Exercize 1

```
clear; close all; clc;
```

Number of Agents in the Network

```
N = 7;
```

Number of possible Topologies of the Network

```
R = 4;
```

Control Gain (the dynamics simulation will be performed with 3 different control gains in order to verify the differences)

```
k1 = 0.5;
k2 = 0.8;
k3 = 1.7;
K = [k1 k2 k3];
```

Sampling Time: empirical rule suggests $0 < T_s < \frac{1}{10 \cdot \lambda_{\max}}$ where $\lambda_{\max}$ is the largest eigenvalue of the combined Laplacians.

```
Ts = 0.1;
```

Final Time

```
Tend = 9;
```

Time Line

```
kend = ceil(Tend/Ts)-1;
Time = 0:Ts:kend*Ts;
```

Switching Sequence

The switching sequence $\sigma(t)$ is a signal that determines which Laplacian matrix is active at time t.

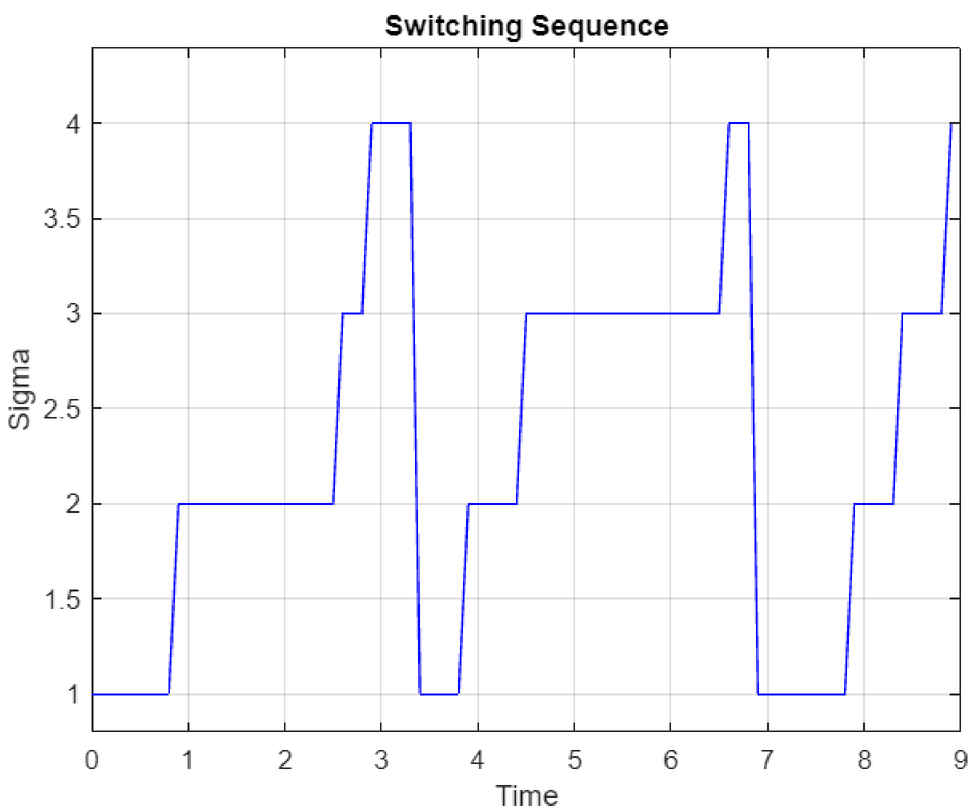
$\sigma : \mathfrak{R}_+ \rightarrow \{1, \ldots, R\}$

```
Tswitch=ceil([1 2.7 3 3.5 4 4.6 6.7 7 8 8.5 9]/Ts);
Sigma = [];
Sigma(1) = 1;
j = 1;

for i=2:kend+1
    Sigma(i)=Sigma(i-1);
    if i==Tswitch(j)
        Sigma(i)=Sigma(i)+1;
```

```
        if Sigma(i)==5
            Sigma(i)=1;
        end
        j=j+1;
    end
end

figure(1)
plot(Time, Sigma, 'b');
grid
ylabel('Sigma');
xlabel('Time');
axis([0 Tend 0.8 4.4]);
title('Switching Sequence')
```

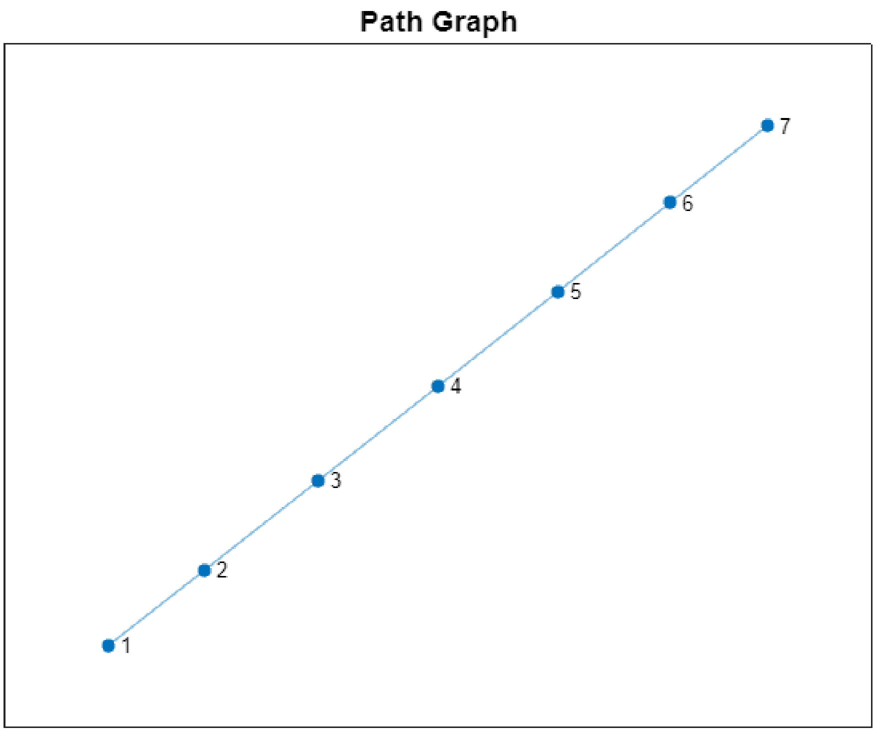


Topologies of the Network

First (Path Graph)

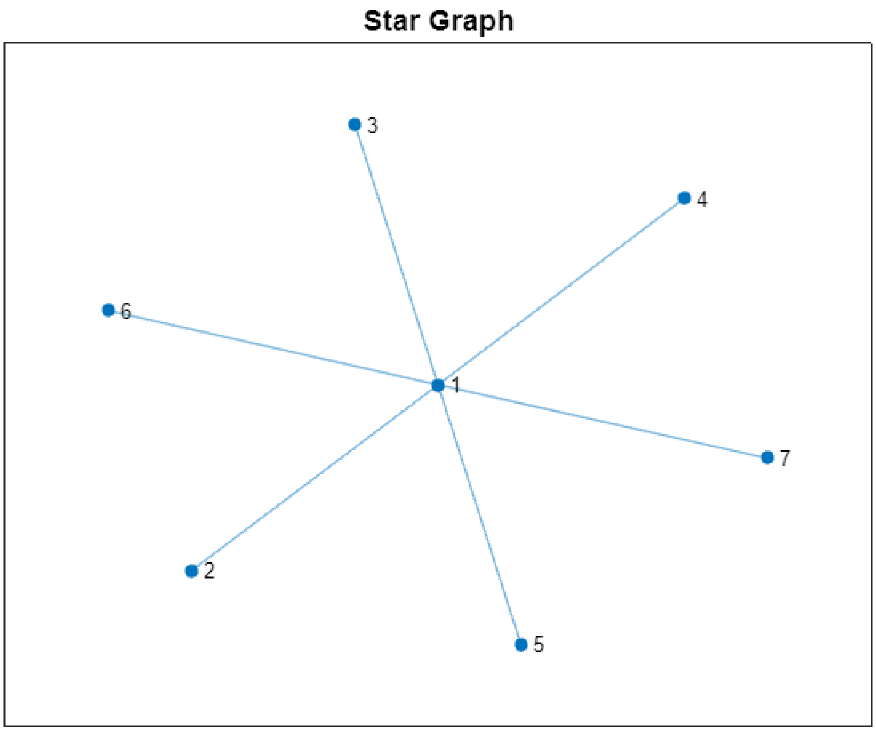
```
% Adjacency matrix
A1 = [0 1 0 0 0 0 0;
      1 0 1 0 0 0 0;
      0 1 0 1 0 0 0;
      0 0 1 0 1 0 0;
      0 0 0 1 0 1 0;
      0 0 0 0 1 0 1;
      0 0 0 0 0 1 0];
```

```
G1 = graph(A1);
figure(2);
plot(G1,'Layout','force')
title('Path Graph');
```



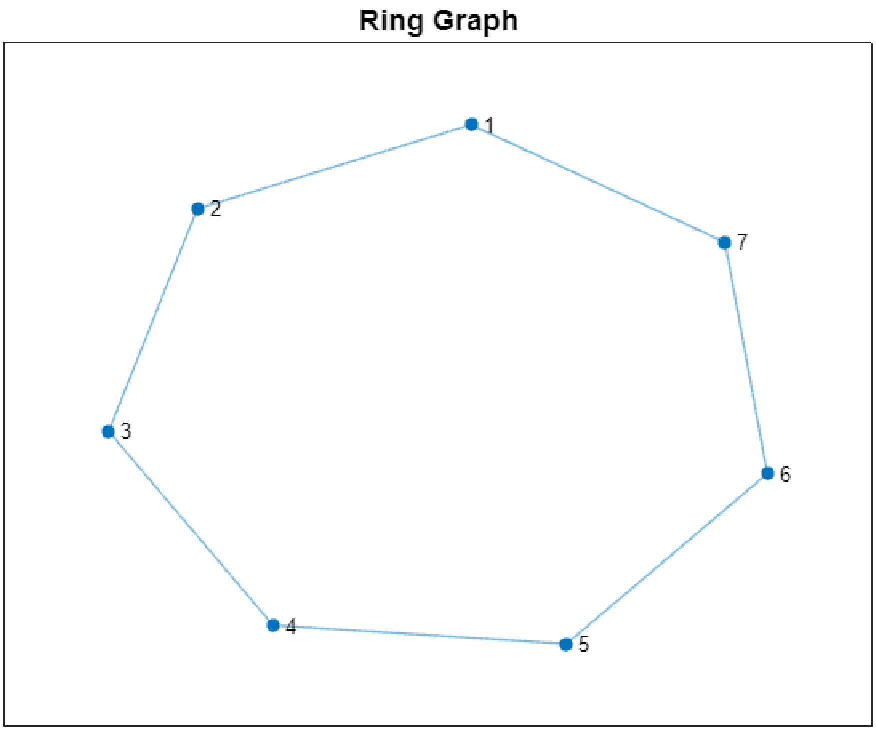
Second (Star Graph)

```
% Adjacency matrix
A2 = [0 1 1 1 1 1 1;
      1 0 0 0 0 0 0;
      1 0 0 0 0 0 0;
      1 0 0 0 0 0 0;
      1 0 0 0 0 0 0;
      1 0 0 0 0 0 0;
      1 0 0 0 0 0 0];
G2 = graph(A2);
figure(3);
plot(G2,'Layout','force')
title('Star Graph');
```



Third (Ring Graph)

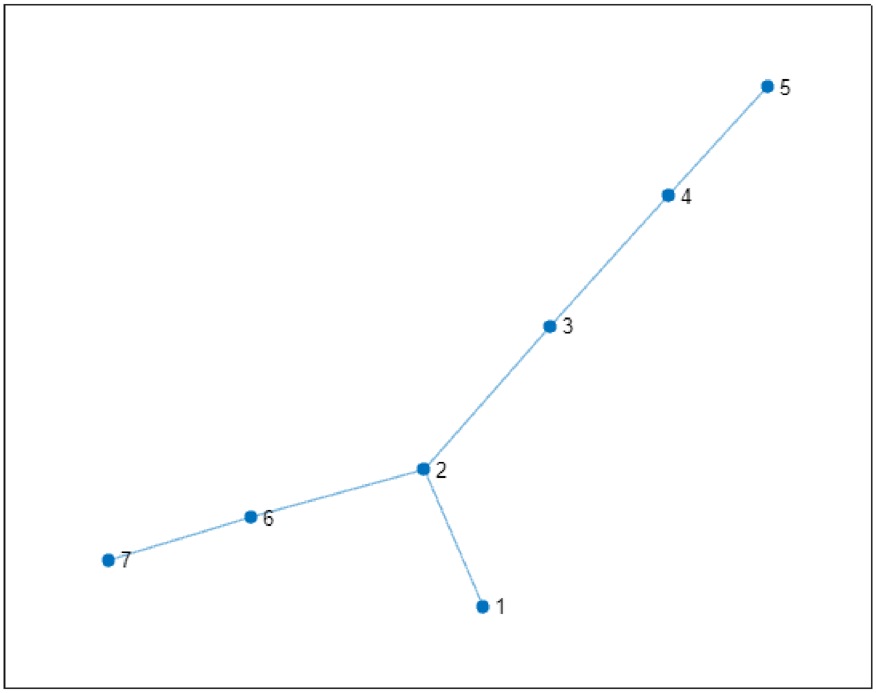
```
% Adjacency matrix
A3 = [0 1 0 0 0 0 1;
      1 0 1 0 0 0 0;
      0 1 0 1 0 0 0;
      0 0 1 0 1 0 0;
      0 0 0 1 0 1 0;
      0 0 0 0 1 0 1;
      1 0 0 0 0 1 0];
G3 = graph(A3);
figure(4);
plot(G3,'Layout','force')
title('Ring Graph');
```



Fourth (Random Connected Graph)

```
% Adjacency matrix
A4 = [0 1 0 0 0 0 0;
      1 0 1 0 0 1 0;
      0 1 0 1 0 0 0;
      0 0 1 0 1 0 0;
      0 0 0 1 0 0 0;
      0 1 0 0 0 0 1;
      0 0 0 0 0 1 0];
G4 = graph(A4);
figure(5);
plot(G4,'Layout','force')
title('Random Connected Graph');
```

Random Connected Graph



Laplacian Matrices

$L := D - A$

where D is a Diagonal Degree Matrix which represents the number of entering edges of each node

$D = \text{diag}(d_1, \ldots, d_N)$ where $d_i = \sum_{j=0}^{N} a_{\text{ij}}$ (*IN-DEGREE*)

We can say that

1) L has a non-negative diagonal

2) The remaining part of L is non-positive (beacase A is a non-negative matrix)

```
L1 = diag(sum(A1))-A1;
L2 = diag(sum(A2))-A2;
L3 = diag(sum(A3))-A3;
L4 = diag(sum(A4))-A4;
```

The **Control Law** for each Agent of the Network

$$u_i(t) = -k \sum_{j \in \mathrm{N}_i^{(\sigma(t))}} [x_i(t) - x_j(t)]$$

where $\mathrm{N}_i^{(\sigma(t))}$ denotes the set of neighbors of agent i in the topology active at time t.

The **Global Dynamics** (vector form) is given by

$\dot{x}(t) = -k\, L^{(\sigma(t))}x(t)$ where $L^{(\sigma(t))}$ is the Laplacian matrix of the active topology at time t.

This system is a **Hybrid System** because there is continuity for $\dot{x}(t)$ and, because of $L^{(\sigma(t))}$, we also have the discrete term.

Simulation

Initial state

```
x0 = [2 -1 0.75 1.1 0.4 0 1.6]';
```

Since we have a **continuous time** dynamics, we have the following dynamic (discretized version):

$$x(\mathrm{kT}_s) = e^{-k \cdot L^{(\sigma(t))} T_s} x((k-1)T_s)$$

About the **consensus value**, it is the average of the initial state because the graphs of the Network are undirected

$$\bar{x} = \frac{1}{N}\sum_{i=0}^{N} x_{0,i}$$

```
bar_x = mean(x0)
```

```
bar_x = 0.6929
```

Loop for the simulation

```
for k = 1:length(K)
    X = [];
    X(1,:) = x0;

    for i=2:kend+1
        switch Sigma(i)
            case 1
                X(i,:) = expm(-K(k)*L1*Ts)*X(i-1,:)';
            case 2
                X(i,:) = expm(-K(k)*L2*Ts)*X(i-1,:)';
            case 3
                X(i,:) = expm(-K(k)*L3*Ts)*X(i-1,:)';
            case 4
                X(i,:) = expm(-K(k)*L4*Ts)*X(i-1,:)';
        end
    end
```

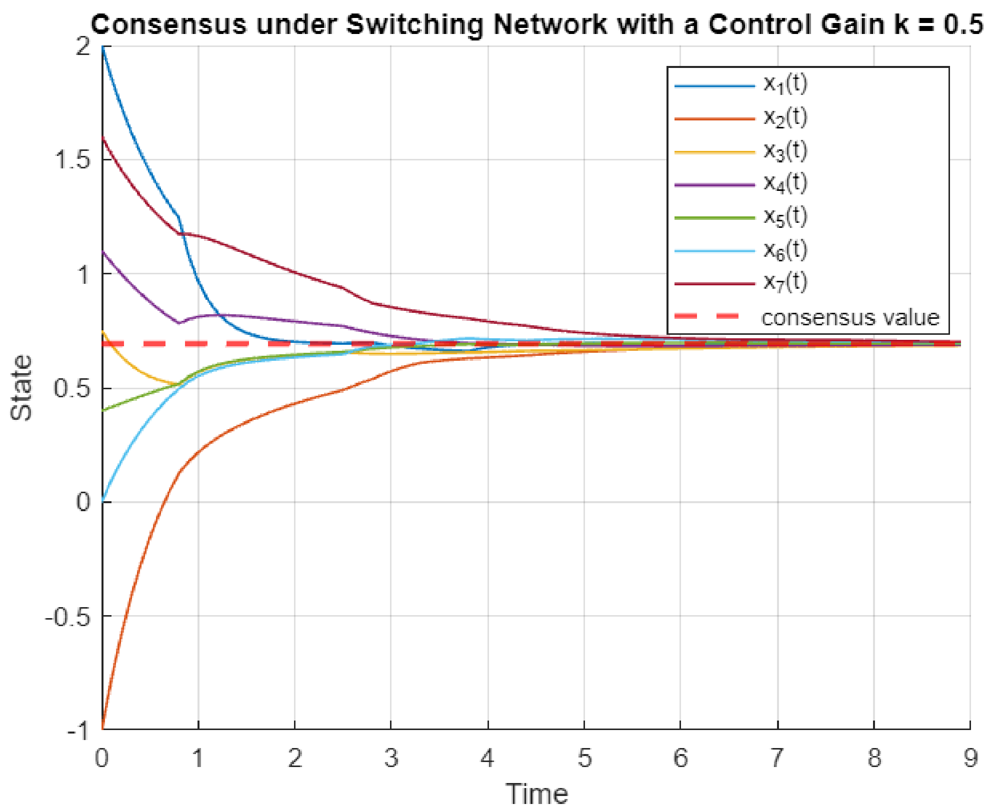
Plot

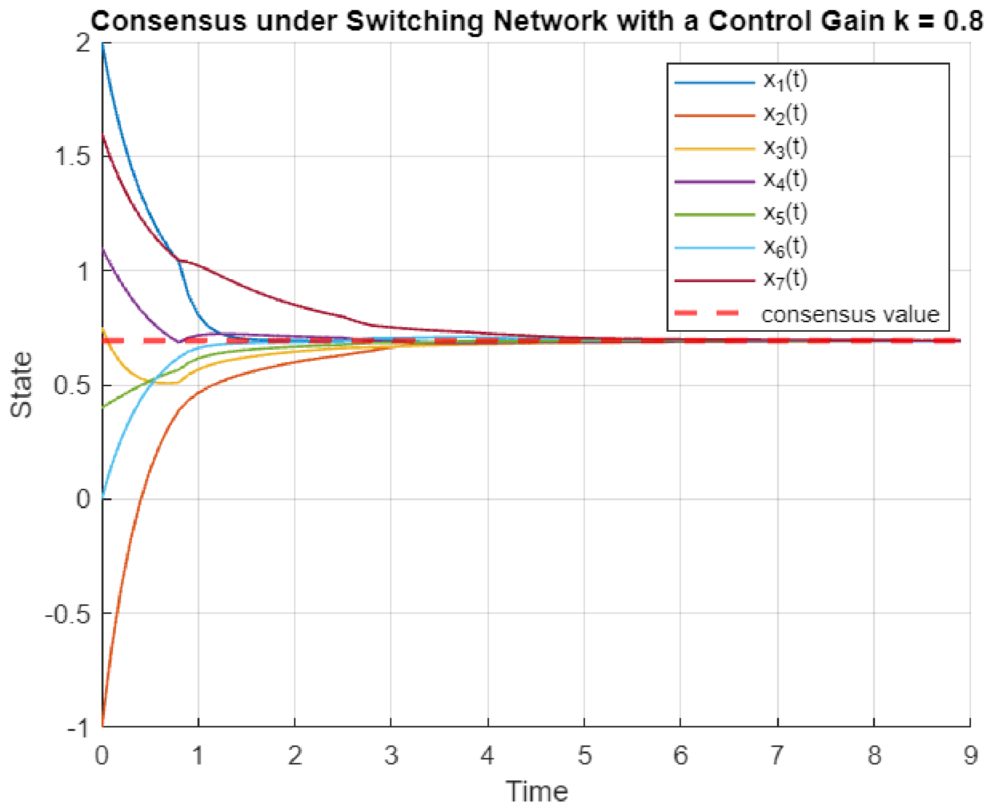
```
    colors = lines(N);
    figure;
```

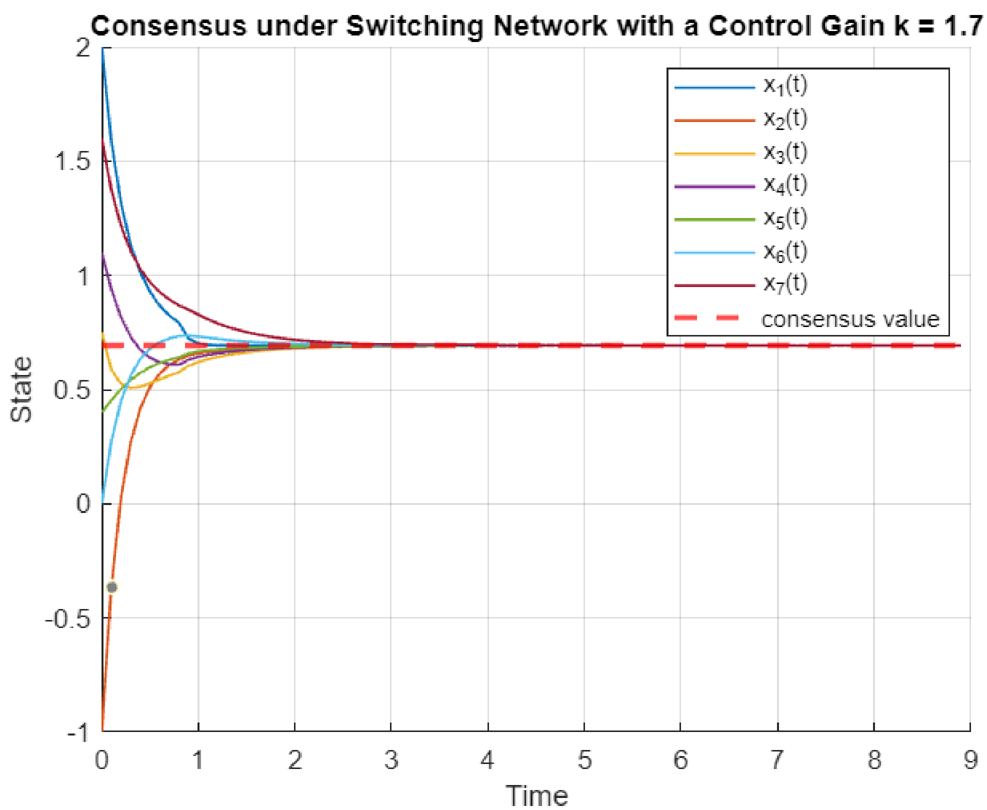
```
    hold on;
    for agent = 1:N
        plot(Time, X(:, agent), 'Color', colors(agent,:), 'LineWidth', 1);
    end

    yline(bar_x, '--r','LineWidth', 2);
    hold off
    xlabel('Time')
    ylabel('State')

legend('x_1(t)','x_2(t)','x_3(t)','x_4(t)','x_5(t)','x_6(t)','x_7(t)','consensus
value')
    title(['Consensus under Switching Network with a Control Gain k = ',
num2str(K(k))])
    grid
end
```







Plot Description

1. ***First plot*** ($k = 0.5$): the convergence to the consensus value is slow (Convergence Time is about $7\,s$); in fact, some agents take longer to approach the common value. Choosing $k = 0.5$ is good to avoid oscillations, but time inefficient.
2. ***Second plot*** ($k = 0.8$): the convergence speed is visibly improved compared to before (Convergence Time is about $5\,s$); it has a higher gain; it improves the consensus speed without introducing instability.
3. ***Third plot*** ($k = 1.7$): the system converges very quickly to the consensus value (Convergence Time is about $2\,s$); there is no visible oscillation, in fact the convergence is stable and clear. The choice of $k = 1.7$ is great for speed, but may be at risk of instability in systems with delays or noise.

Conclusions

- All agents reached a common consensus value, equal to the average of the initial state, thanks to the symmetry and connection of the topologies involved.
- Increasing the gain k accelerates convergence to consensus.