

# Ball Sort Puzzle

Heuristic Search Methods for One Player Solitaire Games

Artificial Intelligence

Master in Informatics and Computing Engineering



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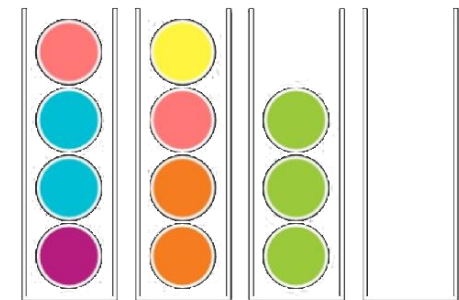
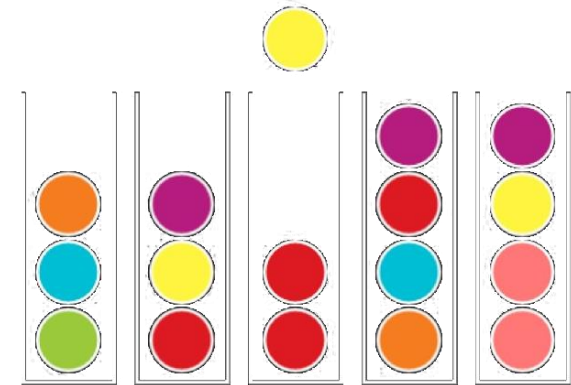
# Specification

The objective of the Ball Sort Puzzle is to sort the colored balls in the tubes until all balls with the same color stay in the same tube.

The player can only move a ball at the top of a tube to:

- an empty tube;
- another tube that has a ball with the same color on top and has enough space;

The player can also undo his moves if he finds himself without moves.



# Formulation of the problem as a search problem

## State Representation:

- Ball:  $[1..\infty]$
- Tube: [Ball, Ball, Ball, Ball] or []
- Game: [Tube, Tube, ...]

## Initial State:

- Game: [Tube, Tube, ...], where every Tube doesn't contain 4 equal numbers

## Objective State:

- Game: [Tube, Tube, ...], where every Tube either contains 4 equal numbers or is empty

## Operators:

- Move(X,Y):
  - PreCond: Tube Y must be empty or have a Ball, on top, just like the one that is moved
  - Effect: move Ball from Tube X to Tube Y
  - Cost: 1 is the general case, it represents a move; C2, evaluation of the number of wrongly placed Balls, based on the Ball at the bottom of a Tube

# Implemented Heuristics

We have implemented BFS, DFS, IDS, Greedy Search and A\*.

Our evaluation function, used in A\*, calculates the number of wrongly placed Balls (Balls with different color of the Ball at the bottom) in a Tube.

Code:

```
def greedy(state: Node, a_star: bool = False, max_depth: int = 5000):
    graph = Graph()
    state.setDist(0)
    stack = [state]
    graph.new_depth()
    graph.add_node(state, 1)

    depth = 1
    while depth != max_depth and len(stack) != 0:
        if a_star:
            stack.sort()
        else:
            stack.sort(key = lambda x: x.cost)
        graph.new_depth()
        node = stack.pop(0)
        if node.gamestate.finished():
            print("Found Goal. Depth:", node.dist)
            return graph, node
        else:
            graph.visit(node)
            expanded = new_states(node, a_star)
            [graph.add_node(x, depth + 1) for x in expanded]
            for children in expanded:
                if children in graph.visited:
                    continue
                if children in stack:
                    if (children.cost + node.dist + 1 < children.cost + children.dist) and a_star:
                        children.setParent(node)
                        children.setDist(node.dist + 1)
                else:
                    stack.append(children)

    print(depth)
    depth += 1
```

```
def better_nWrong_heuristics(self):
    cost = 0

    for tube in self.gamestate.tubes:
        balls = tube.balls.copy()
        idx = next((i for i, v in enumerate(balls) if v != balls[0]), -1)
        if idx == -1:
            continue

        balls = balls[idx:]
        cost += len(balls) * 2

    return cost
```

# Implementation work already carried out

**Programming language:** [Python](#), with visualization using [pygame](#) package

**Development environment:** [VSCode](#)/[IntelliJ](#)

**Data Structures:**

- Lists, for representing the Tubes and the Game
- Nodes and Graphs

**File Structure:** The Project's Repository is available at [Github](#)

**Implemented Work:** All algorithms lectured; Graphical Interface, playable game with hints

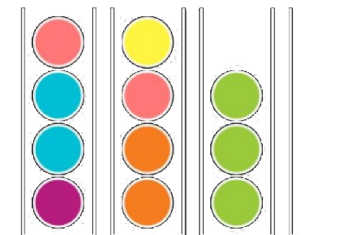
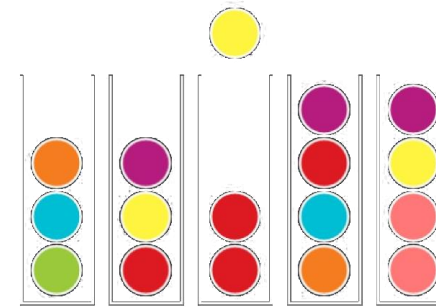
# Approach

As described before, our first heuristic counted the number of wrongly placed balls in a tube. For this second part of the project, we developed two new heuristics: minimum number of moves to fill a tube, assuming we can move any ball in a tube; number of consecutive balls with the same colour. Ex:

1<sup>st</sup> heuristic:  $2+2+1+3+2+3+2 = 15$

2<sup>nd</sup> heuristic:  $(4-3)+(4-2)+(4-2)+(4-2)+(4-1)+(4-1) = 13$

3<sup>rd</sup> heuristic:  $5+5+5+5+5+5+5+15+10 = 60$



# Algorithms

We have implemented all lectured algorithms: BFS, DFS, IDS, Greedy and A\*. For hardware reasons, we implemented a maximum depth, making DFS a DFS-Limited.

We developed a generic “solver” function, that changes the way the nodes are expanded depending on the algorithm. For IDS, it was necessary to make a separate function, since it uses DFS. (See below)

```
def solver(start_node: Node, algorithm: Algorithm, max_depth: int = 5000):
    graph = Graph()
    stack = [start_node]

    graph.new_depth()
    graph.add_node(start_node, 1)

    graph.new_depth()

    while len(stack) != 0:
        node = stack.pop(0)

        visited_node = get_stack_item(graph.visited, node)
        if visited_node is not None and node.dist >= visited_node.dist:
            continue

        graph.visit(node)
        graph.add_node(node, node.dist + 1)

        if algorithm != Algorithm.IDS and node.gamestate.finished():
            print("Found goal!")
            return graph, node

        expanded = expand_node(node, graph.visited, algorithm)

        if node.dist < max_depth - 1:
            stack = add_states_to_stack(stack, expanded, algorithm, node)
        else:
            solution = check_final_depth_solution(expanded)
            if solution is not None:
                return graph, solution
```

```
def ids(start_node: Node, max_depth: int = 5000):
    graph = None
    for depth in range(1, max_depth):
        graph, node = solver(start_node, Algorithm.IDS, depth)
        if node is not None:
            return graph, node
    return graph, None
```

```
def add_states_to_stack(stack, new_states, algorithm: Algorithm, node: Node):
    if algorithm == Algorithm.BFS:
        stack = stack + new_states
    elif algorithm == Algorithm.DFS or algorithm == Algorithm.IDS:
        stack = new_states + stack
    elif algorithm == Algorithm.GREEDY:
        stack = stack + new_states
        stack = sorted(stack, key=lambda x: x.cost, reverse=True)
        #stack.sort(key=lambda x: x.cost)
    elif algorithm == Algorithm.A_STAR:
        for children in new_states:
            stack_node = get_stack_item(stack, children)
            if stack_node is not None:
                if children.getTotalCost() < stack_node.getTotalCost():
                    stack_node.setParent(node)
                    stack_node.setDist(node.dist + 1)
            else:
                stack.append(children)

        stack = sorted(stack, key=lambda x: x.getTotalCost(), reverse=True)
        #stack.sort(key=lambda x: x.getTotalCost())

    return stack
```

# Experimental Results

Initially, we expected that Greedy algorithm was going to have the best performance, time wise, since it tries to find a solution, based on a heuristic, and does not care for optimality.

Space wise, we expected that DFS-Limited would be the best, based on the table below.

Name	Time	Space	Optimal
BFS	$O(b^{d+1})$	$O(b^{d+1})$	YES
DFS-Limited	$O(b^l)$	$O(bl)$	NO
IDS	$O(b^d)$	$O(bd)$	YES
Greedy	$O(b^m)$	$O(b^m)$	NO
A*	$O(b^m)$	$O(b^m)$	YES



# Experimental Results

To analyze and compare the different algorithms, we used 25 pre-existing levels. Below are the results of such tests.

Name	Completed Levels	Expanded Nodes - Avg	Time Execution (s) - Avg
BFS	6	246	0.14152
DFS-Limited	25	54	0.02335
IDS	4	1162	2.77309
Greedy (Wrong Colours)	25	34	0.01437
Greedy (Minimum Moves)	23	84	0.04154
A* (Wrong Colours)	25	852	1.62916
A* (Minimum Moves)	25	1013	2.18084

Notes:

- The algorithms with less completed levels did not find a solution with the given max depth or took too much time
- While the 25 levels have different sizes, size has no correlation with solution size or difficulty and therefore, was not taken into consideration

# Conclusions

As expected, Greedy was the best performing in terms of time, but surprisingly it was also the one that expanded the least nodes, making it the best performing algorithm overall, for non-optimal solutions.

For optimal solutions, as expected, A\* was the best performing algorithm on both ends.

Between our heuristic functions, Wrong Colour was the best one. Minimum Moves is better suited for small solutions.

Unfortunately, after analyzing the results for our third heuristic function, we found out it was not admissible, since it was overestimating the cost. As such, its results were not included, but the code can still be found in the graph.py file.

# References and Materials

- [Link to the Google Play page](#);
- [Link of all existing levels](#);

We used the curricular unit's presentations to guide our implementation of the various algorithms.

- [Python](#), with visualization using [pygame](#) package and [xltw](#) for spreadsheet generation;
- [VSCode](#)/[IntelliJ](#)