Report of Entropy estimates based on NIST SP 800-90B non-IID track

 $2025\text{-}\mathrm{Feb}\text{-}16\ 11:}10:36.204032$

1 Identification information

1.1 Identification of acquisition data from entropy source

Table 1 $\,$ Identification information of acquisition data from entropy source

| URL of the acquisition data | https://github.com/usnistgov/SP800-90B_EntropyAssessment/blob/master/bin/truerand_4bit.bin |
|--|--|
| SHA-256 hash value of the acqui- sition data [hex] | 489bc841 bb364ba8 6da70b16 17138aef 76b25dd9 196ad669 eef40c14 41b6cb88 |

- $\bullet\,$ Name of the submitter of the acquisition data :
- Brief explanation of the acquisition data (or entropy source) :

1.2 Identification of analysis environment

Table 2 Identification information of analysis environment

| Analysis tool | Name | Another entropy estimation tool with extensions |
|----------------------|------------------------|---|
| | Versioning information | 1.0.60 |
| | built as | 64-bit application |
| | built by | Intel C++ Compiler (INTEL_LLVM_COMPILER: 20250004) |
| | linked libraries | Boost C++ 1.87.0 |
| Analysis environment | Hostname | |
| | CPU information | AMD Ryzen |
| | Physical memory size | MiB |
| | OS name | Microsoft Windows 11 Pro |
| | OS version | 10.0.22631 N/A Build 22631 |
| | System type | 64-bit |
| | Username | |

1.3 Identification of analysis conditions

Table 3 $\,$ Identification information of analysis conditions

| Number of samples | 1000000 |
|------------------------|----------------------------------|
| Bits per sample | 4 |
| Byte to bit conversion | Most Significant bit (MSb) first |

1.4 Identification of analysis method

NIST SP 800-90B [1] 6.3 with corrections [2] is applied

2 Executive summary

2.1 Numerical results of min-entropy estimates based on non-IID track

Table 4 Numerical results

| Estimator | $H_{ m original}{}^{ m a}$ | Notes to H_{original} | $H_{ m bitstring}^{ m \ b}$ | Notes to $H_{\text{bitstring}}$ |
|--|----------------------------|--------------------------------|-----------------------------|---------------------------------|
| | [bit / 4 - bit] | | [bit / 1 - bit] | |
| The Most Common Value Estimate | 3.97119 | see 3.1 | 0.99773 | see 4.1 |
| The Collision Estimate | _ | _ | 0.928362 | see 4.2 |
| The Markov Estimate | _ | _ | 0.99947 | see 4.3 |
| The Compression Estimate | _ | _ | 0.900627 | see 4.4 |
| The t-Tuple Estimate | 3.68775 | see 3.2 | 0.929434 | see 4.5 |
| The Longest Repeated Substring (LRS) Estimate | 3.93497 | see 3.3 | 0.986687 | see 4.6 |
| Multi Most Common in Window Prediction Estimate | 3.99229 | see 3.4 | 0.99808 | see 4.7 |
| The Lag Prediction Estimate | 3.97627 | see 3.5 | 0.998649 | see 4.8 |
| The MultiMMC Prediction Estimate | 3.98526 | see 3.6 | 0.998205 | see 4.9 |
| The LZ78Y Prediction Estimate | 3.98428 | see 3.7 | 0.999355 | see 4.10 |
| The intial entropy source estimate [bit / 4 - bit] | | 3.6 | 0251 | |
| $H_I = \min(H_{\text{original}}, 4 \times H_{\text{bitstring}})$ | | | | |

 $[^]a$ $\,$ Entropy estimate of the sequential dataset [source: NIST SP 800-90B [1] 3.1.3]

 $[^]b$ An additional entropy estimation (per bit) for the non-binary sequential dataset [see NIST SP 800-90B [1] 3.1.3]

2.2 Visual comparison of min-entropy estimates from original samples

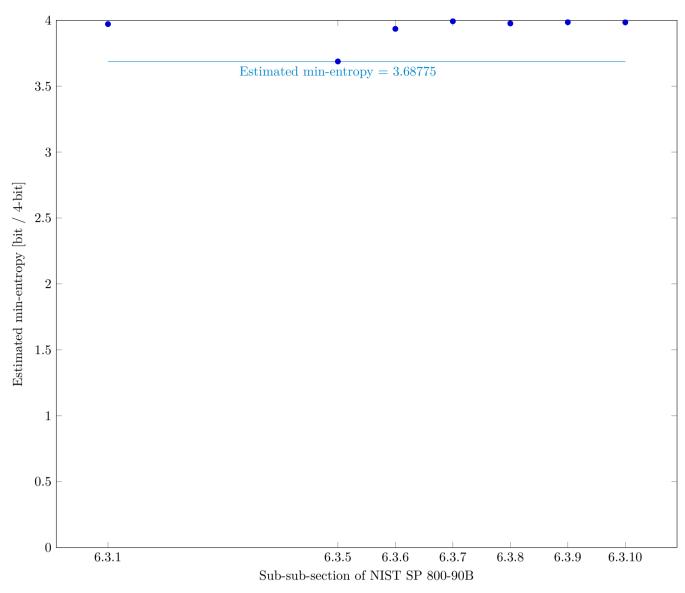


Fig. 1 Estimated Min-Entropy using $\S 6.3$ of NIST SP 800-90B

2.3 Visual comparison of min-entropy estimates by interpreting each sample as bitstring

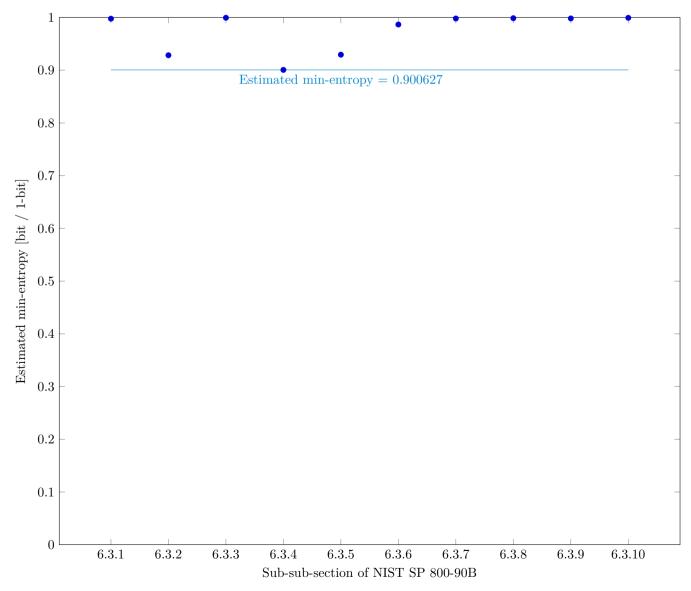


Fig. 2 Estimated Min-Entropy using $\S 6.3$ of NIST SP 800-90B

3 Detailed results of analysis from original samples

3.1 The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)

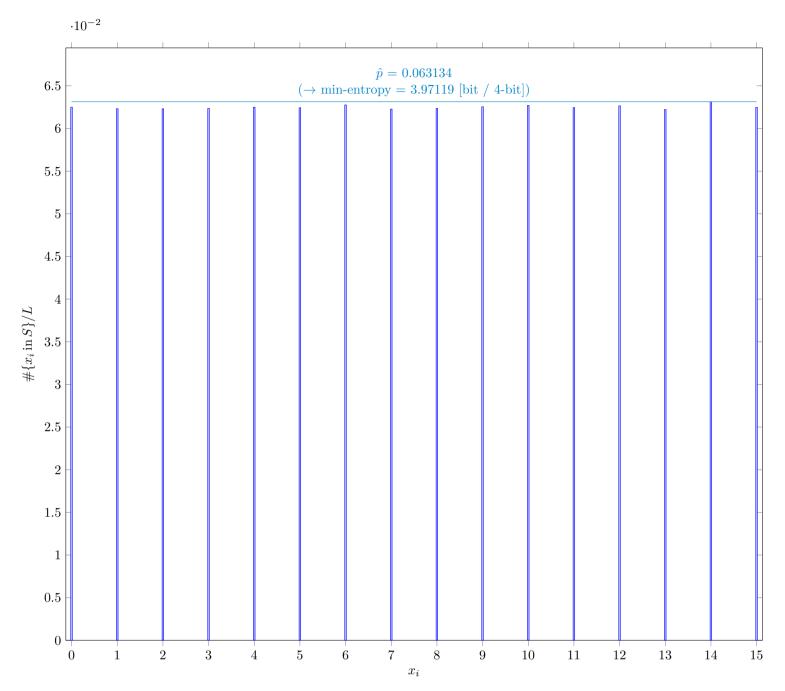


Fig. 3 Distribution of x_i

3.1.1 Supplemental information for traceability

Table 5 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol | Value |
|-----------|-----------|
| mode | 63134 |
| \hat{p} | 0.063134 |
| p_u | 0.0637605 |

3.2 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)



Fig. 4 Intermediate value Q[i] in $\S 6.3.5$ of NIST SP 800-90B

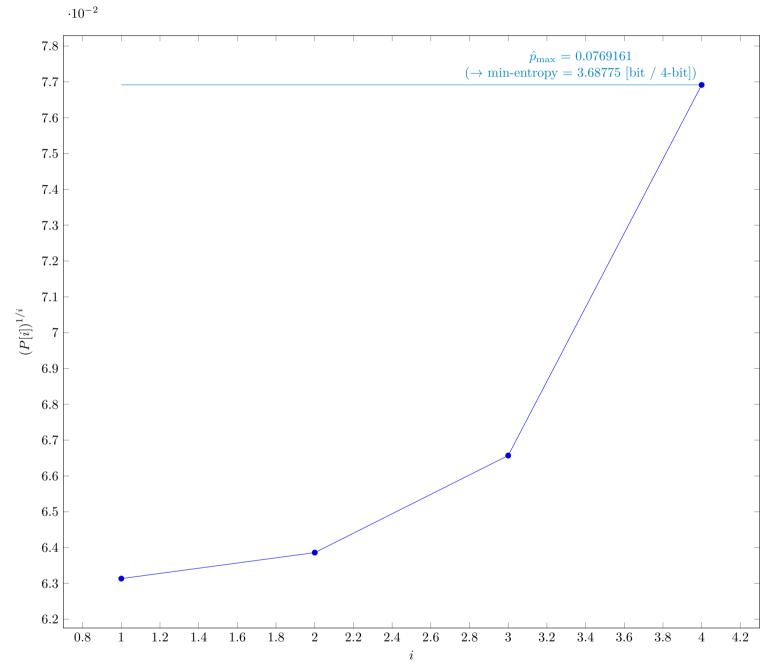


Fig. 5 $P[i]^{1/i}$ in §6.3.5 of NIST SP 800-90B

3.2.1 Supplemental information for traceability

Table 6 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol | Value |
|--------------------------|-----------|
| t | 4 |
| \hat{p}_{max} | 0.0769161 |
| p_u | 0.0776025 |

3.3 The LRS Estimate (NIST SP 800-90B Section 6.3.6)

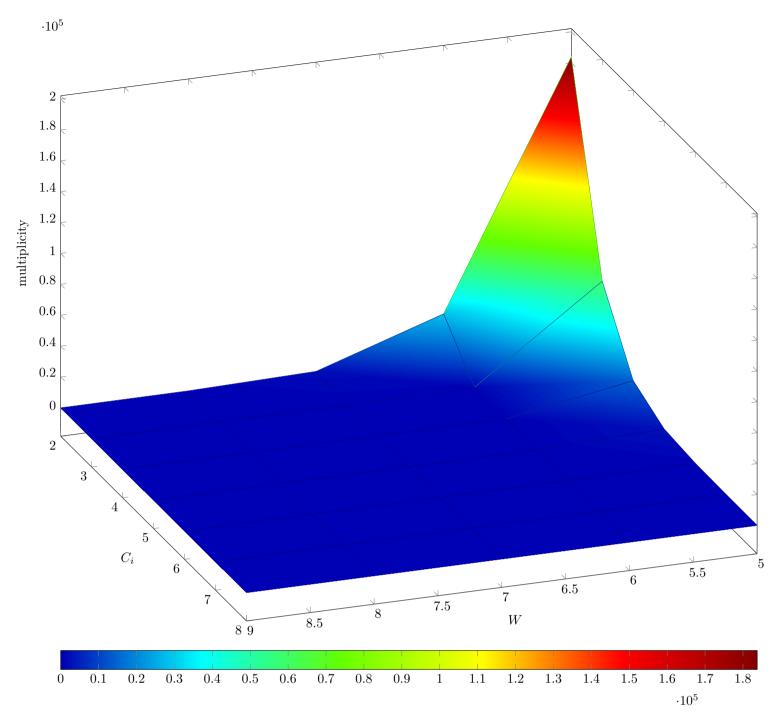
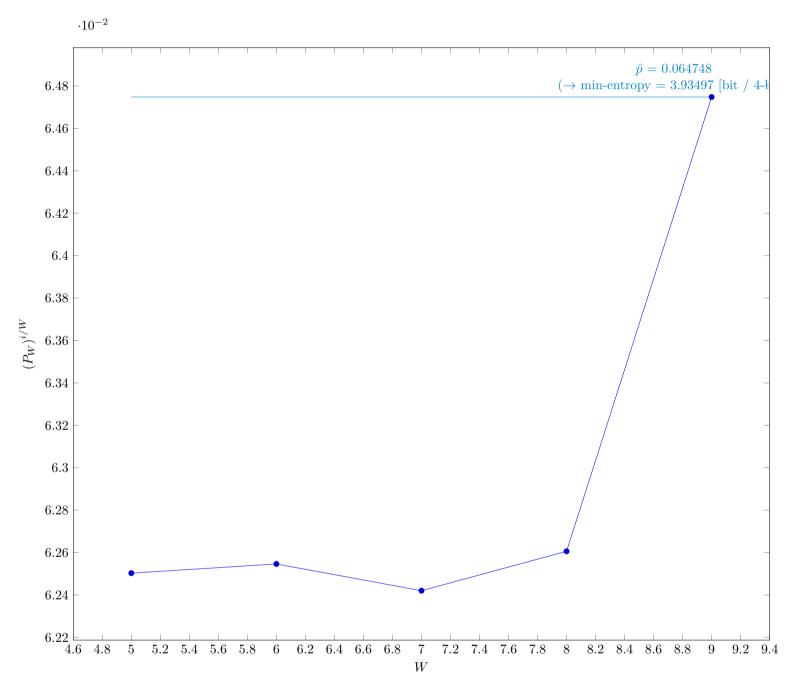


Fig. 6 Estimated W-tuple collision probability in Step 3 of $\S 6.3.6$ of NIST SP 800-90B



 $Fig.~7 \quad Estimated~average~collision~probability~per~string~symbol~in~Step~3~of~\S 6.3.6~of~NIST~SP~800-90B \\$

3.3.1 Supplemental information for traceability

Table 7 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol | Value |
|-----------|-----------|
| u | 5 |
| v | 9 |
| \hat{p} | 0.064748 |
| p_u | 0.0653819 |

3.4 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

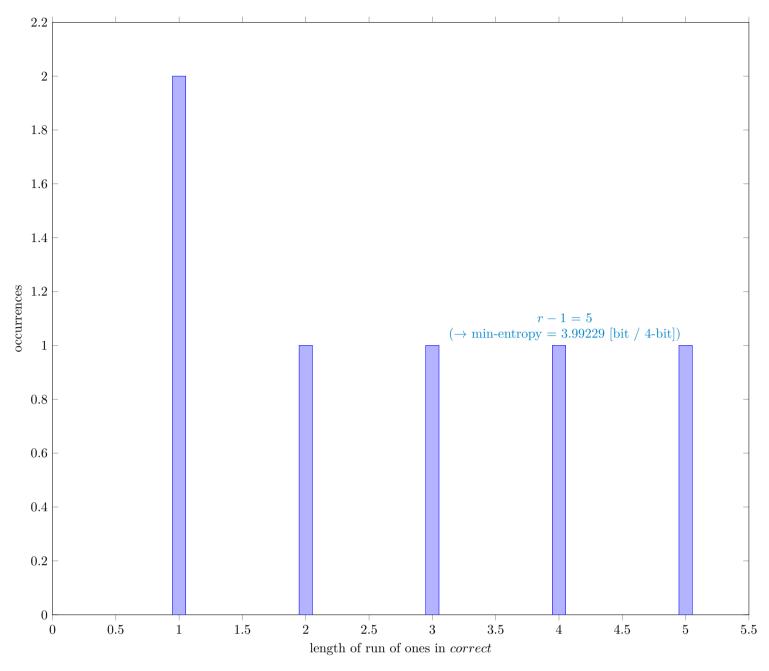


Fig. 8 Distribution of correct

3.4.1 Supplemental information for traceability

Table 8 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol | Value |
|------------------|-----------|
| N | 999937 |
| C | 62209 |
| $P_{ m global}$ | 0.0622129 |
| $P'_{ m global}$ | 0.0628351 |
| r | 6 |
| $P_{ m local}$ | 0.0468281 |

3.5 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

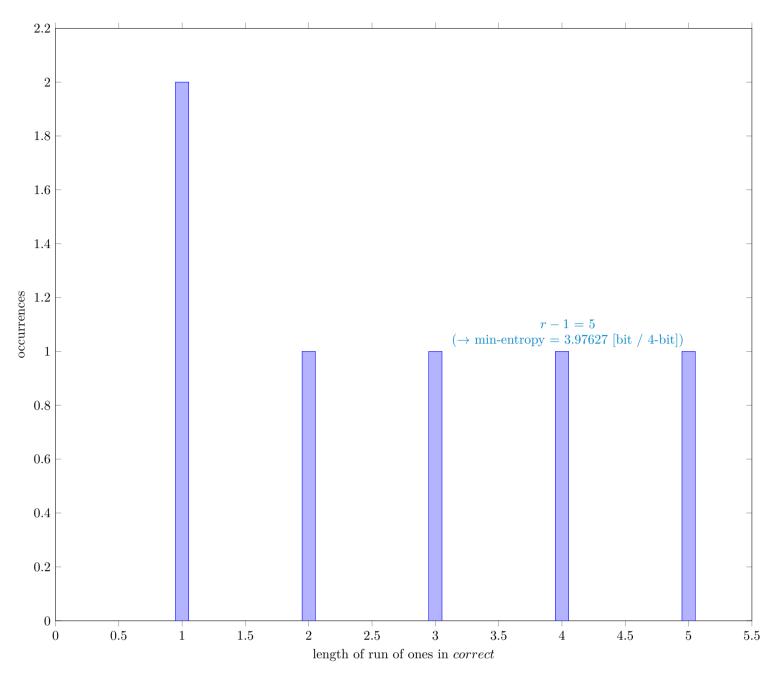


Fig. 9 Distribution of correct

3.5.1 Supplemental information for traceability

Table 9 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol | Value |
|------------------|-----------|
| N | 999999 |
| C | 62911 |
| $P_{ m global}$ | 0.0629111 |
| $P'_{ m global}$ | 0.0635365 |
| r | 6 |
| $P_{ m local}$ | 0.0468276 |

3.6 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

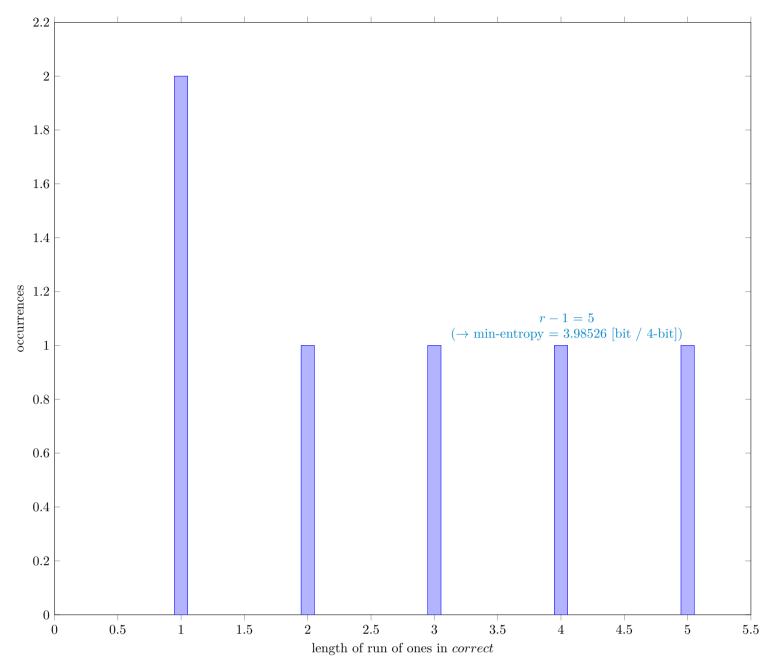


Fig. 10 $\,$ Distribution of correct

3.6.1 Supplemental information for traceability

Table 10 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol | Value |
|-----------------------|-----------|
| N | 999998 |
| C | 62518 |
| P_{global} | 0.0625181 |
| $P'_{ m global}$ | 0.0631417 |
| r | 6 |
| P_{local} | 0.0468276 |

3.7 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

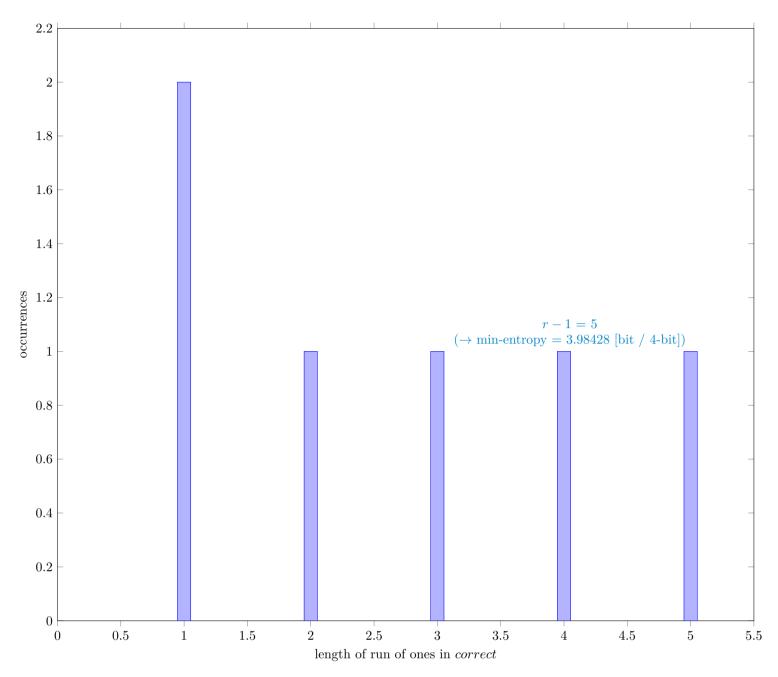


Fig. 11 Distribution of correct

3.7.1 Supplemental information for traceability

Table 11 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol | Value |
|------------------|-----------|
| N | 999983 |
| C | 62560 |
| $P_{ m global}$ | 0.0625611 |
| $P'_{ m global}$ | 0.0631849 |
| r | 6 |
| $P_{ m local}$ | 0.0468277 |

4 Detailed results of analysis by interpreting each sample as bitstrings

4.1 The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)



Fig. 12 Distribution of x_i

4.1.1 Supplemental information for traceability

Table 12 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol | Value |
|-----------|----------|
| mode | 2000573 |
| \hat{p} | 0.500143 |
| p_u | 0.500787 |

4.2 The Collision Estimate (NIST SP 800-90B Section 6.3.2)

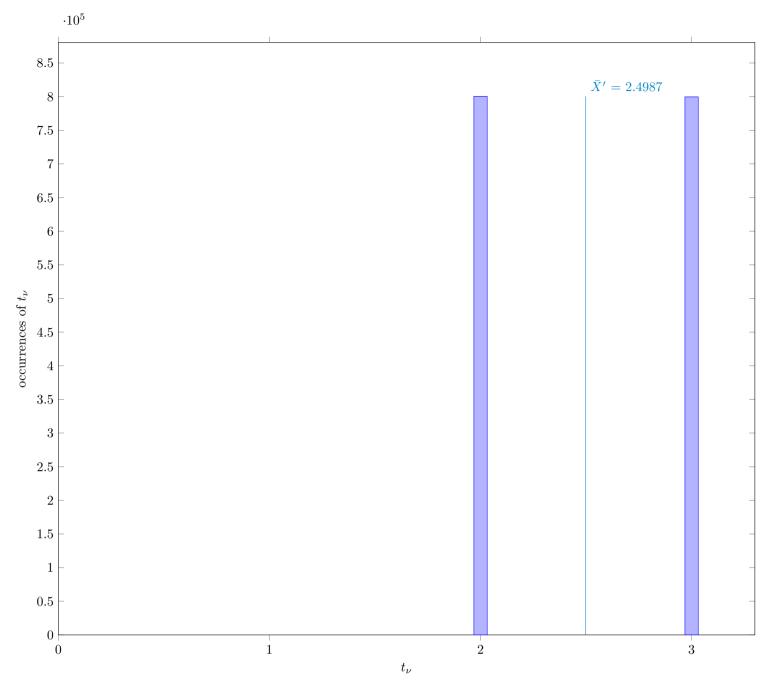


Fig. 13 Distribution of intermediate value t_{ν}



Fig. 14 Solution to the equation in step 7

4.2.1 Supplemental information for traceability

Table 13 Supplemental information for traceability (NIST SP 800-90B Section 6.3.2)

| Symbol | Value |
|----------------|----------|
| p | 0.525455 |
| \bar{X} | 2.49972 |
| $ar{X}'$ | 2.4987 |
| $\hat{\sigma}$ | 0.5 |
| | |

4.3 The Markov Estimate (NIST SP 800-90B Section 6.3.3)

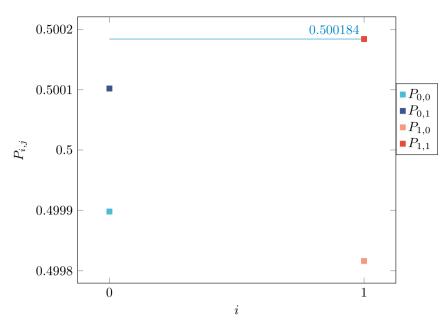


Fig. 15 Transition probability $P_{i,j}$ of §6.3.3 of NIST SP 800-90B

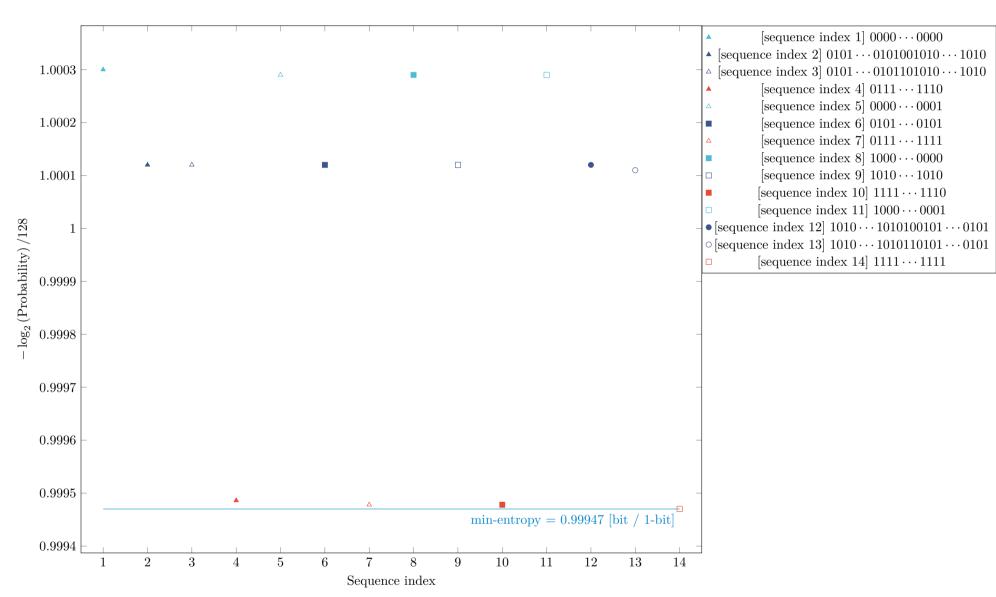


Fig. 16 Estimated Min-Entropy using $\S 6.3.3$ of NIST SP 800-90B

4.4 The Compression Estimate (NIST SP 800-90B Section 6.3.4)

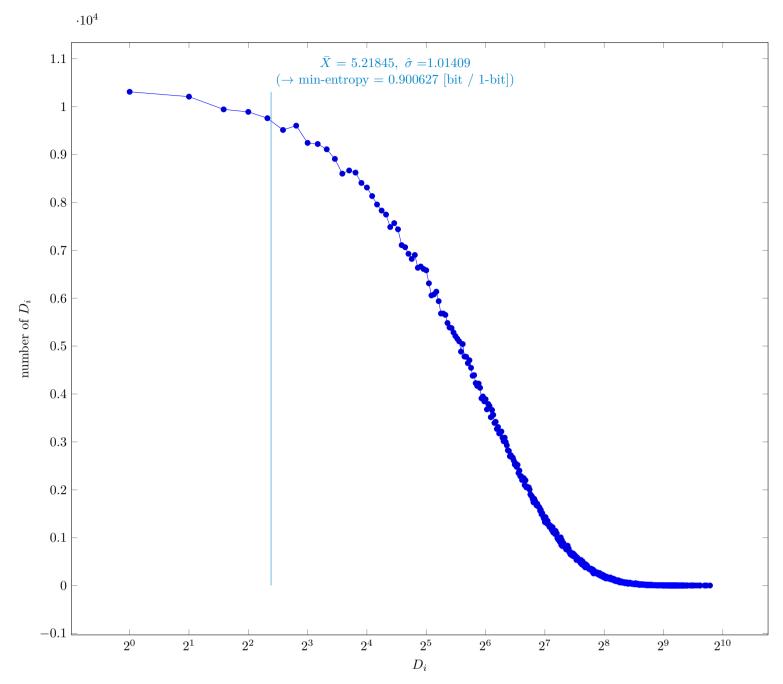


Fig. 17 Distribution of intermediate value D_i

4.4.1 Supplemental information for traceability

Table 14 Supplemental information for traceability (NIST SP 800-90B Section 6.3.4)

| Symbol | Value |
|----------------|-----------|
| p | 0.0236214 |
| \bar{X} | 5.21845 |
| $\hat{\sigma}$ | 1.01409 |
| \bar{X}' | 5.21525 |

4.5 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)



Fig. 18 Intermediate value Q[i] in $\S 6.3.5$ of NIST SP 800-90B



Fig. 19 $P[i]^{1/i}$ in §6.3.5 of NIST SP 800-90B

4.5.1 Supplemental information for traceability

Table 15 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol | Value |
|--------------------------|----------|
| t | 18 |
| \hat{p}_{max} | 0.524421 |
| p_u | 0.525064 |
| | |

4.6 The LRS Estimate (NIST SP 800-90B Section 6.3.6)

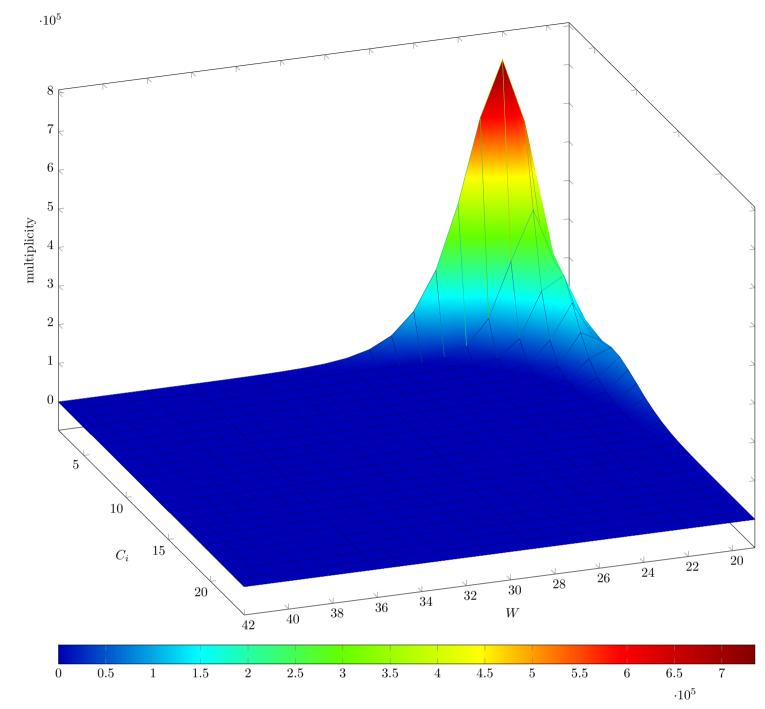


Fig. 20 Estimated W-tuple collision probability in Step 3 of $\S 6.3.6$ of NIST SP 800-90B

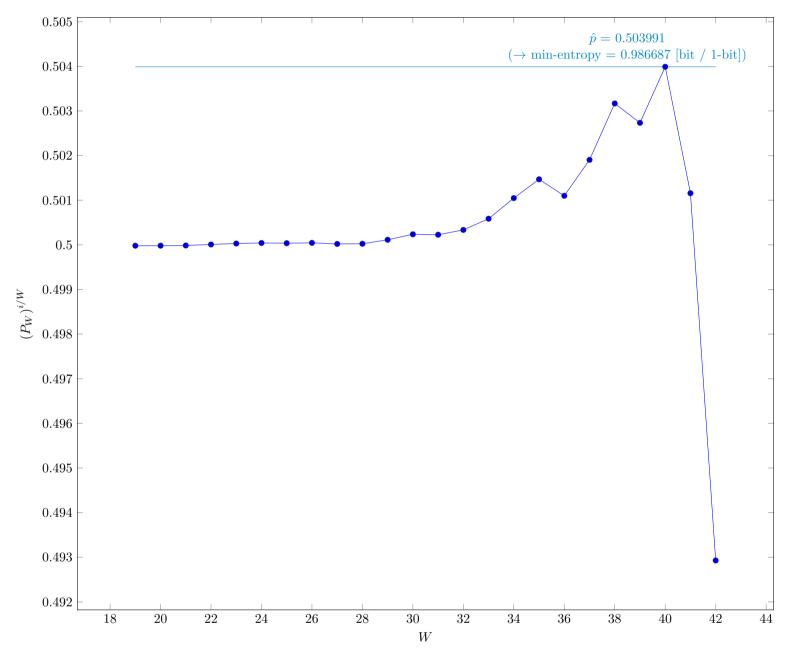


Fig. 21 Estimated average collision probability per string symbol in Step 3 of $\S 6.3.6$ of NIST SP 800-90B

4.6.1 Supplemental information for traceability

Table 16 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol | Value |
|-----------|----------|
| u | 19 |
| v | 42 |
| \hat{p} | 0.503991 |
| p_u | 0.504635 |

4.7 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

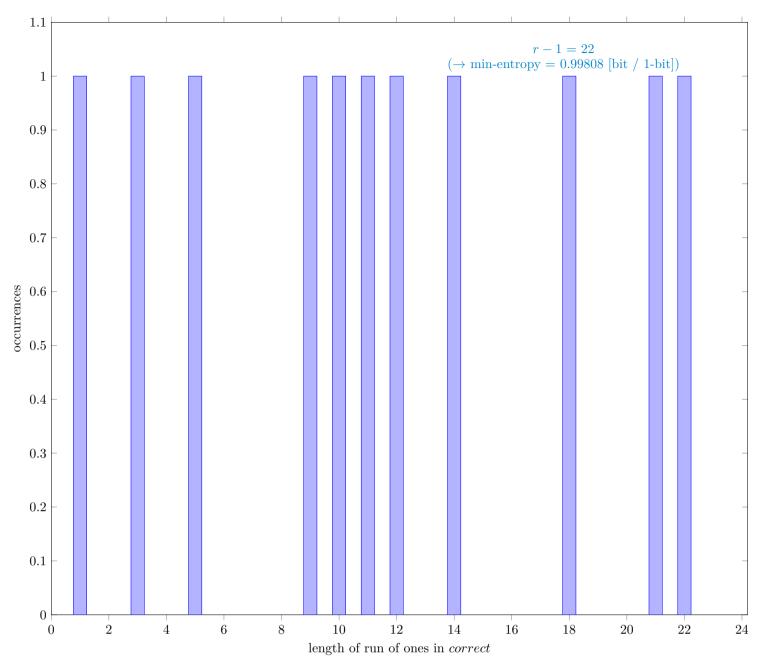


Fig. 22 Distribution of correct

4.7.1 Supplemental information for traceability

Table 17 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol | Value |
|------------------|----------|
| N | 3999937 |
| C | 2000056 |
| $P_{ m global}$ | 0.500022 |
| $P'_{ m global}$ | 0.500666 |
| r | 23 |
| $P_{ m local}$ | 0.433329 |

4.8 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

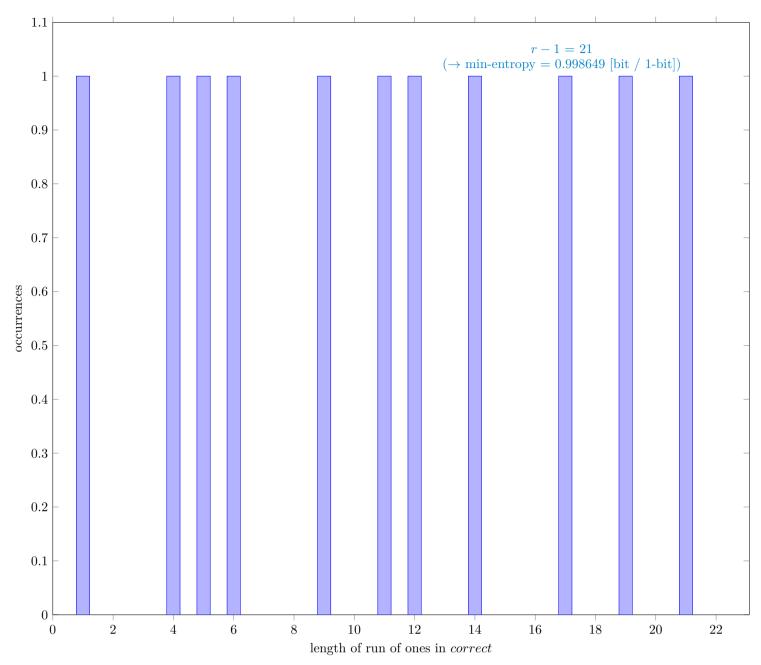


Fig. 23 Distribution of correct

4.8.1 Supplemental information for traceability

Table 18 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol | Value |
|-----------------------|----------|
| N | 3999999 |
| C | 1999298 |
| P_{global} | 0.499825 |
| $P'_{ m global}$ | 0.500469 |
| r | 22 |
| P_{local} | 0.416615 |

4.9 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

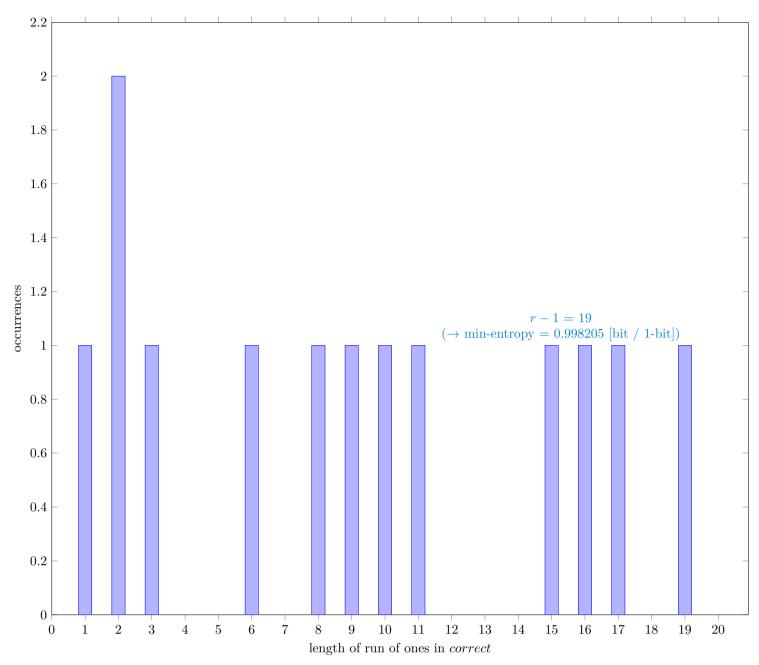


Fig. 24 Distribution of correct

4.9.1 Supplemental information for traceability

Table 19 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol | Value |
|-----------------------|----------|
| N | 3999998 |
| C | 1999913 |
| P_{global} | 0.499978 |
| $P'_{ m global}$ | 0.500622 |
| r | 20 |
| P_{local} | 0.380545 |

4.10 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

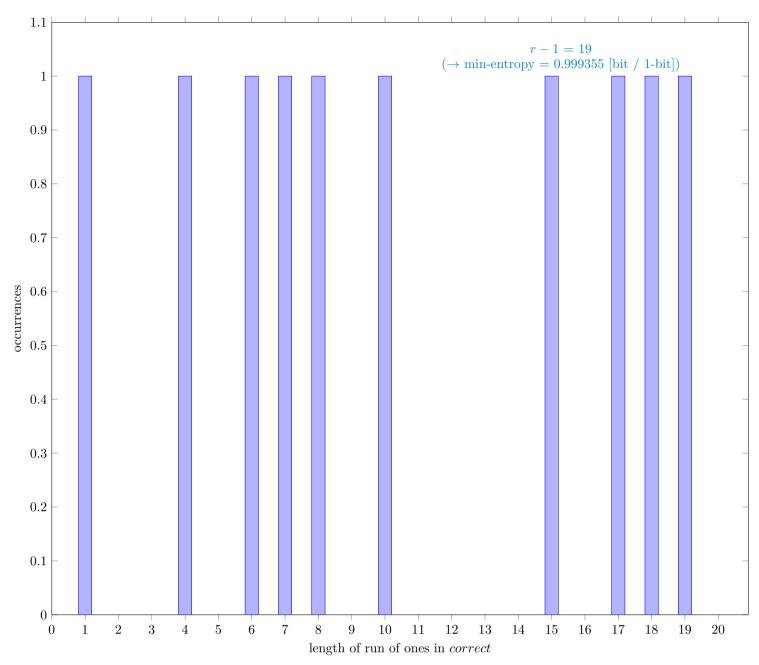


Fig. 25 Distribution of correct

4.10.1 Supplemental information for traceability

Table 20 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol | Value |
|--------------------|----------|
| N | 3999983 |
| C | 1998310 |
| $P_{ m global}$ | 0.49958 |
| $P'_{ m global}$ | 0.500224 |
| r | 20 |
| P_{local} | 0.380545 |

4 References

^[1] Meltem Sönmez Turan, Elaine Barker, John Kelsey, Kerry A. McKay, Mary L. Baish, Mike Boyle, Recommendation for the Entropy Sources Used for Random Bit Generation, NIST Special Publication 800-90B, Jan. 2018 https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-90B.pdf

^[2] G. Sakurai, Proposed list of corrections for NIST SP 800-90B 6.3 Estimators, Dec. 2022 https://github.com/g-g-sakura/AnotherEntropyEstimationTool/blob/main/documentation/ProposedListOfCorrections_SP800-90B.pdf