Theoretical Assignment DeepBayes Summer School 2018 (deepbayes.ru)

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1 Problem 1

By definition

$$P(\xi = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$
$$P(\eta = s|\xi = k) = \binom{k}{s} p^s(1-p)^{k-s}$$

Applying the law of total probability we have

$$P(\eta = s) = \sum_{k=s}^{\infty} P(\eta = s|\xi = k)P(\xi = k) = \sum_{k=s}^{\infty} \frac{k!}{s!(k-s)!} p^s(1-p)^{k-s} \frac{e^{-\lambda}\lambda^k}{k!}$$
$$= \frac{e^{-\lambda}\lambda^s p^s}{s!} \cdot \sum_{k=s}^{\infty} \frac{(\lambda(1-p))^{k-s}}{(k-s)!}$$

The infinite sum is a Maclaurin series of exponential function $e^{\lambda(1-p)}$, so after substitution we get:

$$\frac{e^{-\lambda}\lambda^s p^s}{s!} \cdot \sum_{k=s}^{\infty} \frac{(\lambda(1-p))^{k-s}}{(k-s)!} = \frac{e^{-\lambda}\lambda^s p^s}{s!} \cdot e^{\lambda(1-p)} = \frac{e^{-\lambda p}(\lambda p)^s}{s!} = \frac{e^{-(\lambda p)}(\lambda p)^s}{s!}$$

Resulting probability distribution is Poisson with parameter λp

2 Problem 2

Lets denote i as an index of reviewer: $i = 1$ for a strict reviewer and $i = 2$ for a kind one. By the Bayes rule and the definition of normal distribution:

$$
\begin{aligned}
P(i=2|t_i=10) &= \frac{P(t_i=10|i=2)\cdot P(i=2)}{P(t_i=10|i=1)\cdot P(i=1)+P(t_i=10|i=2)\cdot P(i=2)} \\
&= \frac{\frac{1}{5\sqrt{2\pi}}\cdot e^{\frac{-(10-20)^2}{2\cdot 5^2}}}{\frac{1}{5\sqrt{2\pi}}\cdot e^{\frac{-(10-20)^2}{2\cdot 5^2}}+\frac{1}{10\sqrt{2\pi}}\cdot e^{\frac{-(10-30)^2}{2\cdot 10^2}}} = \frac{e^{-2}}{e^{-2}+\frac{1}{2}e^{-2}} = \frac{2}{3}
\end{aligned}
$$