CSC311 - Final Assignment

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Contents

Part I

Predicting Student Correctness

1 K-Nearest Neighbor

Following parts will refer to the following information:

```python
import knn as knn
k_vals, val_user_acc, val_item_acc = knn.main(data_path="./data")
```

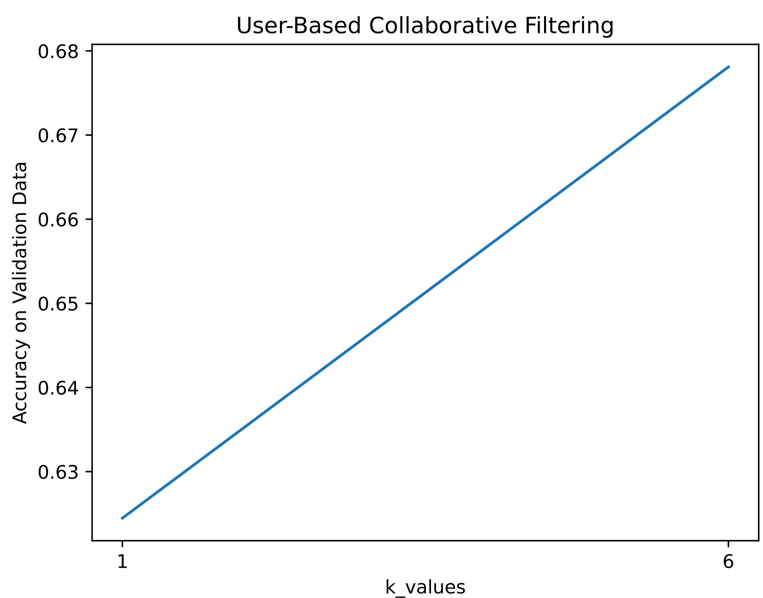
Output:

```
k*: 6 val. acc.: 0.6780976573525261
k*: 6 val. acc.: 0.6542478125882021
User-Based Algorithm Accuracy on Test Data:  0.6646909398814564
Item-Based Algorithm Accuracy on Test Data:  0.6539655659046006
User-based collaborative filtering works better
```

a Complete Main kNN, Plot and Report Accuracy

The implementation of all code is in the part_a/knn.py file. Following are plots of accuracy on validation data as a function of $k \in \{1, 6, 11, 16, 21, 26\}$:

```python
knn.accuracy_plot(k_vals, val_user_acc, "User-Based Collaborative
↪  Filtering")
```



See accuracies in the data output near the beginning of the question.

b Selecting k*

We selected $k = 11$ for user-based collaborative filtering as this resulted in the highest validation accuracy (refer to data output of `main` function near beginning of question for report on final test accuracy).

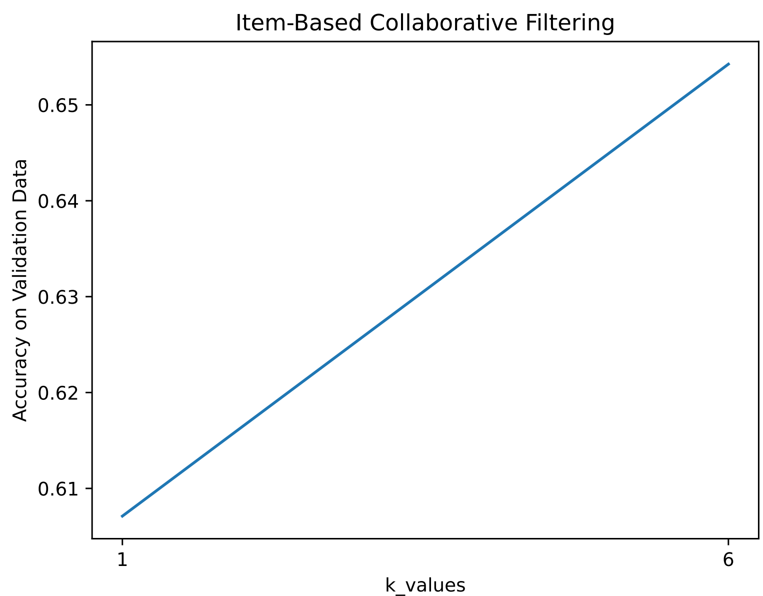
c Implementing Impute by Item

The implementation is in the same file as the user-based version.

Underlying assumption: if answers by certain users to Question A match those of Question B, then As answer correctness corresponding to a specific user matches that of question Y.

Repetition of a) and b) where the data is in the same output box as for user-based collaborative filtering and plot as follows:

```
knn.accuracy_plot(k_vals, val_item_acc, "Item-Based Collaborative
↪  Filtering")
```



d Comparing user and item based Collaborative Filtering

User-Based collaborative filtering performs better on test data. 68.416% accuracy on user-based filtering and 68.162% accuracy on item-based filtering.

e Potential Limitations of kNN in this Context

We can safely assume that there is a high correlation between both question difficulty and student ability on whether or not the question was answered correctly. But, feature importance is not possible for the KNN algorithm (there is no way to define the features which are responsible for the classification), so it will not be able to make accurate inferences based on these two parameters. In the algorithm used in this question, either one of the two parameters (user ability or question difficulty) is focused on, so it has lower validation and test accuracy scores than other algorithms in Part A of this project.

KNN runs slowly. Finding the optimal k-value from the given list of possible k values (1, 6, 11, 21, 26) takes several minutes for each function.

2 Item Response Theory

a Mathematical Derivations for IRT

We are given that $p(c_{ij} = 1|\boldsymbol{\theta}, \boldsymbol{\beta})$. We will assume c_{ij} is a value in $\boldsymbol{C}$ where i and j as coordinates are in set O as defined:

$$O = \{(i, j) : \text{Entry } (i, j) \text{ of matrix } \boldsymbol{C} \text{ and is observed}\}$$

.

Since this c_{ij} is a binary value, we can describe $P(\boldsymbol{C}|\boldsymbol{\theta}, \boldsymbol{\beta})$ with a bernoulli distribution:

$$p(C|\boldsymbol{\theta}, \boldsymbol{\beta}) = \prod_{ij}[\frac{exp(\theta_i - \beta_j)}{1 + exp(\theta_i - \beta_j)}]^{c_{ij}}[\frac{1}{1 + exp(\theta_i - \beta_j)}]^{(1-c_{ij})}$$

Therefore, our Likelihood function is:

$$L(\boldsymbol{\theta}, \boldsymbol{\beta}) = \prod_{ij}[\frac{exp(\theta_i - \beta_j)}{1 + exp(\theta_i - \beta_j)}]^{c_{ij}}[\frac{1}{1 + exp(\theta_i - \beta_j)}]^{(1-c_{ij})}$$

Then, apply log to obtain the log-likelihood where N and M are the number of users and questions respectively:

$$\begin{aligned}
L(\boldsymbol{\theta}, \boldsymbol{\beta}) &= \prod_{ij}[\frac{exp(\theta_i - \beta_j)}{1 + exp(\theta_i - \beta_j)}]^{c_{ij}}[\frac{1}{1 + exp(\theta_i - \beta_j)}]^{(1-c_{ij})} \\
log(L(\boldsymbol{\theta}, \boldsymbol{\beta})) &= \log(\prod_{ij}[\frac{exp(\theta_i - \beta_j)}{1 + exp(\theta_i - \beta_j)}^{c_{ij}}][\frac{1}{1 + exp(\theta_i - \beta_j)}^{1-c_{ij}}] \\
&= \sum_{i=1}^{N}\sum_{j=1}^{M}\log([\frac{exp(\theta_i - \beta_j)}{1 + exp(\theta_i - \beta_j)}^{c_{ij}}][\frac{1}{1 + exp(\theta_i - \beta_j)}^{1-c_{ij}}]) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij}((log(exp(\theta_i - \beta_j)) - log(1 + exp(\theta_i - \beta_j))) \\
&+ (1 - c_{ij})(\log(1) - \log(1 + exp(\theta_i - \beta_j))) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{M}[c_{ij}(\theta_i - \beta_j) - \log(\frac{exp(\theta_i - \beta_j)}{1 + exp(\theta_i - \beta_j)})]
\end{aligned}$$

Then, we solve for the partial derivative with respect to θ_i and β_j respectively:

$$\begin{aligned}
\frac{\delta}{\delta\theta_i} &= \sum_{j=1}^{M}[c_{ij} - \frac{exp(\theta_i - \beta_j)}{1 + exp(\theta_i - \beta_j)}] \\
\frac{\delta}{\delta\beta_j} &= \sum_{i=1}^{N}[-c_{ij} + \frac{exp(\theta_i - \beta_j)}{1 + exp(\theta_i - \beta_j)}]
\end{aligned}$$

b Implementation of IRT

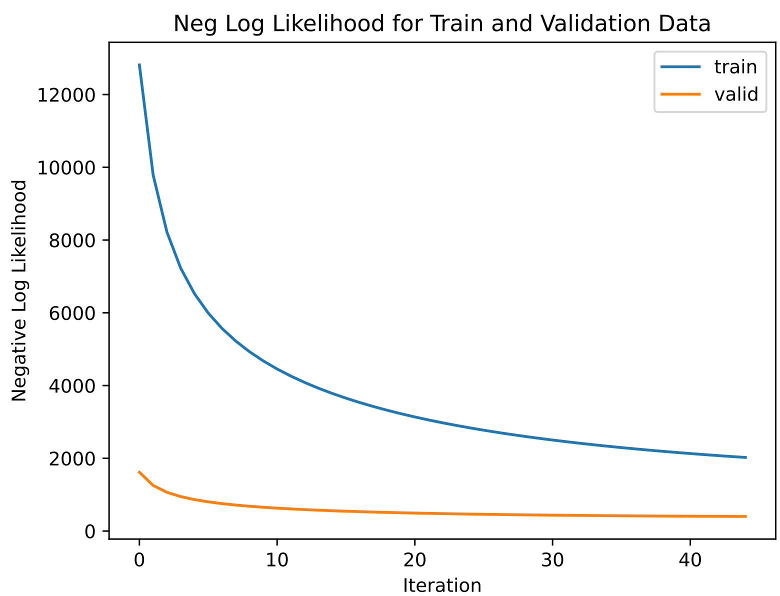
The implementation of IRT is in `part_a/item_response.py`. We chose the hyperparameters α and iterations number by performing multiple combinations of them and seeing which one had the highest validation score (automated, see mentioned code file for this automation). We then manually adjusted the set of tested values and repeated. Doing this a few times resulted in:

```python
import item_response as irt
print()
irt_results = irt.main("./data")
```

```
Training with lr of 0.005 and 45 number of iterations.
Final accuracy: 0.7082980524978831
lr*: 0.005 iterations*: 45 val_acc*: 0.7082980524978831
test accuracy: 0.7044877222692634
```

Which is the best result out of the combinations we tried.

The following is the training curve showing training and validation negative log likelihoods as a function of number of iterations:



c Reporting Validation and Test Accuracies

Validation and test accuracies have been calculated in the previous call to the main function. Implementation is in `part_a/item_response.py`.

Our validation accuracy:

```python
print(irt_results["val_acc"])
```

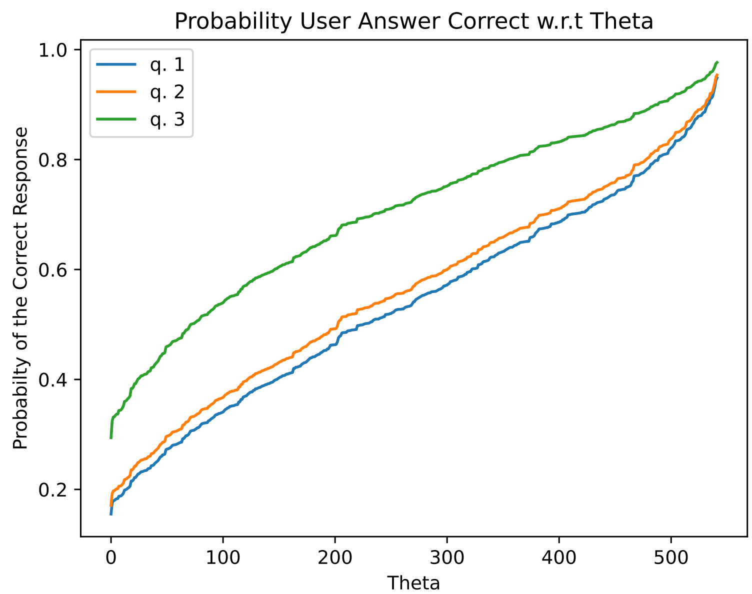
```
0.7082980524978831
```

Test accuracy:

```python
print(irt_results["test_acc"])
```

```
0.7044877222692634
```

d Plots of Questions With Respect to θ and β



From this figure, we can see that there seems to be a sigmoidal shape to all three curves. Since the question difficulty, i.e, β_j, doesn't change, it can be considered a constant for one curve. θ_i, the student's ability, is on the x axis and changing. Note that the probability being calculated is the sigmoid of the difference between the θ_i and β_j. Since the curve is not the sigmoid curve without transformations, and β_j is constant, this must mean that θ when sorted, do not increase linearly. We can thus interpret the curve to mean: For a given question that has a theoretical constant difficulty level, as a user's ability increases, their probability of solving a problem correctly also increases, much more drastically at the lower ability levels (steeper slope), and slowing down near the middle (indicated by the decrease in slope) and then increases dramatically again (steep slope again).

3 Neural Networks

a Differences Between ALS and Neural Networks

1. ALS optimizes 2 variable U and Z, neural net optimize one variable W(with gradient descent),

2. ALS is an optimization algorithm that is incorporated as a part of a machine learning algorithm, while the neural network is a machine learning algorithm that that uses optimization algorithms to achieve learning.

3. In neural net, W is used to manipulate x, while in ALS, W,X is being optimized as one variable U .

4. ALS is essentially measuring the difference between target and product of two value(s), the obtain the two value, train matrix need to be pre-processed with SVD, neural network don't need to do this.

5. neural net don't optimize latent Z directly but achieve Z's optimization with W_1 using $g(W_1 x)$, where as ALS directly optimize Z.

b Implementing AutoEncoder

This part can be found in `part_a/neural_network.py`.

c Tuning and Training NN

```
import neural_network as nn
nn.main(data_path="./data", verbosity = 1)
```

```
Starting hyperparameter combination trials.
        k        Learn. Rate        # of Epochs        Lambda
        10        0.05        10        0.00025

Final training results:

Epoch: 10/10, Loss: 10228.314681, Valid acc.: 0.6741462037821055
        Final Validation Accuracy*        0.6741462037821055
        Final Test Accuracy*        0.6762630539091166
Concluding learning rate, number of epoch and lambda optimizations.
Results:
        Final acc.:        0.6741462037821055
        k*:        10
        Learning rate*:        0.05
        Number of epochs*:        10
        Lambda*:        0.00025
```

d Plotting and Reporting

The following are the plots generated:

e Implementing L_2 Regularization

L_2 regularization has been implemented in the same code file as the other parts of this question (`part_a/neural_network.py`).

```
lamb=0
Final Validation Accuracy*    0.6858594411515665
Final Test Accuracy*    0.6836014676827548
lamb=0.001
Final Validation Accuracy*    0.6869884278859724
Final Test Accuracy*    0.6785210273779283


(optional extra finding):
but with lamb=0.00025
Final accuracy    0.6848715777589613
Final Test Accuracy*    0.6861416878351679
```

There are improvements on the validation accuracy, but not on the test accuracy.

4 Ensemble

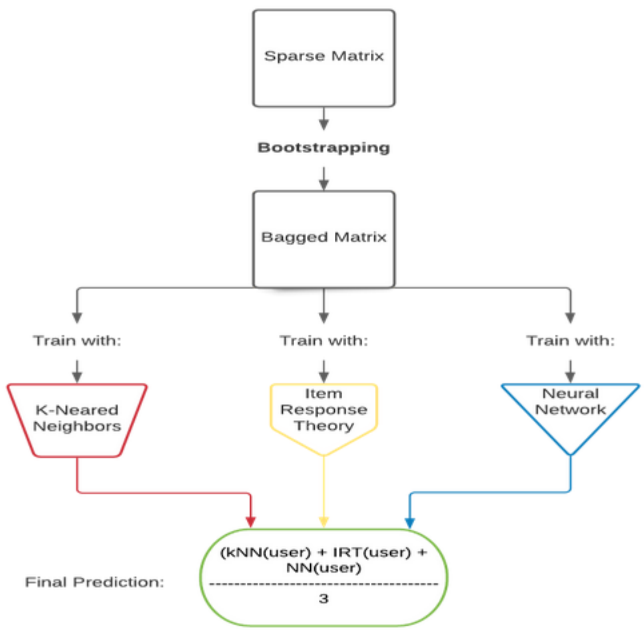
Code for the ensemble is implemented in `part_a/ensembly.py`. We bagged our base neural network, k-Nearest-Neighbor, and Item-Response models with their previously discussed optimal hyper parameters.

```
import ensemble as ensemble
ensemble.evaluate_ensemble(verbosity=1, data_path="./data")
```

Part II
Modifying for Higher Accuracy

5 Formal Description

We are extending the ensemble algorithm (Part A, Question 4). The original ensemble algorithm has 3 phases: bagging (bootstrap aggregating sparse matrix) $\Longrightarrow$ train (training models using the bagged matrix) $\Longrightarrow$ prediction, as demonstrated in the figure below:



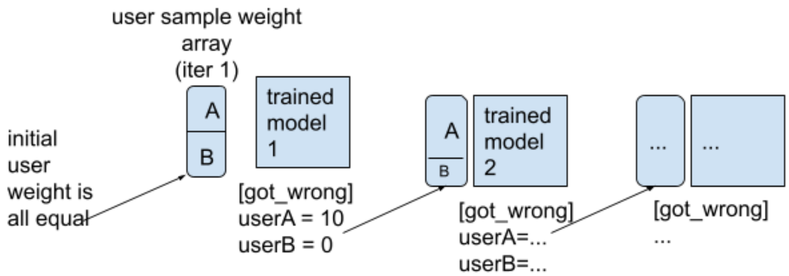
For this project, we gathered inspirations from the AdaBoost algorithm.

AdaBoost is adaptive in the sense that subsequent weak learners are tweaked in favor of those samples misclassified by previous classifiers (in our case, predictors of Item Response Theory and Neural Network algorithms). The individual learners can be weak, but as long as the performance of each one is slightly better than random guessing, the final model can be proven to converge to a strong learner.

We are modifying the bootstrapping phase (that generates a bagged matrix from the sparse matrix), and prediction phase (green box):

1. Instead of bootstrapping with a uniform probability distribution as ensemble in Part A does, we are assigning weights to each user; the weight of each user is determined by the number of questions that the previous model got wrong on this user, except the first bagging which is still uniform probability distribution.

To illustrate an example: if model 1 made 10 wrong predictions on questions done by user A and made 0 wrong prediction on questions done by user B, then in the weighted bootstrapping phase of model 2, the weight that user A will be resampled again will be significantly greater than the the weight of user B:



[Insert formula here] In the general case, After the training of a model, we obtain the number of total training entries; number of training entries that the model scored correctly and wrong , per user . We determine a training accuracy **2**. For the prediction phase, instead of generating 3 predictions and taking the average to be the final prediction, we are assigning specific user weights to each model. This weight is determined by the models performance on their bagged training set of a specific user. The model with the highest weighting of that specific user will be used for all predictions regarding that user.

To illustrate an example, If a model is good at predicting user A and bad at predicting user B and another model that is good at predicting user B and bad at predicting user A, then both model will be selected to predict on users that they are particularly good at and avoid users they dont have a good grasp with.

By focusing on the training errors the models made, we expect this will reduce overfit on users that can be easily predicted, and improve underfit on users that the models struggle to predict.

By picking a single most suited model to predict based on which user is doing the question, we expect this will eliminate the risk of bad performing models having a say on a user that they cant predict well on (which both original ensembles averaging and AdaBoosts weighted majority vote cannot avoid).

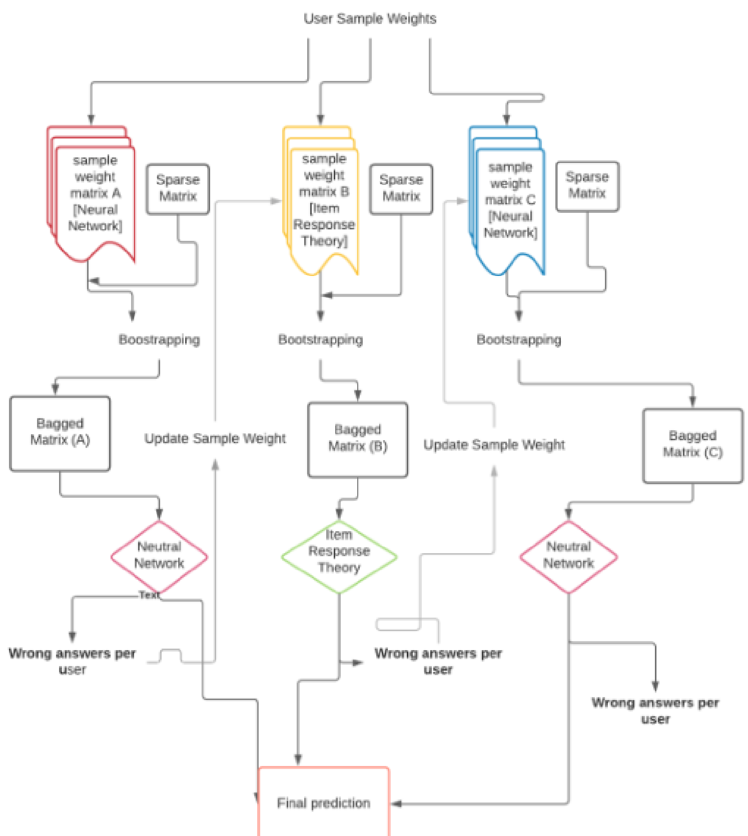
Additionally, our Adaboost Ensemble model does not include k-nn as a weak base model. We have omitted KNN model from our final adaboost ensemble, and instead replaced it with our previously implemented neural network model, with a differently-bagged matrix from the original neural network model. We chose this implementation because the decision surfaces of k-nn classifiers are typically too stable and any multiples of data points in the bootstrap sample do not shift the 'weight' like in many other models. [insert citation]

We believe our modification will perform well in cases such as:

- Individual user's performance is easy to predict by some models and hard to predict by some other models.

- There exist significant patterns to be learned on the errors that the previous model got wrong.

Figure or Diagram: that shows the overall model or idea. The idea is to make your report more accessible, especially to readers who are starting by skimming your report.



6 Figure or Diagram

7 Comparison or Demonstration