Lecture 7: Multifactor Models in Practice

Investments

Overview

I) Introduction

II) Selecting factors

- $\hookrightarrow$ Macroeconomic Factors
- $\hookrightarrow$ Factors based on Characteristics
- $\hookrightarrow$ Factor Analysis / Principal Components

III) Factor Timing

IV) Conclusions

Implementing the APT

- The APT and Multi-factor models can be used in place of the CAPM for
  a) pricing assets
  b) performance evaluation
  c) risk management
- The APT makes fewer assumptions and this comes at a cost:
  - $\hookrightarrow$ unlike the CAPM, the APT does not tell us *which* are the systematic factors driving returns.
- As an asset pricing model, the APT *relies* on a statistical model for returns.
  - $\hookrightarrow$ Its ability to price asset will depend on getting the “right” factors.

Selecting the factors: Three Approaches

1) We treat the factors as observable and specify the $\tilde{f}_j$ directly. *(Macroeconomic Approach)*
   - $\hookrightarrow$ In this case, the factors can be macroeconomic variables like inflation, output.
   - $\hookrightarrow$ We can estimate the loadings via regression.
   - $\hookrightarrow$ We think that these variables will be sufficient to capture the systematic risk in the economy.
   - $\hookrightarrow$ An example of this approach is the Chen, Roll and Ross model.
2) We could treat the *loadings* ($b_{i,j}$'s) as observable and infer the security sensitivities from fundamental information about the securities. *(Fundamental Approach)*
   - $\hookrightarrow$ We could construct indices of some firm characteristics (such as B/M), and treat these risk indices as sensitivities to the factors associated with those characteristics (such as a B/M factor).
   - $\hookrightarrow$ An example of this approach is the Fama-French 3-factor model.

Selecting the factors: Three Approaches

3) Alternatively, we can treat both the factors and the loadings as unobservable. *(Statistical Approach)*

   - ↪ Factor Analysis or Principal Components are statistical techniques designed to jointly estimate the factors and their sensitivities.
   - ↪ This is the method that gave us the level, slope and curvature factors in bond returns.
   - ↪ The resulting factors will be portfolios or linear combinations of the existing securities.

Macroeconomic Variables as Factors

- This approach requires us to specify the factors a priori.
- We should specify the $\tilde{f}_j$'s as a set of macro-economic factors (Why?).
- Once the factors are selected,
  a) use a set of time series regressions of r_i's on $\tilde{f}_j$'s to estimate the $\tilde{b}_{i,j}$'s (first-pass regression)
  b) use a cross-sectional regression of the average firm returns on the $b_{i,j}$'s to estimate λ_i's (second-pass regression).
- The method is similar to that we used when testing the CAPM.
- This is the method of Chen, Roll and Ross that we will discuss

Macroeconomic Variables as Factors

- Each investment firm has its own preferred set of factors and puts a lot of effort into identifying factors and factor sensitivities.
- You want to consider types of risk which you think are being rewarded by the market.
- Some commonly used macro-economic risk factors:
  1. Market Portfolio
  2. Inflation
  3. Interest Rates
  4. Credit Spreads
  5. Business Cycle Variables

Macroeconomic Variables as Factors

- For Business Cycle Variables you would like to use leading indicators

A. Leading indicators

1. Average weekly hours of production workers (manufacturing)
2. Initial claims for unemployment insurance
3. Manufacturers' new orders (consumer goods and materials industrie
4. Vendor performance—slower deliveries diffusion index
5. New orders for nondefense capital goods
6. New private housing units authorized by local building permits
7. Yield curve slope: 10-year Treasury minus federal funds rate
8. Stock prices, 500 common stocks
9. Money supply (M2)
10. Index of consumer expectations

B. Coincident indicators

1. Employees on nonagricultural payrolls
2. Personal income less transfer payments
3. Industrial production
4. Manufacturing and trade sales

C. Lagging indicators

1. Average duration of unemployment
2. Ratio of trade inventories to sales
3. Change in index of labor cost per unit of output
4. Average prime rate charged by banks
5. Commercial and industrial loans outstanding
6. Ratio of consumer installment credit outstanding to personal income
7. Change in consumer price index for services

Macroeconomic Variables as Factors

In order to obtain the factor 'surprises',$(\tilde{f})$ we need to subtract the market expectations from the realized values, $(f - Ef)$. How to get market expectations?

1. Historical Averages
2. Statistical model
3. Analyst surveys

This Week's Calendar

Date	ET	Release	For	Actual	Briefing.com	Consensus	Prior	Revised From
Feb 21	10:00	Leading Indicators	Jan	1.1%	0.6%	0.5%	0.3%	0.1%
Feb 21	14:00	FOMC Minutes	Jan 31					
Feb 22	08:30	Core CPI	Jan	0.2%	0.2%	0.2%	0.1%	0.2%
Feb 22	08:30	CPI	Jan	0.7%	0.4%	0.5%	-0.1%	
Feb 23	08:30	Initial Claims	02/18	278K	285K	300K	298K	297K
Feb 23	10:00	Help-Wanted Index	Jan	37	40	40	38	39
Feb 23	10:30	Crude Inventories	02/17	1121K	NA	NA	4853K	
Feb 24	08:30	Durable Orders	Jan	-10.2%	-4.0%	-2.0%	2.5%	1.3%

Macroeconomic Variables as Factors

- Choose factors that market reacts to:

Updated: 24-Feb-06

Market Snapshot			
Dow		-32.57	(%)
Nasdaq		+1.38	(%)
SP 500		+0.08	(%)
10-yr Note			
NYSE	**Adv** 1731	**Dec** 1424	**Vol** 910.1 mln
Nasdaq	**Adv** 1441	**Dec** 1476	**Vol** 997.2 mln

Industry Watch

Strong: oil & gas refiners, equipment & services, explorers, and drillers; gold, diversified metals & minerals

Weak: specialty consulting services, fertilizer & agricultural chemicals, apparel retail, auto parts & equipment, drug retail, airlines

Moving the Market

- 10-year -03/32 at 4.57%
- Durable Orders fell 10.2% in Jan. (consensus -2.0%), caused by a sharp drop in transportation orders. Excluding transportation, orders were up 0.6%.
- Geopolitical concerns related to Nigeria militants and a foiled attack on a Saudi oil center are affecting energy prices.

Macroeconomic Variables as Factors

Once the factors $k = 1..K$ have been specified

1. Construct the factor surprises $\tilde{f} = f - E(f)$.

2. Estimate the loadings b_{ik} for each security $i = 1..N$. You can use individual stocks or portfolios.

$$R_{i,t} = a_i + b_{i1}\tilde{f}_{1,t} + b_{i2}\tilde{f}_{2,t} + \ldots + b_{iK}\tilde{f}_{K,t} + \varepsilon_{i,t}$$

- $\hookrightarrow$ the $\varepsilon_{i,t}$ is the idiosyncratic news in each security. The APT makes no assumptions on $\varepsilon_{i,t}$ except that it is uncorrelated with $\varepsilon_{j,t}$.
- $\hookrightarrow$ $\hat{b}_{ik}$ is going to be your estimate of the loading of security i on factor k.

Macroeconomic Variables as Factors

3. Estimate the average return of each security in the sample, $\hat{\mu}_i$. That is your estimate of this security's expected return.

4. Estimate the market prices of risk for each factor, $\lambda_1 \ldots \lambda_K$, by a multivariate regression of the N average returns $\hat{\mu}_i$ for the N assets on the $N \times K$ loadings including a constant.

$$\hat{\mu}_i = \lambda_0 + \lambda_1 \hat{b}_{i1} + \lambda_2 \hat{b}_{i2} + \ldots + \lambda_K \hat{b}_{iK} + u_i$$

- ↪ The u_i's are deviations from the APT. If the APT holds then all the u_i's should be zero. This procedure is similar to what we used to test the CAPM.
- ↪ The estimated prices of risk, $\hat{\lambda}_k$, are the reward for bearing factor-k risk. They provide the answer to the question: What is the difference in average returns (cost of capital, discount rates) between two otherwise identical securities that have differential exposure to factor k.

- How should we interpret the sign of the λ's?
- One easy way to think about what the sign of the λ's should be is as follows:
  - ↪ As long as the factors are normally distributed, $Pr(\tilde{f} > 0) = Pr(\tilde{f} < 0) = 1/2$.
  - ↪ Consider two derivative contracts, trade at prices P_H and P_L:
    1. C_H pays $\$1m$ if $\tilde{f} > 0$ and zero otherwise.
    2. C_L pays $\$1m$ if $\tilde{f} < 0$ and zero otherwise.

    - If $P_L > P_H$ then $\lambda > 0$. Examples: Market portfolio, GDP.
    - If $P_H > P_L$ then $\lambda < 0$. Examples: Inflation, Oil Prices.
    - If $P_L = P_H$ then $\lambda = 0$. Factor is not priced because we are indifferent.

Macroeconomic Variables as Factors - Example

Chen, Roll and Ross (1986) specify the factors to be

1. Monthly and annual unanticipated growth in industrial production (MP)
2. Changes in expected inflation, as measured by the change in r_{T-Bill} (DEI).
3. Unexpected inflation (UI)
4. Unanticipated changes in risk premiums, as measured by $r_{Baa} - r_{AAA}$ (UPR)
   - ↪ This is often called the "Default Spread"
5. Unanticipated changes in the slope of the term structure, as measured by $r_{T-Bond} - r_{T-Bill}$. (UTS)
   - ↪ This is often called the "Term Spread"
6. In their pricing equation they also include the return on the equal-weighted (EWNY) and value-weighted (VWNY) NYSE market portfolio.

Macroeconomic Variables as Factors - Example

- Chen, Roll and Ross (1986) find the factor prices λ_k (t-stats in parenthesis).

Table 11.4 Economic Variables and Pricing (percent per month × 10), Multivariate Approach

A	Years	YP	MP	DEI	UI	UPR	UTS	Constant
	1958–84	4.341	13.984	−.111	−.672	7.941	−5.8	4.112
		(.538)	(3.727)	(−1.499)	(−2.052)	(2.807)	(−1.844)	(1.334)
	1958–67	.417	15.760	.014	−.133	5.584	.535	4.868
		(.032)	(2.270)	(.191)	(−.259)	(1.923)	(.240)	(1.156)
	1968–77	1.819	15.645	−.264	−1.420	14.352	−14.329	−2.544
		(.145)	(2.504)	(−3.397)	(−3.470)	(3.161)	(−2.672)	(−.464)
	1978–84	13.549	8.937	−.070	−.373	2.150	−2.941	12.541
		(.774)	(1.602)	(−.289)	(−.442)	(.279)	(−.327)	(1.911)

B	Years	MP	DEI	UI	UPR	UTS	Constant
	1958–84	13.589	−.125	−6.29	7.205	−5.211	4.124
		(3.561)	(−1.640)	(−1.979)	(2.590)	(−1.690)	(1.361)
	1958–67	13.155	.006	−.191	5.560	−.008	4.989
		(1.897)	(.092)	(−.382)	(1.935)	(−.004)	(1.271)
	1968–77	16.966	−.245	−1.353	12.717	−13.142	−1.889
		(2.638)	(−3.215)	(−3.320)	(2.852)	(−2.554)	(−.334)
	1978–84	9.383	−.140	−.221	1.679	−1.312	11.477
		(1.588)	(−.552)	(−.274)	(.221)	(−.149)	(1.747)

Macroeconomic Variables as Factors - Example

- Interestingly, they find that proxies for the market portfolio are not priced.

C	Years	EWNY	MP	DEI	UI	UPR	UTS	Constant
	1958–84	5.021	14.009	−.128	.848	.130	−5.017	6.409
		(1.218)	(3.774)	(−1.666)	(−2.541)	(2.855)	(−1.576)	(1.848)
	1958–67	6.575	14.936	−.005	−.279	5.747	−.146	7.349
		(1.199)	(2.336)	(−.060)	(−.558)	(2.070)	(−.067)	(1.591)
	1968–77	2.334	17.593	−.248	−1.501	12.512	−9.904	3.542
		(.283)	(2.715)	(−3.039)	(−3.366)	(2.758)	(−2.015)	(.558)
	1978–84	6.638	7.563	−.132	−.729	5.273	−4.993	9.164
		(.906)	(1.253)	(−.529)	(−.847)	(.663)	(−.520)	(1.245)

D	Years	VWNY	MP	DEI	UI	UPR	UTS	Constant
	1958–84	−2.403	11.756	−.123	.795	8.274	−5.905	10.713
		(−.633)	(3.054)	(−1.600)	(−2.376)	(2.972)	(−1.879)	(2.755)
	1958–67	1.359	12.394	.005	−.209	5.204	−.086	9.527
		(.277)	(1.789)	(.064)	(−.415)	(1.815)	(−.040)	(1.984)
	1968–77	−5.269	13.466	−.255	−1.421	12.897	−11.708	8.582
		(−.717)	(2.038)	(−3.237)	(−3.106)	(2.955)	(−2.299)	(1.167)
	1978–84	−3.683	8.402	−.116	.739	6.056	−5.928	15.452
		(−.491)	(1.432)	(−.458)	(−.869)	(.782)	(−.644)	(1.867)

Fundamental Approach

- Identify specific firm-characteristics that we a priori think are proxies for differential sensitivity to systematic risk. Example:
  a) membership in specific industries (e.g. durable goods)
  b) book to market (value-growth)
  c) liquidity
  d) leverage
- Form portfolios of stocks sorted based on these characteristics.
  - $\hookrightarrow$ Example: Buy stocks in top quintile of leverage, short stocks in bottom quintile.
- When we want to use portfolio r_k as a factor, we should
  - $\hookrightarrow$ estimate the price of risk by $\lambda_k = Er_k$
  - $\hookrightarrow$ construct the factor "surprise" by $\tilde{f}_k = r_k - Er_k$

Fundamental Approach

- How to select firm characteristics?
  a) differences in the characteristic should be associated with differences in expected returns.
     - Example: Value firms have higher returns than growth firms.
  b) firms that have the same characteristics should also move together:
     - Example: Value firms co-move more with other value firms and less with growth firms
- These portfolios should typically satisfy the following criteria:
  - ↪ They will capture systematic movements in stock returns, i.e. a lot of stocks will have non-zero b_is with these portfolios.
  - ↪ These portfolios will typically be mispriced by the CAPM.

Example: The Fama-French 3 factor Model

- When we looked at the performance of the CAPM in real data, we found that when it failed, it failed in a *systematic* way.
  - $\hookrightarrow$ We saw that value firms and small firms had higher returns than those implied by the CAPM.
- One possible explanation is that value firms and small firms are exposed to sources of systematic risk that are not captured by the existing proxies of the market portfolio, e.g. the S&P 500.
- We saw some possible explanations:
  1. value firms may proxy for "distress".
  2. small firms may proxy for "liquidity" risk.
- Motivated by this, Fama and French in their 1993 *Journal of Financial Economics* paper, specified a version of the APT that is referred to simply as the *Fama-French 3 factor model*.

The Fama-French 3 factor Model

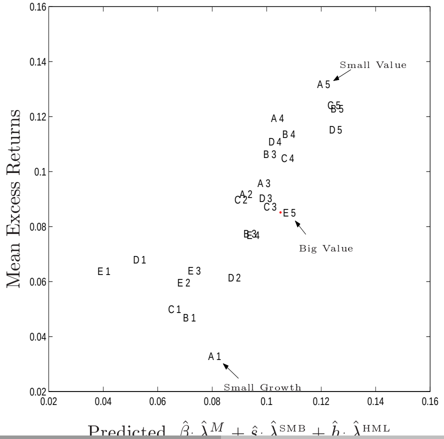
- Fama and French split the universe of stocks into 6 portfolios, based on size (Market Capitalization) and value/growth (Book to Market):

	Book-to-market		
Market Cap	**Low**	**Medium**	**High**
Small	Portfolio 1: Small growth	Portfolio 2: Small core	Portfolio 3: Small value
Large	Portfolio 4: Large growth	Portfolio 5: Large core	Portfolio 6: Large value

- They then constructed 2 factors
  1. Small minus Big (SMB): $(1/2SG + 1/2SV) - (1/2LG + 1/2LV)$
  2. High minus Low (HML): $(1/2SV + 1/2LV) - (1/2SG + 1/2LG)$
- Their version of the APT includes as factors the market portfolio, SMB and HML.

The Fama-French 3 factor Model

- The Fama-French model does a good job explaining the value and size anomalies.
- This is not purely mechanical: it works because there is comovement among value and size stocks.



Statistical Approach

- Principal Components (PCP) is an advanced statistical technique that extracts the common factors from a cross-section of stock returns:

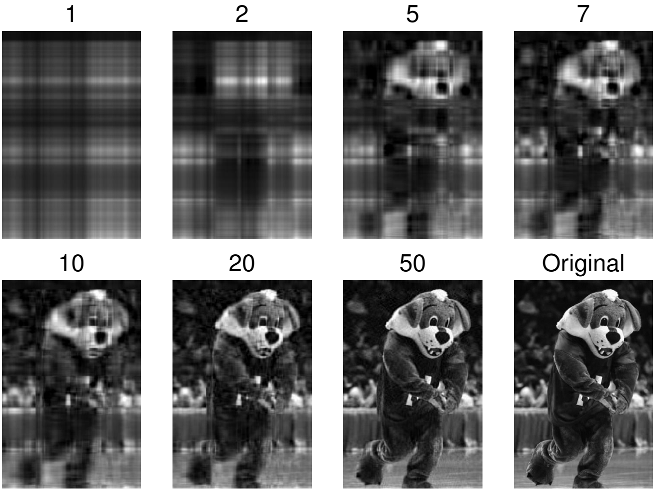
$$R_{i,t} = a_i + b_{i,1} f_{1,t} + b_{i,2} f_{2,t} + ... b_{i,K} f_{K,t} + \varepsilon_{i,t}$$

  - ↪ This is the same technique we used to extract the 'level', 'slope' and 'curvature' factors in the term-structure.
  - ↪ PCP aims to describe the data (R_i) as accurately as possible, using a small number of systematic factors (f_k).
  - ↪ One way to think of PCP is as extracting the 'most relevant', 'pervasive' or systematic information from the data. The remainder (ε) it classifies as 'noise' or un-systematic information.

- Factor Analysis is another related method, which yields similar results.

Principal Components - Intuition

- A small number of factors can reproduce the most important aspects of an image:



Statistical Approach - Example

- Makarov and Papanikolaou (2007) identify four factors from US industry returns
  1. $\approx$ value-weighted market portfolio
  2. $\approx$ capital minus consumption goods producers
  3. $\approx$ cyclical minus non-cyclical industries
  4. $\approx$ input good producers minus everyone else
- The extracted factors
  - $\hookrightarrow$ Are robust across different time and industry sub-samples
  - $\hookrightarrow$ Predict inflation, output and employment.
  - $\hookrightarrow$ Predict asset returns.
  - $\hookrightarrow$ Price the cross-section of stocks.

Extracted factors: Interpretation

- We project the extracted factors on a few industries

factor	Industries					$\rho(\%)$
f_1	MKT 0.851 36.3					85.8
f_2		Business Equip. 0.859 37.4	Telecom. 0.241 6.7	Food -0.598 -27.2	Beer -0.361 -13.0	94.9
f_3		Services -0.271 -4.0	Health -0.543 -11.5	Autos 0.567 9.9	Fab. Prod. 0.722 11.0	72.7
f_4	MKT -1.173 (-24.98)	Steel 0.895 (15.75)	Mines 0.241 (7.52)	Coal 0.168 (5.85)	Oil 0.795 (19.47)	83.7

t-statistics in parenthesis

Extracted factors: Robustness Check

- We repeat the exercise using UK data:

factor	Industries					$\rho(\%)$
f_1	MKT					93.9
	0.936					
	-42.5					
f_2		Business Equip.	Telecom.	Food	Beer	97.8
		0.949	0.049	-0.271	-0.206	
		53.1	2.2	-13.8	-10.5	
f_3		Services	Health	Autos	Fab. Prod.	81.1
		-0.407	-0.387	0.609	0.497	
		-7.4	-9.7	9.6	8.1	
f_4	MKT	Steel	Mines	Coal	Oil	88.3
	-0.965	0.266	0.564	-	0.821	
	(-24.98)	(15.75)	(7.52)	-	(19.47)	

t-statistics in parenthesis

Factor Diagnostics

- The APT, as an asset pricing model, is only as good as the factor model it assumes.
- There are several ways one can obtain the factors. Nevertheless, all approaches should yield factors that:
  1. Explain the common movements in returns. That is, if $\tilde{f}_k$ are the factors, a regression of

     $$R_{i,t} = a_i + \sum b_{i,k} \tilde{f}_k + \varepsilon_i$$

     should yield ε_i's that are more or less uncorrelated amongst securities and $b_{i,k}$'s that are statistically significant.
  2. Differences in factor loadings between assets have to be associated with differences in expected returns.
- Need out of sample tests.
- APT and factor models are very popular in practice, with BARRA the leading provider of factor models.

Factor Tilting

Factor Tilting is more general than market timing because it presumes a multi-factor structure.

- Tilting is forming a portfolio to take advantage of a forecast of a factor, $\tilde{f}$.
- Like in the market=timing example, we assume that in the return generating process equation for our portfolio:

$$\tilde{r}_{p,t} = E[\tilde{r}_{p,t}] + b_{p,1}\tilde{f}_{1,t} + \cdots + b_{p,n}\tilde{f}_{n,t} + \tilde{e}_{p,t}$$

$\hookrightarrow$ $E[\tilde{r}_{i,t}]$ represents the market's expectation

$\hookrightarrow$ $E[\tilde{f}_{1,t}] = 0$ *for the market*

$\hookrightarrow$ But, given our superior information $E[\tilde{f}_{1,t}] \neq 0$ *for us*!

  - That is, we can forecast f better than the market.

Factor Tilting

- The advantage of the multi-factor structure here is that we can express specific macro views, rather than expressing a view or forming a forecast about the whole market portfolio.

- Assuming we have this ability, we can earn superior profits by varying the factor betas/loadings of our portfolio.

  - ↪ We want to increase the loading when we think that the factor is likely to have a positive realization

  - ↪ We want to decrease the loading when we think that the factor is likely to be negative.

Factor Tilting - Example

Your analyst gives you the following information on three securities that are correctly priced according to a 2 factor APT model

$$\begin{aligned}
\tilde{r}_A &= 0.12 + 1 \cdot \tilde{f}_1 + 1 \cdot \tilde{f}_2 + \tilde{e}_A \\
\tilde{r}_B &= 0.12 + 1 \cdot \tilde{f}_1 + 2 \cdot \tilde{f}_2 + \tilde{e}_B \\
\tilde{r}_C &= 0.12 + 3 \cdot \tilde{f}_1 + 2 \cdot \tilde{f}_2 + \tilde{e}_C
\end{aligned}$$

- Factor 1 is a foreign income factor.
- factor 2 is a U.S. earnings price ratio factor.
- The way the model is constructed these factors are uncorrelated.
  - ↪ Here, the $E[r]$'s of 12% are what the market expects - not what we expect!
  - ↪ What are the factor risk premia (the λs)?

Factor Tilting - Example

- You believe very strongly that Japan will finally come out of its recession in the next few months and therefore exports of U.S. produced goods will rise more than the market expects. Moreover, you believe the earnings price ratio factor will not change at all in this time period, consistent with what analysts expect.

- Using the above three securities, construct ANY portfolio that takes advantage of all of these facts.
- What are
  i) the composition of the portfolio
  ii) the b's of the portfolio
  iii) the expected return on the portfolio.

Factor Tilting - Example

- We want to construct a portfolio with a lot of factor 1 exposure and no factor 2 exposure. Therefore, lets assume we want a loading of 10 on factor 1 and 0 on factor 2.
- Therefore, solve the three equations:

$$
\begin{aligned}
1 \cdot w_A + 1 \cdot w_B + 3 \cdot w_C &= 10 \\
1 \cdot w_A + 2 \cdot w_B + 2 \cdot w_C &= 0 \\
1 \cdot w_A + 1 \cdot w_B + 1 \cdot w_C &= 1
\end{aligned}
$$

- (The last equation is the usual restriction that the sum over the weights is one.) We can also write these equations as:

$$
\begin{aligned}
1 \cdot w_A + 1 \cdot w_B + 3 \cdot (1 - w_A - w_B) &= 10 \\
1 \cdot w_A + 2 \cdot w_B + 2 \cdot (1 - w_A - w_B) &= 0
\end{aligned}
$$

Factor Tilting - Example

- The solution to this system of equations are the weights $w_A = 2$, $w_B = -5.5$, and $w_C = 4.5$. Then this portfolio will have a loading of 10 on factor 1 and a loading of 0 on factor 2.
  - $\hookrightarrow$ Alternatively, we could have put 1000% of our wealth into the first factor mimicking portfolio, and -900% of our wealth into the risk-free portfolio.
  - $\hookrightarrow$ Why would this work?
  - $\hookrightarrow$ Would the weights on A, B and C be different?
- Assuming that you believe that the foreign income factor will rise by 2%, the expected return on this portfolio is:

$$2 \cdot 0.12 - 5.5 \cdot 0.12 + 4.5 \cdot 0.12 + 10 \cdot 0.02 = 0.32$$

  - $\hookrightarrow$ How else could you have calculated the expected return of this portfolio, conditional on this factor change?
- Is this the highest Sharpe-Ratio portfolio possible?

Factor Tilting in Practice

- In practice, we have more than three securities available to place a bet on a factor.
- This securities might also be subject to idiosyncratic risk?
- Is an optimal way to construct a portfolio that is a pure bet on a factor?
- We will refer to such portfolios as *factor-mimicking portfolios*.

How to construct a factor mimicking portfolio

1. Start with a relatively small number (5-10) of diversified portfolios.
   - ↪ We will refer to these portfolios as the *basis assets*.

2. Construct factor surprises by regressing realizations of the factor on publicly known information

$$f_t = \underbrace{\gamma_0 + \gamma X_{t-1}}_{\text{predictable part}} + \underbrace{\tilde{f}_t}_{\text{unpredictable part}}$$

   - ↪ The variable X_{t-1} contains information that is known at time $t-1$.

3. Regress realizations of factor surprises $\tilde{f}_t$ on *excess* returns of basis assets

$$\tilde{f}_t = b_0 + b_1 R^e_{1,t} + b_2 R^e_{2,t} + \dots b_N R^e_{N,t} + u_t$$

4. The estimated coefficients $b_1 \dots b_N$ are the weights of the factor mimicking portfolio on the N assets.

How to construct a factor mimicking portfolio - Example

Table 1

Monthly OLS regressions of the form $y_{t+12} = \mathbf{b}\mathbf{R}_{t-1,t} + \mathbf{c}\mathbf{Z}_{t-1} + \varepsilon_{t,t+12}$. y_{t+12} is a macroeconomic variable from month t to month $t+12$, $\mathbf{R}_{t-1,t}$ is a vector of monthly returns in month t, and $\mathbf{Z}_{t-1}$ is a vector of control variables observed in month $t-1$. The sample period is 1947:1–1994:12, except for consumption which is 1959:1–1994:12. Robust standard errors are calculated with Newey-West 24 month lags

		Production growth$_{t,t+12}$		Consumption growth$_{t,t+12}$		Labor income growth$_{t,t+12}$		Inflation$_{t,t+12}$		Excess stock returns$_{t,t+12}$		Excess bond returns$_{t,t+12}$		Nom. T-bill returns$_{t,t+12}$	
		Coef.	SE	Coef.	SE	Coef.	SE	Coef.	SE	Coef.	SE	Coef.	SE	Coef.	SE
	RMRF$_{t-1,t}$ (market)	−0.38	0.33	0.11	0.11	0.20	0.16	−0.40	0.24	2.94	0.73	3.98	0.63	−0.30	0.07
	BASIC INDUSTRIES$_{t-1,t}$	0.03	0.09	−0.02	0.03	−0.08	0.05	0.17	0.07	−0.71	0.35	−0.66	0.15	0.06	0.02
	CAPITAL GOODS$_{t-1,t}$	−0.16	0.14	−0.02	0.04	−0.16	0.08	0.03	0.06	−0.85	0.29	−0.53	0.20	0.03	0.02
	CONSTRUCTION$_{t-1,t}$	0.02	0.08	−0.05	0.03	−0.04	0.04	0.05	0.05	0.14	0.22	−0.32	0.16	0.03	0.02
	CONSUMER GDS$_{t-1,t}$	0.20	0.12	0.02	0.04	0.06	0.07	0.00	0.07	−0.73	0.42	−0.93	0.26	0.08	0.03
	ENERGY$_{t-1,t}$	0.13	0.06	−0.04	0.02	−0.03	0.03	0.11	0.04	−0.42	0.16	−0.79	0.13	0.06	0.01
$\mathbf{R}_{t-1,t}$	FINANCE$_{t-1,t}$	0.14	0.08	0.01	0.04	0.08	0.05	0.04	0.04	−0.37	0.25	−0.35	0.12	0.03	0.01
	TRANSPORTATION$_{t-1,t}$	0.20	0.08	0.02	0.02	0.10	0.04	0.03	0.03	0.00	0.21	−0.36	0.12	0.03	0.02
	UTILITIES$_{t-1,t}$	−0.09	0.10	−0.04	0.05	−0.10	0.05	−0.01	0.07	−0.62	0.26	−0.32	0.19	0.02	0.02
	LONGBOND$_{t-1,t}$	−0.08	0.10	0.03	0.03	−0.06	0.05	−0.01	0.04	−0.19	0.26	−0.68	0.20	0.02	0.03
	INTRMDBOND$_{t-1,t}$	−0.15	0.14	−0.12	0.07	−0.09	0.08	0.13	0.07	0.58	0.64	0.16	0.47	−0.06	0.07
	ONEYRBOND$_{t-1,t}$	1.13	0.63	0.56	0.20	0.85	0.32	−1.10	0.44	4.15	2.11	2.28	1.27	−0.81	0.26
	JUNKBOND$_{t-1,t}$	0.11	0.12	−0.01	0.04	0.05	0.07	0.02	0.05	−0.38	0.28	0.07	0.16	0.00	0.03
	Constant	7.82	2.78	2.76	0.97	4.03	1.78	1.96	1.43	−11.95	6.61	−10.28	4.03	0.69	0.43
	RF$_{t-1,t}$	−11.20	2.95	−4.65	1.59	−6.05	1.76	4.59	1.80	−6.72	5.33	11.08	6.58	9.84	0.74
	TERMLONG$_{t-1}$	−0.05	0.50	−0.20	0.18	−0.12	0.34	−0.29	0.23	2.99	1.15	3.93	0.98	−0.01	0.10
	TERM1YR$_{t-1}$	1.95	0.86	0.29	0.28	0.82	0.48	0.89	0.41	−0.87	1.55	−2.51	1.59	0.62	0.22
$\mathbf{Z}_{t-1}$	DEFBOND$_{t-1}$	3.48	1.56	1.00	0.52	1.92	0.80	−0.98	0.98	−0.97	3.01	−1.04	3.34	0.15	0.43
	DEFCP$_{t-1}$	−1.82	0.86	−0.02	0.27	−0.53	0.45	−0.30	0.42	1.50	2.03	1.34	1.26	0.06	0.20
	DIVYIELD$_{t-1}$	−0.22	0.60	0.75	0.36	0.18	0.32	−0.15	0.40	6.64	1.38	1.38	0.79	−0.16	0.09
	Production growth$_{t-13,t-1}$	−0.30	0.09	−0.02	0.03	−0.06	0.05	0.05	0.03	−0.49	0.21	0.07	0.11	0.01	0.01
	Inflation$_{t-13,t-1}$	−0.32	0.25	−0.20	0.10	−0.29	0.10	0.41	0.14	−1.30	0.30	−0.64	0.31	0.07	0.05
	Excess stock returns$_{t-13,t-1}$	0.05	0.03	0.01	0.01	0.05	0.02	0.00	0.01	−0.22	0.10	−0.14	0.05	0.01	0.01
	R^2	0.45		0.38		0.48		0.54		0.45		0.35		0.91	

Factor-Mimicking Portfolios

- Instead of a regression, sometimes common sense dictates what the appropriate asset is that mimics the factor.
  - ↪ Use oil futures to mimic an oil price factor.
  - ↪ Use TIPS-Treasuries to mimic an inflation factor.
  - ↪ Use REITs to mimic a real-estate price factor.
  - ↪ Use gold futures to mimic a gold price factor.
- Factor mimicking portfolios are useful
  - ↪ For hedging factor risks.
  - ↪ For estimating the risk-premia of the factor. The expected return on a portfolio that mimics factor f (and is uncorrelated with the other APT factors) equals

$$ER_{f,t} = \lambda_f$$

  - ↪ Remember: factor-mimicking portfolios are constructed from excess returns, so they are already zero-investment portfolios.

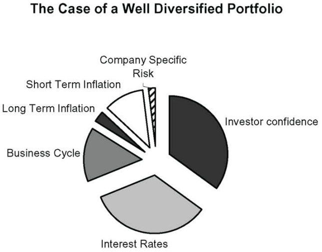
Kellogg School of Management

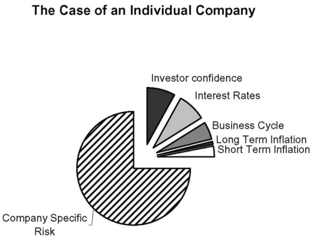
Asset Allocation using the APT

- The APT can be used in our asset allocation decision in place of the CAPM:
  a) Use a multi-factor model to estimate correlations.
  b) Use the APT along with the factor models to determine what expected returns *should* be.
  c) Combine the APT returns with your own views, as in Black-Litterman.
- An example of a firm using the APT to manage money is the *Roll and Ross Asset Management Corporation*.
  ↪ The following diagrams explain their methodology

Asset Allocation using the APT (Roll and Ross)

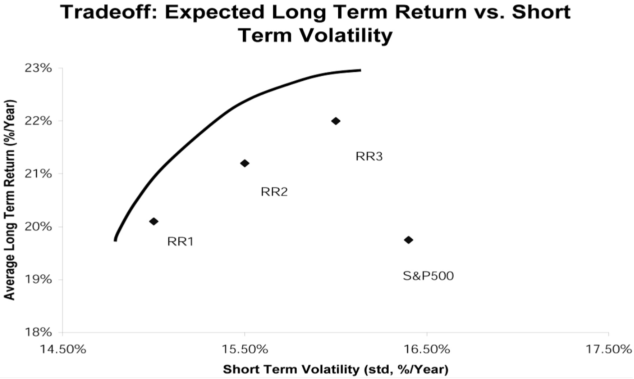
- A portfolio's long-term return and its volatility are completely determined by its factor loadings.





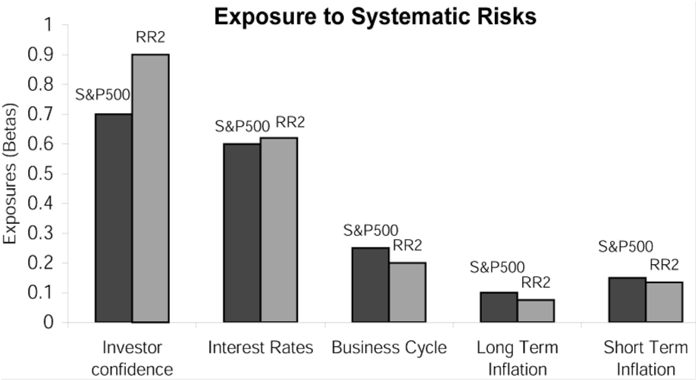
Asset Allocation using the APT (Roll and Ross)

- The benchmark is in general not an “optimal” portfolio



Asset Allocation using the APT (Roll and Ross)

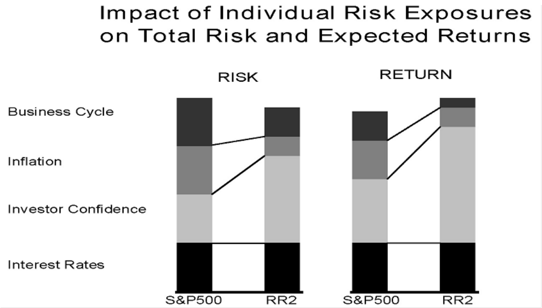
- The factors contribute differently to the aggregate risk and return of the portfolio



Asset Allocation using the APT (Roll and Ross)

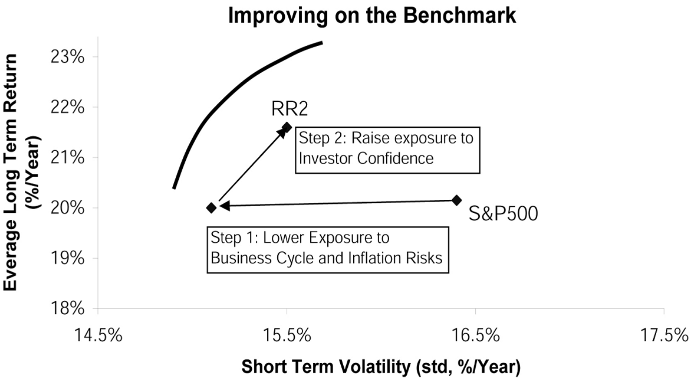
- Assuming that the factors are uncorrelated, the Sharpe ratio of the portfolio is:

$$SR = \frac{\lambda_1 b_1 + \lambda_2 b_2 + \ldots \lambda_k b_k}{\sqrt{\sigma_1^2 b_1^2 + \sigma_2^2 b_2^2 + \ldots \sigma_K^2 b_K^2}}$$



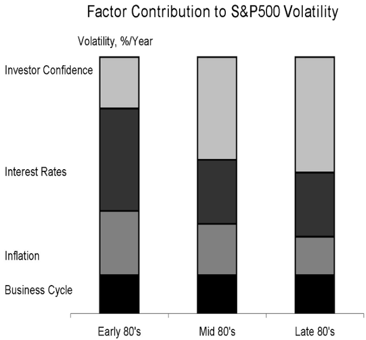
Asset Allocation using the APT (Roll and Ross)

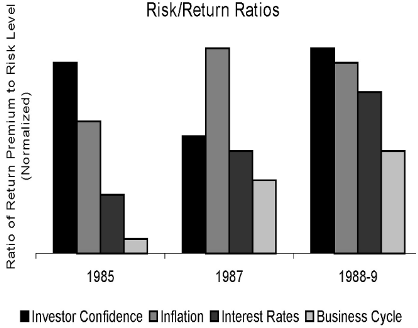
- By measuring and controlling a portfolio's relative systematic risk exposures, the firm of Roll and Ross can produce the highest possible return for a given level of risk.



Asset Allocation using the APT (Roll and Ross)

- Factor volatilities and rewards change over time, so we need to rebalance often.





Risk Management

- Factor models are also a convenient way to manage portfolio risks.
- As a portfolio manager you can control specific risk exposures, customizing the portfolio to the needs of your clients.
- In order to hedge out your exposure to a factor:
  1. Estimate the loading of each asset with respect to the factor, $b_{i,k}$.
  2. Include an additional constraint: The factor loading of your portfolio should equal zero: $b_{p,k} = 0$.

Risk Management - Example

- You believe the following four-factor model holds:

$$R_{it} = a_i + b_{M,i}\tilde{f}_{M,t} + b_{TS,i}\tilde{f}_{TS,t} + b_{CS,i}\tilde{f}_{CS,t} + b_{O,i}\tilde{f}_{O,t} + \varepsilon_{i,t}$$

  where
  - $\hookrightarrow$ $\tilde{f}_{M,t}$ is the surprise return on the market portfolio.
  - $\hookrightarrow$ $\tilde{f}_{TS,t}$ is the surprise change in the slope of the yield curve.
  - $\hookrightarrow$ $\tilde{f}_{CS,t}$ is the surprise change in the credit spreads.
  - $\hookrightarrow$ $\tilde{f}_{O,t}$ is the surprise change in oil prices.
- The covariance matrix of the factors is

Covariance	MKT	TS	YS	OIL
MKT	2.100%			
TS		0.090%		
YS			0.030%	
OIL				0.760%

Risk Management - Example

- Suppose you are choosing between 6 portfolios. You have estimated the following for each portfolio:

No	E (R)	$\sigma(R)$	Factor Loadings			
			market	**TS**	**YS**	**OIL**
SG	14.7%	0.306	1.439	5.487	0.279	0.043
SC	10.7%	0.208	0.872	3.419	-0.018	-0.019
SV	10.2%	0.210	0.786	3.925	1.250	-0.036
LG	11.2%	0.197	1.022	-0.602	2.617	-0.010
LC	9.7%	0.174	0.749	-0.221	-2.134	-0.062
LV	9.3%	0.169	0.672	-0.795	-1.202	-0.113

- What is the optimal portfolio that:
  - ↪ Achieves an expected return of 12% and has zero exposure to Oil risk?
  - ↪ Has the maximum Sharpe ratio ($r_f = 5\%$) and has loading of 1 with the market portfolio?

Risk Management - Example

- First we calculate the correlation matrix implied by the 4-factor model:

	1	2	3	4	5	6
1	1	0.6821	0.6761	0.4662	0.4005	0.3132
2	0.6821	1	0.609	0.4107	0.3605	0.2804
3	0.6761	0.609	1	0.3804	0.2946	0.2203
4	0.4662	0.4107	0.3804	1	0.4221	0.4172
5	0.4005	0.3605	0.2946	0.4221	1	0.3937
6	0.3132	0.2804	0.2203	0.4172	0.3937	1

- Then we plug in the numbers into Markowitz, with some additional constraints on the loadings

- Optimal Portfolio that achieves 12% return and has zero oil exposure.

Number of securities: 6

No	Name	Fraction	Expected Return	Standard Deviation
1	1 (SV)	0.29	14.7%	30.6%
2	2 (SC)	0.12	10.7%	20.8%
3	3 (SG)	-0.03	10.2%	21.0%
4	4 (LV)	0.47	11.2%	19.7%
5	5 (LC)	0.19	9.7%	17.4%
6	6 (LG)	-0.04	9.3%	16.9%
		1.00		

Correlations		2 2 (SC)	3 3 (SG)	4 4 (LV)	5 5 (LC)	6 6 (LG)
1	1 (SV)	0.68	0.68	0.47	0.40	0.31
2	2 (SC)	1.00	0.61	0.41	0.36	0.28
3	3 (SG)		1.00	0.38	0.29	0.22
4	4 (LV)			1.00	0.42	0.42
5	5 (LC)				1.00	0.39
		YES				

Portfolio's Expected Return	12.0%
Portfolio's Standard Deviation	18.4%

Cov(r_p,F_OI)/Var(F_OI) ***(0.00000)***

FACTOR LOADINGS

	market	*term_spread*	*yield spread*	*Oil*
1	1.439	5.487	0.279	0.043
2	0.872	3.419	-0.018	-0.019
3	0.786	3.925	1.250	-0.036
4	1.022	-0.602	2.617	-0.010
5	0.749	-0.221	-2.134	-0.062
6	0.672	-0.795	-1.202	-0.113
PORTFOLIO	***1.097***	***1.614***	***0.922***	***0.000***

Portfolio Factor load =>

Covariance	*market*	*term_spread*	*yield spread*	*Oil inflation*
market	0.021			
term_spread		0.0009		
yield spread			0.0003	
Oil inflation				0.0076

Risk Management - Example

- Optimal Portfolio (i.e. max Sharpe Ratio) with market beta of 1.

Number of securities: 6

No	Name	Fraction	Expected Return	Standard Deviation
1	1 (SV)	0.24	14.7%	30.6%
2	2 (SC)	0.06	10.7%	20.8%
3	3 (SG)	-0.01	10.2%	21.0%
4	4 (LV)	0.33	11.2%	19.7%
5	5 (LC)	0.19	9.7%	17.4%
6	6 (LG)	0.19	9.3%	16.9%
		1.00		

Correlations		2 2 (SC)	3 3 (SG)	4 4 (LV)	5 5 (LC)	6 6 (LG)
1	1 (SV)	0.68	0.68	0.47	0.40	0.31
2	2 (SC)	1.00	0.61	0.41	0.36	0.28
3	3 (SG)		1.00	0.38	0.29	0.22
4	4 (LV)			1.00	0.42	0.42
5	5 (LC)				1.00	0.39
		YES				

Portfolio's Expected Return	11.4%
Portfolio's Standard Deviation	16.2%
Sharpe Ratio	0.393922705
risk free rate	5%

FACTOR LOADINGS

	market	*term_spread*	*yield spread*	*Oil*
1	1.439	5.487	0.279	0.043
2	0.872	3.419	-0.018	-0.019
3	0.786	3.925	1.250	-0.036
4	1.022	-0.602	2.617	-0.010
5	0.749	-0.221	-2.134	-0.062
6	0.672	-0.795	-1.202	-0.113
PORTFOLIO	***1.000***	***1.134***	***0.292***	***-0.026***

Portfolio Factor load =>

market loading ***1.00000***

Covariance	*market*	*term_spread*	*yield spread*	*Oil inflation*
market	0.021			
term_spread		0.0009		
yield spread			0.0003	
Oil inflation				0.0076

Risk Management using Factor-Mimicking Portfolios

- An alternative approach would have been to hedge the risk out by trading in factor-mimicking portfolios.
- The approach would be:
    1. Find the optimal portfolio, without imposing any constraints.
    2. Find the loading of your portfolio with respect to the factor you wish to hedge $(b_{p,f})$.
    3. Sell a fraction $b_{p,f}$ of the portfolio that mimics factor f. Since it is a zero-investment portfolio, no further action is necessary.
- The end result will be the same with either approach if the factor-mimicking portfolio is included in the set of assets you are optimizing over.

Summary

- The APT *does not say* what the systematic factors are.
- In order to implement APT in practice, we need to specify factors.

    a) Factors can be specified a priori: they could be macroeconomic variables (ex inflation, output) that capture the systematic risk in the economy or portfolios proxying for these risks.

    b) Factors can be extracted via Principal Components or Factor Analysis.

- Once the factors are identified, the APT can be used as a performance measurement tool.

    $\hookrightarrow$ See next Lecture.