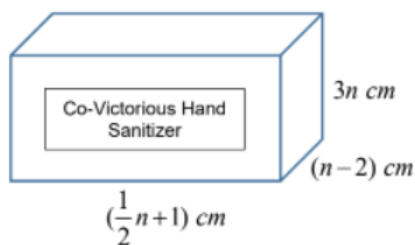


SBGE Paper C (2022)

1. A particular brand of hand sanitiser is in the shape of a cuboid as shown below:

[2]



Find and simplify an expression in terms of n for the volume of the hand sanitiser.

Solution:

$$\begin{aligned}\text{volume of hand sanitiser} &= 3n \left(\frac{1}{2}n + 1 \right) (n - 2) \\ &= \left(\frac{3}{2}n^2 + 3n \right) (n - 2) \\ &= \frac{3}{2}n^3 + 3n^2 - 3n^2 - 6n \\ &= \left(\frac{3}{2}n^3 - 6n \right) \text{cm}^3\end{aligned}$$

2. Factorise completely:

(a) $6x^3 + 2x^2y - 4xy^2$

[2]

Solution:

$$\begin{aligned}6x^3 + 2x^2y - 4xy^2 &= 2x(3x^2 + xy - 2y^2) \\ &= 2x(3x - 2y)(x + y)\end{aligned}$$

(b) $(2x + 1)^2 - (x^2 + x + 1)^2$

[3]

Solution:

$$\begin{aligned}(2x + 1)^2 - (x^2 + x + 1)^2 &= (2x + 1 + x^2 + x + 1)(2x + 1 - x^2 - x - 1) \\ &= (x^2 + 3x + 2)(-x^2 + x) \\ &= -x(x + 1)(x + 2)(x - 1)\end{aligned}$$

3. Simplify:

(a) $\frac{3c}{1-c} + \frac{3}{c-1}$

[3]

Solution:

$$\begin{aligned}\frac{3c}{1-c} + \frac{3}{c-1} &= \frac{3c-3}{1-c} \\ &= -3\end{aligned}$$

(b) $\frac{a^2 - b^2}{(a-b)^2} \div \frac{1}{a^2 + b^2} \times \frac{1}{(a-b)^2 + 2ab}$

[3]

Solution:

$$\begin{aligned}\frac{a^2 - b^2}{(a-b)^2} \div \frac{1}{a^2 + b^2} \times \frac{1}{(a-b)^2 + 2ab} \\ &= \frac{(a+b)(\cancel{a-b})}{(a-b)(\cancel{a-b})} \cdot \frac{\cancel{a^2 + b^2}}{1} \cdot \frac{1}{\cancel{a^2 + b^2}} \\ &= \frac{a+b}{a-b}\end{aligned}$$

4. Make h the subject of the formula: $\sqrt{1-hp} = p$.

[3]

Solution:

$$\begin{aligned}\sqrt{1-hp} &= p \\ 1-hp &= p^2 \\ hp &= 1-p^2 \\ h &= \frac{1-p^2}{p}\end{aligned}$$

5. (a) Solve the equation: $\frac{1}{x} + \frac{2}{3x} = \frac{3}{x+1}$.

[3]

Solution:

$$\begin{aligned}\frac{1}{x} + \frac{2}{3x} &= \frac{3}{x+1} \\ \frac{5}{3x} &= \frac{3}{x+1} \\ 5(x+1) &= 9x \\ 4x &= 5 \\ x &= \frac{5}{4}\end{aligned}$$

(b) Hence or otherwise, solve the equation $\frac{1}{x+1} + \frac{2}{3x+3} = \frac{3}{x+2}$.

[1]

Solution:

$$\begin{aligned}\frac{1}{x+1} + \frac{2}{3x+3} &= \frac{3}{x+2} \\ \frac{1}{x+1} + \frac{2}{3(x+1)} &= \frac{3}{(x+1)+1} \\ \therefore x+1 &= \frac{5}{4} \\ \therefore x &= \frac{1}{4}\end{aligned}$$

6. Given that $a^2 - 49 = 9951$,

(a) find the positive value of a .

[1]

Solution:

$$\begin{aligned}a^2 - 49 &= 9951 \\ a &= \sqrt{9951 + 49} \\ &= 100\end{aligned}$$

(b) Hence, find two factors of 9951 which are between 50 and 200.

[3]

Solution:

$$a^2 - 49 = 9951$$

$$(a + 7)(a - 7) = 9951$$

When $a = 100$, the two factors of 9951 are $a + 7 = 107$ and $a - 7 = 93$.

7. By defining two variables, solve the following problem using SLEs.

[4]

Two books, A and B , have a total of 500 pages. If the number of pages in Book A is 12 less than 3 times the number of pages in Book B , calculate the number of pages in each book.

Solution: Let the number of pages in Books A and B be a and b respectively.

$$a + b = 500 \quad (1)$$

$$a = 3b - 12 \quad (2)$$

Substitute (2) into (1):

$$3b - 12 + b = 500$$

$$4b = 512$$

$$b = 128$$

$$\therefore a = 372$$

The number of pages in Books A and B is 372 and 128 respectively.

8. An ice-cream maker machine can hold 60 L of ice-cream mix. It was discovered that

- (a) when the temperature of the machine is set at -10°C , x L of ice-cream mix can be frozen per min. Write down an expression in terms of x for the time taken to freeze 60 L of ice cream mix at -10°C in min.

[1]

Solution: The time taken would be $\frac{60}{x}$ min.

- (b) when the temperature of the machine was set at -5°C , $(x - 2)$ L of ice cream mix can be frozen per minute. Write down an expression in terms of x for the time taken to freeze 60 L of ice cream mix at -5°C in min.

[1]

Solution: The time taken would be $\frac{60}{x-2}$ min.

- (c) it takes 1.5 min longer to freeze the ice cream mix at a higher temperature. Write down an equation in x , and show that it simplifies to $x^2 - 2x - 80 = 0$. [2]

Solution:

$$\begin{aligned}\frac{60}{x-2} - \frac{60}{x} &= \frac{3}{2} \\ \frac{120}{x(x-2)} &= \frac{3}{2} \\ 3x^2 - 6x &= 240 \\ x^2 - 2x - 80 &= 0\end{aligned}$$

- (d) Solve the equation $x^2 - 2x - 80 = 0$. [2]

Solution:

$$\begin{aligned}x^2 - 2x - 80 &= 0 \\ (x+8)(x-10) &= 0 \\ \therefore x &= -8 \text{ (rej.) or } x = 10\end{aligned}$$

- (e) Explain why you have to reject one of the solutions obtained in (d). [1]

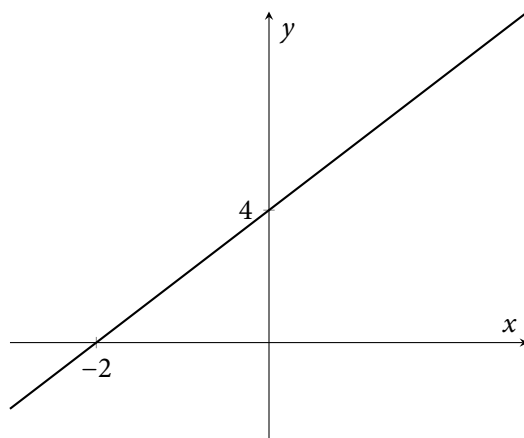
Solution: One of the solutions, $x = -8$, is negative, which cannot represent the number of litres of ice cream frozen per min.

9. The diagram below shows the line L_1 , $y = ax + b$.

- (a) State the values of a and b . [2]

Solution: $a = 2$, $b = 4$.

- (b) Find the equation of another line, L_2 , which passes through the point $(5, 9)$ and is parallel to L_1 . [2]



Solution:

$$y - y_1 = m(x - x_1)$$

$$y - 9 = 2(x - 5)$$

$$y = 2x - 1$$

(c) Does the point $(3, 10)$ lie on line L_2 ? Explain.

[1]

Solution: The point $(3, 10)$ does not lie on L_2 . Substituting the values of $x = 3$ and $y = 10$ into the equation $y = 2x - 1$ makes the equation false.