Problem Of The Day 2022

1. (**27 Jun**) If y varies inversely as x and can be represented by the equation $y = (m-1)x^{m^2-2}$, find the value of constant m.

Solution:

$$y = (m-1)x^{m^2-2} = \frac{k}{x}$$
$$k = (m-1)x^{m^2-1}$$
$$= (m-1)x^{(m+1)(m-1)} \quad (x \neq 0)$$

By definition, $y \neq 0$ as well, hence

$$(m-1)x^{(m+1)(m-1)} \neq 0$$
$$\therefore m \neq 1$$

2. (**28 Jun**) Which of the following is a possible plot of $y = x + m$ and $y = \frac{m}{x}$ on the same axes? (The graphs are not drawn to scale.)

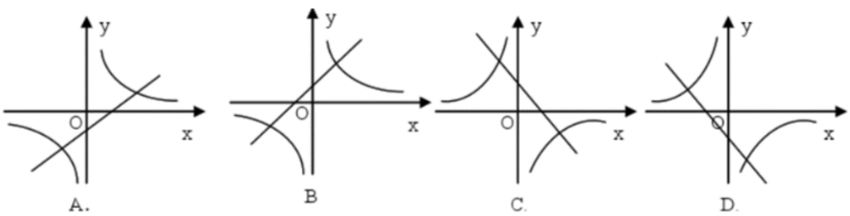

Figure 1: $y = x + m$ and $y = \frac{m}{x}$.

Solution: B.

- The straight line should be increasing, since the coefficient of x is positive. **C** and **D** are eliminated.
- If $m > 0$, the y-intercept of the straight line could not be negative. **A** is eliminated, since the hyperbola in the same graph shows that $m > 0$.

3. (**29 Jun**) Given that points $A(-2, y_1)$, $B(-1, y_2)$, $C(1, y_3)$ are all on the graph of $y = -\frac{1}{x}$, arrange y_1, y_2 and y_3 in ascending order.

Solution: $y_3 < y_1 < y_2$.

Subst. $x = -2$ into $y = -\frac{1}{x}$:

$$\begin{aligned} y_1 &= -\frac{1}{-2} \\ &= \frac{1}{2} \end{aligned}$$

Subst. $x = -1$ into $y = -\frac{1}{x}$:

$$\begin{aligned} y_2 &= -\frac{1}{-1} \\ &= 1 \end{aligned}$$

Subst $x = 1$ into $y = -\frac{1}{x}$:

$$\begin{aligned} y_3 &= -\frac{1}{1} \\ &= -1 \end{aligned}$$

4. (**30 Jun**) Given that points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are all on the graph of $y = \frac{3}{x}$, also $x_1 < x_2 < 0 < x_3$, arrange y_1, y_2 and y_3 in ascending order.

Solution: $y_2 < y_1 < y_3$.

- $y_3 > 0$ since $x_3 > 0$. Hence, y_3 is the greatest.
- $0 > x_2 > x_1$, hence $y_2 < y_1 < 0$.

5. (**1 Jul**) Given that y varies inversely as x such that $y = (a-2)\,x^{a^2-5}$, also when $x > 0$, as x increases, y increases. Find the equation of the hyperbola.

Solution:

$$
\begin{aligned}
\frac{k}{x} &= (a-2)\,x^{a^2-5} \\
k &= (a-2)\,x^{a^2-4} \\
&= (a-2)\,x^{(a+2)(a-2)} \\
\because y &= \frac{(a-2)\,x^{(a+2)(a-2)}}{x} \\
\therefore (a-2)\,x^{(a+2)(a-2)} &< 0 \\
\because a-2 < 0 \text{ and } (a+2)(a-2) &\geq 0 \\
\therefore a+2 &= 0 \\
\therefore a &= -2 \\
\therefore y &= \frac{-4}{x}
\end{aligned}
$$

6. (**5 Jul**) If a straight line $y = (2m-1)x$ and a hyperbola $y = \dfrac{3-m}{x}$ has an intersection point each in Quadrant 1 and 3, what is the range of values of the constant m?

Solution:

$$\because y = \frac{3-m}{x}$$
$$\therefore m < 3$$
$$\because y = x(2m-1)$$
$$\therefore 2m - 1 > 0$$
$$\therefore 0.5 < m < 3$$

7. Points A and B are on the hyperbola $y = \frac{k}{x}$. Right $\triangle AOC$ and $\triangle BOD$ are drawn by perpendicular lines drawn from the two points and connecting the points with the origin. Compare the sizes of the areas of the two triangles.

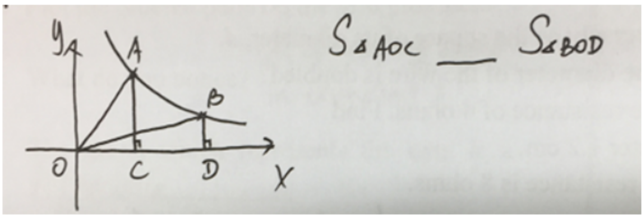

Figure 2: $\triangle AOC$ and $\triangle BOD$.

Solution: $S_{\triangle AOC} < S_{\triangle BOD}$.

As Point B's y-coordinate approaches 0, its x-coordinate approaches infinity, resulting in a triangle which approaches an infinite area. Point A is further from $(\infty, 0)$ than Point B, so $S_{\triangle AOC} < S_{\triangle BOD}$.

8. (**7 Jul**) Points A and B are on the hyperbola $y = \frac{3}{x}$. Rectangles are drawn from the two points as shown. The areas bounded by the two rectangles are labelled S_1 and S_2. If the shaded area is 1 sq. unit, find the sum $S_1 + S_2$.

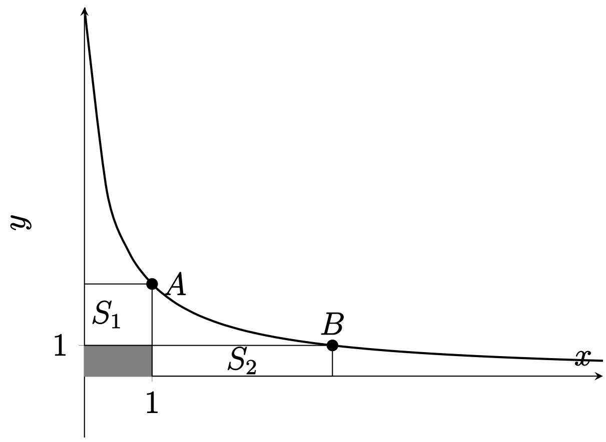

Figure 3: The shaded areas, S_1 and S_2.

Solution:

$$
\begin{aligned}
S_1 &= (1-0) \times (\frac{3}{1} - 1) \\
&= 2 \text{ sq. units} \\
S_2 &= (\frac{3}{3} - 0) \times (3-1) \\
&= 2 \text{ sq. units} \\
S_1 + S_2 &= 2 + 2 \\
&= 4 \text{ sq. units}
\end{aligned}
$$