

Distance and Angle Measurement of Distant Objects on an Oblique Plane Based on Pixel Variation of CCD Image

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Abstract—This paper presents an image-based system for measuring target objects on an oblique plane based on pixel variation of CCD images for digital cameras by referencing to two arbitrarily designated points in image frames. Based on an established relationship between the displacement of the camera movement along the photographing direction and the difference in pixel counts between reference points in the images, photographing distance between the camera and an object on the oblique target plane can be calculated via the proposed method.

Keywords— distance measurement; pixels; oblique plane; incline angle; digital cameras; CCD images; image-based measuring systems.

I. INTRODUCTION

As far as non-contact distance measurement is concerned, ultrasonic-based [1]-[5] and laser-based [6]-[7] techniques are among the most commonly-used methods. Unfortunately, measurement accuracy via the laser- and ultrasonic-based methods heavily depended on surface reflectivity of the object under measurement. If the reflection surface is undesired, the measuring system generally performed poorly or not at all. These methods also have difficulties in recording images of the objects while measuring distance. Alternatively, imaged-based methods have been proposed for distance measurement by using CCD cameras. Thanks to the advantages in providing a rich source of environment information, imaged-based methods are becoming more and more popular in various applications, for example localization of mobile robots. However, traditional vision-based systems generally required two cameras set up at different positions to capture two different pictures for further analysis. As a result, pattern recognition [8]-[12] or image analysis [13]-[15] of a whole image frame were required to extract features from the images for obtaining the distance measurement. Thus, huge amount of storage capacity and high-speed DSP processors are required for system so established, inevitably resulting in disadvantages in terms of system complexity, processing speed, and establishment cost. As a result, performance of real-time

measurement via the pattern recognition or image analysis methods was generally not satisfactory because of the speed constraint. As an attempt to solve this problem, an image-based distance measuring system (IBDMS) [12], [16] was proposed to measure distance and area using two laser projectors and a CCD camera based on a triangular relationship. Unfortunately, the IBDMS is valid only for measuring objects or surfaces that are perfectly perpendicular to the optical axis. Any tilt in the plane formed by the target surface that intersects the two laser beams will have an adverse effect on the determination of pixel counts. Furthermore, measurement accuracy of the IBDMS depended on the distance between the laser projectors. Incorporation of the measuring system into a digital camera might become cumbersome if higher measuring resolution is required. There was also a distance measurement method based on pixel number variation of images [17]. Unfortunately, this approach failed to solve the problem in measuring objects on an oblique plane, similar to the IDBMS method. It is therefore the objective of this paper to propose an image-based system overcoming the above-mentioned difficulties for measuring objects on an oblique plane.

II. DISTANCE MEASUREMENT BASED ON PIXEL NUMBER VARIATION

Figure 1 shows the relationship between distance and variation of pixel counts at different photographing distances of the distance measuring method. To be applicable, the target plane formed by points A, O, B needs to be perpendicular to the optical axis. Based on a simple relationship between the displacement of the camera movement, Δh , and the difference in pixel counts between the reference points in the images at different photographing distances, we can measure the distance of an object on the perpendicular plane. Assume that actual distance between reference points A and B will not change. We have the following relationship between pixel counts and distances:

$$\frac{D(\overline{AB})}{N_1(\overline{AB})} = \frac{D(h_1)}{N_{\max}} \quad (1)$$

$$\frac{D(\overline{AB})}{N_2(\overline{AB})} = \frac{D(h_2)}{N_{\max}} \quad (2)$$

where points A and B can be arbitrarily chosen by users. $N_1(\overline{AB})$ and $N_2(\overline{AB})$ are the pixel counts between points A and B on the image plane at distances $h_1(O)$ and $h_2(O)$, respectively. Δh is the displacement along the photographing direction due to the movement of the camera, which is fixed and known as a priori. h_s is the distance between the optical origin (OP) and the front end of the CCD camera. From (1) and (2), we have:

$$\frac{D(h_1)}{D(h_2)} = \frac{N_2(\overline{AB})}{N_1(\overline{AB})} \quad (3)$$

Because of the displacement, $\Delta h = h_1(O) - h_2(O)$, resulted from the movement of the camera along the photographing direction, two similar isosceles triangles having bases $D(h_1)$ and $D(h_2)$, respectively, are formed as shown in Fig. 1. We have:

$$\frac{N_1(\overline{AB})}{N_2(\overline{AB})} = \frac{D(h_1)}{D(h_2)} = \frac{h_1(O) + h_s}{h_2(O) + h_s} \quad (4)$$

Substituting $h_1(O) = h_2(O) + \Delta h$ into (4), photographing distances $h_1(O)$ and $h_2(O)$ of the object on the perpendicular plane can be obtained as:

$$h_1(O) = \frac{N_2(\overline{AB})}{N_2(\overline{AB}) - N_1(\overline{AB})} \times \Delta h - h_s \quad (5)$$

or

$$h_2(O) = \frac{N_1(\overline{AB})}{N_2(\overline{AB}) - N_1(\overline{AB})} \times \Delta h - h_s \quad (6)$$

Also, distance between points A and B can be obtained as:

$$\begin{aligned} \overline{AB} &= \frac{2(h_2(O) + h_s) \times \tan \theta_H}{N_{H_max}} \times N_2(\overline{AB}) \\ &= \frac{2(h_1(O) + h_s) \times \tan \theta_H}{N_{H_max}} \times N_1(\overline{AB}) \end{aligned} \quad (7)$$

where θ_H is the maximal horizontal view angle of the camera and N_{H_max} is the maximal pixels in a horizontal scan line of an image frame, which is fixed and known as a priori irrelevant of photographing distances. As a result of this relationship, the distance between an arbitrarily-selected point A and the origin O can also be represented as follows:

$$\begin{aligned} \overline{AO} &= \frac{2(h_2(O) + h_s) \times \tan \theta_H}{N_{H_max}} \times N_2(\overline{AO}) \\ &= \frac{2(h_1(O) + h_s) \times \tan \theta_H}{N_{H_max}} \times N_1(\overline{AO}) \end{aligned} \quad (8)$$

Note that the above-mentioned relationship is only valid for target objects or planes that are perfectly perpendicular to the optical axis. If the surface under measurement is not perpendicular to the optical axis, it would have different relationships. Thus, the design objective of this paper is to establish a new relationship between the displacement of the camera movement and the difference in pixel counts to measure the photographing distance and incline angle.

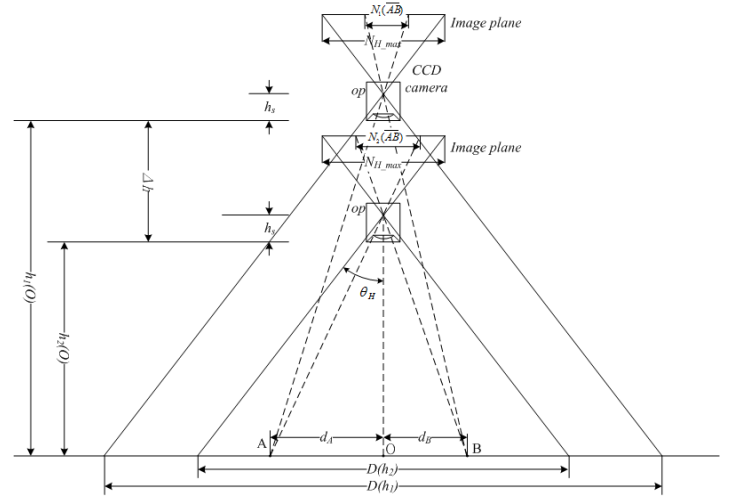


Figure 1. Schematic diagram depicting the relationship between distance and variation of pixel counts at different photographing distances.

III. PROPOSED METHOD FOR MEASURING DISTANCE AND ANGLE

Figure 2 shows the configuration of the proposed image-based system for measuring objects on an oblique surface based on variation of pixel counts, where an incline angle θ_m exists between the oblique and perpendicular planes. Referring to Fig. 2, because point A on the target plane is farther from and point B is closer to the CCD camera, formulas (5) to (8) do not apply in this case.

To solve this problem, virtual planes are introduced to address the problem to measure the incline angle θ_m . Virtual planes, for example planes VP_A and VP_B in Fig. 2, are intangible planes which are perpendicular to the optical axis. Thus, distances between the camera and the virtual planes for different positions can be obtained as:

$$h_1(A) = \frac{N_2(A)}{N_2(A) - N_1(A)} \times \Delta h - h_s \quad (9)$$

$$h_2(A) = \frac{N_1(A)}{N_2(A) - N_1(A)} \times \Delta h - h_s \quad (10)$$

$$h_1(B) = \frac{N_2(B)}{N_2(B) - N_1(B)} \times \Delta h - h_s \quad (11)$$

$$h_2(B) = \frac{N_1(B)}{N_2(B) - N_1(B)} \times \Delta h - h_s \quad (12)$$

where $h_1(A)$ and $h_1(B)$ are distances from the camera at position 1 to virtual plane VP_A and virtual plane VP_B, respectively. $h_2(A)$ and $h_2(B)$ are distances from the camera at position 2 to virtual plane VP_A and virtual plane VP_B, respectively. $N_n(P)$ stands for the pixel counts in the image plane for point P when image is taken at position n .

From (9), (10), (11), and (12), we have:

$$h_1(A) - h_1(B) = h_2(A) - h_2(B) = \Delta k \quad (13)$$

$$\Delta k = (d_A + d_B) \tan \theta_m \quad (14)$$

$$\theta_m = \tan^{-1} \frac{\Delta k}{d_A + d_B} \quad (14)$$

where Δk is the distance between the virtual planes VP_A and VP_B, and θ_m is the incline angle. d_A and d_B represent the distance from point A and point B to the optical axis, respectively, which can be expressed as follows:

$$d_A = \frac{2N_1(A)N_2(A)}{(N_2(A) - N_1(A)) \times N_{H_max}} \times \Delta h \times \tan \theta_H \quad (15)$$

$$d_B = \frac{2N_1(B)N_2(B)}{(N_2(B) - N_1(B)) \times N_{H_max}} \times \Delta h \times \tan \theta_H \quad (16)$$

To this end, the distance from the optical origin at positions 1 and 2 to the oblique plane intersecting the optical axis can be expressed as:

$$h_1(O) = h_1(A) - d_A \tan \theta_m = h_1(B) + d_B \tan \theta_m \quad (17)$$

$$h_2(O) = h_2(A) - d_A \tan \theta_m = h_2(B) + d_B \tan \theta_m$$

In order to determine the position of target objects in the real-world environment, we represent the distance between two arbitrary points A and B as $D(\overline{AB})$, which can be derived as:

$$D(\overline{AB}) = (d_A + d_B) \sec \theta_m \quad (18)$$

$$D(\overline{AB}) = |d_A - d_B| \sec \theta_m \quad (19)$$

Depending on the position of points A and B, different formulas apply. For example, if points A and B lie at different sides of origin O, (18) is used to solve the problem, and (19) is

used to solve the problem when points A and B lie at the same side.

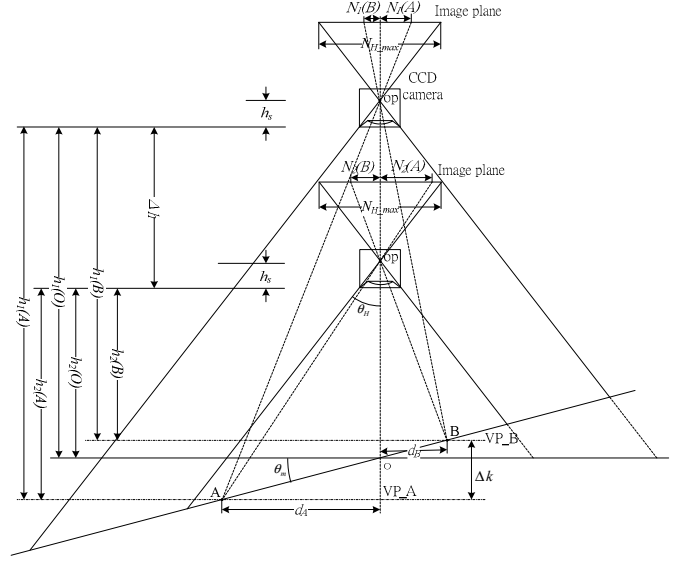


Figure 2. Configuration of the proposed image-based system for measuring objects on an oblique surface based on variation of pixel counts.

IV. EXPERIMENT RESULTS

Figure 3 shows the setup of the proposed image-based system for measuring target objects on an oblique surface based on pixel variation of CCD images. The CCD camera adopted to conduct the experiments in this paper is Canon 400D with $N_{H_max} = 3888$ pixels and $N_{V_max} = 2592$ pixels. Internal parameters of the camera include: $\cot \theta_H = 1.55$, $\cot \theta_V = 2.33$, $h_s = 4.11$ cm, and $\Delta h = 50$ cm, which were obtained by a calibration method in advance. Various photographing distances and incline angles are measured to evaluate the performance of the proposed approach, as shown in Tables 1, 2 and 3. Pictures showing various oblique planes under measurement via the proposed method are shown in Figures 4, 5, 6, and 7, respectively. As demonstrated in these tables, satisfactory measurement of photographing distance and incline angle can be obtained via the proposed method, relaxing the application constraints faced by the existing IBDMS, which is only valid for measuring objects or surfaces that are perfectly perpendicular to the optical axis. In the distance measurement experiment as shown in Table 1, small incline angles do not significantly affect the measurement results. When incline angle become larger, however, the error in measuring photographing distance are dramatically affected. For example, the error can be as high as 12.1% in measuring

photographing distance for incline angle of 60°. As for incline angle measurement and distance measurement between two arbitrary points, the error rates do not show significant effect resulted from different incline angles.

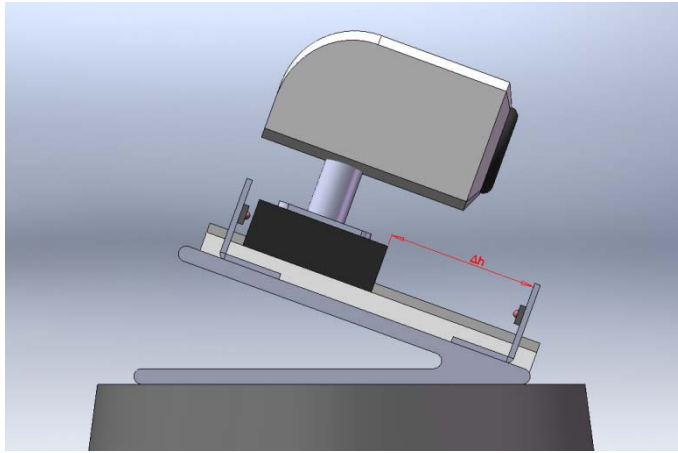


Figure 3. Setup of the proposed image-based system for measuring objects on an oblique surface based on pixel variation of CCD images.

Table 1 Distance measurement at (a) 50cm and (b) 100cm with different incline angles

Incline Angle θ_m	0°	30°	45°	60°
Actual distance (cm)	50	50	50	50
Measured distance $h_1(O)$	50.601	50.614	50.914	57.1164
Error (%)	1.2	1.2	1.8	14.2%

at photographing distance of (a) 50cm

Incline Angle θ_m	0°	30°	45°	60°
Actual distance (cm)	100	100	100	100
Measured distance $h_1(O)$	101.3	101.4	101.8	112.1164
Error (%)	1.3	1.4	1.8	12.1

at photographing distance of (b) 100cm

Table 2 Angle measurement at photographing distance of (a) 50cm and (b) 100cm.

Incline Angle θ_m	0°	30°	45°	60°
Estimated angle θ_m	-0.22	31.14	44.79	60.42
Error (%)	-0.22%	1.14%	-0.47%	0.7%

at photographing distance of (a) 50cm

Incline Angle θ_m	0°	30°	45°	60°
Estimated angle θ_m	0.31	31.16	45.57	62.42
Error (%)	0.31	3.87	1.23	4.03

at photographing distance of (b) 100cm

Table 3 Distance between points A and B at photographing distance of (a) 50cm and (b) 100cm.

Incline Angle θ_m	0°	30°	45°	60°
Actual distance $D(\overline{AB})$ (cm)	30	30	30	30
Estimated distance $D(\overline{AB})$ (cm)	30.8	31.27	30.5771	29.93
Error (%)	2.9	4.2	1.92%	0.22

at photographing distance of (a) 50cm

Incline Angle θ_m	0°	30°	45°	60°
Actual distance $D(\overline{AB})$ (cm)	30	30	30	30
Estimated distance $D(\overline{AB})$ (cm)	31.87	31.13	30.12	31.4
Error (%)	3.42	3.77	0.39	4.7

at photographing distance of (b) 100cm

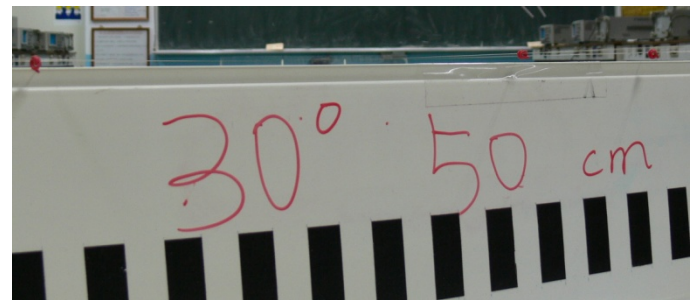


Figure 4. Picture showing the oblique plane under measurement via the proposed method where the incline angle and photographing distance are 30° and 50 cm, respectively.

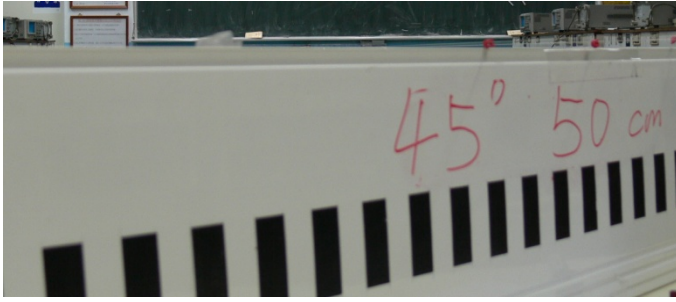


Figure 5. Picture showing the oblique plane under measurement via the proposed method where the incline angle and photographing distance are 45° at 50 cm, respectively.

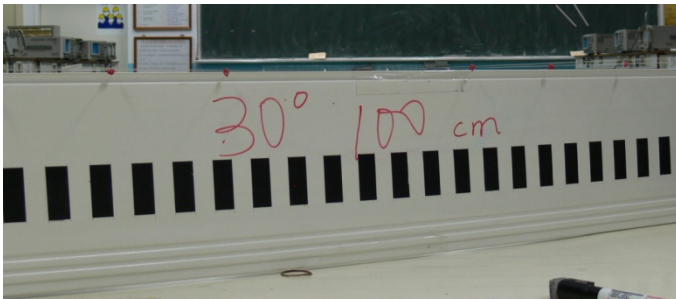


Figure 6. Picture showing the oblique plane under measurement via the proposed method where the incline angle and photographing distance are 30° at 100 cm, respectively.

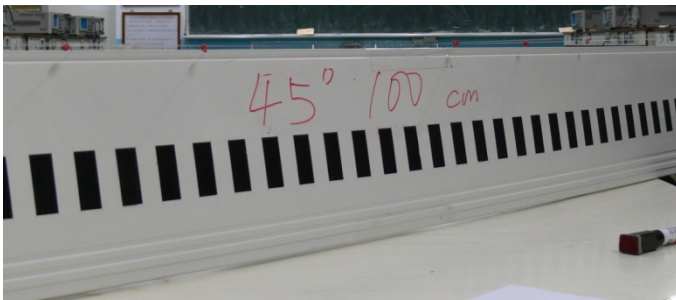


Figure 7. Picture showing the oblique plane under measurement via the proposed method where the incline angle and photographing distance are 45° at 100 cm, respectively.

V. CONCLUSION

This paper presented a novel method to measure the photographing distance of remote objects locating on an oblique plane as well as the incline angle. With the proposed method, distance between any two arbitrary points in the image can be obtained with desired results. As demonstrated in the paper, the proposed measuring system has overcome problems and difficulties encountered by conventional image-based measuring methods and demonstrated itself as a simple yet accurate way in measuring distance and incline angle for objects on an oblique surface while simultaneously recording images.

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