Fama MacBeth Estimation

Prepared by Prof. Jason C. Hsu

For use for the Anderson MFE Program

Two Stage Fama-Macbeth Factor Premium Estimation

The two stage Fama-Macbeth regression estimates the premium rewarded to a particular risk factor exposure by the market. Stated practically, if you have a theory about what particular factors drive portfolio returns, you can use this method to price how much return you would expect to receive for a particular beta exposure to that factor.

Note that many asset pricing models—arbitrage pricing theory (APT), the capital asset pricing model (CAPM), and intertemporal CAPM (ICAPM)—all have the following structure:

$$R = \beta\gamma$$

Where R is the excess return of an asset, β is the vector of factor loadings on a various factors, and γ is the vector of excess returns on those factors. It is all well and good to have a theoretical model to describe returns, but ultimately, we must test this model empirically. The Fama-Macbeth methodology provides a particularly robust way to do so.

Methodology

Assume we have n asset or portfolio returns over T periods with a particular asset's excess return in a particular time denoted $R_{i,t}$. We would like to test whether the m factors $F_{1,t}$, $F_{2,t}$,…, $F_{m,t}$ explain our asset and portfolio returns and the premia awarded to exposure to each factor. To do so, we must run a two-stage regression. The first stage involves a set of regressions equal in number to the number of assets or portfolios one is testing. The second stage is a set of regressions equal in number to the number of time periods. The first stage regressions are a set of time series regression of each asset or portfolio's return on the factors.

$$R_{1,t} = \alpha_1 + \beta_{1,F_1}F_{1,t} + \beta_{1,F_2}F_{2,t} + \cdots + \beta_{1,F_m}F_{m,t} + \epsilon_{1,t}$$

$$R_{2,t} = \alpha_2 + \beta_{2,F_1}F_{1,t} + \beta_{2,F_2}F_{2,t} + \cdots + \beta_{2,F_m}F_{m,t} + \epsilon_{2,t}$$

$$\vdots$$

$$R_{n,t} = \alpha_n + \beta_{n,F_1}F_{1,t} + \beta_{n,F_2}F_{2,t} + \cdots + \beta_{n,F_m}F_{m,t} + \epsilon_{n,t}$$

Expressed in matrix form, the regression for R_n would look as follows:

$$R_n = F\beta_n + \epsilon_n$$

Where R_n is a $t \times 1$ vector of returns, F is a $t \times (m+1)$ matrix of factors where all elements in the first column are 1, β_n is a $(m+1) \times 1$ vector of factor loadings where all elements in the first row are the intercept α_n, and ϵ_n is a $t \times 1$ vector of error terms.

Now, we know to what extent each asset's or portfolio's return is affected by each factor. Note that our regressions do not tell us the premium rewarded to each factor exposure, which will be calculated in the second regression.

Before we look at the second set of regressions, we must first define $\hat{\beta}_{i,F_k}$ as the estimated βs for each asset or portfolio for factor F_k. Note that the only difference between β and $\hat{\beta}$ is that β is the "true" unobservable factor loading while $\hat{\beta}$ is the empirically estimated factor loading. To calculate the factor premiums for each factor, we must run the following set of cross-sectional regressions:

$$R_{i,1} = a_1 + \gamma_{1,1}\hat{\beta}_{i,F_1} + \gamma_{2,1}\hat{\beta}_{i,F_2} + \cdots + \gamma_{m,1}\hat{\beta}_{i,F_m} + e_1$$

$$R_{i,2} = a_2 + \gamma_{1,2}\hat{\beta}_{i,F_1} + \gamma_{2,2}\hat{\beta}_{i,F_2} + \cdots + \gamma_{m,2}\hat{\beta}_{i,F_m} + e_2$$

$$\vdots$$

$$R_{i,T} = a_T + \gamma_{1,T}\hat{\beta}_{i,F_1} + \gamma_{2,T}\hat{\beta}_{i,F_2} + \cdots + \gamma_{m,T}\hat{\beta}_{i,F_m} + e_T$$

The $\gamma_{j,t}$ terms are regression coefficients. Note that the independent variables—$\hat{\beta}_{i,F_k}$ terms—are always exactly the same for every regression. The only variation is in the dependent variables, which are different for each time period. In matrix form, the equation at time t would look as follows:

$$R_t = \hat{\beta}\gamma_t$$

Where R_t is an $n \times 1$ vector of average asset or portfolio returns, $\hat{\beta}$ is an $n \times (m+1)$ vector of factor loadings where all elements in the first column are 1, and γ is an $(m+1) \times 1$ vector of factor premia where all elements in the first row are the intercept a.

To calculate a single risk premium for each factor one must average all the $\gamma_{j,t}$ terms into a single γ_j. To calculate standard errors for the γ_j terms, one must treat each $\gamma_{j,t}$ observation as an independent observation and calculate a t-statistic. The t-stat to test whether γ_j is different from 0 would be as follows:

$$\frac{\gamma_j}{\sigma_{\gamma_j}/\sqrt{T}}$$

Where σ_{γ_j} is the standard deviation of the $\gamma_{j,t}$ terms. Note that the only reason that we can construct standard errors in this fashion is because asset returns are roughly independently and identically distributed.

The γ_k coefficient represents the factor premium for an exposure with a $\hat{\beta}_{i,F_k}$ of 1 to factor F_k. Note that instead of running multiple cross-sectional regressions, we could run a single one that uses the average returns of all assets over time. However, this prevents us from using the clever technique to estimate the factor premia, γ_j. If we were to use a single regression, note that it would look identical to the expectation of our first set of equations.

$$E(R_{i,t}) = E(\alpha_i) + E(\beta_{i,F_1}F_{1,t}) + E(\beta_{i,F_2}F_{2,t}) + \cdots + E(\beta_{i,F_m}F_m)$$

$$R_i = a + \beta_{i,F_1}E(F_{1,t}) + \beta_{i,F_2}E(F_{2,t}) + \cdots + \beta_{i,F_m}E(F_m)$$

$$R_i = a + \gamma_1\beta_{i,F_1} + \gamma_2\beta_{i,F_2} + \cdots + \gamma_m\beta_{i,F_m}$$

Note that we define a as $E(\alpha_i)$, γ_k as $E(F_{k,t})$, and R_i as $E(R_{i,t})$.

To reiterate the intuitive rationale behind the methodology, we believe that some factors can explain portfolio returns. We can regress each portfolio's returns against these factors to determine each portfolio's exposure to these factors. We can then determine what premium is rewarded for a unit exposure to each factor by regressing the return of each portfolio against its factor exposures in each period and averaging the coefficients from this regression over time for each factor. The average coefficients for each factor represent the premium given for a unit factor exposure over time.

Practicalities

Macroeconomic versus Tradable Factors

Your factors can either be macroeconomic factors like unexpected GDP growth or unexpected change in the unemployment rate or tradable factors such as the excess return of high accrual (low cashflow) firms over low accrual (high cashflow) firms.

For a variety of reasons, it is better to use tradable factors.

- First, tradable factors have the nice feature that if the α_i terms from the first set of regressions are frequently significantly positive or negative, it is likely that a factor is missing and the model is poorly specified.
- Second, tradable factors provide investors a portfolio that they can use to hedge the risk of the factor; in other words, to reduce or increase exposure to a factor an investor can short or buy the tradable factor portfolio itself.
- Third, financial data is usually observed with higher precision, therefore the factor-mimicking portfolio might be better than using noisy proxies for the actual factors.
- Lastly, macroeconomic factors are generally less granular with data at weekly to quarterly or even annual intervals. One can find data on tradable factors from daily to minute-by-minute.

The infrequent macroeconomic data make achieving statistical significance less likely. Regressions using macroeconomic data do not share this feature.

If one must use macroeconomic data, it is far better to project those macroeconomic factors onto tradable factors by finding tradable factors that exhibit some correlation with those macroeconomic factors. (See Factor-Mimicking Portfolios.)

Creating Portfolios

Now, imagine that we have 1000 stocks on which we wish to test a hypothesis on factor exposures. We generally would not run 1000 separate regressions against our factors. We avoid this because individual stocks may not have constant loadings on factors across time. If we used individual stocks, we would end up with inappropriate estimates of factor premia. Instead, we create portfolios out of our 1000 stocks, allowing stocks to shift from one portfolio to another based on changing factor exposures.

However, we should not randomly sample the 1000 stocks and put them into 100 portfolios, for example. The reason is that randomly sampling the stocks will result in all portfolios looking very much like the market portfolio, and the only factor that will end up explaining significant returns will likely be the market portfolio. Instead, generally the best approach is to sort by some characteristic that you expect to cause a stock's return to correlate with a factor. Then, create about 100 portfolios out of the sorted stocks such each portfolio has increasing exposure to a factor. In general, one should ensure that there are at least about 10 stocks in each portfolio.

Let's look at an example similar to Fama and French's seminal paper on explaining the cross section of equity returns. Let's say our factors are the market's excess return (r_m), the excess return of the equity of high book-to-market firms over that of low book-to-market firms (HML), and the excess return of the equity of small capitalization firms over that of large firms (SMB). Let us also assume we have 500 months of data. Our returns are the excess returns of 25 portfolios of equities made by the cross section of book-to-market and capitalization quintiles. Using the Fama Macbeth methodology, we would regress the 25 portfolios' excess returns against r_m, HML, and SMB in 25 separate regressions. Then, we would run a second regression with 25 observations of all 25 portfolios' returns against their factor exposures to r_m, HML, and SMB in 500 separate regressions. We would then average our coefficients across all regressions to get our factor premia for each regression.

Why Not Just Take the Mean of the Factor?

You may wonder why we do not simply take the mean of the tradable factor and call this the factor premium. It is true that as your sample size approaches infinity, the sample mean of the tradable factor will approach the true mean of the factor and thus the true expected return of the factor. However, generally, we do not have a sufficiently long history to be confident about the first moment of risk factor premia.

The Fama Macbeth methodology allows us to utilize the covariance of the factor with other assets to determine the factor premia. The second moment of asset returns distributions are much easier to estimate than the first moment, and obtaining more granular data improves estimation considerably, which is not true of mean estimation.

Bibliography

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