

Rotational Kinematics

Definitions:

Angular position:

$$\theta$$

Angular displacement

$$\Delta\theta$$

Average angular velocity:

$$\bar{\omega}_z = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity:

$$\omega_z = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Average angular acceleration:

$$\bar{\alpha}_z = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration:

$$\alpha_z \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Rotational (angular) kinematics:

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

Linear kinematics:

$$v_f = v_i + a \Delta t$$

$$s_f = s_i + v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$v_f^2 = v_i^2 + 2a \Delta s$$

Linear-Angular relationships

Position:

$$s = r \theta$$

Speed:

$$v_t = \frac{ds}{dt} = r \frac{d\theta}{dt} = r\omega$$

Linear acceleration:

$$a_t = \frac{dv}{dt} = r \frac{d\omega}{dt} = r\alpha$$

Centripetal acceleration:

$$a_r = \frac{v^2}{r} = \omega^2 r$$

Constant Angular Acceleration: A Blu-ray disc is slowing to a stop. The disc's angular velocity at $t = 0$ is 27.5 rad/s , and its angular acceleration is a constant -10 rad/s^2 . A line PQ on the disc's surface lies along the $+x$ -axis at $t = 0$.

- What is the disc's angular velocity at $t = 0.300 \text{ s}$?
- What angle does the line PQ make with the $+x$ -axis at this time?

