Some useful Typst for you to use:

- For sets use the function we defined in the source:

$$\{1,2,3\}, \{\emptyset,\{4,5,6\}\}, \left\{\frac{1}{2}, \frac{\alpha}{1+\beta}\right\}$$

  it will format the braces nicely.
- Sometimes it is nice to write ℓ instead of l because it looks nice in formulas.
- For logic, Typst defines the symbols we need:

$$\sim P \quad P \vee Q \quad P \wedge Q \quad P \rightarrow Q \quad P \leftrightarrow Q$$

  We use $\sim$ for negation instead of the default negation symbol $\neg$.
- For a proof we can create a simple proof block:

  Proof: This is my proof. It is just missing a few details, but I'll put in an equation

$$a + b = c$$

  just because I can. $\square$

  Sometimes we want to give the proof a title. Here is a classic false-proof that $2 = 1$.

  Not-quite-a-proof that two equals one: Let x, y be non-zero real numbers so that $x = y$. Then, multiplying by x gives us

$$\begin{aligned} x^2 &= xy && \text{now subtract } y^2 \\ x^2 - y^2 &= xy - y^2 && \text{now factor} \\ (x-y)(x+y) &= y(x-y) && \text{divide by common factor of } (x-y) \\ x + y &= y && \text{since } x = y \\ 2y &= y && \text{now divide by y} \\ 2 &= 1 \end{aligned}$$

  $\square$

- For truth tables you can use the following:

A_1	A_2	A_3	A_4	A_5	A_6
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}

- Remember to check the spelling of your submission.

- Also remember that you should not include your scratchwork unless a question specifically asks for it.
- Finally, please try to make your work look nice and neat and use 12pt font — think about the reader!

Please do not include the above text in your homework solution — we have just included it here to help you write your homework.

Solutions to homework 1:

1. Your answer to question 1.

   (a) The sentence, "2 is even and 11 is prime" is a true statement and can be translated to "$P \wedge Q$", where P: "2 is even", and Q : "11 is prime".

   (b) The sentence, "if n is a multiple of 7 and 4, then it is a multiple of 14" is a false statement and can be translated to "$(P(n) \wedge Q(n)) \Rightarrow R(n)$", where $P(n)$: "n is a multiple of 7", $Q(n)$: "n is a multiple of 4", and $R(n)$: "n is a multiple of 14".

   (c) The sentence if "$5 \leq x \leq 17$" is an open statement, thus it has no truth value. It can be translated to $P(x)$, where $P(x)$: "$5 \leq x \leq 17$".

   (d) The sentence, "a real number x is less than -3 or greater than 3 if its square is greater than or equal to 9" is a true statement and can be translated to "$P(n) \Rightarrow (Q(n) \vee R(n)$", where $P(n)$: "the square of x is greater than 9", $Q(n)$: "x is less than -3", and $R(n)$: "x is greater than 3".

2. Let $a \in \mathbb{Z}$, if $5a + 11$ is odd then $9a + 3$ is odd.

   Proof: Assume that $5a + 11$ is odd, which implies that $5a + 11 = 2k + 1$, for some integer $k \in \mathbb{Z}$.

   $$\begin{aligned} 5a + 11 &= 2k + 1 \\ a + 2(2a + 5) &= 2k \\ a + 2i &= 2k, \qquad i \in \mathbb{Z} \\ a &= 2(k - i) \\ a &= 2j, \qquad j \in \mathbb{Z} \end{aligned}$$

   Thus, it follows that a is an even integer. Now, consider the expression $9a + 3$. It follows that, $9a + 3 = 9(2k) + 3 = 2(9k + 1) + 1$. Knowing that $9k + 1$ is an integer, we can conclude that $9a + 3 = 2\ell + 1, \ell \in \mathbb{Z}$. Hence, $9a + 3$ is an odd integer.

   $\square$

3. Let $n \in \mathbb{R}$. If $3 \mid n - 2$ then $3 \mid n^2 + 2n + 8$.

Proof: Assume that $3 \mid n-2$ such that $n-2=3\ell$ and $\ell \in \mathbb{Z}$. Now consider the expression n^2+2n+8.

$$\begin{aligned} n^2+2n+8 &= n^2+2n-8+16 && \text{add 16 and subtract -8} \\ n^2+2n-8+16 &= (n-2)(n+4)+16 \\ (n-2)(n+4)+16 &= (3\ell)(n+4)+15+1 \\ (3\ell)(n+4)+16 &= 3(\ell(n+4)+5)+1 \\ 3(\ell(n+4)+5)+1 &= 3t && t \in \mathbb{Z} \end{aligned}$$

Thus, we can see that the expression n^2+2n+8 is of the form $3t$ under the assumption that $3 \mid n-2$. Hence, $3 \mid n^2+2n+8$.

4. Let $x, y \in \mathbb{R}$. Show that $xy \leq \frac{1}{2}(x^2+y^2)$.

   Proof: Consider the fact that $q^2 \geq 0, q \in \mathbb{R}$. Now consider the expression $(x-y)^2$ such that $x, y \in \mathbb{R}$. Meaning $(x-y)^2$ is of the form q^2, thus $(x-y)^2 \geq 0$. Expanding the expression, we find $x^2-2xy+y^2 \geq 0$ and after adding $2xy$ and divding both sides by 2 we find the expression $\frac{1}{2}(x^2+y^2) \geq xy$. Hence, $xy \leq \frac{1}{2}(x^2+y^2)$.

5. Your solution to question 5.

6. Your solution to question 6.

7. Your solution to question 7.