We represent the feature correspondences between the images by a partial permutation matrix $A \in \{0,1\}^{n_2 \times n_1}$, $P_1 \in \mathbb{R}^{3 \times n_1}$, $P_2 \in \mathbb{R}^{3 \times n_2}$.

$$\sum_{i=1}^{n_1} \left\| 1^T a_i \left(R p_1^i + t \right) - P_2 a_i \right\|_2^2$$

$$= \| (RP_1 + t1^T) Diag(1^T A) - P_2 A \|_F^2$$

$$= \left\| (RP_1 + t1^T) \odot 1_{3 \times n_2} A - P_2 A \right\|_F^2$$

$$= \left\| RP_1 \odot 1_{3 \times n_2} A + t1^T \odot 1_{3 \times n_2} A - P_2 A \right\|_F^2$$

$$= \left\| R\left(P_1 \odot 1_{3 \times n_2} A\right) + t1^T A - P_2 A \right\|_F^2$$

$$= \| R\bar{P}_1 + t\bar{1}^T - P_2 A \|_F^2$$

Eliminate t for the R subproblem:

$$= tr\left((\bar{1}t^T + \bar{P}_1^T R^T - A^T P_2^T)(t\bar{1}^T + R\bar{P}_1 - P_2 A) \right)$$

$$= tr(\bar{1}^T \bar{1} t^T t) + 2tr(\bar{1}^T \bar{P}_1^T R^T t) - 2tr(\bar{1}^T A^T P_2^T t) + C$$

$$= \bar{n}_0 t^T t + 2(R\bar{P}_1 \bar{1})^T t - 2(P_2 A \bar{1})^T t + C$$

$$\partial = 2\bar{n}_0 t + 2R\bar{P}_1 \bar{1} - 2P_2 A \bar{1} = 0 \Longrightarrow t^* = \frac{1}{\bar{n}_0}(P_2 A - R\bar{P}_1)\bar{1}$$

$$\| R\bar{P}_1 + t^* \bar{1}^T - P_2 A \|_F^2 = \left\| (R\bar{P}_1 - P_2 A) \left(I_{n_0} - \frac{\bar{1}\bar{1}^T}{\bar{n}_0} \right) \right\|_F^2$$

The A subproblem:

$$\left\| (RP_1 + t1^T) \odot 1_{3 \times n_2} A - P_2 A \right\|_F^2 + \mu \left\| A - \tilde{A} \right\|_F^2$$

$$= \left\| \left(Diag\left(vec(RP_1 + t1^T)\right)\left(1_{3 \times n_2} \otimes I_{n1}\right) - (P_2 \otimes I_{n1}) \right) vec(A) \right\|_2^2 + \mu \left\| vec(A) - vec\left(\tilde{A}\right) \right\|_2^2$$

$$= \| Qx \|_2^2 + \mu \left\| Ix - vec\left(\tilde{A}\right) \right\|_2^2 = \left\| \begin{bmatrix} Q \\ \sqrt{\mu} \cdot I \end{bmatrix} x - \begin{bmatrix} 0 \\ \sqrt{\mu} \cdot vec\left(\tilde{A}\right) \end{bmatrix} \right\|_2^2$$

$$= x^T (Q^T Q + \mu I) x - 2\mu \cdot vec^T\left(\tilde{A}\right) x + C$$

$$P_{21} = RP_1 + t1^T$$

$$(Q^TQ + \mu I)_{n_1 \times n_2}.block(i \times n_2, j \times n_2, n_1, n_1)$$

$$= \begin{cases} Diag\left(\begin{bmatrix} {p_{21}^1}^T p_{21}^1 + {p_2^i}^T p_2^j - {p_{21}^1}^T p_2^i - {p_{21}^1}^T p_2^j \\ {p_{21}^2}^T p_{21}^2 + {p_2^i}^T p_2^j - {p_{21}^2}^T p_2^i - {p_{21}^2}^T p_2^j \\ \dots\dots \\ {p_{21}^{n_1}}^T p_{21}^{n_1} + {p_2^i}^T p_2^j - {p_{21}^{n_1}}^T p_2^i - {p_{21}^{n_1}}^T p_2^j \end{bmatrix}\right), & if\ i \neq j \\ Diag\left(\begin{bmatrix} {p_{21}^1}^T p_{21}^1 + {p_2^i}^T p_2^i - 2{p_{21}^1}^T p_2^i \\ {p_{21}^2}^T p_{21}^2 + {p_2^i}^T p_2^i - 2{p_{21}^2}^T p_2^i \\ \dots\dots \\ {p_{21}^{n_1}}^T p_{21}^{n_1} + {p_2^i}^T p_2^i - 2{p_{21}^{n_1}}^T p_2^i \end{bmatrix}\right) + \mu I, & otherwise \end{cases}$$

$$= \begin{cases} Diag\left(\begin{bmatrix} {p_{21}^1}^T p_{21}^1 + {p_2^i}^T p_2^j - {p_{21}^1}^T p_2^i - {p_{21}^1}^T p_2^j \\ {p_{21}^2}^T p_{21}^2 + {p_2^i}^T p_2^j - {p_{21}^2}^T p_2^i - {p_{21}^2}^T p_2^j \\ \dots\dots \\ {p_{21}^{n_1}}^T p_{21}^{n_1} + {p_2^i}^T p_2^j - {p_{21}^{n_1}}^T p_2^i - {p_{21}^{n_1}}^T p_2^j \end{bmatrix}\right), & if\ i \neq j \\ Diag\left(\begin{bmatrix} \left\| p_{21}^1 - p_2^i \right\|_2^2 + \mu \\ \left\| p_{21}^2 - p_2^i \right\|_2^2 + \mu \\ \dots\dots \\ \left\| p_{21}^{n_1} - p_2^i \right\|_2^2 + \mu \end{bmatrix}\right), & otherwise \end{cases}$$