Problem Of The Day 2022

1. (**27 Jun**) If y varies inversely as x and can be represented by the equation $y = (m - 1)x^{m^2-2}$, find the value of constant m.

Solution:

$$y = (m-1)x^{m^2-2} = \frac{k}{x}$$

$$k = (m-1)x^{m^2-1}$$

$$= (m-1)x^{(m+1)(m-1)} \quad (x \neq 0)$$

By definition, $y \neq 0$ as well, hence

$$(m-1)x^{(m+1)(m-1)} \neq 0$$

$$\therefore m \neq 1$$

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2. (**28 Jun**) Which of the following is a possible plot of $y = x + m$ and $y = \dfrac{m}{x}$ on the same axes? (The graphs are not drawn to scale.)
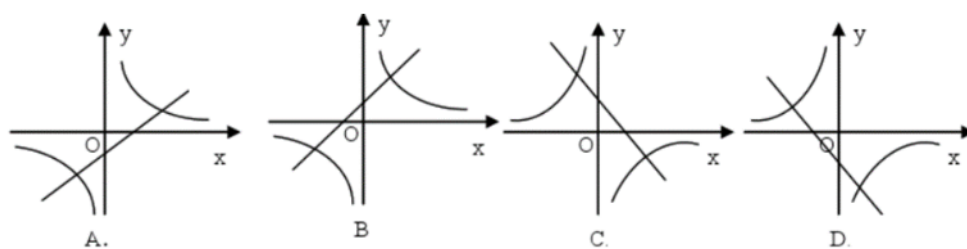

Figure 1: $y = x + m$ and $y = \dfrac{m}{x}$.

Solution: B.

- The straight line should be increasing, since the coefficient of x is positive. **C** and **D** are eliminated.

- If $m > 0$, the y-intercept of the straight line could not be negative. **A** is eliminated, since the hyperbola in the same graph shows that $m > 0$.

3. (**29 Jun**) Given that points $A\left(-2, y_1\right)$, $B\left(-1, y_2\right)$, $C\left(1, y_3\right)$ are all on the graph of $y = -\dfrac{1}{x}$, arrange y_1, y_2 and y_3 in ascending order.

> **Solution:** $y_3 < y_1 < y_2$.
>
> Subst. $x = -2$ into $y = -\dfrac{1}{x}$:
>
> $$y_1 = -\frac{1}{-2}$$
> $$= \frac{1}{2}$$
>
> Subst. $x = -1$ into $y = -\dfrac{1}{x}$:
>
> $$y_2 = -\frac{1}{-1}$$
> $$= 1$$
>
> Subst $x = 1$ into $y = -\dfrac{1}{x}$:
>
> $$y_3 = -\frac{1}{1}$$
> $$= -1$$

4. (**30 Jun**) Given that points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are all on the graph of $y = \dfrac{3}{x}$, also $x_1 < x_2 < 0 < x_3$, arrange y_1, y_2 and y_3 in ascending order.

> **Solution:** $y_2 < y_1 < y_3$.
>
> - $y_3 > 0$ since $x_3 > 0$. Hence, y_3 is the greatest.
>
> - $0 > x_2 > x_1$, hence $y_2 < y_1 < 0$.

5. (**1 Jul**) Given that y varies inversely as x such that $y = (a-2)x^{a^2-5}$, also when $x > 0$, as x increases, y increases. Find the equation of the hyperbola.

Solution:

$$\frac{k}{x} = (a-2)x^{a^2-5}$$

$$k = (a-2)x^{a^2-4}$$

$$= (a-2)x^{(a+2)(a-2)}$$

$$\because y = \frac{(a-2)x^{(a+2)(a-2)}}{x}$$

$$\therefore (a-2)x^{(a+2)(a-2)} < 0$$

$$\because a-2 < 0 \text{ and } (a+2)(a-2) \geq 0$$

$$\therefore a+2 = 0$$

$$\therefore a = -2$$

$$\therefore y = -\frac{(-2-2) \times x^{(-2+2) \times (-2-2)}}{x}$$

$$= -\frac{4}{x}$$

6. (**5 Jul**) If a straight line $y = (2m-1)x$ and a hyperbola $y = \frac{3-m}{x}$ has an intersection point each in Quadrant 1 and 3, what is the range of values of the constant m?

Solution:

$$\because y = \frac{3-m}{x}$$

$$\therefore m < 3$$

$$\because y = x(2m-1)$$

$$\therefore 2m-1 > 0$$

$$\therefore 0.5 < m < 3$$

7. Points A and B are on the hyperbola $y = \frac{k}{x}$. Right $\triangle AOC$ and $\triangle BOD$ are drawn by per-

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pendicular lines drawn from the two points and connecting the points with the origin. Compare the sizes of the areas of the two triangles.
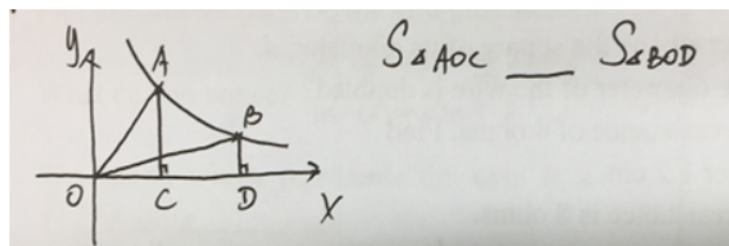

Figure 2: $\triangle AOC$ and $\triangle BOD$.

Solution: $S_{\triangle AOC} < S_{\triangle BOD}$.

As Point B's y-coordinate approaches 0, its x-coordinate approaches infinity, resulting in a triangle which approaches an infinite area. Point A is further from $(\infty, 0)$ than Point B, so $S_{\triangle AOC} < S_{\triangle BOD}$.

8. (**7 Jul**) Points A and B are on the hyperbola $y = \dfrac{3}{x}$. Rectangles are drawn from the two points as shown. The areas bounded by the two rectangles are labelled S_1 and S_2. If the shaded area is 1 sq. unit, find the sum $S_1 + S_2$.
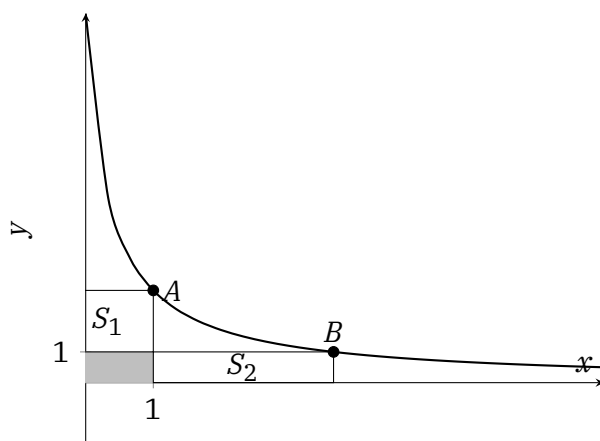

Figure 3: The shaded areas, S_1 and S_2.

Solution:

$$S_1 = (1 - 0) \times \left(\frac{3}{1} - 1 \right)$$

$$= 2 \text{ sq. units}$$

$$S_2 = \left(\frac{3}{3} - 0 \right) \times (3 - 1)$$

$$= 2 \text{ sq. units}$$

$$S_1 + S_2 = 2 + 2$$

$$= 4 \text{ sq. units}$$