T$ represents the camera translation and rotation motion frame by frame in Euclidean coordinates, and p_i^C contains the feature parameters as described in the previous section.

1.2.2 Prediction

(Will be updated to a more complete form)

For a prediction step at time k, the world frame and features parameters are kept unchanged from time $k-1$. The camera parameters are updated using the new inertial measurements: velocity v^C, acceleration a^C, and rate of change in roll/pitch/yaw w^C. The camera motion parameters at time k are then

$$\begin{bmatrix} OX_W^C & c^C & r^C & p_1^C & p_2^C & \vdots & p_n^C \end{bmatrix}_k^T$$
$$= \begin{bmatrix} OX_{W,k-1}^C & c_{measured}^C & r_{measured}^C & p_{1,k-1}^C & p_{2,k-1}^C & \vdots & p_{n,k-1}^C \end{bmatrix}_{k-1}^T \tag{4}$$

Where

$$c_{measured}^C = v_{measured}^C \Delta t + \frac{1}{2} a_{measured}^C \Delta t^2$$

$$r_{measured}^C = r_{k-1}^C + w_{measured}^C$$

1.2.3 Measurement Model

Each observed feature is related to the camera motion through the measurement model (). This relationship enables a correction on the camera motion and features parameters based on the features' locations observed in the image.

For a feature p_i^C, the vector h^Rpointing from the predicted camera location to the feature initialization position is

$$h_k^R = \begin{bmatrix} x_i^C \\ y_i^C \\ z_i^C \end{bmatrix}_k - \begin{bmatrix} c_x^C \\ c_y^C \\ c_z^C \end{bmatrix}_k \tag{5}$$

The normalized vector pointing from the predicted camera position to the feature at time k is then

$$h_k^C = Q^{-1}\left(r_k^C\right)\left(\rho_k h_k^R + m\left(\varphi_k^C, \theta_k^C\right)\right) \tag{6}$$

where $Q^{-1}(r_k^C)$ is the inverse rotation matrix from the camera frame at time $k-1$ to camera frame at time k. From vector h_k^C, the feature location on image plane can be found by

$$h_k^U = \begin{bmatrix} u_k \\ v_k \end{bmatrix} = \begin{bmatrix} \frac{s_x h_{y,k}^C}{h_{x,k}^C} \\ \frac{s_y h_{z,k}^C}{h_{x,k}^C} \end{bmatrix} \tag{7}$$

where s_x and s_y is the scaling factor of the projection, obtained through camera calibration.

1.2.4 Composition Step

Update step corrects the camera motion and feature location in camera frame k-1. To continue to the next cycle of tracking, all parameter must be transform to camera frame k. World reference point coordinate and orientation from k-1 to k is related by

$$\begin{bmatrix} O_x^{C_k} \\ \\ O_y^{C} \\ \\ O_z^{C} \end{bmatrix}_k = R^{-1}(r_k^{C_{k-1}}) \left(\begin{bmatrix} O_x^{C_{k-1}} \\ \\ O_y^{C_{k-1}} \\ \\ O_z^{C_{k-1}} \end{bmatrix}_k - \begin{bmatrix} c_x^{C_{k-1}} \\ \\ c_y^{C_{k-1}} \\ \\ c_z^{C_{k-1}} \end{bmatrix}_k \right) \tag{8}$$

$$\begin{bmatrix} W_x^{C_k} \\ \\ W_y^{C_k} \\ \\ W_z^{C_k} \end{bmatrix}_{k-1} = \begin{bmatrix} W_x^{C_{k-1}} \\ \\ W_y^{C_{k-1}} \\ \\ W_z^{C_{k-1}} \end{bmatrix}_k - r^{C_{k-1}} \tag{9}$$

Feature parameters in new camera frame are related to the previous frame by

$$\begin{bmatrix} x_i^{C_k} \\ \\ y_i^{C_k} \\ \\ z_i^{C_k} \end{bmatrix}_k = Q^{-1}(r^{C_{k-1}}) \left(\begin{bmatrix} x_i^{C_{k-1}} \\ \\ y_i^{C_{k-1}} \\ \\ z_i^{C_{k-1}} \end{bmatrix}_k - \begin{bmatrix} c_x^{C_{k-1}} \\ \\ c_y^{C_{k-1}} \\ \\ c_z^{C_{k-1}} \end{bmatrix}_k \right) \tag{10}$$

$$\begin{bmatrix} \rho_i \\ \\ \varphi_i^{C_k} \\ \\ \theta_i^{C_k} \end{bmatrix}_k = \begin{bmatrix} \rho_{i,k} \\ \\ m^{-1}(R^{-1}(r^{C_{k-1}})m(\varphi_{i,k}^{C_{k-1}}, \theta_{i,k}^{C_{k-1}}) \end{bmatrix} \tag{11}$$

where $m(\varphi_{i,k}^{C_{k-1}}, \theta_{i,k}^{C_{k-1}})$ is the unit vector pointing from the initialization point to the feature seen by the camera at step $k-1$

The covariance matrix is also affected by this transform. Therefore must be updated. The new covariance matrix is related to the old one by

$$P_k^{C_k} = J_{C_{k-1}\to C_k} P_k^{C_{k-1}} J_{C_{k-1}\to C_k}^T \tag{12}$$

The calculation of $J_{C_{k-1}\to C_k}$ is the same as the linearization of prediction matrix in section Method 2.

In order to apply the correction and update the camera reference frame to the new camera position, an additional composition step is necessary. The world reference frame parameters and features parameters are updated by applying reference frame transformation from the camera location at time $k-1$ to camera location at time k. The EKF covariance matrix P_k is also updated through

$$P_k = JP_kJ^T \tag{13}$$

where J is the Jacobian of the composition equations.

1.3 Initialization

1.3.1 Initialize the State Vector

State vectors are initialized at the first frame. The world origin coordinate and orientation, camera motions, and the feature reference points are all initialized to zeros, with variance equals to the smallest machine number. Thus,

$$OX_W^C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

$$c^C = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{15}$$

$$r^C = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{16}$$

$$p_i^C = \begin{bmatrix} 0 & 0 & 0 & \rho_i & \varphi_i & \theta_i \end{bmatrix} \tag{17}$$

The inverse distance ρ of all features are initialized to 0.1 because we are dealing with long distance object. The features elevation-azimuth pair $[\varphi_i^C, \theta_i^C]$ is extracted from features coordinates in image plane. First, a vector pointing from camera optical center to feature can be defined by

$$h^C = \begin{bmatrix} h_x^C \\ h_y^C \\ h_z^C \end{bmatrix} = \begin{bmatrix} 1 \\ u \cdot s_x \\ v \cdot s_y \end{bmatrix} \tag{18}$$

Where $[uv]$ is the feature coordinate in the image, and $[s_x s_y]$is the scaling factor of the projection from the scene to image plane. The elevation-azimuth pair $[\varphi_i^C, \theta_i^C]$ can then be directly calculated from h^C

$$\varphi = arctan \left(\frac{h_z^C}{\sqrt{h_x^{C2} + h_y^{C2}}} \right) \tag{19}$$

$$\theta = arctan \left(\frac{h_y^C}{h_x^C} \right) \tag{20}$$

1.3.2 Initialized the State Covariance Matrix

Because the world origin is defined at the first frame, it enables initializing the filter with minimum variance, which helps reducing the lower bound of the filter error. The covariance matrix of the world coordinate and orientation, and the camera motion is

$$P = I_{12\times 12} \cdot \epsilon \tag{21}$$

where I is a 12×12 identity matrix, and ϵ is the lowest significant bit (LSB) of a machine.

The covariance of features is added one by one as there is correlation between them. For every new feature added, the new covariance matrix becomes

$$P_{new} = J \begin{bmatrix} P_{old} & 0 \\ 0 & R \end{bmatrix} J^T \tag{22}$$

where P_{old} isthe covariance matrix of the existing state vector, and the initial P_{old} is defined in . Matrix R is the covariance matrix of the variable in features initialization.

$$R = \begin{bmatrix} \sigma_{x_i^C} & & & & & \\ & \sigma_{y_i^C} & & 0 & & \\ & & \sigma_{z_i^C} & & & \\ & & & \sigma_\rho & & \\ & 0 & & & \sigma_{image} & \\ & & & & & \sigma_{image} \end{bmatrix} = \begin{bmatrix} \epsilon & & & & & \\ & \epsilon & & 0 & & \\ & & \epsilon & & & \\ & & & 0.1 & & \\ & 0 & & & 1 & \\ & & & & & 1 \end{bmatrix} \tag{23}$$

where$[\sigma_{x_i^C} \sigma_{y_i^C} \sigma_{z_i^C}]$ is the uncertainty of the camera optical center position, initialized to ϵ.$\sigma_{image}$is the image plane pixel variance, set to 1. $\sigma_\rho$is the uncertainty of the inverse distance. Because the filter mainly deals with distance feature, σ_ρ is initialized to 0.1 to cover any distance from 50 meters to infinity.

J in equation?? is the Jacobian matrix for the initialization equation.

$$J = \begin{bmatrix} & & & & & & 0 \\ & & & I & & & \vdots \\ & & & & & & 0 \\ \frac{\partial p_i}{\partial OX_W^C} & \frac{\partial p_i}{\partial c^C} & \frac{\partial p_i}{\partial r^C} & 0 & \dots & 0 & \frac{\partial p_i}{\partial g_i} \end{bmatrix} \tag{24}$$

Whenever a new feature is added, it's initial position is always $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ in the camera centric coordinate, and the $\begin{bmatrix} \rho & \varphi & \theta \end{bmatrix}$ parameters are not a function of OX_W^C, c^C,orr^C, therefore

$$\frac{\partial p_i}{\partial OX_W^C} = 0_{6\times 6} \tag{25}$$

$$\frac{\partial p_i}{\partial c^C} = 0_{6\times 3} \frac{\partial p_i}{\partial r^C} = 0_{6\times 3} \tag{26}$$

J can then be simplified as

$$J = \begin{bmatrix} I & 0 & \\ 0 & \frac{\partial p_i}{\partial g_i} & \end{bmatrix} \tag{27}$$

whereg_i includes the variable in matrix R: $g_i = [x_i^C \quad y_i^C \quad z_i^C \quad \rho_i \quad u_i \quad v_i]$. Then

$$
\frac{\partial p_i}{\partial g_i} = \begin{bmatrix} I_{3\times3} & & 0_{3\times3} & \\ & \frac{\partial \rho_i}{\partial \rho_i} & \frac{\partial \rho_i}{\partial u_i} & \frac{\partial \rho_i}{\partial v_i} \\ 0_{3\times3} & \frac{\partial \varphi_i^C}{\partial \rho_i} & \frac{\partial \varphi_i^C}{\partial u_i} & \frac{\partial \varphi_i^C}{\partial v_i} \\ & \frac{\partial \theta_i^C}{\partial \rho_i} & \frac{\partial \theta_i^C}{\partial u_i} & \frac{\partial \theta_i^C}{\partial v_i} \end{bmatrix} = \begin{bmatrix} I_{3\times3} & & 0_{3\times3} & \\ & 1 & 0 & 0 \\ 0_{3\times3} & 0 & \frac{\partial \varphi_i^C}{\partial u_i} & \frac{\partial \varphi_i^C}{\partial v_i} \\ & 0 & \frac{\partial \theta_i^C}{\partial u_i} & \frac{\partial \theta_i^C}{\partial v_i} \end{bmatrix} \tag{28}
$$

Based on the rule of derivation,

$$
\frac{\partial \varphi_i^C}{\partial u_i} = \frac{\partial \varphi_i^C}{\partial h^C} \frac{\partial h^C}{\partial u_i}
$$

$$
\frac{\partial \theta_i^C}{\partial u_i} = \frac{\partial \theta_i^C}{\partial h^C} \frac{\partial h^C}{\partial u_i}
$$

$$
\frac{\partial \varphi_i^C}{\partial v_i} = \frac{\partial \varphi_i^C}{\partial h^C} \frac{\partial h^C}{\partial v_i}
$$

$$
\frac{\partial \theta_i^C}{\partial v_i} = \frac{\partial \theta_i^C}{\partial h^C} \frac{\partial h^C}{\partial v_i}
$$

and

$$
\frac{\partial \varphi_i^C}{\partial h^C} = \begin{bmatrix} \frac{\partial \varphi_i^C}{\partial h_x^C} & \frac{\partial \varphi_i^C}{\partial h_y^C} & \frac{\partial \varphi_i^C}{\partial h_z^C} \end{bmatrix} = \begin{bmatrix} \frac{-h_x^C \cdot h_z^C}{({h_x^C}^2 + {h_y^C}^2 + {h_z^C}^2)\sqrt{{h_x^C}^2 + {h_y^C}^2}} \\ \frac{-h_y^W \cdot h_z^W}{({h_x^C}^2 + {h_y^C}^2 + {h_z^C}^2)\sqrt{{h_x^C}^2 + {h_y^C}^2}} \\ \frac{\sqrt{{h_x^C}^2 + {h_y^C}^2}}{({h_x^C}^2 + {h_y^C}^2 + {h_z^C}^2)} \end{bmatrix}^T \tag{29}
$$

$$
\frac{\partial \theta_i^C}{\partial h^C} = \begin{bmatrix} \frac{\partial \theta_i^C}{\partial h_x^C} & \frac{\partial \theta_i^C}{\partial h_y^C} & \frac{\partial \theta_i^C}{\partial h_z^C} \end{bmatrix} = \begin{bmatrix} -\frac{h_y^C}{{h_x^C}^2 + {h_y^C}^2} & \frac{h_x^C}{{h_x^C}^2 + {h_y^C}^2} & 0 \end{bmatrix} \tag{30}
$$

$$\frac{\partial h^C}{\partial u_i} = \begin{bmatrix} \frac{\partial h_x^C}{\partial u_i} \\ \frac{\partial h_y^C}{\partial u_i} \\ \frac{\partial h_z^C}{\partial u_i} \end{bmatrix} = \begin{bmatrix} 0 \\ s_x \\ 0 \end{bmatrix} \tag{31}$$

$$\frac{\partial h^C}{\partial v_i} = \begin{bmatrix} \frac{\partial h_x^C}{\partial v_i} \\ \frac{\partial h_y^C}{\partial v_i} \\ \frac{\partial h_z^C}{\partial v_i} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ s_y \end{bmatrix} \tag{32}$$

Linearization of Measurement Model

The measurement model of the algorithm is not linear, and must be linearized by evaluating first derivative of the measurement equation at the predicted camera location. Since the measurement model is not a function of world origin coordinate and orientation, the Jacobian matrix of the measurement model is

$$\begin{aligned}\frac{\partial h_k^U(x)}{\partial x} &= \begin{bmatrix} 0_{2n\times 6} & \frac{\partial h_k^U(x)}{\partial c^C} & \frac{\partial h_k^U(x)}{\partial r^C} & \frac{\partial h_k^U(x)}{\partial p_1} & \dots & \frac{\partial h_k^U(x)}{\partial p_n} \end{bmatrix} = \\ &\begin{bmatrix} 0_{2n\times 6} & \frac{\partial h_k^U}{\partial h_k^C}\cdot\frac{\partial h_k^C}{\partial c^C} & \frac{\partial h_k^U}{\partial h_k^C}\cdot\frac{\partial h_k^C}{\partial r^C} & \frac{\partial h_k^U}{\partial h_k^C}\cdot\frac{\partial h_k^C}{\partial p_1} & \dots & \frac{\partial h_k^U}{\partial h_k^C}\cdot\frac{\partial h_k^C}{\partial p_n} \end{bmatrix}\end{aligned} \tag{33}$$

Elements in the matrix above are given in - . The formula for calculating Q^{-1}in - can be found in

$$\frac{\partial h_k^U}{\partial h_k^C} = \begin{bmatrix} -\frac{s_x h_{y,k}^C}{{h_{x,k}^C}^2} & \frac{s_x}{h_{x,k}^C} & 0 \\ -\frac{s_y h_{z,k}^C}{{h_{x,k}^C}^2} & 0 & \frac{s_y}{h_{x,k}^C} \end{bmatrix} \tag{34}$$

$$\frac{\partial h^C}{\partial c^C} = -R^{-1}(r_k^C)\rho_k \tag{35}$$

$$\frac{\partial h(x)}{\partial r^C} = \begin{bmatrix} \frac{\partial h^C}{\partial r_x^C} & \frac{\partial h^C}{\partial r_y^C} & \frac{\partial h^C}{\partial r_z^C} \end{bmatrix} \tag{36}$$

$$\frac{\partial h^C}{\partial r_x} = \begin{bmatrix} 0 \\ h_x^C(sin(r_x)sin(r_z) + cos(r_x)cos(r_z)sin(r_y)) + h_y^C(-cos(r_z)sin(r_x) + cos(r_x)sin(r_y)sin(r_z)) \\ h_x^C(cos(r_x)\sin(r_z) - cos(r_z)sin(r_x)sin(r_y)) + h_y^C(-cos(r_x)cos(r_z) - sin(r_x)sin(r_y)sin(r_z)) \end{bmatrix} \tag{37}$$

$$\frac{\partial h^C}{\partial r_y} = \begin{bmatrix} -h_z^C\cos(r_y) - h_x^C\cos(r_z)sin(r_y) - h_y^C sin(r_y)sin(r_z) \\ -h_z^C sin(r_x)sin(r_y) + h_x^C cos(r_y)cos(r_z)sin(r_x) + h_y^C cos(r_y)sin(r_x)sin(r_z) \\ -h_z^C cos(r_x)sin(r_y) + h_x^C cos(r_x)cos(r_y)cos(r_z) + h_y^C cos(r_x)cos(r_y)sin(r_z) \end{bmatrix} \tag{38}$$

$$\frac{\partial h^C}{\partial r_z} = \begin{bmatrix} h_y^C cos(r_y)cos(r_z) - h_x^C cos(r_y)sin(r_z) \\ h_x^C(-cos(r_x)\cos(r_z) - sin(r_x)\sin(r_y)\sin(r_z)) + h_y^C(-cos(r_x)sin(r_z) + cos(r_z)sin(r_x)sin(r_y \\ h_x^C(cos(r_z)sin(r_x) - cos(r_x)sin(r_y)sin(r_z)) + h_y^C(sin(r_x)sin(r_z) + cos(r_x)cos(r_z)sin(r_y)) \end{bmatrix} \tag{39}$$

$$\frac{\partial h^C}{\partial p_i} = \begin{bmatrix} \frac{\partial h^C}{\partial x_i^C} & \frac{\partial h^C}{\partial y_i^C} & \frac{\partial h^C}{\partial z_i^C} & \frac{\partial h^C}{\partial \rho_i} & \frac{\partial h^C}{\partial \theta_i} & \frac{\partial h^C}{\partial \varphi_i} \end{bmatrix} \tag{40}$$

Equation above is a 3x6 matrix

$$\begin{bmatrix} \frac{\partial h^C}{\partial x_i^C} & \frac{\partial h^C}{\partial y_i^C} & \frac{\partial h^C}{\partial z_i^C} \end{bmatrix} = Q^{-1}(r_k^C)\rho_k \tag{41}$$

$$\frac{\partial h^C}{\partial \rho_i} = -Q^{-1}(r_k^C) \cdot c_k \tag{42}$$

$$\frac{\partial h^C}{\partial \theta_i} = Q^{-1}(r_k^C) \begin{bmatrix} -cos(\varphi_i)\sin(\theta_i) \\ \\ \cos(\varphi_i)cos(\theta_i) \\ \\ 0 \end{bmatrix} \tag{43}$$

$$\frac{\partial h^C}{\partial \varphi_i} = Q^{-1}(r_k^C) \begin{bmatrix} -cos(\varphi_i)sin(\theta_i) \\ \\ -sin(\varphi_i)sin(\theta_i) \\ \\ cos?(\varphi_i) \end{bmatrix} \tag{44}$$

Jacobian Matrix of Composition Equations

$$J_{C_{k-1} \to C_k} = \frac{\partial x_k}{\partial x_{k-1}} = \begin{bmatrix} \frac{\partial x_k}{\partial OX_{W,k-1}^C} & \frac{\partial x_k}{\partial c_{k-1}} & \frac{\partial x_k}{\partial r_{k-1}} & \frac{\partial x_k}{\partial p_{1,k-1}^C} & \frac{\partial x_k}{\partial p_{2,k-1}^C} & \dots \end{bmatrix} \tag{45}$$

$\frac{\partial x_k}{\partial OX_{W,k-1}^C}$is given by:

$$\frac{\partial x_k}{\partial OX^C_{W,k-1}} = \begin{bmatrix} \frac{\partial O^C_{W,k}}{\partial O^C_{W,k-1}} & \frac{\partial O^C_{W,k}}{\partial W^C_{W,k-1}} \\ \frac{\partial W^C_{W,k}}{\partial O^C_{W,k-1}} & \frac{\partial W^C_{W,k}}{\partial W^C_{W,k-1}} \\ \frac{\partial c_k}{\partial O^C_{W,k-1}} & \frac{\partial c_k}{\partial W^C_{W,k-1}} \\ \frac{\partial r_k}{\partial O^C_{W,k-1}} & \frac{\partial r_k}{\partial W^C_{W,k-1}} \\ \frac{\partial p^C_i}{\partial O^C_{W,k-1}} & \frac{\partial p^C_i}{\partial W^C_{W,k-1}} \end{bmatrix} = \begin{bmatrix} Q^{-1}(r^C) & 0 \\ 0 & I_{3\times 3} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{46}$$

Derivatives of $x_k$with respect to c and $p$are given in -

$$\begin{bmatrix} \frac{\partial O^C_{W,k}}{\partial c_{k-1}} & \frac{\partial O^C_{W,k}}{\partial r_{k-1}} \\ \frac{\partial W^C_{W,k}}{\partial c_{k-1}} & \frac{\partial W^C_{W,k}}{\partial r_{k-1}} \\ \frac{\partial c_k}{\partial c_{k-1}} & \frac{\partial c_k}{\partial r_{k-1}} \\ \frac{\partial r_k}{\partial c_{k-1}} & \frac{\partial r_k}{\partial r_{k-1}} \\ \frac{\partial p(0:2)^C_i}{\partial c_{k-1}} & \frac{\partial p(0:2)^C_i}{\partial r_{k-1}} \\ \frac{\partial p(3:5)^C_i}{\partial c_{k-1}} & \frac{\partial p(3:5)^C_i}{\partial r_{k-1}} \end{bmatrix} = \begin{bmatrix} -Q^{-1}(r_k^{C_{k-1}}) & \frac{\partial Q^{-1}(r_k^{C_{k-1}})}{\partial r_{k-1}} \\ 0_{3\times 3} & -I_{3\times 3} \\ I_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & I_{3\times 3} \\ -Q^{-1}(r_k^{C_{k-1}}) & \frac{\partial Q^{-1}(r_k^{C_{k-1}})}{\partial r_{k-1}} \\ 0_{3\times 3} & pr \end{bmatrix} \tag{47}$$

where

$$pr = \begin{bmatrix} 1 & 0 & 0 \\ & \frac{\partial \varphi^C_{i,k}}{\partial r_{k-1}} & \\ & \frac{\partial \theta^C_{i,k}}{\partial r_{k-1}} & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ & \frac{\partial m^{-1}(Q^{-1}(r_k^{C_{k-1}})m(\varphi^{C_{k-1}}_{i,k}, \theta^{C_{k-1}}_{i,k}))}{\partial r_{k-1}} & \end{bmatrix} \tag{48}$$

Let $M = Q^{-1}(r_k^{C_{k-1}})m(\varphi^{C_{k-1}}_{i,k}, \theta^{C_{k-1}}_{i,k})$, and $m^{-1} = m^{-1}(M)$, then

$$\frac{\partial m^{-1}}{\partial r_k^{C_{k-1}}} = \frac{\partial m^{-1}}{\partial M} \cdot \frac{\partial M}{\partial r_k^{C_{k-1}}} \tag{49}$$

where

$$\frac{\partial M}{\partial r_k^{C_{k-1}}} = \frac{\partial Q^{-1}(r_k^{C_{k-1}})}{\partial r_{k-1}} \cdot m(\varphi_{i,k}^{C_{k-1}}, \theta_{i,k}^{C_{k-1}}) \tag{50}$$

as m^{-1} isthe function that calculate $\begin{bmatrix} \varphi & \theta \end{bmatrix}$ from a vector, $\frac{\partial m^{-1}}{\partial M}$ is the same as and .

Derivatives of x_k with respect to feature parameters p_i^C are given in

$$\frac{\partial x_k^{C_k}}{\partial p_{i,k}^{C_{k-1}}} = \begin{bmatrix} \frac{\partial O_{W,k}^{C_k}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial W_{W,k}^{C_k}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial c_k^{C_k}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial r_k^{C_k}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial p_{i,k}^{C}}{\partial p_{i,k}^{C_{k-1}}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\partial p_{i,k}^{C_k}}{\partial p_{i,k}^{C_{k-1}}} \end{bmatrix} \tag{51}$$

$$\frac{\partial p_{i,k}^{C_k}}{\partial p_{i,k}^{C_{k-1}}} = \begin{bmatrix} \frac{\partial x_{i,k}^{C_k}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial y_{i,k}^{C}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial z_{i,k}^{C}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial \rho_{i,k}^{C}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial \varphi_{i,k}^{C}}{\partial p_{i,k}^{C_{k-1}}} \\ \frac{\partial \theta_{i,k}^{C}}{\partial p_{i,k}^{C_{k-1}}} \end{bmatrix} = \begin{bmatrix} Q^{-1}(r_k^{C_{k-1}}) & 0_{3\times 3} \\ 0_{3\times 3} & \frac{\partial p(3{:}5)_{i,k}^{C_k}}{\partial p(3{:}5)_{i,k}^{C_{k-1}}} \end{bmatrix} \tag{52}$$

$$\frac{\partial p(3:5)_{i,k}^{C_k}}{\partial p(3:5)_{i,k}^{C_{k-1}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\partial \varphi_{i,k}^{C_k}}{\partial \varphi_{i,k}^{C_{k-1}}} & \frac{\partial \varphi_{i,k}^{C_k}}{\partial \theta_{i,k}^{C_{k-1}}} \\ 0 & \frac{\partial \theta_{i,k}^{C_k}}{\partial \varphi_{i,k}^{C_{k-1}}} & \frac{\partial \theta_{i,k}^{C_k}}{\partial \theta_{i,k}^{C_{k-1}}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\partial \varphi_{i,k}^{C_k}}{\partial M} \frac{\partial M}{\partial \varphi_{i,k}^{C_{k-1}}} & \frac{\partial \varphi_{i,k}^{C_k}}{\partial M} \frac{\partial M}{\partial \theta_{i,k}^{C_{k-1}}} \\ 0 & \frac{\partial \theta_{i,k}^{C_k}}{\partial M} \frac{\partial M}{\partial \varphi_{i,k}^{C_{k-1}}} & \frac{\partial \theta_{i,k}^{C_k}}{\partial M} \frac{\partial M}{\partial \theta_{i,k}^{C_{k-1}}} \end{bmatrix} \tag{53}$$

in which $\frac{\partial \varphi_{i,k}^{C_k}}{\partial M}$ is the same as equation **??**. $\frac{\partial \theta_{i,k}^{C_k}}{\partial M}$ is the same as equation **??** . $\frac{\partial M}{\partial \varphi_{i,k}^{C_{k-1}}}$ and $\frac{\partial M}{\partial \theta_{i,k}^{C_{k-1}}}$ is given below.

$$\frac{\partial M}{\partial \varphi_{i,k}^{C_{k-1}}} = R^{-1}(r^C) \begin{bmatrix} -\cos(\theta_{i,k}^{C_{k-1}}) \sin(\varphi_{i,k}^{C_{k-1}}) \\ -\sin(\varphi_{i,k}^{C_{k-1}}) \sin(\theta_{i,k}^{C_{k-1}}) \\ \cos(\varphi_{i,k}^{C_{k-1}}) \end{bmatrix} \tag{54}$$

$$\frac{\partial M}{\partial \theta_{i,k}^{C_{k-1}}} = R^{-1}(r^C) \begin{bmatrix} -\cos(\varphi_{i,k}^{C_{k-1}}) \sin(\theta_{i,k}^{C_{k-1}}) \\ \cos(\varphi_{i,k}^{C_{k-1}}) \cos(\theta_{i,k}^{C_{k-1}}) \\ 0 \end{bmatrix} \tag{55}$$

Adding and Deleting Features

Features that move out of the camera's FOI are removed from the filter. The removal is done by directly deleting the parameters from the state vector, and the related rows and columns from the state covariance matrix. The parameters of the deleted features are still recorded into the database, and remained unchanged for all iteration after the removal unless a GPS bundle correction occurs.

To maintain sufficient amount of features to generate map, new features are acquired and added to the filter if a feature deletion procedure occurred, or number of tracked feature is lower than a threshold. The feature addition procedure occurs after the composition step of an iteration, and follows the same procedure as described in section .

Bundle Correction with GPS (not yet implemented)

Apply overall correction on the map at a sparser time interval using the GPS positions.

Processing for Comparison to Ground Truth Data

The accuracy of the result was analyzed by comparing to the digital elevation models (DEM) .To make the comparison, the features coordinate and the DEM data must be brought to the same coordinate system. For ease of viewing, the world coordinate is used. The features' positions can be converted to Euclidean representation in world frame by:

$$\begin{bmatrix} X_i^W \\ Y_i^W \\ Z_i^W \end{bmatrix} = Q^{-1}(O^c_{XYZ}, W^c_{XYZ})(\begin{bmatrix} x_i^C \\ y_i^C \\ z_i^C \end{bmatrix} + \frac{1}{\rho_i} m(\varphi_i^C, \theta_i^C)) \tag{56}$$

The DEM is converted into the world frame using the UAV's initial GPS location $\begin{bmatrix} Latitude_{init} & Longitude_{init} & Height_{init} \end{bmatrix}$ and orientation $\begin{bmatrix} Roll_{init} & Pitch_{init} & Azimuth_{init} \end{bmatrix}$. First, the UAV's GPS latitude and longitude is converted into UTM representation $\begin{bmatrix} Northing_{init} & Easting_{init} & Height_{init} \end{bmatrix}$. Then, a transformation matrix can be defined as followed:

$$Q_{Roll}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(Roll_{init}) & \sin(Roll_{init}) & 0 \\ 0 & -sin(Roll_{init}) & \cos(Roll_{init}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{57}$$

$$Q_{Pitch}^{-1} = \begin{bmatrix} \cos(Pitch_{init} + \pi) & 0 & -\sin(Pitcch_{init} + \pi) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(Pitch_{init} + \pi) & 0 & \cos(Pitch_{init} + \pi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{58}$$

$$Q_{Azimuth}^{-1} = \begin{bmatrix} \cos(Azimuth_{init} + \frac{\pi}{2}) & \sin(Azimuth_{init} + \frac{\pi}{2}) & 0 & 0 \\ -sin(Azimuth_{init} + \frac{\pi}{2}) & \cos(Azimuth_{init} + \frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{59}$$

$$Q_{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -Northing_{init} \\ 0 & 1 & 0 & -Easting_{init} \\ 0 & 0 & 1 & -Height_{init} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{60}$$

$$Q^{-1} = Q_{T}^{-1} \cdot Q_{Roll}^{-1} \cdot Q_{Pitch}^{-1} \cdot Q_{Azimuth}^{-1} \tag{61}$$

At last, the DEM in world coordinate is given by

$DEM_{World} = Q^{-1} \cdot DEM_{UTM}$

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