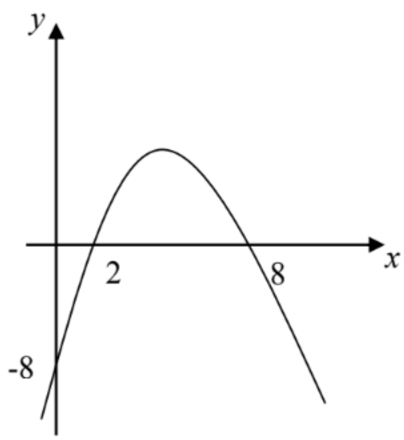
Revision Practice

T2W10 HBL

26 May 2022

1. The diagram shows a quadratic curve which can be expressed in the form of $y = ax^2 + bx + c$. Given that the curve cuts the x-axis at 2 and 8 and the y-axis at -8, find the values of a, b and c.



Solution:

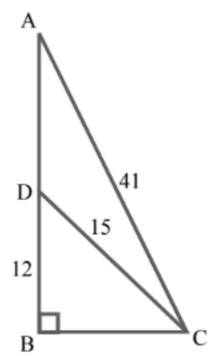
$$0 = a(x-2)(x-8)$$
$$y = a(x-2)(x-8)$$

Using the fact that the graph intercepts $(0, -8)$,

$$\begin{aligned} -8 &= a(0-2)(0-8) \\ 16a &= -8 \\ a &= -\frac{1}{2} \\ \therefore\ y &= -\frac{1}{2}(x-2)(x-8) \\ &= -\frac{1}{2}x^2 + 5x - 8 \end{aligned}$$

$$\therefore\ a = -\frac{1}{2},\ b = 5,\ c = -8$$

2. In the diagram, $\angle ABC = 90°$, $AC = 41$ cm. D is on AB such that $CD = 15$ cm, $BD = 12$ cm. Calculate the value of BC and of AD.



Solution:

$$\begin{aligned} BC &= \sqrt{15^2 - 12^2} \\ &= 9\text{ cm} \\ AD &= \sqrt{41^2 - BC^2} - BD \\ &= \sqrt{41^2 - 9^2} - 12 \\ &= 28\text{ cm} \end{aligned}$$

3. Solve for x in the following equations.

(a) $2006^{x^2-9x+20} - 1 = 0$

Solution:

$$2006^{x^2-9x+20} - 1 = 0$$
$$x^2 - 9x + 20 = 0$$
$$(x-4)(x-5) = 0$$
$$x = 4 \text{ or } 5$$

(b) $4^x\left(5^{2x}\right) = 10$

Solution:

$$4^x\left(5^{2x}\right) = 10$$
$$4^x \cdot \left(5^2\right)^x = 10$$
$$4^x \cdot 25^x = 10$$
$$100^x = 10$$
$$\left(10^2\right)^x = 10^1$$
$$2x = 1$$
$$x = \frac{1}{2}$$

(c) $3^{14}\left(9^{1-x}\right) = \left(3^3\right)^{2x}$

Solution:

$$3^{14}\left(9^{1-x}\right) = \left(3^3\right)^{2x}$$
$$3^{14} \cdot 3^{2-2x} = 3^{6x}$$
$$16 - 2x = 6x$$
$$x = 2$$

(d) $25^{x+2} = 125^{4-x}$

Solution:

$$
\begin{aligned}
25^{x+2} &= 125^{4-x} \\
5^{2x+4} &= 5^{12-3x} \\
2x+4 &= 12-3x \\
x &= \frac{8}{5}
\end{aligned}
$$

(e) $\sqrt{m\sqrt{m\sqrt{m}}} = m^{x-1}$

Solution:

$$
\begin{aligned}
\sqrt{m\sqrt{m\sqrt{m}}} &= m^{x-1} \\
\sqrt{m\sqrt{m^{\frac{3}{2}}}} &= m^{x-1} \\
\sqrt{m^{\frac{7}{4}}} &= m^{x-1} \\
m^{\frac{7}{8}} &= m^{x-1} \\
x-1 &= \frac{7}{8} \\
x &= \frac{15}{8}
\end{aligned}
$$

4. It is given that Newton's Law of Universal Gravitation is defined by the formula $F = \frac{GMm}{r^2}$.

   (a) Make r the subject of the formula.

Solution:

$$
\begin{aligned}
F &= \frac{GMm}{r^2} \\
r^2 &= \frac{GMm}{F} \\
r &= \pm\sqrt{\frac{GMm}{F}}
\end{aligned}
$$

   (b) Find the positive value of r (correct to the nearest whole number) if $G = 6.67 \times 10^{-11}$,

$M = 6.6 \times 10^{21}$, $m = 1.5 \times 10^{2}$ and $F = 1.43$.

Solution:

$$\begin{aligned} r &= \sqrt{\frac{GMm}{F}} \\ &= \sqrt{\frac{6.67 \times 10^{-11} \times 6.6 \times 10^{21} \times 1.5 \times 10^{2}}{1.43}} \\ &= \sqrt{\frac{66.033 \times 10^{12}}{1.43}} \\ &= 6\,795\,360 \text{ (nearest whole number)} \end{aligned}$$

5. Simplify the following expressions.

(a) $\left(a^2b^{-3}\right)^3 \times \dfrac{ab^{-2}}{a^3}$

Solution:

$$\begin{aligned} \left(a^2b^{-3}\right)^3 \times \frac{ab^{-2}}{a^3} &= \frac{a^6b^{-9} \times ab^{-2}}{a^3} \\ &= \frac{a^7}{a^3b^{11}} \\ &= \frac{a^4}{b^{11}} \end{aligned}$$

(b) $\left(a^3b\right)^{-2} \div \left(a^2b^{-5}\right) \times \dfrac{a^3}{b^7}$

Solution:

$$\begin{aligned} \left(a^3b\right)^{-2} \div \left(a^2b^{-5}\right) \times \frac{a^3}{b^7} &= \frac{a^{-6}b^{-2}}{a^2b^{-5}} \times \frac{a^3}{b^7} \\ &= \frac{b^3}{a^8} \times \frac{a^3}{b^7} \\ &= \frac{1}{a^5b^4} \end{aligned}$$

(c) $\left(3a^{-2}b^2\right)^3 \times \left(6a^3b^{-2}\right)^{-2}$

Solution:

$$\begin{aligned}\left(3a^{-2}b^{2}\right)^{3}\times\left(6a^{3}b^{-2}\right)^{-2} &= 27a^{-6}b^{6}\times\frac{1}{36}a^{-6}b^{4}\\ &= \frac{27a^{-12}b^{10}}{36}\\ &= \frac{3b^{10}}{4a^{12}}\end{aligned}$$

(d) $\left(5a^{-4}b^{5}\right)^{-1}\times 6\left(a^{2}b\right)^{-3}$

Solution:

$$\begin{aligned}\left(5a^{-4}b^{5}\right)^{-1}\times 6\left(a^{2}b\right)^{-3} &= \frac{a^{4}}{5b^{5}}\times\frac{6}{a^{6}b^{3}}\\ &= \frac{6}{5a^{2}b^{8}}\end{aligned}$$

(e) $\dfrac{7m^{\frac{5}{3}}n^{3}}{2p}\div\dfrac{21m^{\frac{2}{3}}n^{4}}{6p^{2}}$

Solution:

$$\begin{aligned}\frac{7m^{\frac{5}{3}}n^{3}}{2p}\div\frac{21m^{\frac{2}{3}}n^{4}}{6p^{2}} &= \frac{7m^{\frac{5}{3}}n^{3}}{2p}\times\frac{6p^{2}}{21m^{\frac{2}{3}}n^{4}}\\ &= m\times\frac{3p}{3n}\\ &= \frac{mp}{n}\end{aligned}$$

(f) $4^{x}\times 8^{x+2}\times 6^{2x-2}$

Solution:

$$\begin{aligned}4^{x}\times 8^{x+2}\times 6^{2x-2} &= 2^{2x+3x+6}\times 2^{2x-2}\times 3^{2x-2}\\ &= 2^{7x+4}\times 3^{2x-2}\end{aligned}$$

(g) $\dfrac{\sqrt{x^{-2}y^{4}}}{\left(\dfrac{x}{y}\right)^{-2}}$

Solution:

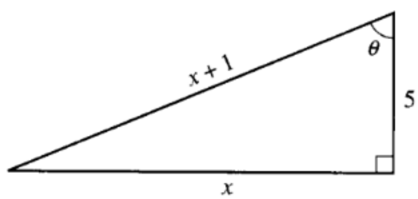
$$\frac{\sqrt{x^{-2}y^4}}{\left(\frac{x}{y}\right)^{-2}} = \frac{x^{-1}y^2}{\frac{y^2}{x^2}}$$
$$= \frac{xy^2}{y^2}$$
$$= x$$

(h) $\dfrac{(2p^4q^3)^3}{4p^2q^{15}}$

Solution:

$$\frac{(2p^4q^3)^3}{4p^2q^{15}} = \frac{2p^{12}q^9}{p^2q^{15}}$$
$$= \frac{2p^{10}}{q^6}$$

6. The figure below shows a right-angled triangle with sides x cm, 5 cm and $(x+1)$ cm respectively. Write down an equation in x, and, hence, find the value of x.



Solution:

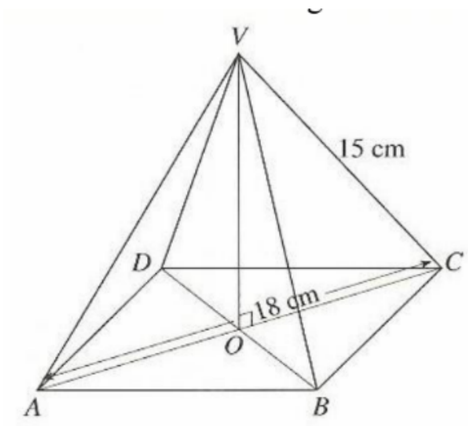
$$5^2 + x^2 = (x+1)^2$$
$$x^2 + 25 = x^2 + 2x + 1$$
$$2x = 24$$
$$x = 12$$

7. Evaluate $\dfrac{9.016\times10^3+6.292\times10^4}{5.673\times10^{-2}-\sqrt{2.490\times10^{-5}}}$ using a calculator, leaving your answer in standard form, correct to three significant figures.

Solution:

$$\begin{aligned}\frac{9.016\times10^3+6.292\times10^4}{5.673\times10^{-2}-\sqrt{2.490\times10^{-5}}} &\approx 1\,390\,336.028\\ &\approx 1.39\times10^6 \text{ (3 s.f.)}\end{aligned}$$

8. The diagonal of the square base of the right pyramid below is 18 cm and the slant edge, VC is 15 cm. Calculate



(a) the height of the pyramid,

Solution:

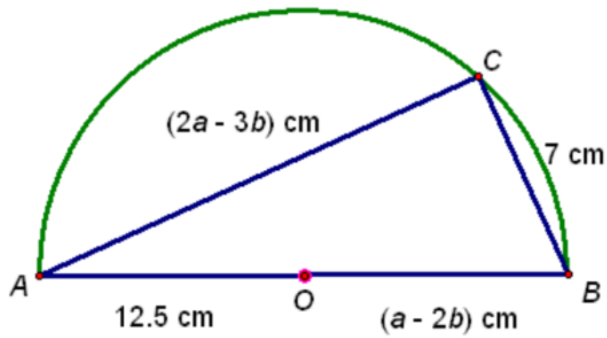
$$\begin{aligned}\text{height of pyramid} &= \sqrt{15^2-\left(\frac{18}{2}\right)^2}\\ &= 12\text{ cm}\end{aligned}$$

(b) the volume of the pyramid.

Solution:

$$\begin{aligned}\text{volume of pyramid} &= \frac{1}{3}\times\left(2\times\frac{1}{2}\times18\times\frac{18}{2}\right)\times12\\ &= 648\text{ cm}^3\end{aligned}$$

9. The diagram below shows a semicircle with O as centre and AB as diameter. C is a point on the circumference such that $BC = 7\,\text{cm}$ and $AC = (2a - 3b)$ cm. Given that $OA = 12.5\,\text{cm}$ and $OB = (a - 2b)$ cm,



(a) By considering the diameter of the circle, write down an equation involving a and b.

> **Solution:** $a - 2b = 12.5$

(b) By using the Pythagoras Theorem, form another equation involving a and b.

> **Solution:**
>
> $$\begin{aligned} 7^2 + (2a - 3b)^2 &= [2(a - 2b)]^2 \\ 49 + 4a^2 - 12ab + 9b^2 &= (2a - 4b)^2 \\ 4a^2 - 12ab + 9b^2 + 49 &= 4a^2 - 16ab + 16b^2 \\ 4ab + 49 &= 7b^2 \end{aligned}$$

(c) Find the values of a and b by solving the equations obtained in **(a)** and **(b)** simultaneously.

> **Solution:**
>
> $$a - 2b = 12.5 \quad (1)$$
> $$4ab + 49 = 7b^2 \quad (2)$$
>
> From (2):
>
> $$\begin{aligned} 4ab + 49 &= 7b^2 \\ 4ab &= 7b^2 - 49 \\ a &= \frac{7b^2 - 49}{4b} \quad (3) \end{aligned}$$

Substitute (3) into (1):

$$\frac{7b^2-49}{4b}-2b=12.5$$
$$7b^2-49-8b^2=50b$$
$$b^2+50b+49=0(b+49)(b+1) \qquad =0$$
$$\therefore b=-1 \text{ or } b=-49$$

Substitute $b=-1$ into (1):

$$a+2=12.5$$
$$\therefore a=10.5$$

Substitute $b=-49$ into (1):

$$a+98=12.5$$
$$\therefore a=-85.5 \text{ (rej.)}$$

$$\therefore \begin{cases} a & =10.5 \\ b & =-1 \end{cases}$$

10. Evaulate

(a) $27^{\frac{3}{5}} \div 27^{\frac{2}{5}} \times 27^{\frac{2}{15}}$

Solution:

$$27^{\frac{3}{5}} \div 27^{\frac{2}{5}} \times 27^{\frac{2}{15}} = 27^{\frac{3}{5}-\frac{2}{5}+\frac{1}{15}}$$
$$= 27^{\frac{4}{15}}$$

(b) $(-2)^{-3}$

Solution:

$$(-2)^{-3} = \frac{1}{(-2)^3}$$
$$= -\frac{1}{8}$$

(c) $\left(\frac{343}{64}\right)^{-\frac{2}{3}}$

Solution:

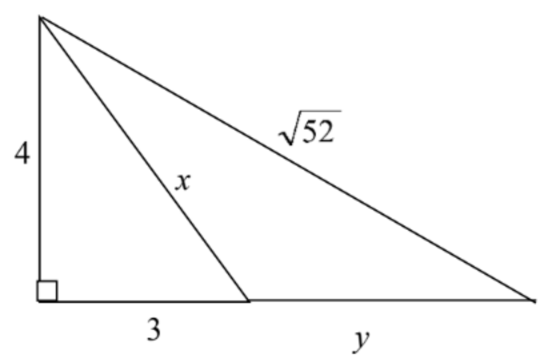
$$\begin{aligned}\left(\frac{343}{64}\right)^{-\frac{2}{3}} &= \left(\frac{64}{343}\right)^{\frac{2}{3}} \\ &= \sqrt[3]{\left(\frac{64}{343}\right)^2} \\ &= \sqrt[3]{\frac{2^{12}}{7^6}} \\ &= \frac{2^4}{7^2} \\ &= \frac{16}{49}\end{aligned}$$

(d) $\left(\frac{1}{2}\right)^{-2} \times 2^0$

Solution:

$$\begin{aligned}\left(\frac{1}{2}\right)^{-2} \times 2^0 &= 2^2 \\ &= 4\end{aligned}$$

11. For the given figure, find



(a) x,

Solution:

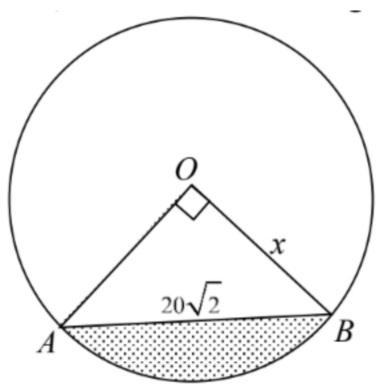
$$x = \sqrt{3^2 + 4^2}$$
$$= 5$$

(b) y.

Solution:

$$y = \sqrt{\left(\sqrt{52}\right)^2 - 4^2} - 3$$
$$= 3$$

12. A circle, centre O, radius x cm, has a chord AB of length $20\sqrt{2}$ cm. $\angle AOB = 90°$. Find



(a) the value of x,

Solution:

$$x = \sqrt{\frac{\left(20\sqrt{2}\right)^2}{2}}$$
$$= 20$$

(b) the area of the shaded region correct to the nearest 5 cm^2.

Solution:

$$\begin{aligned}\text{area of the shaded region} &= \frac{1}{4}\times\pi\times 20^2 - \frac{1}{2}\times 20^2\\ &= 114\,\text{cm}^2\\ &= 115\,\text{cm}^2\ (\text{nearest } 5\,\text{cm}^2)\end{aligned}$$