

Exercise 1

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$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \rightarrow H$$

$$H_1 \times H_2 = H$$

H_1 is 2×2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

H_2 is 2×2

$$\begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

H_2 has to be 2×2 as well, so when the slide happens:

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 1 & 0 & 1 \\ \hline a_3 & a_4 & a_5 \\ \hline a_6 & a_7 & a_8 \\ \hline \end{array}$$

We know that row 1 and 3 are opposite, therefore, row 1 and 2 of H_2 are opposite as well.

$$H_2 = \begin{bmatrix} e & f \\ -e & -f \end{bmatrix}$$

$$ae = -1 \wedge cf + be = -2 \wedge bf = -1$$

$$\begin{aligned} -ae + ce = 0 \wedge -af - be + cf + de = 0 \wedge -bf + df = 0 \Rightarrow & \begin{cases} e(-a+c) = 0 \wedge f(-b+d) = 0, e \neq 0 \wedge f \neq 0 \\ -ce = 1 \wedge -cf - de = 2 \wedge -df = 1 \end{cases} \end{aligned}$$

$$H_1 = \begin{bmatrix} a & b \\ a & b \end{bmatrix} \rightarrow a=c \wedge b=d$$

From this we can get that $a = -1/e$, $b = -1/f$, $f = 2$ → So, for $e = -1$ we get H