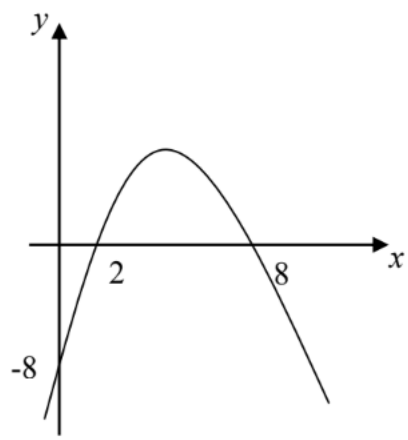
Revision Practice

T2W10 HBL

26 May 2022

1. The diagram shows a quadratic curve which can be expressed in the form of $y = ax^2 + bx + c$. Given that the curve cuts the x-axis at 2 and 8 and the y-axis at -8, find the values of a, b and c.



Solution:

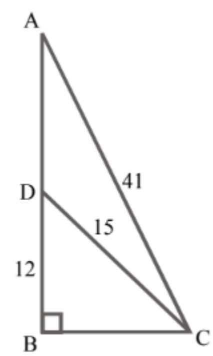
$$0 = a(x-2)(x-8)$$
$$y = a(x-2)(x-8)$$

Using the fact that the graph intercepts $(0, -8)$,

$$\begin{aligned} -8 &= a(0-2)(0-8) \\ 16a &= -8 \\ a &= -\frac{1}{2} \\ \therefore\ y &= -\frac{1}{2}(x-2)(x-8) \\ &= -\frac{1}{2}x^2 + 5x - 8 \end{aligned}$$

$$\therefore\ a = -\frac{1}{2},\ b = 5,\ c = -8$$

2. In the diagram, $\angle ABC = 90°$, $AC = 41$ cm. D is on AB such that $CD = 15$ cm, $BD = 12$ cm. Calculate the value of BC and of AD.



Solution:

$$\begin{aligned} BC &= \sqrt{15^2 - 12^2} \\ &= 9\text{ cm} \\ AD &= \sqrt{41^2 - BC^2} - BD \\ &= \sqrt{41^2 - 9^2} - 12 \\ &= 28\text{ cm} \end{aligned}$$

3. Solve for x in the following equations.

(a) $2006^{x^2-9x+20} - 1 = 0$

Solution:

$$\begin{aligned}2006^{x^2-9x+20}-1&=0\\x^2-9x+20&=0\\(x-4)(x-5)&=0\\x&=4\text{ or }5\end{aligned}$$

(b) $4^x\left(5^{2x}\right)=10$

Solution:

$$\begin{aligned}4^x\left(5^{2x}\right)&=10\\4^x\cdot\left(5^2\right)^x&=10\\4^x\cdot 25^x&=10\\100^x&=10\\\left(10^2\right)^x&=10^1\\2x&=1\\x&=\frac{1}{2}\end{aligned}$$

(c) $3^{14}\left(9^{1-x}\right)=\left(3^3\right)^{2x}$

Solution:

$$\begin{aligned}3^{14}\left(9^{1-x}\right)&=\left(3^3\right)^{2x}\\3^{14}\cdot 3^{2-2x}&=3^{6x}\\16-2x&=6x\\x&=2\end{aligned}$$

(d) $25^{x+2}=125^{4-x}$

Solution:

$$
\begin{aligned}
25^{x+2} &= 125^{4-x} \\
5^{2x+4} &= 5^{12-3x} \\
2x + 4 &= 12 - 3x \\
x &= \frac{8}{5}
\end{aligned}
$$

(e) $\sqrt{m\sqrt{m\sqrt{m}}} = m^{x-1}$

Solution:

$$
\begin{aligned}
\sqrt{m\sqrt{m\sqrt{m}}} &= m^{x-1} \\
\sqrt{m\sqrt{m^{\frac{3}{2}}}} &= m^{x-1} \\
\sqrt{m^{\frac{7}{4}}} &= m^{x-1} \\
m^{\frac{7}{8}} &= m^{x-1} \\
x - 1 &= \frac{7}{8} \\
x &= \frac{15}{8}
\end{aligned}
$$

4. It is given that Newton's Law of Universal Gravitation is defined by the formula $F = \frac{GMm}{r^2}$.

(a) Make r the subject of the formula.

Solution:

$$
\begin{aligned}
F &= \frac{GMm}{r^2} \\
r^2 &= \frac{GMm}{F} \\
r &= \pm\sqrt{\frac{GMm}{F}}
\end{aligned}
$$

(b) Find the positive value of r (correct to the nearest whole number) if $G = 6.67\times10^{-11}$,

$M = 6.6 \times 10^{21}$, $m = 1.5 \times 10^{2}$ and $F = 1.43$.

Solution:

$$\begin{aligned} r &= \sqrt{\frac{GMm}{F}} \\ &= \sqrt{\frac{6.67 \times 10^{-11} \times 6.6 \times 10^{21} \times 1.5 \times 10^{2}}{1.43}} \\ &= \sqrt{\frac{66.033 \times 10^{12}}{1.43}} \\ &= 6\,795\,360 \text{ (nearest whole number)} \end{aligned}$$

5. Simplify the following expressions.

(a) $\left(a^{2}b^{-3}\right)^{3} \times \dfrac{ab^{-2}}{a^{3}}$

Solution:

$$\begin{aligned} \left(a^{2}b^{-3}\right)^{3} \times \frac{ab^{-2}}{a^{3}} &= \frac{a^{6}b^{-9} \times ab^{-2}}{a^{3}} \\ &= \frac{a^{7}}{a^{3}b^{11}} \\ &= \frac{a^{4}}{b^{11}} \end{aligned}$$

(b) $\left(a^{3}b\right)^{-2} \div \left(a^{2}b^{-5}\right) \times \dfrac{a^{3}}{b^{7}}$

Solution:

$$\begin{aligned} \left(a^{3}b\right)^{-2} \div \left(a^{2}b^{-5}\right) \times \frac{a^{3}}{b^{7}} &= \frac{a^{-6}b^{-2}}{a^{2}b^{-5}} \times \frac{a^{3}}{b^{7}} \\ &= \frac{b^{3}}{a^{8}} \times \frac{a^{3}}{b^{7}} \\ &= \frac{1}{a^{5}b^{4}} \end{aligned}$$

(c) $\left(3a^{-2}b^{2}\right)^{3} \times \left(6a^{3}b^{-2}\right)^{-2}$

Solution:

$$
\begin{aligned}
\left(3a^{-2}b^{2}\right)^{3} \times \left(6a^{3}b^{-2}\right)^{-2} &= 27a^{-6}b^{6} \times \frac{1}{36}a^{-6}b^{4} \\
&= \frac{27a^{-12}b^{10}}{36} \\
&= \frac{3b^{10}}{4a^{12}}
\end{aligned}
$$

(d) $\left(5a^{-4}b^{5}\right)^{-1} \times 6\left(a^{2}b\right)^{-3}$

Solution:

$$
\begin{aligned}
\left(5a^{-4}b^{5}\right)^{-1} \times 6\left(a^{2}b\right)^{-3} &= \frac{a^{4}}{5b^{5}} \times \frac{6}{a^{6}b^{3}} \\
&= \frac{6}{5a^{2}b^{8}}
\end{aligned}
$$

(e) $\dfrac{7m^{\frac{5}{3}}n^{3}}{2p} \div \dfrac{21m^{\frac{2}{3}}n^{4}}{6p^{2}}$

Solution:

$$
\begin{aligned}
\frac{7m^{\frac{5}{3}}n^{3}}{2p} \div \frac{21m^{\frac{2}{3}}n^{4}}{6p^{2}} &= \frac{7m^{\frac{5}{3}}n^{3}}{2p} \times \frac{6p^{2}}{21m^{\frac{2}{3}}n^{4}} \\
&= m \times \frac{3p}{3n} \\
&= \frac{mp}{n}
\end{aligned}
$$

(f) $4^{x} \times 8^{x+2} \times 6^{2x-2}$

Solution:

$$
\begin{aligned}
4^{x} \times 8^{x+2} \times 6^{2x-2} &= 2^{2x+3x+6} \times 2^{2x-2} \times 3^{2x-2} \\
&= 2^{7x+4} \times 3^{2x-2}
\end{aligned}
$$

(g) $\dfrac{\sqrt{x^{-2}y^{4}}}{\left(\dfrac{x}{y}\right)^{-2}}$

Solution:

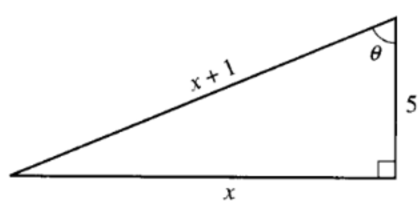
$$\frac{\sqrt{x^{-2}y^4}}{\left(\frac{x}{y}\right)^{-2}} = \frac{x^{-1}y^2}{\frac{y^2}{x^2}}$$
$$= \frac{xy^2}{y^2}$$
$$= x$$

(h) $\frac{\left(2p^4q^3\right)^3}{4p^2q^{15}}$

Solution:

$$\frac{\left(2p^4q^3\right)^3}{4p^2q^{15}} = \frac{2p^{12}q^9}{p^2q^{15}}$$
$$= \frac{2p^{10}}{q^6}$$

6. The figure below shows a right-angled triangle with sides x cm, 5 cm and $(x+1)$ cm respectively. Write down an equation in x, and, hence, find the value of x.



Solution:

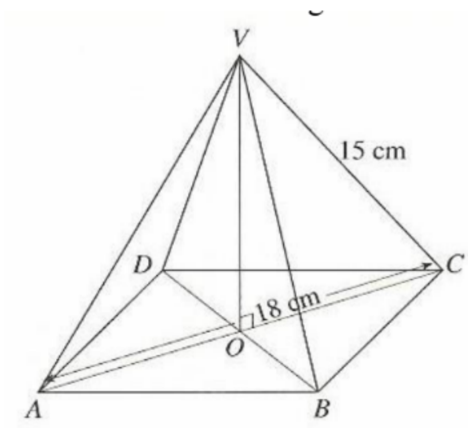
$$5^2 + x^2 = (x+1)^2$$
$$x^2 + 25 = x^2 + 2x + 1$$
$$2x = 24$$
$$x = 12$$

7. Evaluate $\dfrac{9.016 \times 10^3 + 6.292 \times 10^4}{5.673 \times 10^{-2} - \sqrt{2.490 \times 10^{-5}}}$ using a calculator, leaving your answer in standard form, correct to three significant figures.

Solution:

$$\begin{aligned}\frac{9.016 \times 10^3 + 6.292 \times 10^4}{5.673 \times 10^{-2} - \sqrt{2.490 \times 10^{-5}}} &\approx 1\,390\,336.028 \\ &\approx 1.39 \times 10^6 \text{ (3 s.f.)}\end{aligned}$$

8. The diagonal of the square base of the right pyramid below is 18 cm and the slant edge, VC is 15 cm. Calculate



(a) the height of the pyramid,

Solution:

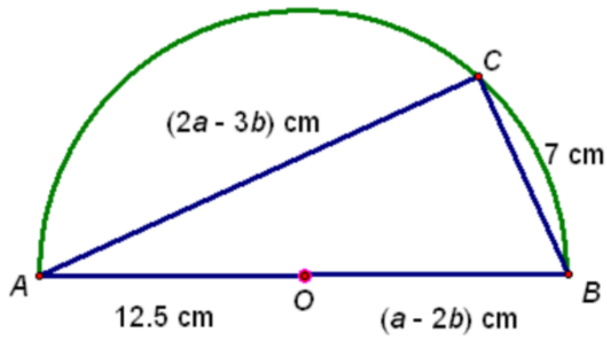
$$\begin{aligned}\text{height of pyramid } &= \sqrt{15^2 - \left(\frac{18}{2}\right)^2} \\ &= 12\text{ cm}\end{aligned}$$

(b) the volume of the pyramid.

Solution:

$$\begin{aligned}\text{volume of pyramid } &= \frac{1}{3} \times \left(2 \times \frac{1}{2} \times 18 \times \frac{18}{2}\right) \times 12 \\ &= 648\text{ cm}^3\end{aligned}$$

9. The diagram below shows a semicircle with O as centre and AB as diameter. C is a point on the circumference such that $BC = 7$ cm and $AC = (2a - 3b)$ cm. Given that $OA = 12.5$ cm and $OB = (a - 2b)$ cm,



(a) By considering the diameter of the circle, write down an equation involving a and b.

Solution: $a - 2b = 12.5$

(b) By using the Pythagoras Theorem, form another equation involving a and b.

Solution:

$$
\begin{aligned}
7^2 + (2a - 3b)^2 &= [2(a - 2b)]^2 \\
49 + 4a^2 - 12ab + 9b^2 &= (2a - 4b)^2 \\
4a^2 - 12ab + 9b^2 + 49 &= 4a^2 - 16ab + 16b^2 \\
4ab + 49 &= 7b^2
\end{aligned}
$$

(c) Find the values of a and b by solving the equations obtained in **(a)** and **(b)** simultaneously.

Solution:

$$a - 2b = 12.5 \qquad (1)$$

$$4ab + 49 = 7b^2 \qquad (2)$$

From (2):

$$
\begin{aligned}
4ab + 49 &= 7b^2 \\
4ab &= 7b^2 - 49 \\
a &= \frac{7b^2 - 49}{4b} \qquad (3)
\end{aligned}
$$

Substitute (3) into (1):

$$\begin{aligned}\frac{7b^2 - 49}{4b} - 2b &= 12.5\\ 7b^2 - 49 - 8b^2 &= 50b\\ b^2 + 50b + 49 &= 0\\ (b + 49)(b + 1) &= 0\\ \therefore b &= -1 \text{ or } b = -49\end{aligned}$$

Substitute $b = -1$ into (1):

$$\begin{aligned}a + 2 &= 12.5\\ \therefore a &= 10.5\end{aligned}$$

Substitute $b = -49$ into (1):

$$\begin{aligned}a + 98 &= 12.5\\ \therefore a &= -85.5 \text{ (rej.)}\end{aligned}$$

$$\therefore \begin{cases} a &= 10.5\\ b &= -1\end{cases}$$

10. Evaulate

(a) $27^{\frac{3}{5}} \div 27^{\frac{2}{5}} \times 27^{\frac{2}{15}}$

Solution:

$$\begin{aligned}27^{\frac{3}{5}} \div 27^{\frac{2}{5}} \times 27^{\frac{2}{15}} &= 27^{\frac{3}{5}-\frac{2}{5}+\frac{1}{15}}\\ &= 27^{\frac{4}{15}}\end{aligned}$$

(b) $(-2)^{-3}$

Solution:

$$\begin{aligned}(-2)^{-3} &= \frac{1}{(-2)^3}\\ &= -\frac{1}{8}\end{aligned}$$

(c) $\left(\dfrac{343}{64}\right)^{-\frac{2}{3}}$

Solution:

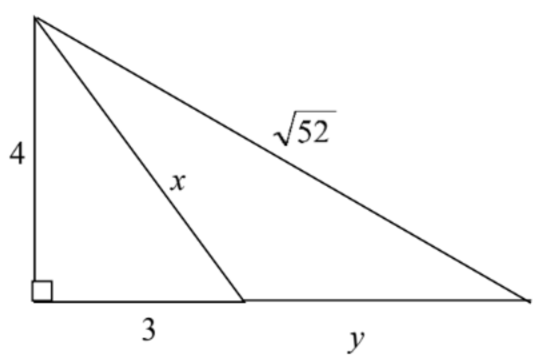
$$\begin{aligned}\left(\frac{343}{64}\right)^{-\frac{2}{3}} &= \left(\frac{64}{343}\right)^{\frac{2}{3}}\\ &= \sqrt[3]{\left(\frac{64}{343}\right)^2}\\ &= \sqrt[3]{\frac{2^{12}}{7^6}}\\ &= \frac{2^4}{7^2}\\ &= \frac{16}{49}\end{aligned}$$

(d) $\left(\dfrac{1}{2}\right)^{-2} \times 2^0$

Solution:

$$\begin{aligned}\left(\frac{1}{2}\right)^{-2} \times 2^0 &= 2^2\\ &= 4\end{aligned}$$

11. For the given figure, find



(a) x,

Solution:

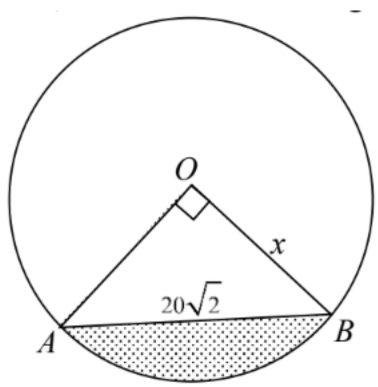
$$\begin{aligned} x &= \sqrt{3^2 + 4^2} \\ &= 5 \end{aligned}$$

(b) y.

Solution:

$$\begin{aligned} y &= \sqrt{\left(\sqrt{52}\right)^2 - 4^2} - 3 \\ &= 3 \end{aligned}$$

12. A circle, centre O, radius x cm, has a chord AB of length $20\sqrt{2}$ cm. $\angle AOB = 90°$. Find



(a) the value of x,

Solution:

$$\begin{aligned} x &= \sqrt{\frac{\left(20\sqrt{2}\right)^2}{2}} \\ &= 20 \end{aligned}$$

(b) the area of the shaded region correct to the nearest 5 cm^2.

Solution:

$$\begin{aligned}\text{area of the shaded region } &= \frac{1}{4} \times \pi \times 20^2 - \frac{1}{2} \times 20^2 \\ &= 114\,\text{cm}^2 \\ &= 115\,\text{cm}^2 \text{ (nearest } 5\,\text{cm}^2\text{)}\end{aligned}$$