(a) We write in component form:

$$\begin{aligned}\mathbf{x}^t\mathbf{K}\mathbf{x} &= \sum_{j=1}^{d}\sum_{i=1}^{d} x_i K_{ij} x_j \\ &= \sum_{i=1}^{d} x_i^2 K_{ii} + \sum_{i\neq j}^{d} x_i x_j K_{ij}.\end{aligned}$$

The derivatives are, then,

$$\begin{aligned}\frac{d}{dx_r}\mathbf{x}^t\mathbf{K}\mathbf{x} &= \frac{d}{dx_r}\left[\sum_{i=1}^{d} x_i^2 K_{ii} + \sum_{i\neq j}^{d} x_i x_j K_{ij}\right] \\ &= \frac{d}{dx_r}\Bigg[\sum_{i\neq r} x_i^2 K_{ii} + x_r^2 K_{rr} \\ &\qquad + \sum_{i}^{d}\sum_{j}^{d} x_i x_j K_{ij} + x_r \sum_{j\neq r} x_j K_{rj} + x_r \sum_{i\neq r} x_i K_{ir}\Bigg] \\ &= 0 + 2x_r K_{rr} + 0 + \sum_{j\neq r}^{d} x_j K_{rj} + \sum_{i\neq r}^{d} x_i K_{ir} \\ &= \sum_{j=1}^{d} x_j K_{rj} + \sum_{i=1}^{d} x_i K_{ir} \\ &= (\mathbf{K}\mathbf{x})_{r\text{th element}} + (\mathbf{K}^t\mathbf{x})_{t\text{th element}}\end{aligned}$$

This implies

$$\frac{d}{d\mathbf{x}}\mathbf{x}^t\mathbf{K}\mathbf{x} = \mathbf{K}\mathbf{x} + \mathbf{K}^t\mathbf{x} = (\mathbf{K} + \mathbf{K}^t)\mathbf{x}.$$

(b) So, when $\mathbf{K}$ is symmetric, we have

$$\frac{d}{d\mathbf{x}}\mathbf{x}^t\mathbf{K}\mathbf{x} = (\mathbf{K} + \mathbf{K}^t)\mathbf{x} = 2\mathbf{K}\mathbf{x}.$$