SBGE Paper B (2022)

1. The latest hand sanitiser bottle is in the shape of a cone. The circumference of the circular base is $(44x + 44)$ and the height of the cone is $\left(3x - \frac{6}{7}\right)$. Taking $\pi = \frac{22}{7}$, and the volume of the cone is $\frac{1}{3}\pi r^2 h$, expand and simplify the volume of the hand sanitiser. [3]

Solution:

$$\begin{aligned}
\text{radius of cone} &= \frac{44x + 44}{2 \times \frac{22}{7}} \\
&= 7x + 7 \\
\text{volume of cone} &= \frac{1}{3} \times \frac{22}{7} \times (7x + 7)^2 \times \frac{21x - 6}{7} \\
&= \frac{1}{3} \times 22(7x + 7)(x + 1) \times \frac{21x - 6}{7} \\
&= 22(x + 1)^2 \times (7x - 2) \\
&= 22\left(x^2 + 2x + 1\right)(7x - 2) \\
&= 22\left[7x^3 + 14x^2 + 7x - \left(2x^2 + 4x + 2\right)\right] \\
&= 22\left(7x^3 + 12x^2 + 3x - 2\right) \\
&= 154x^3 + 264x^2 + 66x - 44
\end{aligned}$$

2. Factorise the following **completely**:

   (a) $27p^3 - 36p^2 + 12p$ [2]

Solution:

$$\begin{aligned}
27p^3 - 36p^2 + 12p &= 3p\left(9p^2 - 12p + 4\right) \\
&= 3p\,(3p - 2)^2
\end{aligned}$$

   (b) $de^2 - e^2 f - 4df^2 + 4f^3$ [3]

Solution:

$$\begin{aligned} de^2 - e^2 f - 4df^2 + 4f^3 &= e^2(d-f) - 4f^2(d-f) \\ &= (d-f)\left(e^2 - 4f^2\right) \\ &= (d-f)(e+2f)(e-2f) \end{aligned}$$

3. Simplify the following algebraic expressions.

(a) $\dfrac{3y}{y^2-1} + \dfrac{3}{1-y}$ [3]

Solution:

$$\begin{aligned} \frac{3y}{y^2-1} + \frac{3}{1-y} &= \frac{3y - 3(y+1)}{y^2-1} \\ &= -\frac{3}{y^2-1} \end{aligned}$$

(b) $\dfrac{a^2-3ab+2b^2}{(a-b)^2} \div \dfrac{2a^2-ab-6b^2}{a^2-b^2}$ [4]

Solution:

$$\begin{aligned} \frac{a^2-3ab+2b^2}{(a-b)^2} \div \frac{2a^2-ab-6b^2}{a^2-b^2} &= \frac{\cancel{(a-b)}\cancel{(a-2b)}}{\cancel{(a-b)}^{1}} \cdot \frac{(a+b)\cancel{(a-b)}}{\cancel{(a-2b)}(2a+3b)} \\ &= \frac{a+b}{2a+3b} \end{aligned}$$

4. Make h the subject of the formula: $\sqrt{\dfrac{h^3mp}{h^3+p}} = mp$. [4]

Solution:

$$\sqrt{\frac{h^3mp}{h^3+p}} = mp$$
$$\frac{h^3mp}{h^3+p} = m^2p^2$$
$$h^3mp = h^3m^2p^2 + m^2p^3$$
$$h^3mp - h^3m^2p^2 = m^2p^3$$
$$h^3(mp - m^2p^2) = m^2p^3$$
$$h^3 = \frac{m^{\cancel{2}1}p^{\cancel{3}2}}{\cancel{mp}(1-mp)}$$
$$h = \sqrt[3]{\frac{mp^2}{1-mp}}$$

5. (a) Solve the equation: $\frac{3x}{x+1} - \frac{2x}{x-1} = 1$. [2]

Solution:

$$\frac{3x}{x+1} - \frac{2x}{x-1} = 1$$
$$3x(x-1) - 2x(x+1) = x^2 - 1$$
$$x^2 - 5x = x^2 - 1$$
$$-5x = -1$$
$$x = \frac{1}{5}$$

(b) Hence or otherwise, solve the equation: $\frac{3x-3}{x} + \frac{2x-2}{x-2} = 1$. [1]

Solution:

$$\frac{3x-3}{x} + \frac{2x-2}{x-2} = 1$$
$$\frac{3x-3}{(x-1)+1} + \frac{2x-2}{(x-1)-1} = 1$$
$$x - 1 = \frac{1}{5}$$
$$x = \frac{6}{5}$$

6. Given that $a^2 - 121 = 9879$,

(a) Find the positive value of a. [1]

Solution:

$$a^2 - 121 = 9879$$
$$a^2 = 10000$$
$$a = 100$$

(b) Hence, find two factors of 9879 which are between 50 and 200. [3]

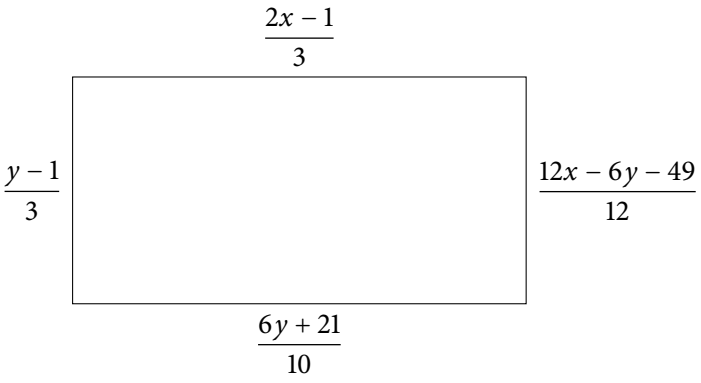
Solution:

$$a^2 - 121 = 9879$$
$$(a + 11)(a - 11) = 9879$$

Taking $a = 100$, the factors of 9879 are $100 + 11 = 111$ and $100 - 11 = 89$.

7. Solve the entirety of this question using Simultaneous Linear Equations. [4]

In every school, a space is required to be set aside for students who may exhibit any symptoms of cough or cold. In one particular school, this space is in the form of a rectangle of the following dimensions (in m). Find the length of the rectangle.



Solution:

$$\frac{2x-1}{3} = \frac{6y+21}{10} \tag{1}$$

$$\frac{y-1}{3} = \frac{12x-6y-49}{12} \tag{2}$$

Cross-multiply (1):

$$\frac{2x-1}{3} = \frac{6y+21}{10}$$

$$10(2x-1) = 3(6y+21)$$

$$20x-10 = 18y+63$$

$$20x-73 = 18y$$

$$y = \frac{20x-73}{18} \tag{3}$$

Cross-multiply (2):

$$\frac{y-1}{3} = \frac{12x-6y-49}{12}$$

$$12(y-1) = 3(12x-6y-49)$$

$$4y-4 = 12x-6y-49$$

$$10y+45 = 12x \tag{4}$$

Substitute (3) into (4):

$$10\left(\frac{20x-73}{18}\right)+45 = 12x$$

$$100x-365+405 = 108x$$

$$\therefore x = \frac{405-365}{8}$$

$$= 5$$

$$\therefore y = \frac{20x-73}{18}$$

$$= \frac{20\times 5-73}{18}$$

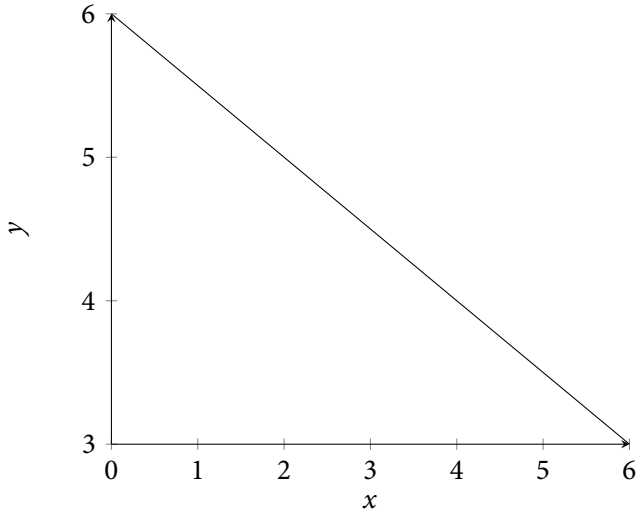
$$= \frac{3}{2}$$

$$\therefore \frac{2x-1}{3} = \frac{6\times\frac{3}{2}+21}{10}$$

$$= 3 \text{ units}$$

The length of the rectangle is **3 units**.

8. The diagram below shows the line l_1, $y = ax + b$.



(a) State the values of a and b. [2]

Solution: $a = -\frac{1}{2}$, $b = 6$.

(b) Find the equation of another line, l_2, which is parallel to l_1 and passes through the point $(2, 3)$. [2]

Solution: On l_2,

$$y_2 - y_1 = m(x_2 - x_1)$$
$$y_2 - 3 = -\frac{1}{2}(x_2 - 2)$$
$$y_2 = -\frac{1}{2}x_2 + 1 + 3$$

The equation of l_2 is $y = -\frac{1}{2}x + 4$.

9. **Attempt the whole of this question on the graph paper provided.**

The variables x and y are connected by the equation $y + 2 = 3x$.

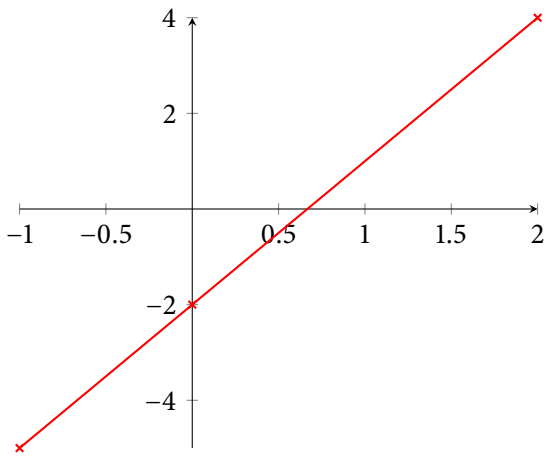
(a) Copy and complete the following table. [1]

x	-1	0	2
y			

Solution:

x	-1	0	2
y	-5	-2	4

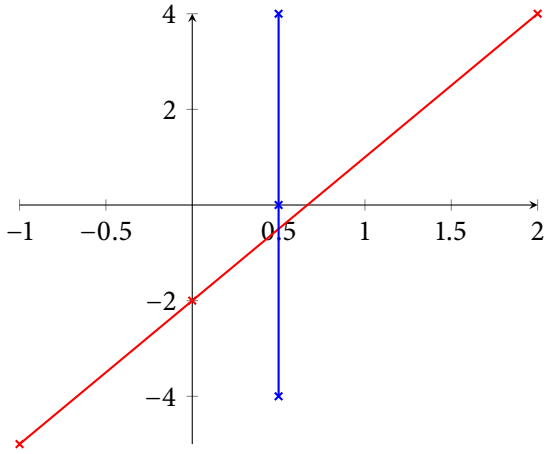
(b) Using a scale of 4 cm to represent 1 unit on the x-axis and a scale of 2 cm to represent 1 unit on the y-axis, draw the graph of $y + 2 = 3x$ for $-1 \leq x \leq 2$. [2]



(c) **Read from your graph** the value of x when $y = 2.3$. [1]

Solution: 1.4 *(from graph). [Actual:* $1\frac{13}{30}$*]*

(d) On the same axes as in (b), draw the graph of $x = 0.5$. [1]



(e) Given the graphs you have drawn in (b) and (d), explain how to find the solution to the simultaneous equations $y + 2 = 3x$ and $x = 0.5$. [1]

Solution: Find the coordinates of the point of intersection between the two lines. The x- and y-coordinates of the point will correspond with the solutions to x and y in the simultaneous equations.