Problem Of The Day 2022

1. (**21 Mar**) Simplify the algebraic fraction $\dfrac{a^4 - a^2b^2}{(a-b)^2} \div \dfrac{a(a+b)}{b^2} \times \dfrac{b}{a}$.

> **Solution:**
>
> $$\begin{aligned}
> &\frac{a^4 - a^2b^2}{(a-b)^2} \div \frac{a(a+b)}{b^2} \times \frac{b^2}{a} \\
> &= \frac{a^2\cancel{(a+b)(a-b)}}{(a-b)^{\cancel{2}}} \times \frac{b^2}{\cancel{a(a+b)}} \times \frac{b^2}{\cancel{a}} \\
> &= \frac{b^4}{a-b}
> \end{aligned}$$

2. (**22 Mar**) Factorise $a^4 + a^2b^2 + b^2$.

> **Solution:**
>
> $$\begin{aligned}
> a^4 + a^2b^2 + b^4 &= a^4 + 2a^2b^2 + b^4 - a^2b^2 \\
> &= (a^2 + b^2)^2 - (ab)^2 \\
> &= (a^2 - ab + b^2)(a^2 + ab + b^2)
> \end{aligned}$$

3. (**23 Mar**) Simplify $\dfrac{1}{a-x} - \dfrac{1}{a+x} - \dfrac{2x}{a^2+x^2} - \dfrac{4x^3}{a^4+x^4} + \dfrac{8x^7}{a^8-x^8}$.

> **Solution:**
>
> $$\begin{aligned}
> &\frac{1}{a-x} - \frac{1}{a+x} - \frac{2x}{a^2+x^2} - \frac{4x^3}{a^4+x^4} + \frac{8x^7}{a^8-x^8} \\
> &= \frac{2x}{a^2-x^2} - \frac{2x}{a^2+x^2} - \frac{4x^3}{a^4+x^4} + \frac{8x^7}{a^8-x^8} \\
> &= \frac{4x^3}{a^4-x^4} - \frac{4x^3}{a^4+x^4} + \frac{8x^7}{a^8-x^8} \\
> &= \frac{8x^7}{a^8-x^8} + \frac{8x^7}{a^8-x^8} \\
> &= \frac{16x^7}{a^8-x^8}
> \end{aligned}$$

4. (**24 Mar**) Factorise completely $64x^6 - y^{12}$.

Solution:

$$\begin{aligned} 64x^6 - y^{12} &= (8x^3 + y^6)(8x^3 - y^6) \\ &= (2x + y^2)(2x - y^2)(4x^2 + 2xy^2 + y^4)(4x^2 - 2xy^2 + y^4) \end{aligned}$$

5. (**25 Mar**) Factorise completely $x^2(x-1)^2 + 32(x - x^2) + 60$.

Solution:

$$\begin{aligned} &x^2(x-1)^2 + 32(x - x^2) + 60 \\ &= x^2(x-1)^2 - 32x(x-1) + 60 \\ &= [x(x-1)]^2 - 32[x(x-1)] + 16^2 - 14^2 \\ &= [x(x-1) - 16]^2 - 14^2 \\ &= (x^2 - x - 2)(x^2 - x - 30) \\ &= (x-6)(x-2)(x+1)(x+5) \end{aligned}$$

6. (**28 Mar**) Simplify $\dfrac{x^2-4}{x^2-4x+4} + \dfrac{2-x}{x+2}$.

Solution:

$$\begin{aligned} &\frac{x^2-4}{x^2-4x+4} + \frac{2-x}{x+2} \\ &= \frac{(x+2)\cancel{(x-2)}}{(x-2)^{\cancel{2}}} + \frac{-(x-2)}{x+2} \\ &= \frac{(x+2)^2 - (x-2)^2}{(x+2)(x-2)} \\ &= \frac{8x}{x^2-4} \end{aligned}$$

7. (**29 Mar**) An equation in x, $\dfrac{m}{x-1} + \dfrac{3}{1-x} = 1$, has a positive solution. Find the possible range of values for m.

Solution:

$$\begin{aligned}\frac{m}{x-1}+\frac{3}{1-x}&=1\\ m-3&=x-1\\ x&=m-2\\ \because x&>0,\\ \therefore m&>2\end{aligned}$$

8. (**30 Mar**) Given that $\frac{1}{x}+\frac{1}{y}=3$, find the value of $\frac{3x+4xy+3y}{x+2xy+y}$.

Solution:

$$\begin{aligned}\because \frac{1}{x}+\frac{1}{y}&=3\\ \therefore x+y&=3xy\end{aligned}$$

$$\begin{aligned}\frac{3x+4xy+3y}{x+2xy+y}&=\frac{\frac{13}{3}(x+y)}{\frac{5}{3}(x+y)}\\ &=\frac{13}{5}\end{aligned}$$