

SBGE Paper B (2022)

1. The latest hand sanitiser bottle is in the shape of a cone. The circumference of the circular base is $(44x + 44)$ and the height of the cone is $\left(3x - \frac{6}{7}\right)$. Taking $\pi = \frac{22}{7}$, and the volume of the cone is $\frac{1}{3}\pi r^2 h$, expand and simplify the volume of the hand sanitiser. [3]

Solution:

$$\begin{aligned}\text{radius of cone} &= \frac{44x + 44}{2 \times \frac{22}{7}} \\ &= 7x + 7 \\ \text{volume of cone} &= \frac{1}{3} \times \frac{22}{7} \times (7x + 7)^2 \times \frac{21x - 6}{7} \\ &= \frac{1}{3} \times 22(7x + 7)(x + 1) \times \frac{21x - 6}{7} \\ &= 22(x + 1)^2 \times (7x - 2) \\ &= 22(x^2 + 2x + 1)(7x - 2) \\ &= 22[7x^3 + 14x^2 + 7x - (2x^2 + 4x + 2)] \\ &= 22(7x^3 + 12x^2 + 3x - 2) \\ &= 154x^3 + 264x^2 + 66x - 44\end{aligned}$$

2. Factorise the following **completely**:

(a) $27p^3 - 36p^2 + 12p$

[2]

Solution:

$$\begin{aligned}27p^3 - 36p^2 + 12p &= 3p(9p^2 - 12p + 4) \\ &= 3p(3p - 2)^2\end{aligned}$$

(b) $de^2 - e^2f - 4df^2 + 4f^3$

[3]

Solution:

$$\begin{aligned} de^2 - e^2f - 4df^2 + 4f^3 &= e^2(d - f) - 4f^2(d - f) \\ &= (d - f)(e^2 - 4f^2) \\ &= (d - f)(e + 2f)(e - 2f) \end{aligned}$$

3. Simplify the following algebraic expressions.

(a) $\frac{3y}{y^2 - 1} + \frac{3}{1 - y}$

[3]

Solution:

$$\begin{aligned} \frac{3y}{y^2 - 1} + \frac{3}{1 - y} &= \frac{3y - 3(y + 1)}{y^2 - 1} \\ &= -\frac{3}{y^2 - 1} \end{aligned}$$

(b) $\frac{a^2 - 3ab + 2b^2}{(a - b)^2} \div \frac{2a^2 - ab - 6b^2}{a^2 - b^2}$

[4]

Solution:

$$\begin{aligned} \frac{a^2 - 3ab + 2b^2}{(a - b)^2} \div \frac{2a^2 - ab - 6b^2}{a^2 - b^2} &= \frac{(a - b)(a - 2b)}{(a - b)^2} \cdot \frac{(a + b)(a - b)}{(a - 2b)(2a + 3b)} \\ &= \frac{a + b}{2a + 3b} \end{aligned}$$

4. Make h the subject of the formula: $\sqrt{\frac{h^3mp}{h^3 + p}} = mp$.

[4]

Solution:

$$\begin{aligned}\sqrt{\frac{h^3 mp}{h^3 + p}} &= mp \\ \frac{h^3 mp}{h^3 + p} &= m^2 p^2 \\ h^3 mp &= h^3 m^2 p^2 + m^2 p^3 \\ h^3 mp - h^3 m^2 p^2 &= m^2 p^3 \\ h^3 (mp - m^2 p^2) &= m^2 p^3 \\ h^3 &= \frac{m^2 p^3}{mp(1 - mp)} \\ h &= \sqrt[3]{\frac{mp^2}{1 - mp}}\end{aligned}$$

5. (a) Solve the equation: $\frac{3x}{x+1} - \frac{2x}{x-1} = 1$.

[2]

Solution:

$$\begin{aligned}\frac{3x}{x+1} - \frac{2x}{x-1} &= 1 \\ 3x(x-1) - 2x(x+1) &= x^2 - 1 \\ x^2 - 5x &= x^2 - 1 \\ -5x &= -1 \\ x &= \frac{1}{5}\end{aligned}$$

- (b) Hence or otherwise, solve the equation: $\frac{3x-3}{x} + \frac{2x-2}{x-2} = 1$.

[1]

Solution:

$$\begin{aligned}\frac{3x-3}{x} + \frac{2x-2}{x-2} &= 1 \\ \frac{3x-3}{(x-1)+1} + \frac{2x-2}{(x-1)-1} &= 1 \\ x-1 &= \frac{1}{5} \\ x &= \frac{6}{5}\end{aligned}$$

6. Given that $a^2 - 121 = 9879$,

(a) Find the positive value of a .

[1]

Solution:

$$a^2 - 121 = 9879$$

$$a^2 = 10000$$

$$a = 100$$

(b) Hence, find two factors of 9879 which are between 50 and 200.

[3]

Solution:

$$a^2 - 121 = 9879$$

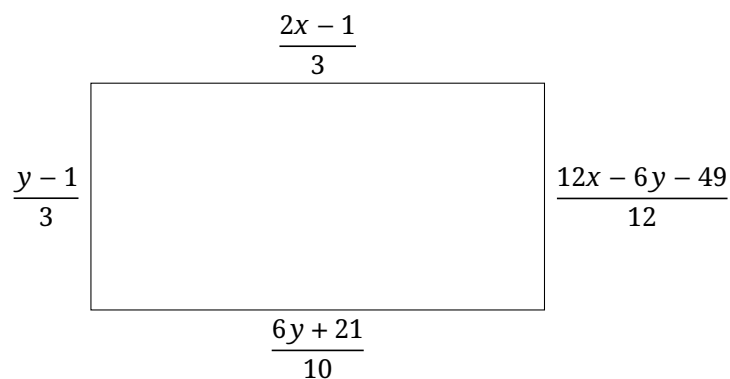
$$(a + 11)(a - 11) = 9879$$

Taking $a = 100$, the factors of 9879 are $100 + 11 = 111$ and $100 - 11 = 89$.

7. Solve the entirety of this question using Simultaneous Linear Equations.

[4]

In every school, a space is required to be set aside for students who may exhibit any symptoms of cough or cold. In one particular school, this space is in the form of a rectangle of the following dimensions (in m). Find the length of the rectangle.



Solution:

$$\frac{2x - 1}{3} = \frac{6y + 21}{10} \quad (1)$$

$$\frac{y - 1}{3} = \frac{12x - 6y - 49}{12} \quad (2)$$

Cross-multiply (1):

$$\begin{aligned} \frac{2x - 1}{3} &= \frac{6y + 21}{10} \\ 10(2x - 1) &= 3(6y + 21) \\ 20x - 10 &= 18y + 63 \\ 20x - 73 &= 18y \\ y &= \frac{20x - 73}{18} \end{aligned} \quad (3)$$

Cross-multiply (2):

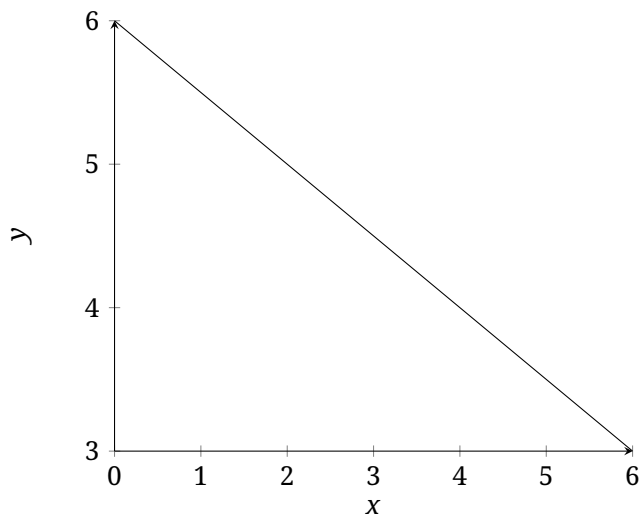
$$\begin{aligned} \frac{y - 1}{3} &= \frac{12x - 6y - 49}{12} \\ 12(y - 1) &= 3(12x - 6y - 49) \\ 4y - 4 &= 12x - 6y - 49 \\ 10y + 45 &= 12x \end{aligned} \quad (4)$$

Substitute (3) into (4):

$$\begin{aligned} 10 \left(\frac{20x - 73}{18} \right) + 45 &= 12x \\ 100x - 365 + 405 &= 108x \\ \therefore x &= \frac{405 - 365}{8} \\ &= 5 \\ \therefore y &= \frac{20x - 73}{18} \\ &= \frac{20 \times 5 - 73}{18} \\ &= \frac{3}{2} \\ \therefore \frac{2x - 1}{3} &= \frac{6 \times \frac{3}{2} + 21}{10} \\ &= 3 \text{ units} \end{aligned}$$

The length of the rectangle is **3 units**.

8. The diagram below shows the line l_1 , $y = ax + b$.



- (a) State the values of a and b .

[2]

Solution: $a = -\frac{1}{2}$, $b = 6$.

- (b) Find the equation of another line, l_2 , which is parallel to l_1 and passes through the point $(2, 3)$.

[2]

Solution: On l_2 ,

$$y_2 - y_1 = m(x_2 - x_1)$$

$$y_2 - 3 = -\frac{1}{2}(x_2 - 2)$$

$$y_2 = -\frac{1}{2}x_2 + 1 + 3$$

The equation of l_2 is $y = -\frac{1}{2}x + 4$.

9. Attempt the whole of this question on the graph paper provided.

The variables x and y are connected by the equation $y + 2 = 3x$.

- (a) Copy and complete the following table.

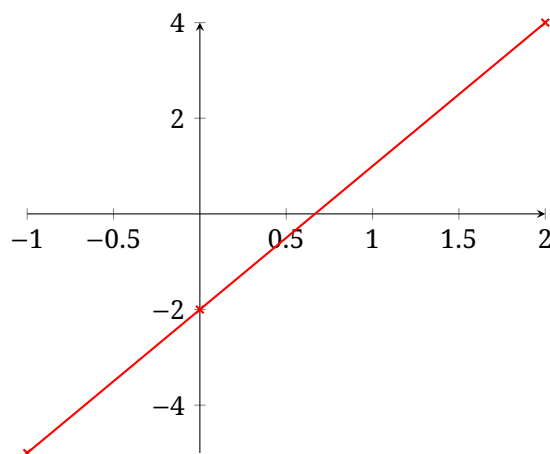
x	-1	0	2
y			

[1]

Solution:

x	-1	0	2
y	-5	-2	4

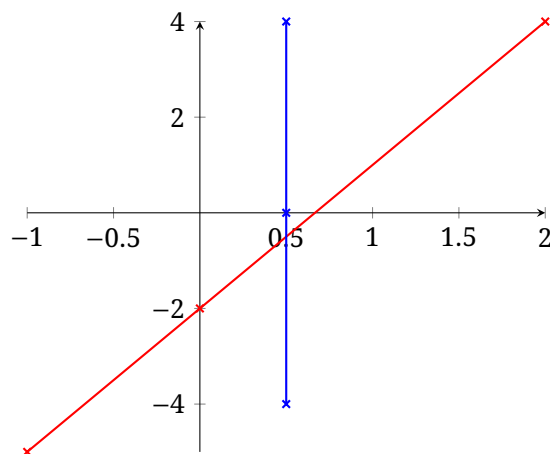
- (b) Using a scale of 4 cm to represent 1 unit on the x -axis and a scale of 2 cm to represent 1 unit on the y -axis, draw the graph of $y + 2 = 3x$ for $-1 \leq x \leq 2$. [2]



- (c) **Read from your graph** the value of x when $y = 2.3$. [1]

Solution: 1.4 (from graph). [Actual: $1\frac{13}{30}$]

- (d) On the same axes as in (b), draw the graph of $x = 0.5$. [1]



- (e) Given the graphs you have drawn in (b) and (d), explain how to find the solution to the simultaneous equations $y + 2 = 3x$ and $x = 0.5$. [1]

Solution: Find the coordinates of the point of intersection between the two lines. The x - and y -coordinates of the point will correspond with the solutions to x and y in the simultaneous equations.