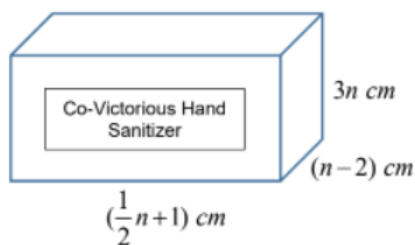


## SBGE Paper C (2022)

1. A particular brand of hand sanitiser is in the shape of a cuboid as shown below:

[2]



Find and simplify an expression in terms of  $n$  for the volume of the hand sanitiser.

**Solution:**

$$\begin{aligned}\text{volume of hand sanitiser} &= 3n \left( \frac{1}{2}n + 1 \right) (n - 2) \\ &= \left( \frac{3}{2}n^2 + 3n \right) (n - 2) \\ &= \frac{3}{2}n^3 + 3n^2 - 3n^2 - 6n \\ &= \left( \frac{3}{2}n^3 - 6n \right) \text{cm}^3\end{aligned}$$

2. Factorise completely:

(a)  $6x^3 + 2x^2y - 4xy^2$

[2]

**Solution:**

$$\begin{aligned}6x^3 + 2x^2y - 4xy^2 &= 2x(3x^2 + xy - 2y^2) \\ &= 2x(3x - 2y)(x + y)\end{aligned}$$

(b)  $(2x + 1)^2 - (x^2 + x + 1)^2$

[3]

**Solution:**

$$\begin{aligned}(2x + 1)^2 - (x^2 + x + 1)^2 &= (2x + 1 + x^2 + x + 1)(2x + 1 - x^2 - x - 1) \\ &= (x^2 + 3x + 2)(-x^2 + x) \\ &= -x(x + 1)(x + 2)(x - 1)\end{aligned}$$

3. Simplify:

(a)  $\frac{3c}{1-c} + \frac{3}{c-1}$

[3]

**Solution:**

$$\begin{aligned}\frac{3c}{1-c} + \frac{3}{c-1} &= \frac{3c-3}{1-c} \\ &= -3\end{aligned}$$

(b)  $\frac{a^2 - b^2}{(a-b)^2} \div \frac{1}{a^2 + b^2} \times \frac{1}{(a-b)^2 + 2ab}$

[3]

**Solution:**

$$\begin{aligned}\frac{a^2 - b^2}{(a-b)^2} \div \frac{1}{a^2 + b^2} \times \frac{1}{(a-b)^2 + 2ab} \\ &= \frac{(a+b)(\cancel{a-b})}{(a-b)(\cancel{a-b})} \cdot \frac{\cancel{a^2 + b^2}}{1} \cdot \frac{1}{\cancel{a^2 + b^2}} \\ &= \frac{a+b}{a-b}\end{aligned}$$

4. Make  $h$  the subject of the formula:  $\sqrt{1-hp} = p$ .

[3]

**Solution:**

$$\begin{aligned}\sqrt{1-hp} &= p \\ 1-hp &= p^2 \\ hp &= 1-p^2 \\ h &= \frac{1-p^2}{p}\end{aligned}$$

5. (a) Solve the equation:  $\frac{1}{x} + \frac{2}{3x} = \frac{3}{x+1}$ .

[3]

**Solution:**

$$\begin{aligned}\frac{1}{x} + \frac{2}{3x} &= \frac{3}{x+1} \\ \frac{5}{3x} &= \frac{3}{x+1} \\ 5(x+1) &= 9x \\ 4x &= 5 \\ x &= \frac{5}{4}\end{aligned}$$

(b) Hence or otherwise, solve the equation  $\frac{1}{x+1} + \frac{2}{3x+3} = \frac{3}{x+2}$ .

[1]

**Solution:**

$$\begin{aligned}\frac{1}{x+1} + \frac{2}{3x+3} &= \frac{3}{x+2} \\ \frac{1}{x+1} + \frac{2}{3(x+1)} &= \frac{3}{(x+1)+1} \\ \therefore x+1 &= \frac{5}{4} \\ \therefore x &= \frac{1}{4}\end{aligned}$$

6. Given that  $a^2 - 49 = 9951$ ,

(a) find the positive value of  $a$ .

[1]

**Solution:**

$$\begin{aligned}a^2 - 49 &= 9951 \\ a &= \sqrt{9951 + 49} \\ &= 100\end{aligned}$$

(b) Hence, find two factors of 9951 which are between 50 and 200.

[3]

**Solution:**

$$a^2 - 49 = 9951$$

$$(a + 7)(a - 7) = 9951$$

When  $a = 100$ , the two factors of 9951 are  $a + 7 = 107$  and  $a - 7 = 93$ .

7. By defining two variables, solve the following problem using SLEs.

[4]

Two books,  $A$  and  $B$ , have a total of 500 pages. If the number of pages in Book  $A$  is 12 less than 3 times the number of pages in Book  $B$ , calculate the number of pages in each book.

**Solution:** Let the number of pages in Books  $A$  and  $B$  be  $a$  and  $b$  respectively.

$$a + b = 500 \quad (1)$$

$$a = 3b - 12 \quad (2)$$

Substitute (2) into (1):

$$3b - 12 + b = 500$$

$$4b = 512$$

$$b = 128$$

$$\therefore a = 372$$

The number of pages in Books  $A$  and  $B$  is 372 and 128 respectively.

8. An ice-cream maker machine can hold 60 L of ice-cream mix. It was discovered that

- (a) when the temperature of the machine is set at  $-10^\circ\text{C}$ ,  $x$  L of ice-cream mix can be frozen per min. Write down an expression in terms of  $x$  for the time taken to freeze 60 L of ice cream mix at  $-10^\circ\text{C}$  in min.

[1]

**Solution:** The time taken would be  $\frac{60}{x}$  min.

- (b) when the temperature of the machine was set at  $-5^\circ\text{C}$ ,  $(x - 2)$  L of ice cream mix can be frozen per minute. Write down an expression in terms of  $x$  for the time taken to freeze 60 L of ice cream mix at  $-5^\circ\text{C}$  in min.

[1]

**Solution:** The time taken would be  $\frac{60}{x-2}$  min.

- (c) it takes 1.5 min longer to freeze the ice cream mix at a higher temperature. Write down an equation in  $x$ , and show that it simplifies to  $x^2 - 2x - 80 = 0$ . [2]

**Solution:**

$$\begin{aligned}\frac{60}{x-2} - \frac{60}{x} &= \frac{3}{2} \\ \frac{120}{x(x-2)} &= \frac{3}{2} \\ 3x^2 - 6x &= 240 \\ x^2 - 2x - 80 &= 0\end{aligned}$$

- (d) Solve the equation  $x^2 - 2x - 80 = 0$ . [2]

**Solution:**

$$\begin{aligned}x^2 - 2x - 80 &= 0 \\ (x+8)(x-10) &= 0 \\ \therefore x &= -8 \text{ (rej.) or } x = 10\end{aligned}$$

- (e) Explain why you have to reject one of the solutions obtained in (d). [1]

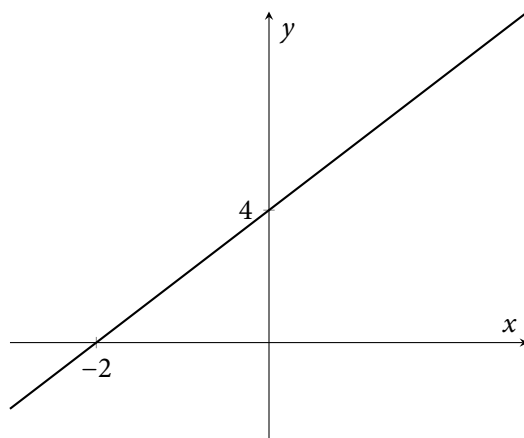
**Solution:** One of the solutions,  $x = -8$ , is negative, which cannot represent the number of litres of ice cream frozen per min.

9. The diagram below shows the line  $L_1$ ,  $y = ax + b$ .

- (a) State the values of  $a$  and  $b$ . [2]

**Solution:**  $a = 2$ ,  $b = 4$ .

- (b) Find the equation of another line,  $L_2$ , which passes through the point  $(5, 9)$  and is parallel to  $L_1$ . [2]



**Solution:**

$$y - y_1 = m(x - x_1)$$

$$y - 9 = 2(x - 5)$$

$$y = 2x - 1$$

(c) Does the point  $(3, 10)$  lie on line  $L_2$ ? Explain.

[1]

**Solution:** The point  $(3, 10)$  does not lie on  $L_2$ . Substituting the values of  $x = 3$  and  $y = 10$  into the equation  $y = 2x - 1$  makes the equation false.