SBGE Paper B (2022)

1. The latest hand sanitiser bottle is in the shape of a cone. The circumference of the circular base is $(44x+44)$ and the height of the cone is $\left(3x - \frac{6}{7}\right)$. Taking $\pi = \frac{22}{7}$, and the volume of the cone is $\frac{1}{3}\pi r^2 h$, expand and simplify the volume of the hand sanitiser. [3]

Solution:

$$\begin{aligned}
\text{radius of cone} &= \frac{44x + 44}{2 \times \frac{22}{7}} \\
&= 7x + 7 \\
\text{volume of cone} &= \frac{1}{3} \times \frac{22}{7} \times (7x+7)^2 \times \frac{21x - 6}{7} \\
&= \frac{1}{3} \times 22(7x+7)(x+1) \times \frac{21x-6}{7} \\
&= 22(x+1)^2 \times (7x-2) \\
&= 22\left(x^2 + 2x + 1\right)(7x - 2) \\
&= 22\left[7x^3 + 14x^2 + 7x - \left(2x^2 + 4x + 2\right)\right] \\
&= 22\left(7x^3 + 12x^2 + 3x - 2\right) \\
&= 154x^3 + 264x^2 + 66x - 44
\end{aligned}$$

2. Factorise the following **completely**:

(a) $27p^3 - 36p^2 + 12p$ [2]

Solution:

$$\begin{aligned}
27p^3 - 36p^2 + 12p &= 3p\left(9p^2 - 12p + 4\right) \\
&= 3p\,(3p-2)^2
\end{aligned}$$

(b) $de^2 - e^2 f - 4df^2 + 4f^3$ [3]

Solution:

$$\begin{aligned}
de^2 - e^2 f - 4df^2 + 4f^3 &= e^2(d-f) - 4f^2(d-f) \\
&= (d-f)\left(e^2 - 4f^2\right) \\
&= (d-f)(e+2f)(e-2f)
\end{aligned}$$

3. Simplify the following algebraic expressions.

(a) $\dfrac{3y}{y^2-1}+\dfrac{3}{1-y}$ [3]

Solution:

$$\begin{aligned}\frac{3y}{y^2-1}+\frac{3}{1-y} &= \frac{3y-3(y+1)}{y^2-1}\\ &= -\frac{3}{y^2-1}\end{aligned}$$

(b) $\dfrac{a^2-3ab+2b^2}{(a-b)^2}\div\dfrac{2a^2-ab-6b^2}{a^2-b^2}$ [4]

Solution:

$$\begin{aligned}\frac{a^2-3ab+2b^2}{(a-b)^2}\div\frac{2a^2-ab-6b^2}{a^2-b^2} &= \frac{\cancel{(a-b)}\cancel{(a-2b)}}{\cancel{(a-b)^{2}}}\cdot\frac{(a+b)\cancel{(a-b)}}{\cancel{(a-2b)}(2a+3b)}\\ &= \frac{a+b}{2a+3b}\end{aligned}$$

4. Make h the subject of the formula: $\sqrt{\dfrac{h^3mp}{h^3+p}}=mp.$ [4]

Solution:

$$\begin{aligned}\sqrt{\frac{h^3mp}{h^3+p}} &= mp\\ \frac{h^3mp}{h^3+p} &= m^2p^2\\ h^3mp &= h^3m^2p^2+m^2p^3\\ h^3mp-h^3m^2p^2 &= m^2p^3\\ h^3(mp-m^2p^2) &= m^2p^3\\ h^3 &= \frac{m^{\cancel{2}}p^{\cancel{3}2}}{\cancel{m}\cancel{p}(1-mp)}\\ h &= \sqrt[3]{\frac{mp^2}{1-mp}}\end{aligned}$$

5. (a) Solve the equation: $\dfrac{3x}{x+1}-\dfrac{2x}{x-1}=1.$ [2]

Solution:

$$\frac{3x}{x+1} - \frac{2x}{x-1} = 1$$
$$3x(x-1) - 2x(x+1) = x^2 - 1$$
$$x^2 - 5x = x^2 - 1$$
$$-5x = -1$$
$$x = \frac{1}{5}$$

(b) Hence or otherwise, solve the equation: $\frac{3x-3}{x} + \frac{2x-2}{x-2} = 1$. [1]

Solution:

$$\frac{3x-3}{x} + \frac{2x-2}{x-2} = 1$$
$$\frac{3x-3}{(x-1)+1} + \frac{2x-2}{(x-1)-1} = 1$$
$$x - 1 = \frac{1}{5}$$
$$x = \frac{6}{5}$$

6. Given that $a^2 - 121 = 9879$,

(a) Find the positive value of a. [1]

Solution:

$$a^2 - 121 = 9879$$
$$a^2 = 10000$$
$$a = 100$$

(b) Hence, find two factors of 9879 which are between 50 and 200. [3]

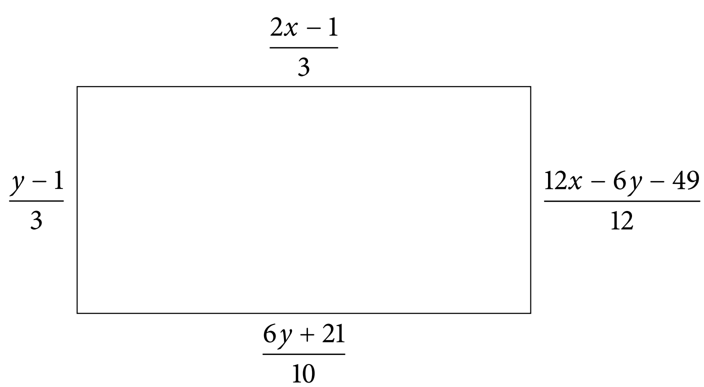
Solution:

$$a^2 - 121 = 9879$$
$$(a+11)(a-11) = 9879$$

Taking $a = 100$, the factors of 9879 are $100 + 11 = 111$ and $100 - 11 = 89$.

7. <u>Solve the entirety of this question using Simultaneous Linear Equations.</u> [4]

In every school, a space is required to be set aside for students who may exhibit any symptoms of cough or cold. In one particular school, this space is in the form of a rectangle of the following dimensions (in m). Find the length of the rectangle.



Solution:

$$\frac{2x-1}{3} = \frac{6y+21}{10} \qquad (1)$$
$$\frac{y-1}{3} = \frac{12x-6y-49}{12} \qquad (2)$$

Cross-multiply (1):

$$\frac{2x-1}{3} = \frac{6y+21}{10}$$
$$10(2x-1) = 3(6y+21)$$
$$20x - 10 = 18y + 63$$
$$20x - 73 = 18y$$
$$y = \frac{20x-73}{18} \qquad (3)$$

Cross-multiply (2):

$$
\begin{aligned}
\frac{y-1}{3} &= \frac{12x-6y-49}{12}\\
12(y-1) &= 3(12x-6y-49)\\
4y-4 &= 12x-6y-49\\
10y+45 &= 12x \qquad (4)
\end{aligned}
$$

Substitute (3) into (4):

$$
\begin{aligned}
10\left(\frac{20x-73}{18}\right)+45 &= 12x\\
100x-365+405 &= 108x\\
\therefore x &= \frac{405-365}{8}\\
&= 5\\
\therefore y &= \frac{20x-73}{18}\\
&= \frac{20\times 5-73}{18}\\
&= \frac{3}{2}\\
\therefore \frac{2x-1}{3} &= \frac{6\times\frac{3}{2}+21}{10}\\
&= 3 \text{ units}
\end{aligned}
$$

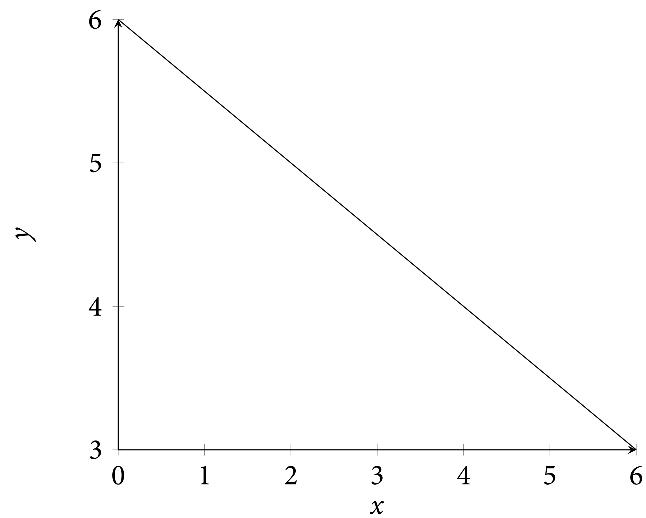
The length of the rectangle is **3 units**.

8. The diagram below shows the line l_1, $y = ax + b$.

(a) State the values of a and b. [2]

Solution: $a = -\frac{1}{2}$, $b = 6$.

(b) Find the equation of another line, l_2, which is parallel to l_1 and passes through the point $(2, 3)$. [2]



Solution: On l_2,

$$
\begin{aligned}
y_2 - y_1 &= m(x_2 - x_1) \\
y_2 - 3 &= -\frac{1}{2}(x_2 - 2) \\
y_2 &= -\frac{1}{2}x_2 + 1 + 3
\end{aligned}
$$

The equation of l_2 is $y = -\frac{1}{2}x + 4$.

9. **Attempt the whole of this question on the graph paper provided.**

The variables x and y are connected by the equation $y + 2 = 3x$.

(a) Copy and complete the following table. [1]

x	-1	0	2
y			

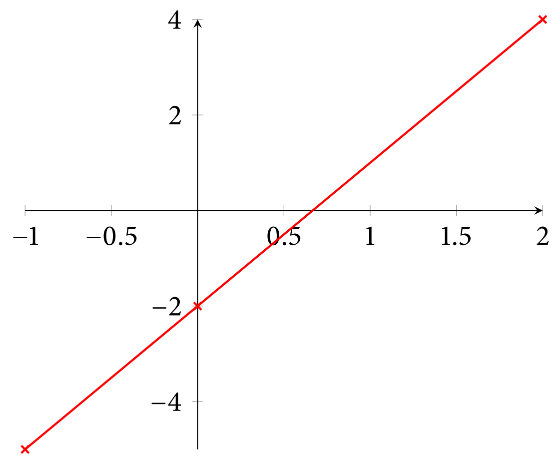
Solution:

x	-1	0	2
y	-5	-2	4

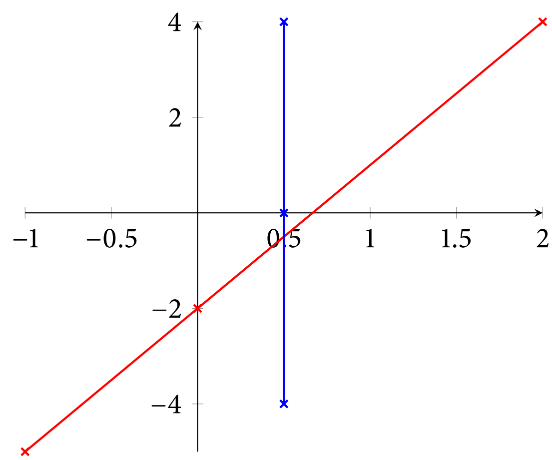
(b) Using a scale of 4 cm to represent 1 unit on the x-axis and a scale of 2 cm to represent 1 unit on the y-axis, draw the graph of $y + 2 = 3x$ for $-1 \le x \le 2$. [2]

(c) **Read from your graph** the value of x when $y = 2.3$. [1]

Solution: 1.4 *(from graph)*. *[Actual: $1\frac{13}{30}$]*



(d) On the same axes as in (b), draw the graph of $x = 0.5$. [1]



(e) Given the graphs you have drawn in (b) and (d), explain how to find the solution to the simultaneous equations $y + 2 = 3x$ and $x = 0.5$. [1]

Solution: Find the coordinates of the point of intersection between the two lines. The x- and y-coordinates of the point will correspond with the solutions to x and y in the simultaneous equations.