Problem Of The Day 2022

1. (**27 Jun**) If y varies inversely as x and can be represented by the equation $y = (m-1)x^{m^2-2}$, find the value of constant m.

Solution:

$$y = (m-1)x^{m^2-2} = \frac{k}{x}$$
$$k = (m-1)x^{m^2-1}$$
$$= (m-1)x^{(m+1)(m-1)} \ (x \neq 0)$$

By definition, $y \neq 0$ as well, hence

$$(m-1)x^{(m+1)(m-1)} \neq 0$$
$$\therefore m \neq 1$$

2. (**28 Jun**) Which of the following is a possible plot of $y = x + m$ and $y = \frac{m}{x}$ on the same axes? (The graphs are not drawn to scale.)

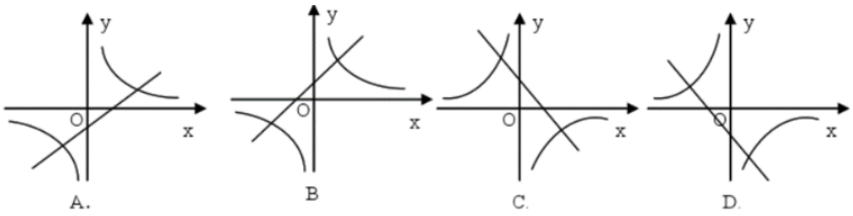

Figure 1: $y = x + m$ and $y = \frac{m}{x}$.

Solution: B.

- The straight line should be increasing, since the coefficient of x is positive. **C** and **D** are eliminated.
- If $m > 0$, the y-intercept of the straight line could not be negative. **A** is eliminated, since the hyperbola in the same graph shows that $m > 0$.

3. (**29 Jun**) Given that points $A(-2, y_1)$, $B(-1, y_2)$, $C(1, y_3)$ are all on the graph of $y = -\dfrac{1}{x}$, arrange y_1, y_2 and y_3 in ascending order.

Solution: $y_3 < y_1 < y_2$.

Subst. $x = -2$ into $y = -\dfrac{1}{x}$:

$$\begin{aligned} y_1 &= -\frac{1}{-2} \\ &= \frac{1}{2} \end{aligned}$$

Subst. $x = -1$ into $y = -\dfrac{1}{x}$:

$$\begin{aligned} y_2 &= -\frac{1}{-1} \\ &= 1 \end{aligned}$$

Subst $x = 1$ into $y = -\dfrac{1}{x}$:

$$\begin{aligned} y_3 &= -\frac{1}{1} \\ &= -1 \end{aligned}$$

4. (**30 Jun**) Given that points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are all on the graph of $y = \dfrac{3}{x}$, also $x_1 < x_2 < 0 < x_3$, arrange y_1, y_2 and y_3 in ascending order.

Solution: $y_2 < y_1 < y_3$.

- $y_3 > 0$ since $x_3 > 0$. Hence, y_3 is the greatest.
- $0 > x_2 > x_1$, hence $y_2 < y_1 < 0$.