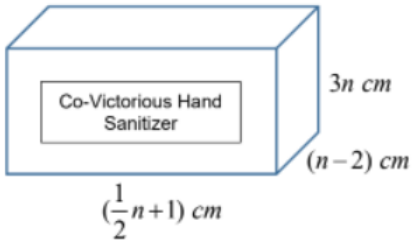
SBGE Paper C (2022)

1. A particular brand of hand sanitiser is in the shape of a cuboid as shown below: [2]



Find and simplify an expression in terms of n for the volume of the hand sanitiser.

Solution:

$$\begin{aligned}\text{volume of hand sanitiser} &= 3n\left(\frac{1}{2}n+1\right)(n-2)\\ &= \left(\frac{3}{2}n^2+3n\right)(n-2)\\ &= \frac{3}{2}n^3+3n^2-3n^2-6n\\ &= \left(\frac{3}{2}n^3-6n\right)\text{cm}^3\end{aligned}$$

2. Factorise completely:

(a) $6x^3+2x^2y-4xy^2$ [2]

Solution:

$$\begin{aligned}6x^3+2x^2y-4xy^2 &= 2x\left(3x^2+xy-2y^2\right)\\ &= 2x(3x-2y)(x+y)\end{aligned}$$

(b) $(2x+1)^2-\left(x^2+x+1\right)^2$ [3]

Solution:

$$\begin{aligned}(2x+1)^2-\left(x^2+x+1\right)^2 &= \left(2x+1+x^2+x+1\right)\left(2x+1-x^2-x-1\right)\\ &= \left(x^2+3x+2\right)\left(-x^2+x\right)\\ &= -x(x+1)(x+2)(x-1)\end{aligned}$$

3. Simplify:

(a) $\dfrac{3c}{1-c} + \dfrac{3}{c-1}$ [3]

Solution:

$$\frac{3c}{1-c} + \frac{3}{c-1} = \frac{3c-3}{1-c}$$
$$= -3$$

(b) $\dfrac{a^2-b^2}{(a-b)^2} \div \dfrac{1}{a^2+b^2} \times \dfrac{1}{(a-b)^2+2ab}$ [3]

Solution:

$$\frac{a^2-b^2}{(a-b)^2} \div \frac{1}{a^2+b^2} \times \frac{1}{(a-b)^2+2ab}$$
$$= \frac{(a+b)\cancel{(a-b)}}{(a-b)\cancel{(a-b)}} \cdot \frac{\cancel{a^2+b^2}}{1} \cdot \frac{1}{\cancel{a^2+b^2}}$$
$$= \frac{a+b}{a-b}$$

4. Make h the subject of the formula: $\sqrt{1-hp} = p$. [3]

Solution:

$$\sqrt{1-hp} = p$$
$$1-hp = p^2$$
$$hp = 1-p^2$$
$$h = \frac{1-p^2}{p}$$

5. (a) Solve the equation: $\dfrac{1}{x} + \dfrac{2}{3x} = \dfrac{3}{x+1}$. [3]

Solution:

$$\frac{1}{x} + \frac{2}{3x} = \frac{3}{x+1}$$
$$\frac{5}{3x} = \frac{3}{x+1}$$
$$5(x+1) = 9x$$
$$4x = 5$$
$$x = \frac{5}{4}$$

(b) Hence or otherwise, solve the equation $\frac{1}{x+1} + \frac{2}{3x+3} = \frac{3}{x+2}$. [1]

Solution:

$$\frac{1}{x+1} + \frac{2}{3x+3} = \frac{3}{x+2}$$
$$\frac{1}{x+1} + \frac{2}{3(x+1)} = \frac{3}{(x+1)+1}$$
$$\therefore x+1 = \frac{5}{4}$$
$$\therefore x = \frac{1}{4}$$

6. Given that $a^2 - 49 = 9951$,

(a) find the positive value of a. [1]

Solution:

$$a^2 - 49 = 9951$$
$$a = \sqrt{9951 + 49}$$
$$= 100$$

(b) Hence, find two factors of 9951 which are between 50 and 200. [3]

Solution:

$$a^2 - 49 = 9951$$
$$(a+7)(a-7) = 9951$$

When $a = 100$, the two factors of 9951 are $a + 7 = 107$ and $a - 7 = 93$.

7. By defining two variables, solve the following problem using SLEs. [4]

Two books, A and B, have a total of 500 pages. If the number of pages in Book A is 12 less than 3 times the number of pages in Book B, calculate the number of pages in each book.

Solution: Let the number of pages in Books A and B be a and b respectively.

$$a + b = 500 \quad (1)$$
$$a = 3b - 12 \quad (2)$$

Substitute (2) into (1):

$$3b - 12 + b = 500$$
$$4b = 512$$
$$b = 128$$
$$\therefore a = 372$$

The number of pages in Books A and B is 372 and 128 respectively.

8. An ice-cream maker machine can hold 60 L of ice-cream mix. It was discovered that

(a) when the temperature of the machine is set at $-10°$C, x L of ice-cream mix can be frozen per min. [1]
Write down an expression in terms of x for the time taken to freeze 60 L of ice cream mix at $-10°$C in min.

Solution: The time taken would be $\frac{60}{x}$ min.

(b) when the temperature of the machine was set at $-5°$C, $(x - 2)$ L of ice cream mix can be frozen per minute. Write down an expression in terms of x for the time taken to freeze 60 L of ice cream mix at $-5\,°$C in min. [1]

Solution: The time taken would be $\frac{60}{x-2}$ min.

(c) it takes 1.5 min longer to freeze the ice cream mix at a higher temperature. Write down an equation in x, and show that it simplifies to $x^2 - 2x - 80 = 0$. [2]

Solution:

$$\frac{60}{x-2} - \frac{60}{x} = \frac{3}{2}$$
$$\frac{120}{x(x-2)} = \frac{3}{2}$$
$$3x^2 - 6x = 240$$
$$x^2 - 2x - 80 = 0$$

(d) Solve the equation $x^2 - 2x - 80 = 0$. [2]

Solution:

$$x^2 - 2x - 80 = 0$$
$$(x+8)(x-10) = 0$$
$$\therefore x = -8 \text{ (rej.) or } x = 10$$

(e) Explain why you have to reject one of the solutions obtained in (d). [1]

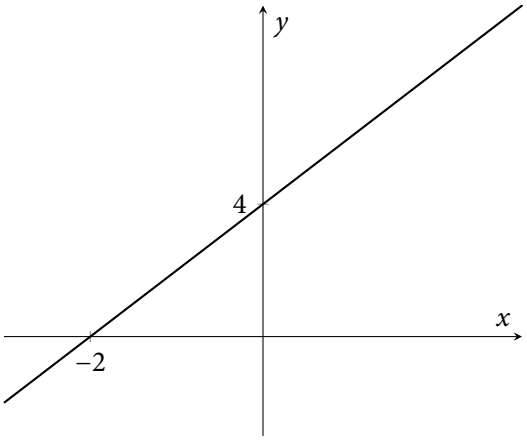
Solution: One of the solutions, $x = -8$, is negative, which cannot represent the number of litres of ice cream frozen per min.

9. The diagram below shows the line L_1, $y = ax + b$.

(a) State the values of a and b. [2]

Solution: $a = 2$, $b = 4$.

(b) Find the equation of another line, L_2, which passes through the point $(5, 9)$ and is parallel to L_1. [2]



Solution:

$$y - y_1 = m(x - x_1)$$
$$y - 9 = 2(x - 5)$$
$$y = 2x - 1$$

(c) Does the point $(3, 10)$ lie on line L_2? Explain. [1]

Solution: The point $(3, 10)$ does not lie on L_2. Substituting the values of $x = 3$ and $y = 10$ into the equation $y = 2x - 1$ makes the equation false.