# SBGE Paper B (2022)

1. The latest hand sanitiser bottle is in the shape of a cone. The circumference of the circular base is $(44x + 44)$ and the height of the cone is $\left(3x - \frac{6}{7}\right)$. Taking $\pi = \frac{22}{7}$, and the volume of the cone is $\frac{1}{3}\pi r^2 h$, expand and simplify the volume of the hand sanitiser. [3]

**Solution:**

$$\begin{aligned}
\text{radius of cone} &= \frac{44x + 44}{2 \times \frac{22}{7}} \\
&= 7x + 7 \\
\text{volume of cone} &= \frac{1}{3} \times \frac{22}{7} \times (7x + 7)^2 \times \frac{21x - 6}{7} \\
&= \frac{1}{3} \times 22(7x + 7)(x + 1) \times \frac{21x - 6}{7} \\
&= 22(x + 1)^2 \times (7x - 2) \\
&= 22\left(x^2 + 2x + 1\right)(7x - 2) \\
&= 22\left[7x^3 + 14x^2 + 7x - \left(2x^2 + 4x + 2\right)\right] \\
&= 22\left(7x^3 + 12x^2 + 3x - 2\right) \\
&= 154x^3 + 264x^2 + 66x - 44
\end{aligned}$$

2. Factorise the following **completely**:

(a) $27p^3 - 36p^2 + 12p$ [2]

**Solution:**

$$\begin{aligned}
27p^3 - 36p^2 + 12p &= 3p\left(9p^2 - 12p + 4\right) \\
&= 3p\,(3p - 2)^2
\end{aligned}$$

(b) $de^2 - e^2 f - 4df^2 + 4f^3$ [3]

**Solution:**

$$\begin{aligned} de^2 - e^2 f - 4df^2 + 4f^3 &= e^2(d-f) - 4f^2(d-f) \\ &= (d-f)\left(e^2 - 4f^2\right) \\ &= (d-f)(e+2f)(e-2f) \end{aligned}$$

3. Simplify the following algebraic expressions.

(a) $\dfrac{3y}{y^2-1} + \dfrac{3}{1-y}$ [3]

**Solution:**

$$\begin{aligned} \frac{3y}{y^2-1} + \frac{3}{1-y} &= \frac{3y - 3(y+1)}{y^2-1} \\ &= -\frac{3}{y^2-1} \end{aligned}$$

(b) $\dfrac{a^2 - 3ab + 2b^2}{(a-b)^2} \div \dfrac{2a^2 - ab - 6b^2}{a^2 - b^2}$ [4]

**Solution:**

$$\begin{aligned} \frac{a^2 - 3ab + 2b^2}{(a-b)^2} \div \frac{2a^2 - ab - 6b^2}{a^2 - b^2} &= \frac{\cancel{(a-b)}\cancel{(a-2b)}}{\cancel{(a-b)^2}} \cdot \frac{(a+b)\cancel{(a-b)}}{\cancel{(a-2b)}(2a+3b)} \\ &= \frac{a+b}{2a+3b} \end{aligned}$$

4. Make $h$ the subject of the formula: $\sqrt{\dfrac{h^3 mp}{h^3 + p}} = mp.$ [4]

**Solution:**

$$
\begin{aligned}
\sqrt{\frac{h^3mp}{h^3+p}} &= mp \\
\frac{h^3mp}{h^3+p} &= m^2p^2 \\
h^3mp &= h^3m^2p^2 + m^2p^3 \\
h^3mp - h^3m^2p^2 &= m^2p^3 \\
h^3(mp - m^2p^2) &= m^2p^3 \\
h^3 &= \frac{m^{\cancel{2}}p^{\cancel{3}2}}{\cancel{mp}\,(1-mp)} \\
h &= \sqrt[3]{\frac{mp^2}{1-mp}}
\end{aligned}
$$

5. (a) Solve the equation: $\dfrac{3x}{x+1} - \dfrac{2x}{x-1} = 1.$ [2]

**Solution:**

$$
\begin{aligned}
\frac{3x}{x+1} - \frac{2x}{x-1} &= 1 \\
3x(x-1) - 2x(x+1) &= x^2 - 1 \\
x^2 - 5x &= x^2 - 1 \\
-5x &= -1 \\
x &= \frac{1}{5}
\end{aligned}
$$

(b) Hence or otherwise, solve the equation: $\dfrac{3x-3}{x} + \dfrac{2x-2}{x-2} = 1.$ [1]

**Solution:**

$$
\begin{aligned}
\frac{3x-3}{x} + \frac{2x-2}{x-2} &= 1 \\
\frac{3x-3}{(x-1)+1} + \frac{2x-2}{(x-1)-1} &= 1 \\
x - 1 &= \frac{1}{5} \\
x &= \frac{6}{5}
\end{aligned}
$$

6. Given that $a^2 - 121 = 9879$,

(a) Find the positive value of $a$. [1]

**Solution:**

$$a^2 - 121 = 9879$$
$$a^2 = 10000$$
$$a = 100$$

(b) Hence, find two factors of 9879 which are between 50 and 200. [3]

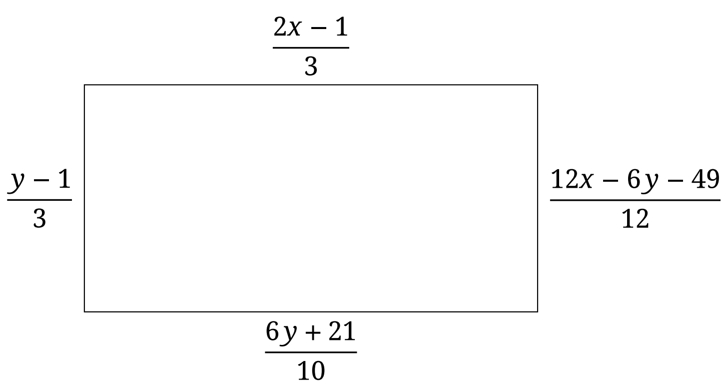
**Solution:**

$$a^2 - 121 = 9879$$
$$(a+11)(a-11) = 9879$$

Taking $a = 100$, the factors of 9879 are $100 + 11 = 111$ and $100 - 11 = 89$.

7. Solve the entirety of this question using Simultaneous Linear Equations. [4]

In every school, a space is required to be set aside for students who may exhibit any symptoms of cough or cold. In one particular school, this space is in the form of a rectangle of the following dimensions (in m). Find the length of the rectangle.



**Solution:**

$$\frac{2x-1}{3} = \frac{6y+21}{10} \tag{1}$$
$$\frac{y-1}{3} = \frac{12x-6y-49}{12} \tag{2}$$

Cross-multiply (1):

$$\begin{aligned}
\frac{2x-1}{3} &= \frac{6y+21}{10} \\
10(2x-1) &= 3(6y+21) \\
20x-10 &= 18y+63 \\
20x-73 &= 18y \\
y &= \frac{20x-73}{18} \qquad (3)
\end{aligned}$$

Cross-multiply (2):

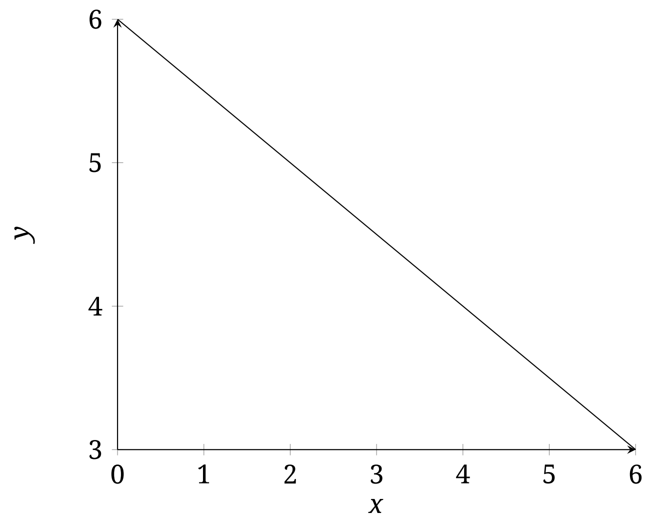
$$\begin{aligned}
\frac{y-1}{3} &= \frac{12x-6y-49}{12} \\
12(y-1) &= 3(12x-6y-49) \\
4y-4 &= 12x-6y-49 \\
10y+45 &= 12x \qquad (4)
\end{aligned}$$

Substitute (3) into (4):

$$\begin{aligned}
10\left(\frac{20x-73}{18}\right)+45 &= 12x \\
100x-365+405 &= 108x \\
\therefore x &= \frac{405-365}{8} \\
&= 5 \\
\therefore y &= \frac{20x-73}{18} \\
&= \frac{20\times 5-73}{18} \\
&= \frac{3}{2} \\
\therefore \frac{2x-1}{3} &= \frac{6\times\frac{3}{2}+21}{10} \\
&= 3 \text{ units}
\end{aligned}$$

The length of the rectangle is **3 units**.

8. The diagram below shows the line $l_1$, $y = ax + b$.




(a) State the values of $a$ and $b$. [2]

**Solution:** $a = -\frac{1}{2}$, $b = 6$.

(b) Find the equation of another line, $l_2$, which is parallel to $l_1$ and passes through the point $(2, 3)$. [2]

**Solution:** On $l_2$,

$$\begin{aligned} y_2 - y_1 &= m(x_2 - x_1) \\ y_2 - 3 &= -\frac{1}{2}(x_2 - 2) \\ y_2 &= -\frac{1}{2}x_2 + 1 + 3 \end{aligned}$$

The equation of $l_2$ is $y = -\frac{1}{2}x + 4$.

9. **Attempt the whole of this question on the graph paper provided.**

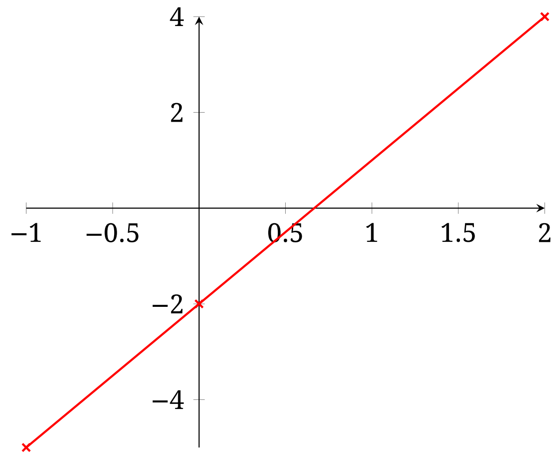
The variables $x$ and $y$ are connected by the equation $y + 2 = 3x$.

(a) Copy and complete the following table. [1]

| $x$ | $-1$ | $0$ | $2$ |
|---|---|---|---|
| $y$ | | | |

| | | | |
|---|---|---|---|
| **Solution:** $x$ | $-1$ | $0$ | $2$ |
| $y$ | $-5$ | $-2$ | $4$ |

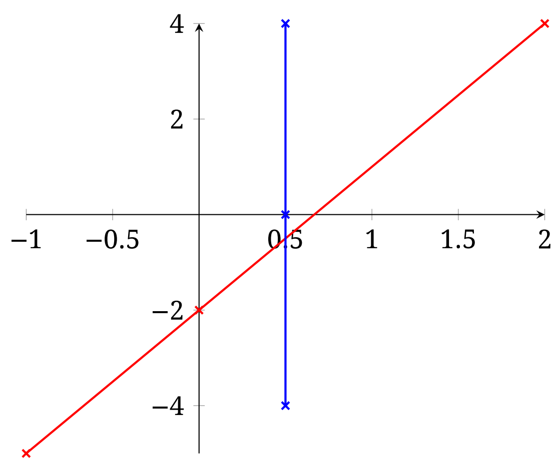
(b) Using a scale of 4 cm to represent 1 unit on the $x$-axis and a scale of 2 cm to represent 1 unit on the $y$-axis, draw the graph of $y + 2 = 3x$ for $-1 \le x \le 2$. [2]




(c) **Read from your graph** the value of $x$ when $y = 2.3$. [1]

**Solution:** 1.4 *(from graph). [Actual:* $1\frac{13}{30}$*]*

(d) On the same axes as in (b), draw the graph of $x = 0.5$. [1]




(e) Given the graphs you have drawn in (b) and (d), explain how to find the solution to the simultaneous equations $y + 2 = 3x$ and $x = 0.5$. [1]

**Solution:** Find the coordinates of the point of intersection between the two lines. The $x$- and $y$-coordinates of the point will correspond with the solutions to $x$ and $y$ in the simultaneous equations.