Revision Exercise

T2W10 HBL

25 May 2022

1. The figure below shows two right-angled triangles. Given that $PQ = x$ cm , $PR = (x + 8)$ cm , $QR = (x + 4)$ cm , and $SQ = 5$ cm , find the length of PS.

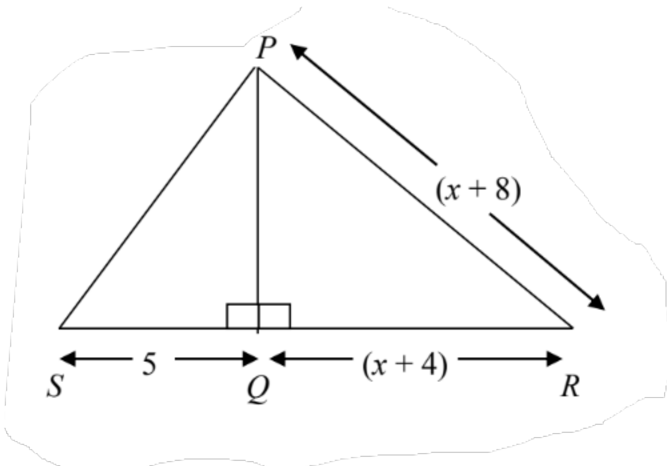

Figure 1: Two right-angled triangles.

Solution:

$$
\begin{aligned}
x^2 + (x+4)^2 &= (x+8)^2 \\
2x^2 + 8x + 16 &= x^2 + 16x + 64 \\
x^2 - 8x - 48 &= 0 \\
(x+4)(x-12) &= 0 \\
x &= -4 \text{ (rej.) or } 12 \\
\therefore PS &= \sqrt{x^2 + 5^2} \\
&= \sqrt{12^2 + 5^2} \\
&= 13
\end{aligned}
$$

The length of PS is 13 cm .

2. The diagram shows part of the graph $y = 20 + 3x - 2x^2$. The graph cuts the axis at P and R and the y-axis at Q.

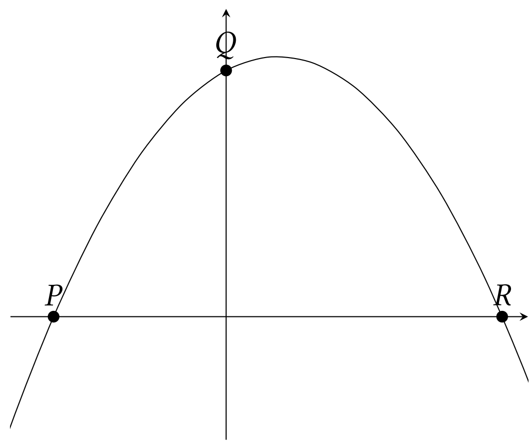

Figure 2: The graph of $y = 20 + 3x - 2x^2$.

(a) Find the coordinates of P, Q and R.

Solution:

$$\begin{aligned} Q &= (0, 20) \\ y &= 20 + 3x - 2x^2 \\ &= -(2x+5)(x-4) \\ -(2x+5)(x-4) &= 0 \\ x &= -\frac{5}{2} \text{ or } 4 \\ \therefore P &= \left(-\frac{5}{2}, 0\right) \\ \therefore R &= (4, 0) \end{aligned}$$

(b) Write down the equation of the line of symmetry of the graph $y = 20 + 3x - 2x^2$.

Solution:

$$\begin{aligned} x &= \frac{-\frac{5}{2} + 4}{2} \\ &= \frac{3}{4} \end{aligned}$$

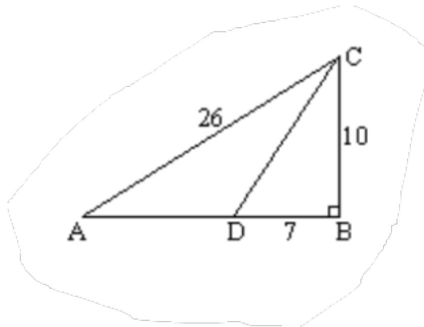

Figure 3: Question 3.

3. In the diagram, $AC = 26$ cm , $BD = 7$ cm , $BC = 10$ cm , and $\angle ABC = 90°$. Calculate

   (a) AD,

   Solution:

   $$AD = \sqrt{26^2 - 10^2} - 7$$
   $$= 17 \text{ cm}$$

   (b) the area of ΔACD.

   Solution:

   $$\Delta ACD = \frac{17 \times 10}{2}$$
   $$= 85 \text{ cm}^2$$

4. $VABCD$ is a pyramid with a square base $ABCD$ and a height VN. Given that the height, VN, is 12 cm and the volume of the pyramid is 400 cm^3, calculate the

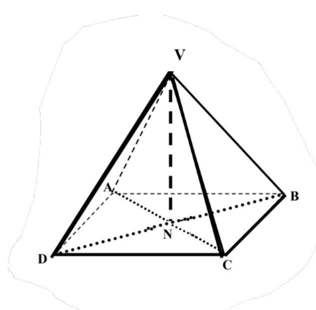

Figure 4: Pyramid $VABCD$.

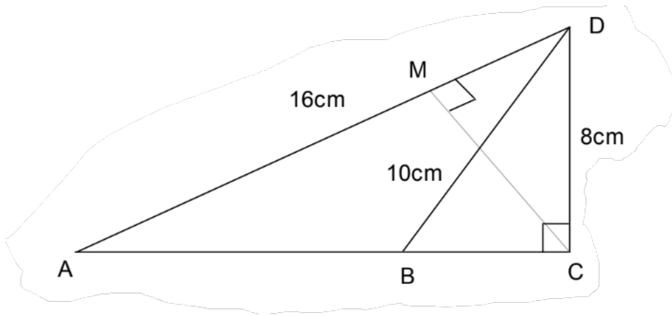

Figure 5: Question 5.

(a) length of the side of the square base,

Solution:

$$\text{length of the side of the square base} = \sqrt{\frac{400 \div \frac{1}{3}}{12}}$$
$$= 10 \text{ cm}$$

(b) total surface area of the pyramid.

Solution:

$$\text{total surface area} = 10^2 + 4 \times \frac{1}{2} \times 10 \times \sqrt{\left(\frac{10}{2}\right)^2 + 12^2}$$
$$= 100 + 2 \times 130$$
$$= 360 \text{ cm}^2$$

5. In the given diagram, $AD = 16$ cm , $BD = 10$ cm , and $CD = 8$ cm . M lies on the line AD. Find

(a) BC.

Solution:

$$BC = \sqrt{10^2 - 8^2}$$
$$= 6 \text{ cm}$$

(b) AB.

Solution:

$$\begin{aligned} AB &= \sqrt{16^2 - 8^2} - 6 \\ &= \left(8\sqrt{3} - 6\right) \text{ cm} \\ &= 7.86 \text{ cm} \ \ (3 \text{ s.f.}) \end{aligned}$$

(c) the area of ΔACD.

Solution:

$$\begin{aligned} \text{area of } \Delta ACD &= 8\sqrt{3} \times 8 \times \frac{1}{2} \\ &= 32\sqrt{3} \text{ cm}^2 \\ &= 55.4 \text{ cm}^2 \ (3 \text{ s.f.}) \end{aligned}$$

(d) CM.

Solution:

$$\begin{aligned} \text{area of } \Delta ACD &= \frac{1}{2} \times CM \times 16 \\ CM &= 32\sqrt{3} \div 8 \\ &= 4\sqrt{3} \\ &= 6.93 \text{cm} \ (3 \text{ s.f.}) \end{aligned}$$

The length of CM is 6.93 cm (3 s.f.).

6. Simplify the expressions, expressing your answer in positive indices.

(a) $g^3 \left(\dfrac{h^2}{g^4}\right)^2 \div \left(\dfrac{g^{-2}h^3}{g^2 h}\right)^{-3}$

Solution:

$$
\begin{aligned}
g^3\left(\frac{h^2}{g^4}\right)^2 \div \left(\frac{g^{-2}h^3}{g^2h}\right)^{-3} &= g^3\left(\frac{h^4}{g^8}\right) \div \left(\frac{h^2}{g^4}\right)^{-3} \\
&= \frac{g^3h^4}{g^8} \times \left(\frac{h^2}{g^4}\right)^3 \\
&= \frac{h^4}{g^5} \times \frac{h^6}{g^{12}} \\
&= \frac{h^{10}}{g^{17}}
\end{aligned}
$$

(b) $\left(-\frac{4}{5}xy^3\right)^2 \times \left(-2x^2y\right)^{-3}$

Solution:

$$
\begin{aligned}
\left(-\frac{4}{5}xy^3\right)^2 \times \left(-2x^2y\right)^{-3} &= \frac{16}{25}x^2y^6 \cdot \frac{1}{\left(-2x^2y\right)^3} \\
&= \frac{\frac{16}{25}x^2y^6}{-8x^6y^3} \\
&= -\frac{2y^3}{25x^4}
\end{aligned}
$$

(c) $\left(16a^4\right)^{\frac{1}{4}} \times \left(\frac{1}{1000a^3}\right)^{\frac{1}{3}}$

Solution:

$$
\begin{aligned}
\left(16a^4\right)^{\frac{1}{4}} \times \left(\frac{1}{1000a^3}\right)^{\frac{1}{3}} &= \left(2^4 \cdot a^4\right)^{\frac{1}{4}} \cdot \left(\frac{1}{10^3 \cdot a^3}\right)^{\frac{1}{3}} \\
&= 2a \cdot \frac{1}{10a} \\
&= \frac{1}{5}
\end{aligned}
$$

(d) $\left(m^3 \times m^{-4}\right)^{-2} + 2m^{\frac{1}{2}} \times m^{\frac{1}{4}} \times m^{1\frac{1}{4}}$

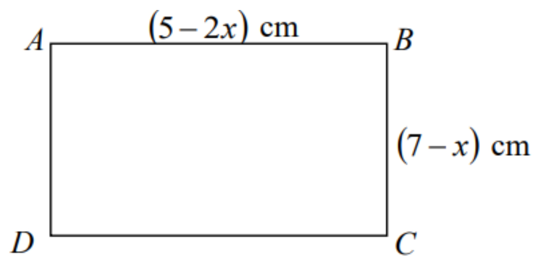

Figure 6: Rectangle $ABCD$.

Solution:

$$\left(m^3 \times m^{-4}\right)^{-2} + 2m^{\frac{1}{2}} \times m^{\frac{1}{4}} \times m^{1\frac{1}{4}} = \left(\frac{1}{m}\right)^{-2} + 2m^2$$
$$= m^2 + 2m^2$$
$$= 3m^2$$

7. Rectangle $ABCD$ has length $AB = (5 - 2x)$ cm , and breadth $BC = (7 - x)$ cm .
   (a) Write down an expression for the perimeter of the rectangle, leaving your answer in terms of x.

Solution:

$$\text{perimeter of rect.} = 2\left[(5 - 2x) + (7 - x)\right]$$
$$= 2(-3x + 12)$$
$$= (-6x + 24)\text{ cm}$$

   (b) Given that the rectangle has an area of $110\,\text{cm}^2$, write down an equation in x and show that it reduces to $2x^2 - 19x - 75 = 0$.

Solution:

$$(5 - 2x)(7 - x) = 110$$
$$35 - 14x - 5x + 2x^2 = 110$$
$$2x^2 - 19x + 35 = 110$$
$$2x^2 - 19x - 75 = 0$$

   (c) Expand $(2x - 25)(x + 3)$.

Solution:

$$\begin{aligned}(2x-25)(x+3) &= 2x^2 - 25x + 6x - 75 \\ &= 2x^2 + 19x - 75\end{aligned}$$

(d) Solve the equation $2x^2 + 19x - 75 = 0$.

Solution:

$$\begin{aligned}2x^2 + 19x - 75 &= 0 \\ (2x-25)(x+3) &= 0 \\ x &= \frac{25}{2} \text{ or } -3\end{aligned}$$

(e) Substitute both values of x into your expression in **(a)** and explain why you need to reject one of the values.

Solution:
Case 1:

$$\begin{aligned}-6x + 24 &= -6 \times \frac{25}{2} + 24 \\ &= -51 \textbf{ (rej.)}\end{aligned}$$

Case 2:

$$\begin{aligned}-6x + 24 &= -6 \times (-3) + 24 \\ &= 42\end{aligned}$$

The solution of $x = \frac{25}{2}$ needs to be rejected as it yields a negative result when substituted into the expression for the rectangle's perimeter, since a rectangle's perimeter cannot be of negative length.

8. Solve for x in the following equations.

(a) $2^x \div 4^{x-3} \times 8^{3x+1} = 0.25$.

Solution:

$$\begin{aligned} 2^x \div 4^{x-3} \times 8^{3x+1} &= 0.25 \\ 2^x \div 2^{2x-6} \times 2^{9x+3} &= 2^{-2} \\ x - (2x-6) + (9x+3) &= -2 \\ 8x + 9 &= -2 \\ x &= -\frac{11}{8} \end{aligned}$$

(b) $8^{-2} \times 2^{2x} = \sqrt{4^{3x+5}}$.

Solution:

$$\begin{aligned} 8^{-2} \times 2^{2x} &= \sqrt{4^{3x+5}} \\ 2^{2x-6} &= 2^{3x+5} \\ 2x - 6 &= 3x + 5 \\ x + 5 &= -6 \\ x &= -11 \end{aligned}$$

(c) $2 \times 9^{2000} + 9^{2000} = 3^x$

Solution:

$$\begin{aligned} 2 \times 9^{2000} + 9^{2000} &= 3^x \\ 3^1 \times 3^{4000} &= 3^x \\ x &= 4001 \end{aligned}$$

(d) $8^{3x+2} = 0.03125$

Solution:

$$\begin{aligned} 8^{3x+2} &= 0.03125 \\ 2^{9x+6} &= 2^{-5} \\ 9x + 6 &= -5 \\ x &= -\frac{11}{9} \end{aligned}$$

(e) $2^{2008}+2^{2008}+2^{2008}+2^{2008}+2^{2008}+2^{2008}+2^{2008}+2^{2008}=2^x$

Solution:

$$\begin{aligned}2^{2008}+2^{2008}+2^{2008}+2^{2008}+2^{2008}+2^{2008}+2^{2008}+2^{2008} &= 2^x\\ 2^3\times 2^{2008} &= 2^x\\ x &= 2011\end{aligned}$$

(f) $8^{3x+2}=\dfrac{2^x}{32}$

Solution:

$$\begin{aligned}8^{3x+2} &= \frac{2^x}{32}\\ 2^{9x+6} &= 2^{x-5}\\ 9x+6 &= x-5\\ x &= -\frac{11}{8}\end{aligned}$$

(g) $81^{3x-7}=243^x\div 27^{x-4}$

Solution:

$$\begin{aligned}81^{3x-7} &= 243^x\div 27^{x-4}\\ 3^{12x-28} &= 3^{5x}\div 3^{3x-12}\\ 12x-28 &= 2x+12\\ 10x &= 40\\ x &= 4\end{aligned}$$

9. Given that $p=4\times 10^2$ and $q=2\times 10^{-4}$, evaluate, leaving your answer in standard form,

(a) $\dfrac{1}{q}+2p$,

Solution:

$$\begin{aligned}\frac{1}{q} + 2p &= \frac{1}{2 \times 10^{-4}} + 8 \times 10^2 \\ &= \frac{1}{2} \times 10^4 + 8 \times 10^2 \\ &= 5 \times 10^3 + 8 \times 10^2 \\ &= 5.8 \times 10^3\end{aligned}$$

(b) $\frac{p}{q}$

Solution:

$$\begin{aligned}\frac{p}{q} &= \frac{4 \times 10^2}{2 \times 10^{-4}} \\ &= 2 \times 10^6\end{aligned}$$

10. (a) Express $2.05\ \text{cm}$ in km, giving your answer in standard form.

Solution: $2.05\ \text{cm} = 2.05 \times 10^{-5}\ \text{km}$

(b) Evaluate $2.4 \times 10^{-3} - 7.8 \times 10^{-2}$, giving your answer in standard form.

Solution:

$$\begin{aligned}2.4 \times 10^{-3} - 7.8 \times 10^{-2} &= 0.24 \times 10^{-2} - 7.8 \times 10^{-2} \\ &= -7.56 \times 10^{-2}\end{aligned}$$

11. Identify the errors in the solution given below by rewriting the correct solution, and showing all the necessary workings.

Evaluate the expression $5.86 \times 10^{-2} - 9.2 \times 10^{-3}$*, giving your answer in standard form.*

$$\begin{aligned}5.86 \times 10^{-2} - 9.2 \times 10^{-3} &= (5.86 - 9.2 \times 10) \times 10^{-2} \\ &= (5.86 - 92) \times 10^{-2} \\ &= -86.14 \times 10^{-2}\end{aligned}$$

Solution:

$$\begin{aligned} 5.86 \times 10^{-2} - 9.2 \times 10^{-3} &= (5.86 - 9.2 \div 10) \times 10^{-2} \\ &= (5.86 - 0.92) \times 10^{-2} \\ &= 4.94 \times 10^{-2} \end{aligned}$$