

## Problem Of The Day 2022

1. (27 Jun) If  $y$  varies inversely as  $x$  and can be represented by the equation  $y = (m - 1)x^{m^2-2}$ , find the value of constant  $m$ .

**Solution:**

$$\begin{aligned} y &= (m - 1)x^{m^2-2} = \frac{k}{x} \\ k &= (m - 1)x^{m^2-1} \\ &= (m - 1)x^{(m+1)(m-1)} \quad (x \neq 0) \end{aligned}$$

By definition,  $y \neq 0$  as well, hence

$$\begin{aligned} (m - 1)x^{(m+1)(m-1)} &\neq 0 \\ \therefore m &\neq 1 \end{aligned}$$

2. (28 Jun) Which of the following is a possible plot of  $y = x + m$  and  $y = \frac{m}{x}$  on the same axes? (The graphs are not drawn to scale.)

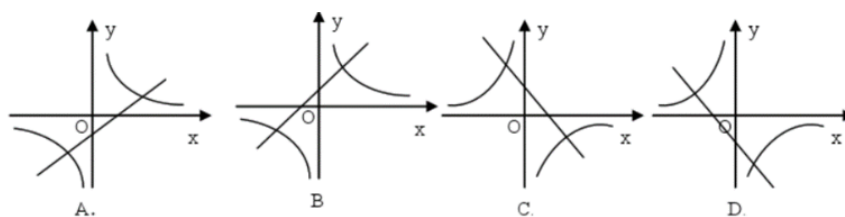


Figure 1:  $y = x + m$  and  $y = \frac{m}{x}$ .

**Solution: B.**

- The straight line should be increasing, since the coefficient of  $x$  is positive. **C** and **D** are eliminated.
- If  $m > 0$ , the y-intercept of the straight line could not be negative. **A** is eliminated, since the hyperbola in the same graph shows that  $m > 0$ .

3. **(29 Jun)** Given that points  $A(-2, y_1)$ ,  $B(-1, y_2)$ ,  $C(1, y_3)$  are all on the graph of  $y = -\frac{1}{x}$ , arrange  $y_1$ ,  $y_2$  and  $y_3$  in ascending order.

**Solution:**  $y_3 < y_1 < y_2$ .

Subst.  $x = -2$  into  $y = -\frac{1}{x}$ :

$$\begin{aligned} y_1 &= -\frac{1}{-2} \\ &= \frac{1}{2} \end{aligned}$$

Subst.  $x = -1$  into  $y = -\frac{1}{x}$ :

$$\begin{aligned} y_2 &= -\frac{1}{-1} \\ &= 1 \end{aligned}$$

Subst  $x = 1$  into  $y = -\frac{1}{x}$ :

$$\begin{aligned} y_3 &= -\frac{1}{1} \\ &= -1 \end{aligned}$$

4. **(30 Jun)** Given that points  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  are all on the graph of  $y = \frac{3}{x}$ , also  $x_1 < x_2 < 0 < x_3$ , arrange  $y_1$ ,  $y_2$  and  $y_3$  in ascending order.

**Solution:**  $y_2 < y_1 < y_3$ .

- $y_3 > 0$  since  $x_3 > 0$ . Hence,  $y_3$  is the greatest.
- $0 > x_2 > x_1$ , hence  $y_2 < y_1 < 0$ .

5. (1 Jul) Given that  $y$  varies inversely as  $x$  such that  $y = (a - 2)x^{a^2-5}$ , also when  $x > 0$ , as  $x$  increases,  $y$  increases. Find the equation of the hyperbola.

**Solution:**

$$\frac{k}{x} = (a - 2)x^{a^2-5}$$

$$k = (a - 2)x^{a^2-4}$$

$$= (a - 2)x^{(a+2)(a-2)}$$

$$\therefore y = \frac{(a - 2)x^{(a+2)(a-2)}}{x}$$

$$\therefore (a - 2)x^{(a+2)(a-2)} < 0$$

$$\therefore a - 2 < 0 \text{ and } (a + 2)(a - 2) \geq 0$$

$$\therefore a + 2 = 0$$

$$\therefore a = -2$$

$$\therefore y = \frac{-4}{x}$$

6. (5 Jul) If a straight line  $y = (2m - 1)x$  and a hyperbola  $y = \frac{3 - m}{x}$  has an intersection point each in Quadrant 1 and 3, what is the range of values of the constant  $m$ ?

**Solution:**

$$\therefore y = \frac{3-m}{x}$$

$$\therefore m < 3$$

$$\therefore y = x(2m-1)$$

$$\therefore 2m-1 > 0$$

$$\therefore 0.5 < m < 3$$

7. Points  $A$  and  $B$  are on the hyperbola  $y = \frac{k}{x}$ . Right  $\triangle AOC$  and  $\triangle BOD$  are drawn by perpendicular lines drawn from the two points and connecting the points with the origin. Compare the sizes of the areas of the two triangles.

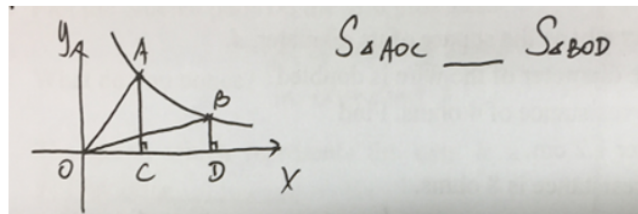


Figure 2:  $\triangle AOC$  and  $\triangle BOD$ .

**Solution:**  $S_{\triangle AOC} < S_{\triangle BOD}$ .

As Point  $B$ 's  $y$ -coordinate approaches 0, its  $x$ -coordinate approaches infinity, resulting in a triangle which approaches an infinite area. Point  $A$  is further from  $(\infty, 0)$  than Point  $B$ , so  $S_{\triangle AOC} < S_{\triangle BOD}$ .