Solving Torque and Rotational Equilibrium Problems:

CONCEPT: **Torque** ("moment" in engineering) is the rotational counterpart of force. The torque about a point P is given by the cross product between the vectors ***r*** and ***F***.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

MAGNITUDE: The *magnitude* of the torque about a pivot point is given by:

$$\boldsymbol{\tau} = r\, F \sin \phi$$

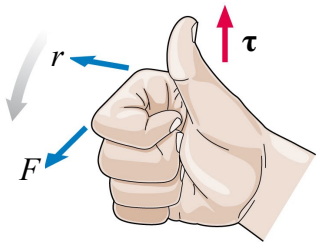
where:

r is the distance from the pivot to the point of application of the force,
F is the magnitude of the force, and
ϕ is the angle between the tails of ***r*** and ***F***.

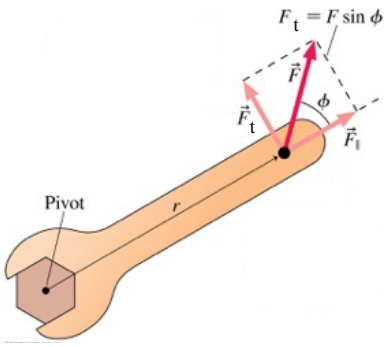
DIRECTION: By convention, the *direction* of the torque is considered:

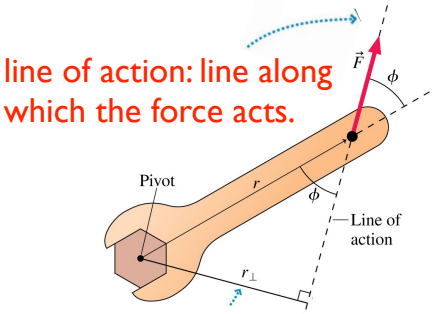
- ⊕ **positive** if it causes a counterclockwise (ccw) rotation, and
- ⊖ **negative** if it causes a clockwise (cw) rotation.

Use the right-hand rule to determine direction.



VISUALIZING TORQUE: There are two ways of visualizing torque:





(1) due to the tangential (perpendicular) component of the force w.r.t. r, $F_\perp$:

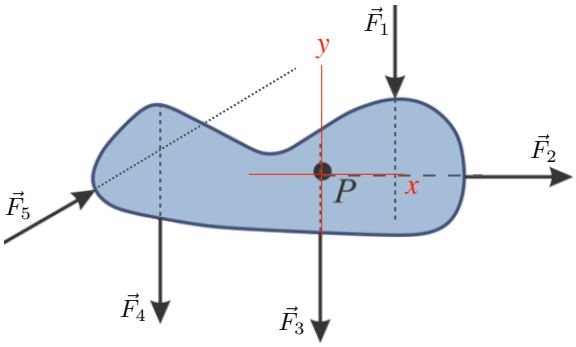
$$\boldsymbol{\tau} = r\,(F_\perp)$$

(2) due to the moment arm $r_\perp$ multiplied by the force:

$$\boldsymbol{\tau} = (r_\perp)\, F$$

Recall: When the line of action of the force crosses the pivot, the torque exerted by that force zero.

Setup Stage: Sketch a Free-Body Diagram.
Because the distance from the pivot to the point of application of the force is important, you should provide some idea of the geometry of the object of interest; do not represent the object as a dot at the origin of a coordinate system for torque problems.



Analysis Stage:

- Write the fundamental principle or statement
  i.e: Corollary of Newton's 1st Law ($\Sigma\tau = 0$)
  and indicate the pivot point

  example:
  $\sum \vec{\tau} = 0$ (about point P)

- Write a vector torque term for every force in the problem.
  If the line of action of the force crosses the pivot, the torque is zero.

  $\vec{\tau}_1 + \cancel{\vec{\tau}_2} + \cancel{\vec{\tau}_3} + \vec{\tau}_4 + \vec{\tau}_5 = 0$

- Use the right-hand rule to determine the direction of the torques. Represent the direction with +/– signs.
  Note: the terms are now magnitudes (no arrow hats); the direction is represented by the +/– signs.

  $-\tau_1 + \tau_4 - \tau_5 = 0$

- Write an expression for each torque term.

  $\tau_1 = r_1 F_1 \sin\phi_1$ or $\tau_1 = r_{\perp 1} F_1$
  $\tau_4 = r_4 F_4 \sin\phi_4$ or $\tau_4 = r_{\perp 4} F_4$

- Identify and solve for the desired quantity.