

Problem Of The Day 2022

1. (21 Mar) Simplify the algebraic fraction $\frac{a^4 - a^2b^2}{(a-b)^2} \div \frac{a(a+b)}{b^2} \times \frac{b}{a}$.

Solution:

$$\begin{aligned} & \frac{a^4 - a^2b^2}{(a-b)^2} \div \frac{a(a+b)}{b^2} \times \frac{b}{a} \\ &= \frac{\cancel{a^2(a+b)}(a-b)}{(a-b)^2} \times \frac{b^2}{\cancel{a(a+b)}} \times \frac{b}{\cancel{a}} \\ &= \frac{b^4}{a-b} \end{aligned}$$

2. (22 Mar) Factorise $a^4 + a^2b^2 + b^4$.

Solution:

$$\begin{aligned} a^4 + a^2b^2 + b^4 &= a^4 + 2a^2b^2 + b^4 - a^2b^2 \\ &= (a^2 + b^2)^2 - (ab)^2 \\ &= (a^2 - ab + b^2)(a^2 + ab + b^2) \end{aligned}$$

3. (23 Mar) Simplify $\frac{1}{a-x} - \frac{1}{a+x} - \frac{2x}{a^2+x^2} - \frac{4x^3}{a^4+x^4} + \frac{8x^7}{a^8-x^8}$.

Solution:

$$\begin{aligned} & \frac{1}{a-x} - \frac{1}{a+x} - \frac{2x}{a^2+x^2} - \frac{4x^3}{a^4+x^4} + \frac{8x^7}{a^8-x^8} \\ &= \frac{2x}{a^2-x^2} - \frac{2x}{a^2+x^2} - \frac{4x^3}{a^4+x^4} + \frac{8x^7}{a^8-x^8} \\ &= \frac{4x^3}{a^4-x^4} - \frac{4x^3}{a^4+x^4} + \frac{8x^7}{a^8-x^8} \\ &= \frac{8x^7}{a^8-x^8} + \frac{8x^7}{a^8-x^8} \\ &= \frac{16x^7}{a^8-x^8} \end{aligned}$$