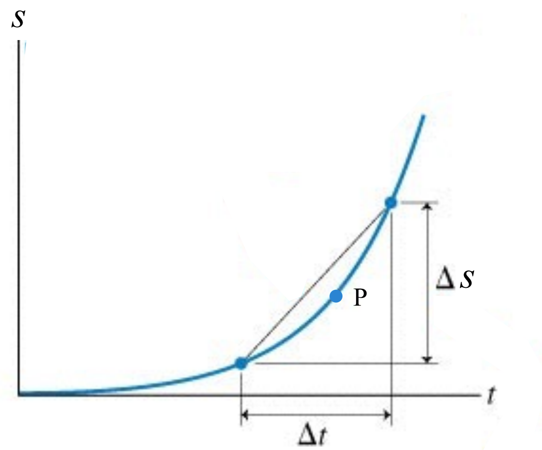
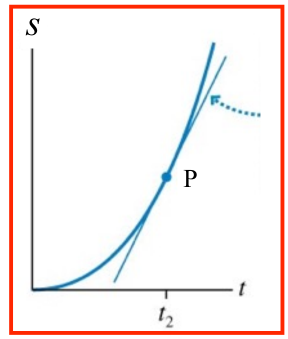
# Instantaneous Velocity (1D)

Consider the motion of an accelerating object (i.e. its velocity is changing).




"Zoom in", take limit $\Delta t \to 0$



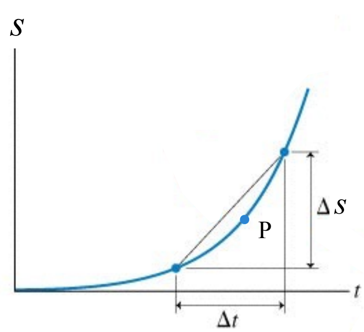

The instantaneous velocity at a point P is the slope of the line tangent to the position-vs-time graph at that point.

$$v_s = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

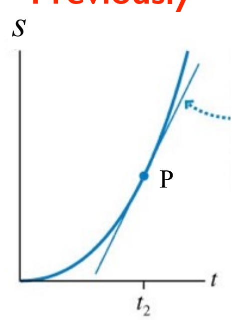
"derivative"

# Velocity vs Time

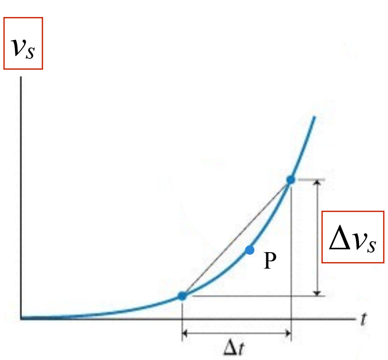
The same tools developed when analyzing *velocity* from a position vs time graph can be used to study *acceleration* from velocity vs time graphs.



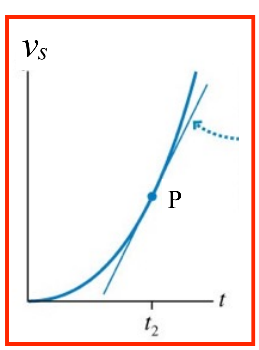

**Previously**




$$v_s = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$



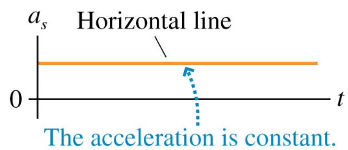

**NOW**



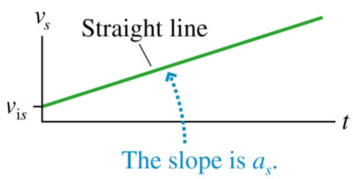

$$a_s = \lim_{\Delta t \to 0} \frac{\Delta v_s}{\Delta t} = \frac{dv_s}{dt}$$

The instantaneous *acceleration* at a point P is the slope of the line tangent to the velocity-vs-time graph at that point.

# Motion with Constant Acceleration

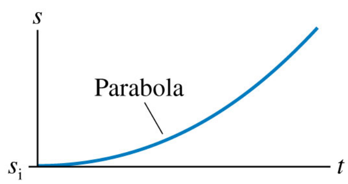



If acceleration is constant, then: $a_s = \dfrac{dv_s}{dt}$




From this expression and we get:

$$\boxed{v_f = v_i + a\,\Delta t}$$




by integrating the above, we get:

$$\boxed{s_f = s_i + v_i\,\Delta t + \frac{1}{2}a\,(\Delta t)^2}$$

using equations above we get:

$$\boxed{v_f^2 = v_i^2 + 2a\,\Delta s}$$

Three equations to rule all of (constant acceleration) kinematics

# Motion with Constant Acceleration

Displacement: $\Delta s = s_f - s_i$  Velocity: $v_s = \dfrac{ds}{dt}$  Acceleration: $a_s = \dfrac{dv_s}{dt}$

If acceleration is constant, then:

| 3 Equations | 5 Variables |
|---|---|
| $v_f = v_i + a\,\Delta t$ | $\Delta s = s_f - s_i$ |
| $s_f = s_i + v_i\,\Delta t + \frac{1}{2}a\,(\Delta t)^2$ | $v_f$ |
| $v_f^2 = v_i^2 + 2a\,\Delta s$ | $v_i$ |
| | $a$ |
| | $\Delta t$ |

Which equation(s) you need in a given problem depend on the information you are given. But if you are given 3 variables, you can always get the other 2.