University of Toronto

CO2 emission & Global temperature analysis

YiFei Gu

1 CO2

1.1 Introduction:

The following report provides a brief discussion of atmospheric Carbon Dioxide concentrations and whether they were impacted by some meaninigful events happened worldwide. For example: the signing of the Paris Agreement on 12 December 2015, intended to limit CO2 emissions.

1.2 Methods:

For the analysis, the following model is used:

$$Y_i \sim \text{Gamma}(\theta, \frac{\lambda_i}{\theta})$$

With

$$\lambda_i = \beta_0 + \beta_1 \text{sin}(2\pi X_i) + \beta_2 \text{sin}(4\pi X_i) + \beta_3 \text{cos}(2\pi X_i) + \beta_4 \cos(4\pi X_i) + U(t_i) + V_i$$

with $\sin(4\pi X_i), \cos(4\pi X_i), \sin(2\pi X_i), \cos(2\pi X_i)$ being annual and semi-annual seasonal effects.
Note that $U(t_i)$ is the random effect, with

$$[U_1, ..., U_T]^T \sim RW2(0, \sigma_U^2)$$

And

$$V_i \sim N(0, \sigma_V^2)$$

As one can identify from the graph showing the time random effect on ppm from figure 1, the linear trend does not show clearly the change of ppm of CO2 emissions during specific time frame. Further studies need to be accessed, addressing the impact of specific event on CO2 emissions.

1.3 Results:

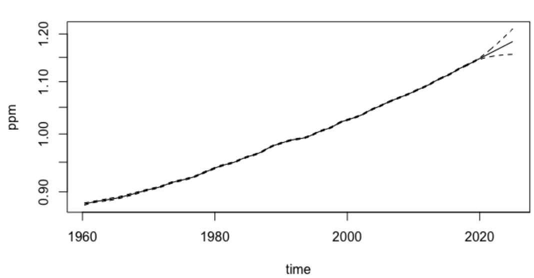

Figure 1: Estimated Smoothed Trend of CO2 data

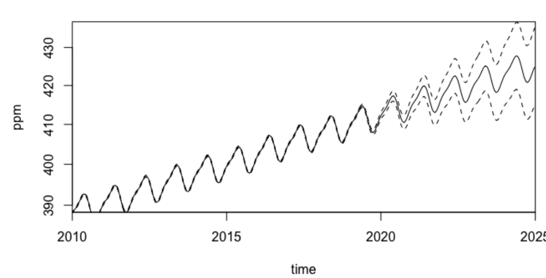

Figure 2: Predicted Trend of CO2 emission

Figure 2 shows the predicted trend of CO2 emissions, with dashed lines above and below the solid lines being its credible interval.
The following figure shows the derivative of logged ppm and time.

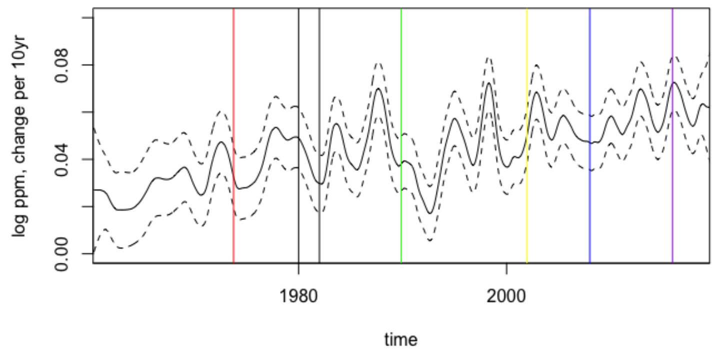

Figure 3: Derivative of logged estimated smoothed trend of CO2 data

Each vertical line indicates a specific time frame, with their interaction being the direction of change of log ppm. The events below are indicated by the same color as the vertical lines, from left to right. The dashed black lines are the credible interval of trend of CO2 emission.

- the OPEC oil embargo which began in October 1973: the intersection between red vertical line and log ppm change per 10 yr has a downward slope indicating a decrease of CO2 emission at 1973.
- **the global economic recessions around 1980-1982:** Between two black lines from 1980 to 1982, it is clearly that the trend of CO2 emission has decreased during this period, indicated by the downward slope within two black lines.
- the fall of the Berlin wall: During the specific time frame there is not a significant change of the trend of CO2 emission, however, one can identify that over a ten year time frame there is a decrease of trend indicated by the downward slope.
- China joining the WTO on 11 December 2001: The upward slope intersected with yellow lines, which represents trend of CO2 emission is increasing along with the rapid growth in industrial production by China after joining WTO.
- The bankruptcy of Lehman Brothers: Resulting the global financial crisis, the vertical line intersected with the plot appeared to be rather flat, however, the following years has an obvious upward slope indicates the increase of trend of CO2 emission.
- The signing of the Paris Agreement: As indicated by the intersection with purple vertical line, the following downward slope indicates the decrease of the trend of CO2 emission, which shows a positive results from the agreement that aimed to limit CO2 emission.

2 Heat

2.1 Introduction

The following reports briefly analyze whether the data from Sable Island can support the statement from IPCC, that Human activities are estimated to have caused approximately 1.0°C of global warming above preindustrial levels, with a likely range of 0.8°C to 1.2°C. In addititon, global warming is likely to reach 1.5°C between 2030 and 2052 if it continues to increase at the current rate. To test whether the statement is responsible or not, temperature data recorded on Sable Island, off the coast of Nova Scotia will by analyzed, in addition, estimated time trend and posterior samples of the trend will also be analyzed.

2.2 Methods

Since estimation of trends are needed to be acessed, non-parametric model is suitable in this case, with Y being the daily maximum temperature, s as a positive fixed coefficient, ν_{ij} the linear predictor. The penalized prior $P(\frac{1}{\sqrt{\tau}} > 1.75) = 0.5$.

$$\sqrt{sT}(Y_{ij} - \nu_{ij}) \sim T_v$$

with:

$$\nu_{ij} = \beta_0 + \beta_1 \sin(4\pi X_{ij}) + \beta_2 \cos(4\pi X_{ij}) + \beta_3 \sin(2\pi X_{ij}) + \beta_4 \cos(2\pi X_{ij}) + U(t_{ij}) + V_{ij} + W_i$$

Note that $\sin(4\pi X_{ij}), \cos(4\pi X_{ij}), \sin(2\pi X_{ij}), \cos(2\pi X_{ij})$ are annual and semi-annual seasonal effects. $U(t_{ij})$ is the second order random walk, with:

$$[U_1, ..., U_T]^T \sim RW2(0, \sigma_U^2)$$

And V_{ij} being weeks, W_i being year:

$$V_{ij} \sim N(0, \sigma_V^2)$$

$$W_i \sim N(0, \sigma_W^2)$$

The prior for V_{ij} is $P(\frac{1}{\sigma_V} > 1) = 0.5$, and the prior for W_i is $P(\frac{1}{\sigma_W} > 0.7 = 0.5)$

2.3 Results:

The following figure indicates the change of maximum temperature exclusively in summer from 1900 to 2020, with credible interval shown as dashed line above and below the solid line.

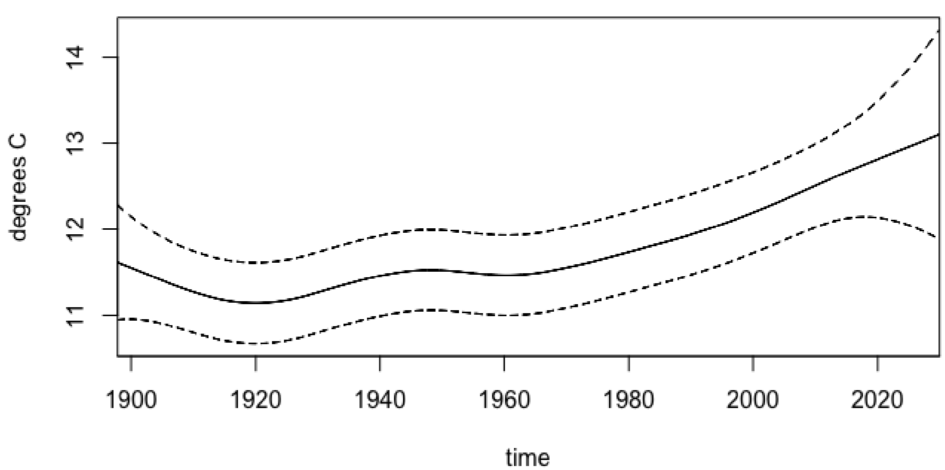

Figure 4: Estimated time trend of temperature in Sable Island

From 1990 to 2020, the temperature increased from approximately 11.6 degree to 13 degree, with a change of 1.4 degree. From Figure 4., one can definitely see an upward trend of maximum temperature. However, it is not rigorous to conclude that, Global warming is likely to reach 1.5°C between 2030 and 2052 if it continues to increase at the current rate. As specifically, all possible outcomes that are within the credible interval does not indicate a definite increase, more specifically, it can be illustrated even more clearly with the Figure 5. shown below:

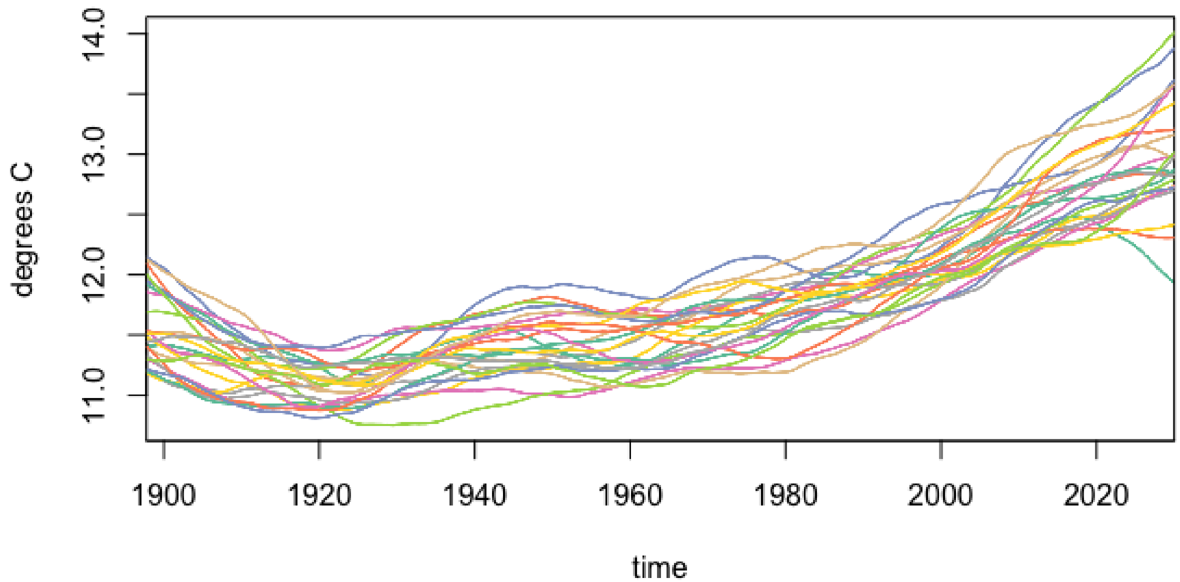

Figure 5: Posterior samples of estimated time trend of temperature

An apparent example is that the pink line below, from 1920 to 1950, the temperature remained the same, which contradicts to the estimated time trend of temperatures, as shown in Figure 4, which shows an upward trend during that time frame. It is true that human did cause a temperature increase of 1 degree, but one can only conclude the fact that, if the temperature keeps increasing in the current rate, the global warming will reach a 1.5 degree temperature difference.

3 Appendix

```
co2s = read.csv("co2s.csv")
co2s$date = strptime(paste(co2s$day, co2s$time), format = "%Y-%m-%d
,→ %H:%M", tz = "UTC")
# remove low-quality measurements
co2s[co2s$quality >= 1, "co2"] = NA
plot(co2s$date, co2s$co2, log = "y", cex = 0.3, col = "#00000040",
xlab = "time", ylab = "ppm")
plot(co2s[co2s$date > ISOdate(2015, 3, 1, tz = "UTC"),
c("date", "co2")], log = "y", type = "o", xlab = "time",
ylab = "ppm", cex = 0.5)
timeOrigin = ISOdate(1980, 1, 1, 0, 0, 0, tz = "UTC")
co2s$days = as.numeric(difftime(co2s$date, timeOrigin, units = "days"))
co2s$cos12 = cos(2 * pi * co2s$days/365.25)
co2s$sin12 = sin(2 * pi * co2s$days/365.25)
co2s$cos6 = cos(2 * 2 * pi * co2s$days/365.25)
co2s$sin6 = sin(2 * 2 * pi * co2s$days/365.25)
cLm = lm(co2 ~ days + cos12 + sin12 + cos6 + sin6, data = co2s)
```

```
summary(cLm)$coef[, 1:2]
newX = data.frame(date = seq(ISOdate(1990, 1, 1, 0, 0, 0, tz = "UTC"), by
,→ = "1 days", length.out = 365 * 30))
newX$days = as.numeric(difftime(newX$date, timeOrigin, units = "days"))
newX$cos12 = cos(2 * pi * newX$days/365.25)
newX$sin12 = sin(2 * pi * newX$days/365.25)
newX$cos6 = cos(2 * 2 * pi * newX$days/365.25)
newX$sin6 = sin(2 * 2 * pi * newX$days/365.25)
coPred = predict(cLm, newX, se.fit = TRUE)
coPred = data.frame(est = coPred$fit, lower = coPred$fit - 2 *
,→ coPred$se.fit, upper = coPred$fit + 2 * coPred$se.fit)
plot(newX$date, coPred$est, type = "l")
matlines(as.numeric(newX$date), coPred[, c("lower", "upper", "est")], lty
,→ = 1, col = c("yellow", "yellow","black"))
newX = newX[1:365, ]
newX$days = 0
plot(newX$date, predict(cLm, newX))
# time random effect
timeBreaks = seq(min(co2s$date), ISOdate(2025, 1, 1, tz = "UTC"), by =
,→ "14 days")
timePoints = timeBreaks[-1]
co2s$timeRw2 = as.numeric(cut(co2s$date, timeBreaks))
# derivatives of time random effect
D = Diagonal(length(timePoints)) - bandSparse(length(timePoints), k = -1)
derivLincomb = inla.make.lincombs(timeRw2 = D[-1, ])
names(derivLincomb) = gsub("^lc", "time", names(derivLincomb))
# seasonal effect
StimeSeason = seq(ISOdate(2009, 9, 1, tz = "UTC"), ISOdate(2011, 3, 1, tz
,→ = "UTC"), len = 1001)
StimeYear = as.numeric(difftime(StimeSeason, timeOrigin,\ "days"))/365.35
6
seasonLincomb = inla.make.lincombs(sin12 = sin(2 * pi * StimeYear), cos12
= cos(2 * pi * StimeYear), sin6 = sin(2 * 2 * pi * StimeYear), cos6 =
cos(2 * 2 * pi * StimeYear))
,→
,→
names(seasonLincomb) = gsub("^lc", "season", names(seasonLincomb))
# predictions
StimePred = as.numeric(difftime(timePoints, timeOrigin, units =
,→ "days"))/365.35
predLincomb = inla.make.lincombs(timeRw2 = Diagonal(length(timePoints)),
`(Intercept)` = rep(1, length(timePoints)), sin12 = sin(2 *pi *
StimePred), cos12 = cos(2 * pi * StimePred), sin6 = sin(2 * 2 * pi *
StimePred), cos6 = cos(2 * 2 * pi * StimePred))
,→
,→
,→
names(predLincomb) = gsub("^lc", "pred", names(predLincomb))
StimeIndex = seq(1, length(timePoints))
timeOriginIndex = which.min(abs(difftime(timePoints, timeOrigin)))
# disable some error checking in INLA
```

```
library("INLA")
mm = get("inla.models", INLA:::inla.get.inlaEnv())
if(class(mm) == 'function') mm = mm()
mm$latent$rw2$min.diff = NULL
assign("inla.models", mm, INLA:::inla.get.inlaEnv())
co2res = inla(co2 ~ sin12 + cos12 + sin6 + cos6 + f(timeRw2, model =
'rw2',values = StimeIndex, prior='pc.prec', param = c(log(1.01)/26,
0.5)),
,→
,→
data = co2s, family='gamma', lincomb = c(derivLincomb, seasonLincomb,
predLincomb),control.family =
list(hyper=list(prec=list(prior='pc.prec', param=c(2, 0.5)))),
,→
,→
matplot(timePoints, exp(co2res$summary.random$timeRw2[,c("0.5quant",
,→ "0.025quant", "0.975quant")]), type = "l",
col = "black", lty = c(1, 2, 2), log = "y", xaxt = "n",
xlab = "time", ylab = "ppm")
xax = pretty(timePoints)
axis(1, xax, format(xax, "%Y"))
derivPred =
co2res$summary.lincomb.derived[grep("time",rownames(co2res$summary.lincomb.derived)),
c("0.5quant", "0.025quant", "0.975quant")]
,→
,→
scaleTo10Years = (10 * 365.25/as.numeric(diff(timePoints,units =
,→ "days")))
matplot(timePoints[-1], scaleTo10Years * derivPred,type = "l", col =
"black", lty = c(1, 2, 2), ylim = c(0,0.1), xlim =
range(as.numeric(co2s$date)),xaxs = "i", xaxt = "n", xlab = "time",
ylab = "log ppm, change per 10yr")
,→
,→
,→
axis(1, xax, format(xax, "%Y"))
abline(v = ISOdate(1973, 10, 1, tz = "UTC"), col = "red")
abline(v = ISOdate(1980, 1, 1, tz = "UTC"), col = "orange")
abline(v = ISOdate(1982, 12, 30, tz = "UTC"), col = "orange")
abline(v = ISOdate(1989, 1, 1, tz = "UTC"), col = "yellow")
abline(v = ISOdate(2001, 12, 11, tz = "UTC"), col = "green")
abline(v = ISOdate(2008, 9, 15, tz = "UTC"), col = "blue")
7
abline(v = ISOdate(2015, 12, 12, tz = "UTC"), col = "purple")
matplot(StimeSeason,exp(co2res$summary.lincomb.derived[grep("season",rownames(co2res$summary.lincomb.de
c("0.5quant","0.025quant", "0.975quant")]), type = "l", col =
"black",lty = c(1, 2, 2), log = "y", xaxs = "i", xaxt = "n",xlab =
"time",ylab = "relative ppm")
,→
,→
,→
```

```
xaxSeason = seq(ISOdate(2009, 9, 1, tz = "UTC"), by = "2 months", len =
,→ 20)
axis(1, xaxSeason, format(xaxSeason, "%b"))
timePred =
co2res$summary.lincomb.derived[grep("pred",rownames(co2res$summary.lincomb.derived)),
c("0.5quant","0.025quant", "0.975quant")]
,→
,→
matplot(timePoints, exp(timePred), type = "l", col = "black",lty = c(1,
2, 2), log = "y", xlim = ISOdate(c(2010,2025), 1, 1, tz = "UTC"),
ylim = c(390, 435),xaxs = "i", xaxt = "n", xlab = "time", ylab =
"ppm")
,→
,→
,→
xaxPred = seq(ISOdate(2010, 1, 1, tz = "UTC"), by = "5 years",
len = 20)
axis(1, xaxPred, format(xaxPred, "%Y"))
##Q2
heatUrl = "http://pbrown.ca/teaching/appliedstats/data/sableIsland.rds"
heatFile = tempfile(basename(heatUrl))
download.file(heatUrl, heatFile)
x = readRDS(heatFile)
x$month = as.numeric(format(x$Date, "%m"))
xSub = x[x$month %in% 5:10 & !is.na(x$Max.Temp...C.),]
weekValues = seq(min(xSub$Date), ISOdate(2030, 1, 1, 0, 0, 0, tz =
,→ "UTC"), by = "7 days")
xSub$week = cut(xSub$Date, weekValues)
xSub$weekIid = xSub$week
xSub$day = as.numeric(difftime(xSub$Date, min(weekValues), units =
,→ "days"))
xSub$cos12 = cos(xSub$day * 2 * pi/365.25)
xSub$sin12 = sin(xSub$day * 2 * pi/365.25)
xSub$cos6 = cos(xSub$day * 2 * 2 * pi/365.25)
xSub$sin6 = sin(xSub$day * 2 * 2 * pi/365.25)
xSub$yearFac = factor(format(xSub$Date, "%Y"))
lmStart = lm(Max.Temp...C. ~ sin12 + cos12 + sin6 + cos6, data = xSub)
startingValues = c(lmStart$fitted.values, rep(lmStart$coef[1],
nlevels(xSub$week)),
rep(0,nlevels(xSub$weekIid)+nlevels(xSub$yearFac)), lmStart$coef[-1])
,→
,→
INLA::inla.doc('^t$')
mm = get("inla.models", INLA:::inla.get.inlaEnv())
if(class(mm) == 'function') mm = mm()
mm$latent$rw2$min.diff = NULL
assign("inla.models", mm, INLA:::inla.get.inlaEnv())
sableRes = INLA::inla(
8
Max.Temp...C. ~ 0 + sin12 + cos12 + sin6 + cos6 + f(week,
model='rw2',constr=FALSE,prior='pc.prec',param = c(0.1/(52*100),
```

```
0.05)) +
,→
,→
f(weekIid, model='iid', prior='pc.prec', param = c(1, 0.5)) +
f(yearFac, model='iid', prior='pc.prec', param = c(1, 0.5)),
family='T', control.family = list(
hyper = list(prec = list(prior='pc.prec', param=c(1, 0.5)), dof =
,→ list(prior='pc.dof', param=c(10, 0.5)))),
control.mode = list(theta = c(-1,2,20,0,1), x = startingValues,
,→ restart=TRUE),
control.compute=list(config = TRUE),
control.inla = list(strategy='gaussian', int.strategy='eb'), data =
,→ xSub, verbose=TRUE)
sableRes$summary.hyper[, c(4, 3, 5)]
mySample = inla.posterior.sample(n = 24, result = sableRes, num.threads =
8, selection = list(week =
seq(1,nrow(sableRes$summary.random$week)))
,→
,→
length(mySample)
names(mySample[[1]])
weekSample = do.call(cbind, lapply(mySample, function(xx) xx$latent))
dim(weekSample)
head(weekSample)
plot(x$Date, x$Max.Temp...C., col = mapmisc::col2html("black", 0.3))
forAxis = ISOdate(2016:2020, 1, 1, tz = "UTC")
plot(x$Date, x$Max.Temp...C., xlim = range(forAxis),xlab = "time", ylab =
,→ "degrees C", col = "red",xaxt = "n")
points(xSub$Date, xSub$Max.Temp...C.)
axis(1, forAxis, format(forAxis, "%Y"))
matplot(weekValues[-1], sableRes$summary.random$week[,paste0(c(0.5,
0.025, 0.975), "quant")], type = "l", lty = c(1, 2, 2), xlab =
"time", ylab = "degrees C", xaxt = "n", col = "black", xaxs = "i")
,→
,→
forXaxis2 = ISOdate(seq(1880, 2040, by = 20), 1, 1, tz = "UTC")
axis(1, forXaxis2, format(forXaxis2, "%Y"))
myCol = mapmisc::colourScale(NA, breaks = 1:8, style = "unique",col =
,→ "Set2", opacity=0.3)$col
matplot(weekValues[-1], weekSample, type = "l", lty = 1, col = myCol,
,→ xlab = "time", ylab = "degrees C", xaxt = "n", xaxs = "i")
axis(1, forXaxis2, format(forXaxis2, "%Y"))
```