Solutions to homework 5:

1. **Solutions**

- (a) $u_1 = \frac{4}{3}, u_2 = \frac{10}{7}, u_3 = \frac{24}{17}$
- (b) *Proof*

  Proof by Induction:

  Base Case: $n = 0$, $u_0 = 2$ so $1 \leq u_0 \leq 2$. Base case holds.

  I.H. for $k \geq 0$, $1 \leq u_k \leq 2$

  I.S.

  - Lower Bound

$$
\begin{aligned}
u_k &\geq 1 && \text{By I.H.} \\
u_k + 2 &\geq u_k + 1 \\
\frac{u_k + 2}{u_k + 1} &\geq 1 && \text{By I.H. } u_k + 1 > 0 \\
u_{k+1} &\geq 1
\end{aligned}
$$

  - Upper Bound

$$
\begin{aligned}
u_k &\leq 2 && \text{By I.H.} \\
u_k + 2 &\leq 2u_k + 2 \\
\frac{u_k + 2}{u_k + 1} &\leq 2 && \text{By I.H. } u_k + 1 > 0 \\
u_{k+1} &\leq 2
\end{aligned}
$$

  Hence, by induction $1 \leq u_k \leq 2$.

2. **Solution**

- **Proof by Induction:**

  Base Case: