SBGE Paper B (2022)

1. The latest hand sanitiser bottle is in the shape of a cone. The circumference of the circular base is (44x+44) and the height of the cone is $(3x - \frac{6}{7})$. Taking $\pi = \frac{22}{7}$, and the volume of the cone is $\frac{1}{3}\pi r^2 h$, expand and simplify the volume of the hand sanitiser.

Solution:

radius of cone =
$$\frac{44x + 44}{2 \times \frac{22}{7}}$$

= $7x + 7$
volume of cone = $\frac{1}{3} \times \frac{22}{7} \times (7x + 7)^2 \times \frac{21x - 6}{7}$
= $\frac{1}{3} \times 22(7x + 7)(x + 1) \times \frac{21x - 6}{7}$
= $22(x + 1)^2 \times (7x - 2)$
= $22(x^2 + 2x + 1)(7x - 2)$
= $22[7x^3 + 14x^2 + 7x - (2x^2 + 4x + 2)]$
= $22(7x^3 + 12x^2 + 3x - 2)$
= $154x^3 + 264x^2 + 66x - 44$

2. Factorise the following **completely**:

(a)
$$27p^3 - 36p^2 + 12p$$

Solution:

$$27p^{3} - 36p^{2} + 12p = 3p(9p^{2} - 12p + 4)$$
$$= 3p(3p - 2)^{2}$$

(b)
$$de^2 - e^2 f - 4df^2 + 4f^3$$

Solution:

$$de^{2} - e^{2}f - 4df^{2} + 4f^{3} = e^{2}(d - f) - 4f^{2}(d - f)$$
$$= (d - f)(e^{2} - 4f^{2})$$
$$= (d - f)(e + 2f)(e - 2f)$$

3. Simplify the following algebraic expressions.

(a)
$$\frac{3y}{y^2-1} + \frac{3}{1-y}$$

Solution:

$$\frac{3y}{y^2 - 1} + \frac{3}{1 - y} = \frac{3y - 3(y + 1)}{y^2 - 1}$$
$$= -\frac{3}{y^2 - 1}$$

(b)
$$\frac{a^2 - 3ab + 2b^2}{(a - b)^2} \div \frac{2a^2 - ab - 6b^2}{a^2 - b^2}$$
 [4]

Solution:

$$\frac{a^2 - 3ab + 2b^2}{(a - b)^2} \div \frac{2a^2 - ab - 6b^2}{a^2 - b^2} = \frac{(a - b)(a - 2b)}{(a - b)^{\frac{3}{4}}} \cdot \frac{(a + b)(a - b)}{(a - 2b)(2a + 3b)}$$
$$= \frac{a + b}{2a + 3b}$$

4. Make *h* the subject of the formula:
$$\sqrt{\frac{h^3mp}{h^3+p}} = mp$$
. [4]

Solution:

$$\sqrt{\frac{h^{3}mp}{h^{3}+p}} = mp$$

$$\frac{h^{3}mp}{h^{3}+p} = m^{2}p^{2}$$

$$h^{3}mp = h^{3}m^{2}p^{2} + m^{2}p^{3}$$

$$h^{3}mp - h^{3}m^{2}p^{2} = m^{2}p^{3}$$

$$h^{3}(mp - m^{2}p^{2}) = m^{2}p^{3}$$

$$h^{3} = \frac{m^{7}p^{3}}{mp(1-mp)}$$

$$h = \sqrt[3]{\frac{mp^{2}}{1-mp}}$$

5. (a) Solve the equation:
$$\frac{3x}{x+1} - \frac{2x}{x-1} = 1.$$

Solution:

$$\frac{3x}{x+1} - \frac{2x}{x-1} = 1$$

$$3x(x-1) - 2x(x+1) = x^2 - 1$$

$$x^2 - 5x = x^2 - 1$$

$$-5x = -1$$

$$x = \frac{1}{5}$$

(b) Hence or otherwise, solve the equation: $\frac{3x-3}{x} + \frac{2x-2}{x-2} = 1$.

[1]

Solution:

$$\frac{3x-3}{x} + \frac{2x-2}{x-2} = 1$$

$$\frac{3x-3}{(x-1)+1} + \frac{2x-2}{(x-1)-1} = 1$$

$$x-1 = \frac{1}{5}$$

$$x = \frac{6}{5}$$

- 6. Given that $a^2 121 = 9879$,
 - (a) Find the positive value of *a*.

[1]

Solution:

$$a^2 - 121 = 9879$$
$$a^2 = 10000$$
$$a = 100$$

(b) Hence, find two factors of 9879 which are between 50 and 200.

[3]

Solution:

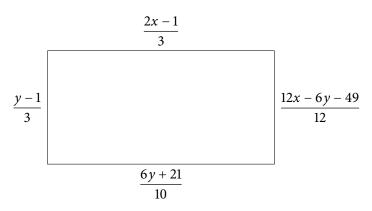
$$a^2 - 121 = 9879$$

 $(a + 11)(a - 11) = 9879$

Taking a = 100, the factors of 9879 are 100 + 11 = 111 and 100 - 11 = 89.

7. Solve the entirety of this question using Simultaneous Linear Equations.

In every school, a space is required to be set aside for students who may exhibit any symptoms of cough or cold. In one particular school, this space is in the form of a rectangle of the following dimensions (in m). Find the length of the rectangle.



Solution:

$$\frac{2x-1}{3} = \frac{6y+21}{10}$$

$$\frac{y-1}{3} = \frac{12x-6y-49}{12}$$
(2)

[4]

$$\frac{y-1}{3} = \frac{12x - 6y - 49}{12} \tag{2}$$

Cross-multiply (1):

$$\frac{2x-1}{3} = \frac{6y+21}{10}$$

$$10(2x-1) = 3(6y+21)$$

$$20x-10 = 18y+63$$

$$20x-73 = 18y$$

$$y = \frac{20x-73}{18}$$
(3)

Cross-multiply (2):

$$\frac{y-1}{3} = \frac{12x - 6y - 49}{12}$$

$$12(y-1) = 3(12x - 6y - 49)$$

$$4y - 4 = 12x - 6y - 49$$

$$10y + 45 = 12x$$
(4)

Substitute (3) into (4):

$$10\left(\frac{20x - 73}{18}\right) + 45 = 12x$$

$$100x - 365 + 405 = 108x$$

$$\therefore x = \frac{405 - 365}{8}$$

$$= 5$$

$$\therefore y = \frac{20x - 73}{18}$$

$$= \frac{20 \times 5 - 73}{18}$$

$$= \frac{3}{2}$$

$$\therefore \frac{2x - 1}{3} = \frac{6 \times \frac{3}{2} + 21}{10}$$

$$= 3 \text{ units}$$

The length of the rectangle is **3 units**.

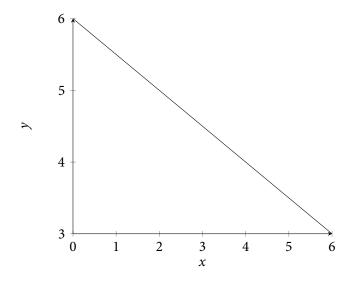
- 8. The diagram below shows the line l_1 , y = ax + b.
 - (a) State the values of *a* and *b*.

Solution:
$$a = -\frac{1}{2}, b = 6.$$

(b) Find the equation of another line, l_2 , which is parallel to l_1 and passes through the point (2,3).

[2]

[2]



Solution: On l_2 ,

$$y_2 - y_1 = m(x_2 - x_1)$$
$$y_2 - 3 = -\frac{1}{2}(x_2 - 2)$$
$$y_2 = -\frac{1}{2}x_2 + 1 + 3$$

The equation of l_2 is $y = -\frac{1}{2}x + 4$.

9. Attempt the whole of this question on the graph paper provided.

The variables x and y are connected by the equation y + 2 = 3x.

(a) Copy and complete the following table.

ole.	х	-1	0	2
	y			

[1]

[2]

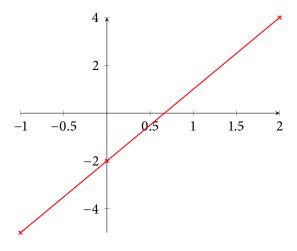
[1]

Solution:

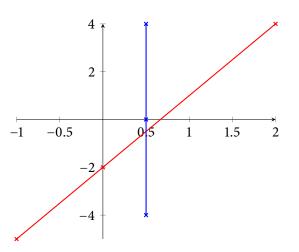
x	-1	0	2
y	-5	-2	4

- (b) Using a scale of 4 cm to represent 1 unit on the *x*-axis and a scale of 2 cm to represent 1 unit on the *y*-axis, draw the graph of y + 2 = 3x for $-1 \le x \le 2$.
- (c) **Read from your graph** the value of x when y = 2.3.

Solution: 1.4 (from graph). [Actual: $1\frac{13}{30}$]



(d) On the same axes as in (b), draw the graph of x = 0.5.



[1]

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(e) Given the graphs you have drawn in (b) and (d), explain how to find the solution to the simultaneous equations y + 2 = 3x and x = 0.5.

Solution: Find the coordinates of the point of intersection between the two lines. The x- and y-coordinates of the point will correspond with the solutions to x and y in the simultaneous equations.