# Report of Entropy estimates based on NIST SP 800-90B non-IID track

 $2025\text{-}\mathrm{Feb}\text{-}16\ 10\text{:}41\text{:}52.521439$ 

# 1 Identification information

### 1.1 Identification of acquisition data from entropy source

Table 1 Identification information of acquisition data from entropy source

| URL of the acquisition data                        | https://github.com/usnistgov/SP800-90B_EntropyAssessment/blob/master/bin/rand4_short.bin |
|----------------------------------------------------|------------------------------------------------------------------------------------------|
| SHA-256 hash value of the acqui- sition data [hex] | a9e2169c b1accc78 cd23892d 793a232b 84b0cd13 ccc39235 26e0b207 62bd77ac                  |

- $\bullet\,$  Name of the submitter of the acquisition data :
- Brief explanation of the acquisition data (or entropy source) :

### 1.2 Identification of analysis environment

Table 2 Identification information of analysis environment

| Analysis tool        | Name                   | Another entropy estimation tool with extensions     |
|----------------------|------------------------|-----------------------------------------------------|
|                      | Versioning information | 1.0.60                                              |
|                      | built as               | 64-bit application                                  |
|                      | built by               | Intel C++ Compiler (INTEL_LLVM_COMPILER: 20250004 ) |
|                      | linked libraries       | Boost C++ 1.87.0                                    |
| Analysis environment | Hostname               |                                                     |
|                      | CPU information        | AMD Ryzen                                           |
|                      | Physical memory size   | MiB                                                 |
|                      | OS name                | Microsoft Windows 11 Pro                            |
|                      | OS version             | 10.0.22631 N/A Build 22631                          |
|                      | System type            | 64-bit                                              |
|                      | Username               |                                                     |

### 1.3 Identification of analysis conditions

Table 3 Identification information of analysis conditions

| Number of samples      | 10000                            |
|------------------------|----------------------------------|
| Bits per sample        | 4                                |
| Byte to bit conversion | Most Significant bit (MSb) first |

# 1.4 Identification of analysis method

NIST SP 800-90B [1] 6.3 with corrections [2] is applied

# 2 Executive summary

### 2.1 Numerical results of min-entropy estimates based on non-IID track

Table 4 Numerical results

| Estimator                                                        | $H_{ m original}{}^{ m a}$ | Notes to $H_{\text{original}}$ | $H_{ m bitstring}^{ m \ b}$ | Notes to $H_{\text{bitstring}}$ |
|------------------------------------------------------------------|----------------------------|--------------------------------|-----------------------------|---------------------------------|
|                                                                  | [bit / 4 - bit]            |                                | [bit / 1 - bit]             |                                 |
| The Most Common Value Estimate                                   | 3.79004                    | see 3.1                        | 0.979189                    | see 4.1                         |
| The Collision Estimate                                           | _                          | _                              | 0.89818                     | see 4.2                         |
| The Markov Estimate                                              | _                          | _                              | 0.990617                    | see 4.3                         |
| The Compression Estimate                                         | _                          | _                              | 0.803872                    | see 4.4                         |
| The t-Tuple Estimate                                             | 3.56747                    | see 3.2                        | 0.898777                    | see 4.5                         |
| The Longest Repeated Substring (LRS) Estimate                    | 3.83353                    | see 3.3                        | 0.932969                    | see 4.6                         |
| Multi Most Common in Window Prediction Estimate                  | 3.86695                    | see 3.4                        | 0.986561                    | see 4.7                         |
| The Lag Prediction Estimate                                      | 3.78365                    | see $3.5$                      | 0.982642                    | see 4.8                         |
| The MultiMMC Prediction Estimate                                 | 3.88466                    | see 3.6                        | 0.977697                    | see 4.9                         |
| The LZ78Y Prediction Estimate                                    | 3.8825                     | see $3.7$                      | 0.980145                    | see 4.10                        |
| The intial entropy source estimate [bit / 4 - bit]               |                            | 3.2                            | 1549                        |                                 |
| $H_I = \min(H_{\text{original}}, 4 \times H_{\text{bitstring}})$ |                            |                                |                             |                                 |

 $<sup>^</sup>a$   $\,$  Entropy estimate of the sequential dataset [source: NIST SP 800-90B [1] 3.1.3]

 $<sup>^</sup>b$  An additional entropy estimation (per bit) for the non-binary sequential dataset [see NIST SP 800-90B [1] 3.1.3]

### 2.2 Visual comparison of min-entropy estimates from original samples

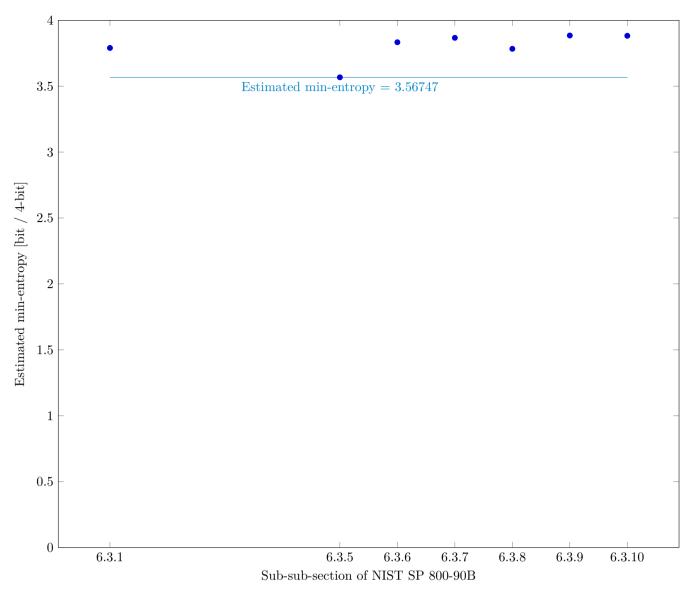


Fig. 1 Estimated Min-Entropy using  $\S 6.3$  of NIST SP 800-90B

### 2.3 Visual comparison of min-entropy estimates by interpreting each sample as bitstring



Fig. 2 Estimated Min-Entropy using  $\S 6.3$  of NIST SP 800-90B

# 3 Detailed results of analysis from original samples

### 3.1 The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)

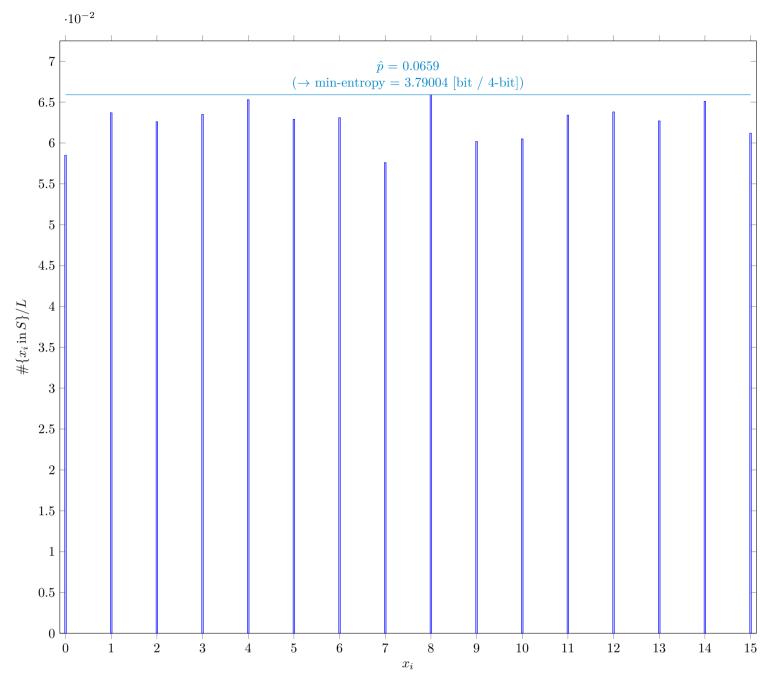


Fig. 3 Distribution of  $x_i$ 

### 3.1.1 Supplemental information for traceability

Table 5 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol    | Value     |
|-----------|-----------|
| mode      | 659       |
| $\hat{p}$ | 0.0659    |
| $p_u$     | 0.0722911 |

# 3.2 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)

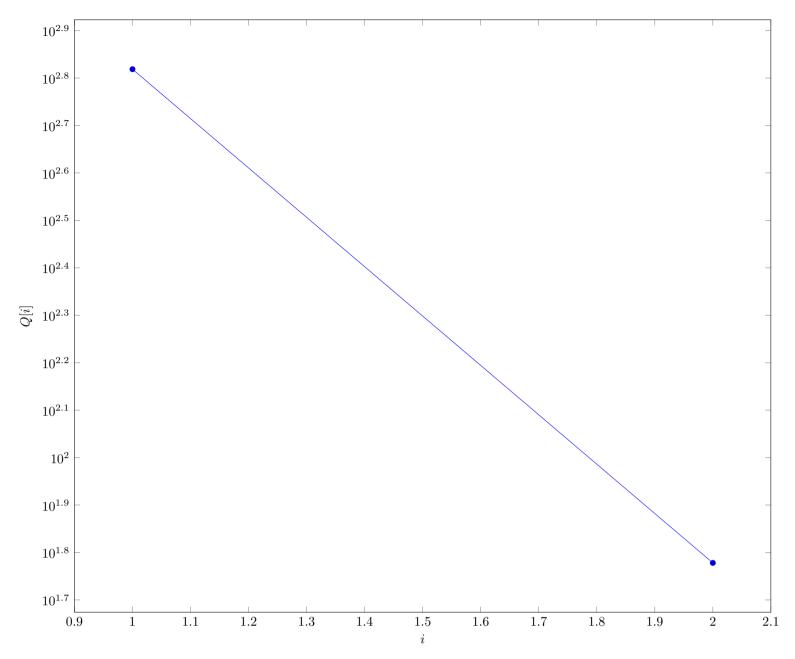


Fig. 4 Intermediate value Q[i] in  $\S 6.3.5$  of NIST SP 800-90B

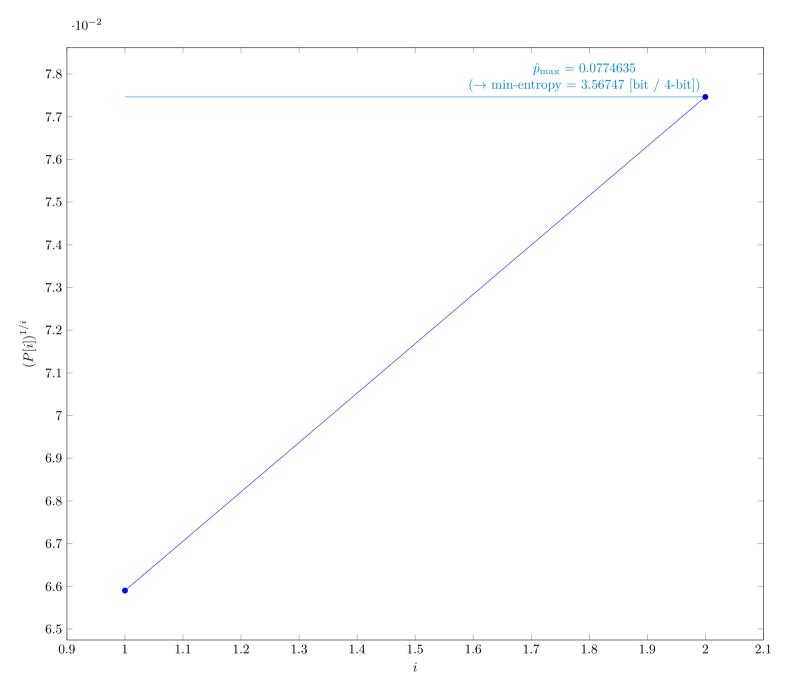


Fig. 5  $P[i]^{1/i}$  in  $\S 6.3.5$  of NIST SP 800-90B

### 3.2.1 Supplemental information for traceability

Table 6 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol                   | Value     |
|--------------------------|-----------|
| t                        | 2         |
| $\hat{p}_{\mathrm{max}}$ | 0.0774635 |
| $p_u$                    | 0.0843497 |

# 3.3 The LRS Estimate (NIST SP 800-90B Section 6.3.6)



Fig. 6 Estimated W-tuple collision probability in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B

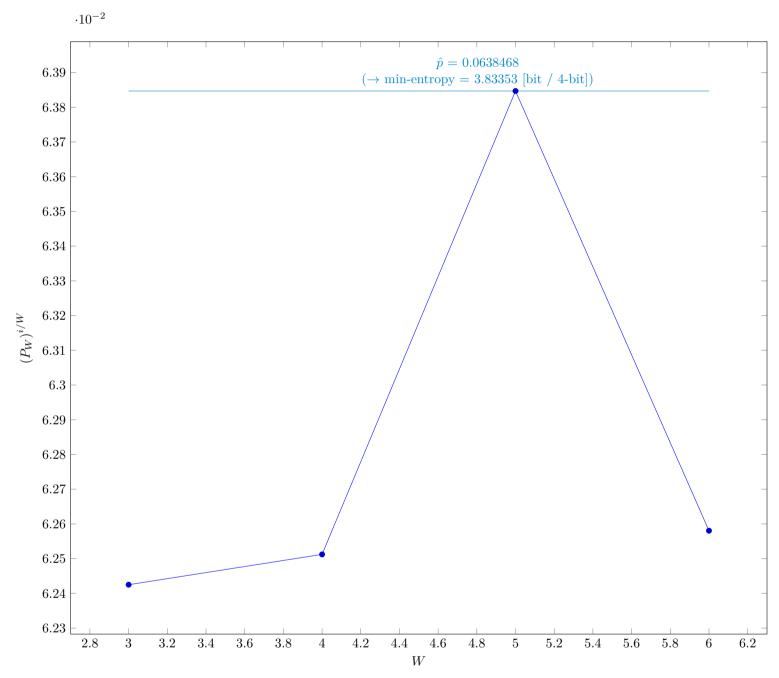


Fig. 7 Estimated average collision probability per string symbol in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B

### 3.3.1 Supplemental information for traceability

Table 7 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol    | Value     |
|-----------|-----------|
| u         | 3         |
| v         | 6         |
| $\hat{p}$ | 0.0638468 |
| $p_u$     | 0.0701445 |

### 3.4 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

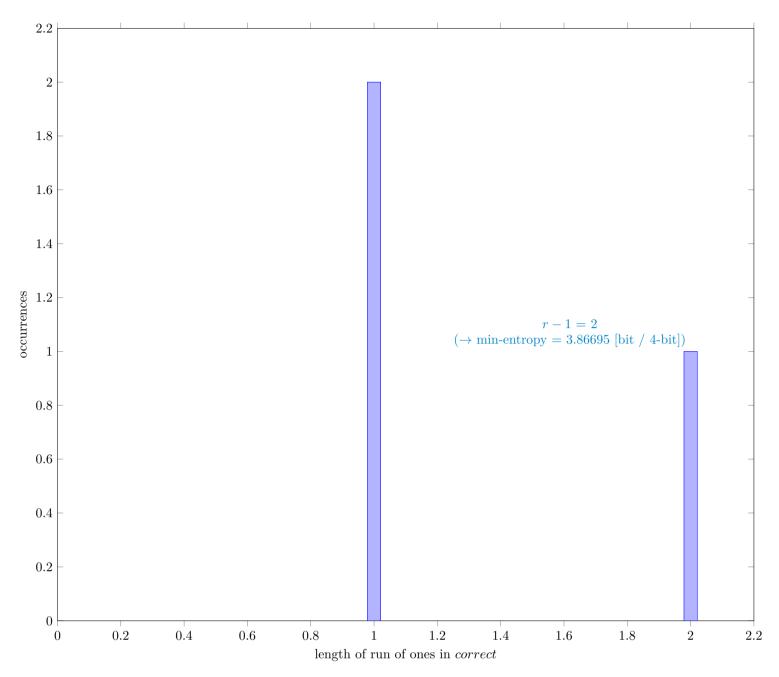


Fig. 8 Distribution of correct

### 3.4.1 Supplemental information for traceability

Table 8 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol           | Value     |
|------------------|-----------|
| N                | 9937      |
| C                | 619       |
| $P_{ m global}$  | 0.0622924 |
| $P'_{ m global}$ | 0.0685379 |
| r                | 3         |
| $P_{ m local}$   | 0.0100725 |

### 3.5 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

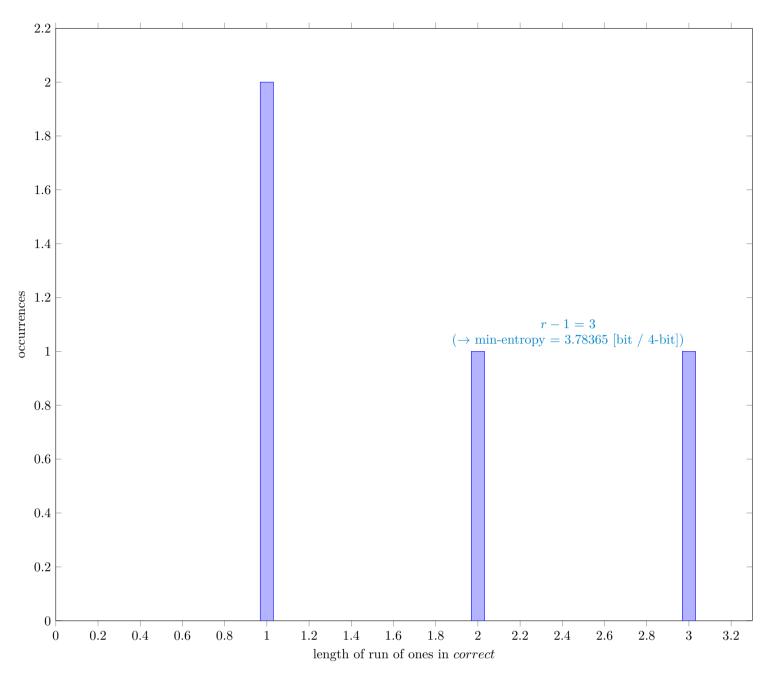


Fig. 9 Distribution of correct

### 3.5.1 Supplemental information for traceability

Table 9 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol           | Value     |
|------------------|-----------|
| N                | 9999      |
| C                | 662       |
| $P_{ m global}$  | 0.0662066 |
| $P'_{ m global}$ | 0.0726119 |
| r                | 4         |
| $P_{ m local}$   | 0.0319235 |

### 3.6 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

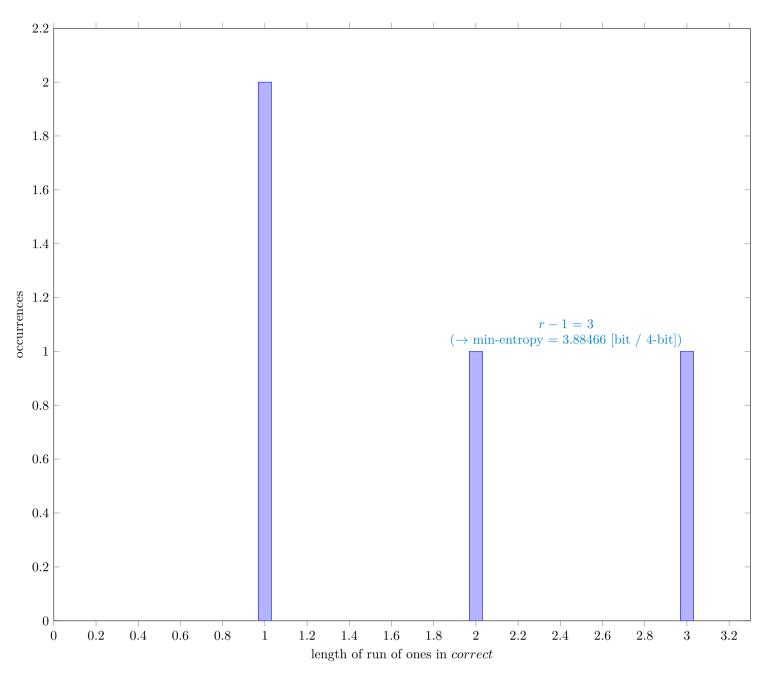


Fig. 10  $\,$  Distribution of correct

### 3.6.1 Supplemental information for traceability

Table 10 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol                | Value     |
|-----------------------|-----------|
| N                     | 9998      |
| C                     | 615       |
| $P_{\mathrm{global}}$ | 0.0615123 |
| $P'_{ m global}$      | 0.0677021 |
| r                     | 4         |
| $P_{\mathrm{local}}$  | 0.0319243 |

### 3.7 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

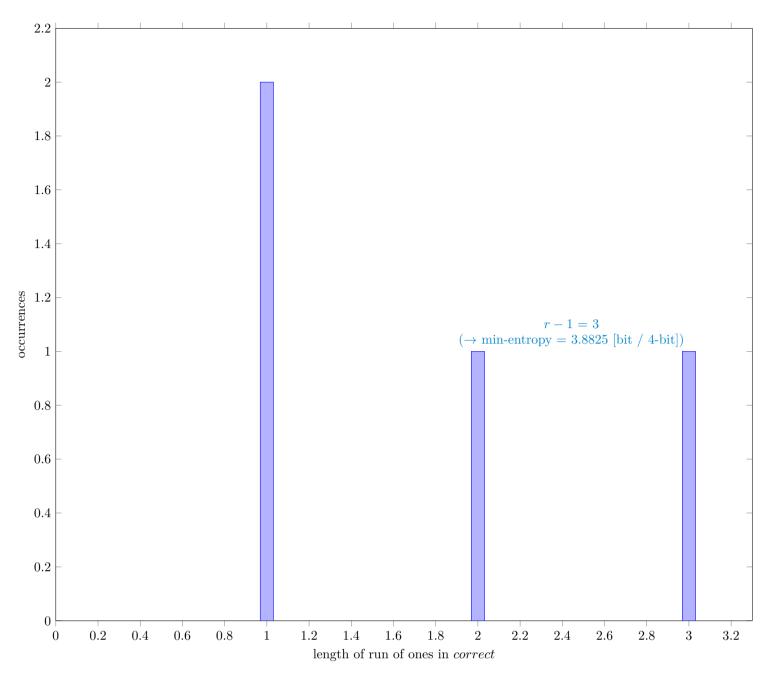


Fig. 11 Distribution of correct

### 3.7.1 Supplemental information for traceability

Table 11 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol               | Value     |
|----------------------|-----------|
| N                    | 9983      |
| C                    | 615       |
| $P_{ m global}$      | 0.0616047 |
| $P'_{ m global}$     | 0.0678035 |
| r                    | 4         |
| $P_{\mathrm{local}}$ | 0.0319364 |

# 4 Detailed results of analysis by interpreting each sample as bitstrings

### 4.1 The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)

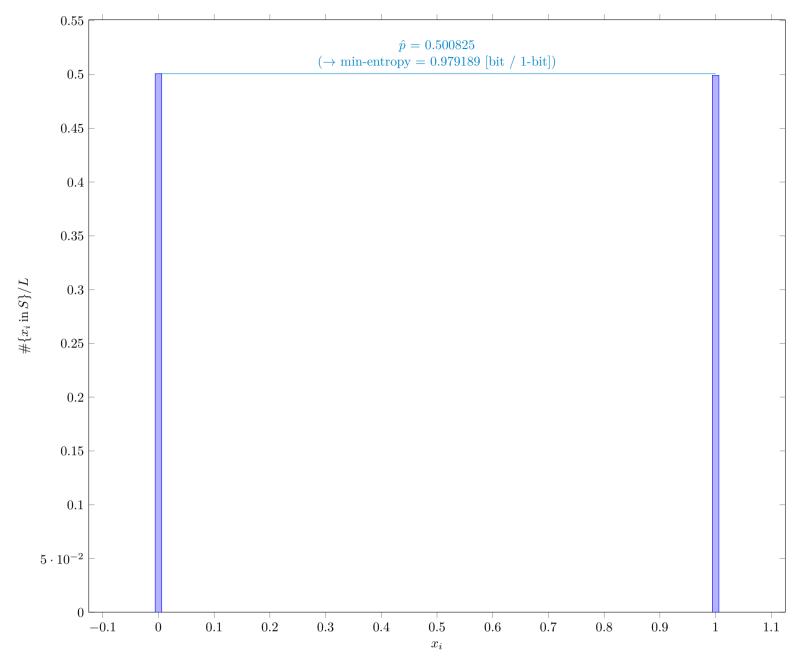


Fig. 12 Distribution of  $x_i$ 

#### 4.1.1 Supplemental information for traceability

Table 12 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol    | Value    |
|-----------|----------|
| mode      | 20033    |
| $\hat{p}$ | 0.500825 |
| $p_u$     | 0.507265 |

# 4.2 The Collision Estimate (NIST SP 800-90B Section 6.3.2)

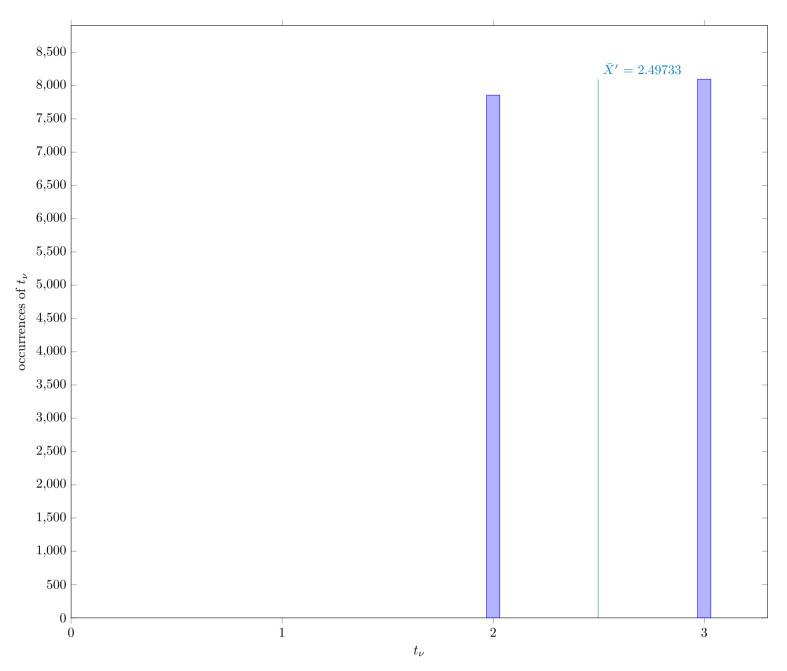


Fig. 13 Distribution of intermediate value  $t_{\nu}$ 

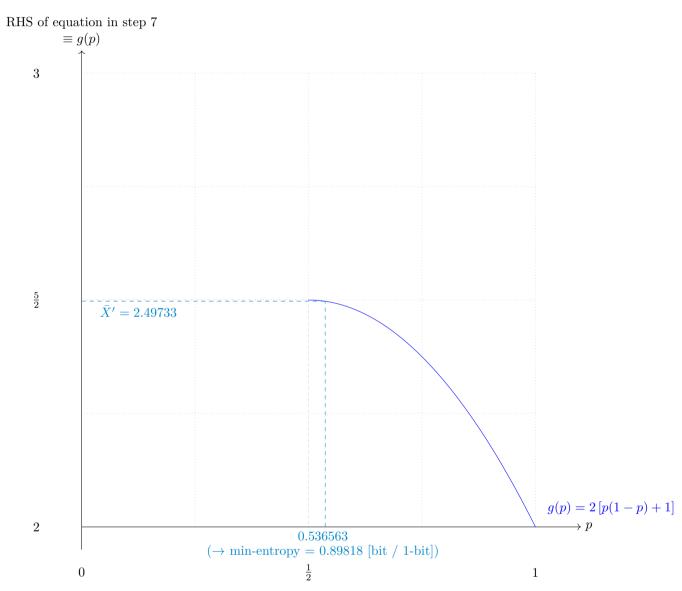


Fig. 14 Solution to the equation in step 7

### 4.2.1 Supplemental information for traceability

Table 13 Supplemental information for traceability (NIST SP 800-90B Section 6.3.2)

| Symbol         | Value    |
|----------------|----------|
| p              | 0.536563 |
| $\bar{X}$      | 2.50752  |
| $ar{X}'$       | 2.49733  |
| $\hat{\sigma}$ | 0.499959 |

### 4.3 The Markov Estimate (NIST SP 800-90B Section 6.3.3)

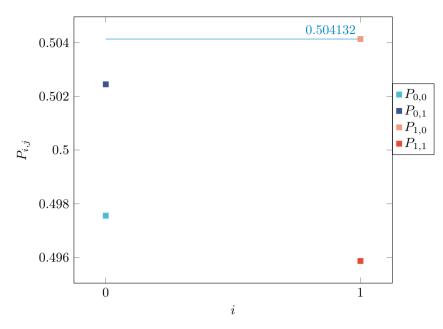


Fig. 15 Transition probability  $P_{i,j}$  of §6.3.3 of NIST SP 800-90B

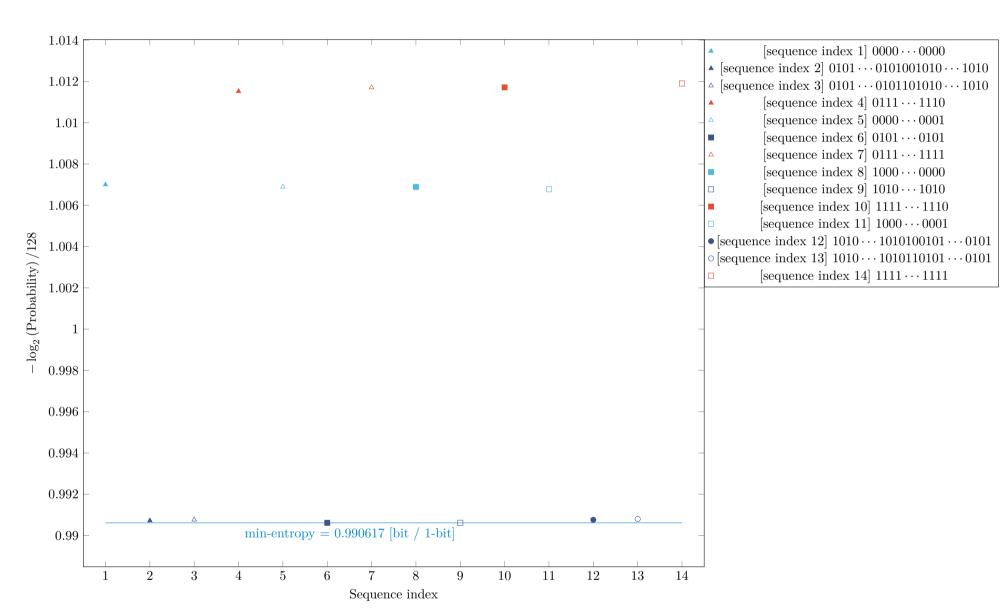


Fig. 16 Estimated Min-Entropy using  $\S 6.3.3$  of NIST SP 800-90B

### 4.4 The Compression Estimate (NIST SP 800-90B Section 6.3.4)

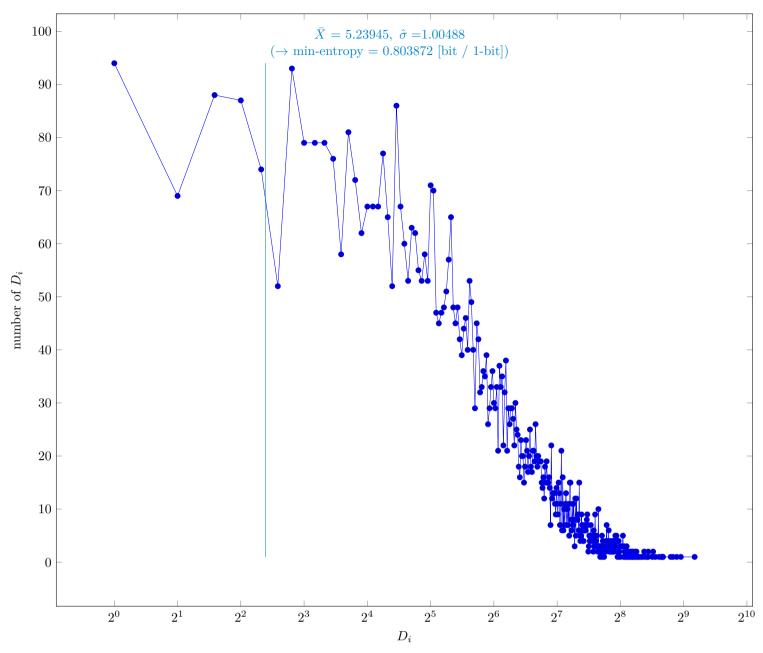


Fig. 17 Distribution of intermediate value  $D_i$ 

#### 4.4.1 Supplemental information for traceability

Table 14 Supplemental information for traceability (NIST SP 800-90B Section 6.3.4)

| Symbol         | Value     |
|----------------|-----------|
| p              | 0.0353234 |
| $\bar{X}$      | 5.23945   |
| $\hat{\sigma}$ | 1.00488   |
| $\bar{X}'$     | 5.20506   |

# 4.5 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)

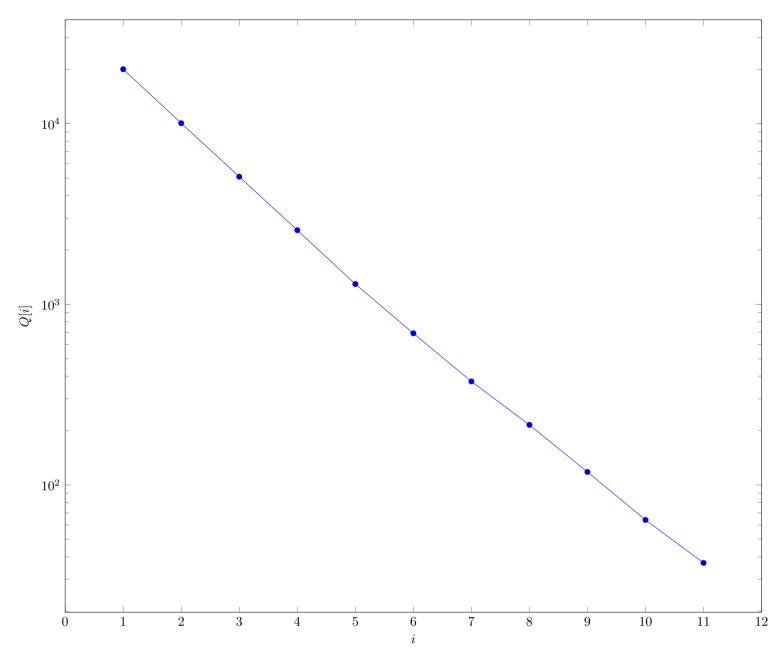


Fig. 18 Intermediate value Q[i] in  $\S 6.3.5$  of NIST SP 800-90B

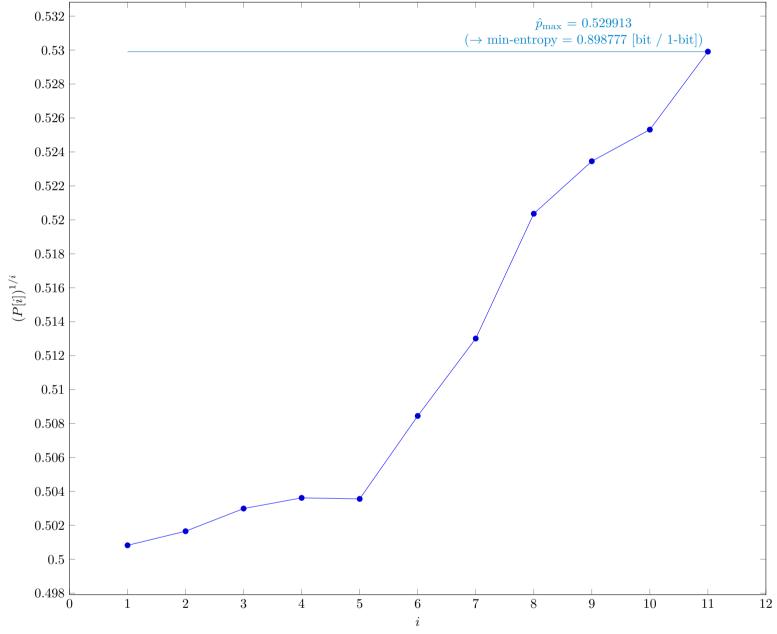


Fig. 19  $P[i]^{1/i}$  in §6.3.5 of NIST SP 800-90B

### 4.5.1 Supplemental information for traceability

Table 15 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol                   | Value    |
|--------------------------|----------|
| t                        | 11       |
| $\hat{p}_{\mathrm{max}}$ | 0.529913 |
| $p_u$                    | 0.536341 |
| 20                       |          |

# 4.6 The LRS Estimate (NIST SP 800-90B Section 6.3.6)



Fig. 20 Estimated W-tuple collision probability in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B



Fig. 21 Estimated average collision probability per string symbol in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B

### 4.6.1 Supplemental information for traceability

Table 16 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol    | Value    |
|-----------|----------|
| u         | 12       |
| v         | 31       |
| $\hat{p}$ | 0.517343 |
| $p_u$     | 0.523779 |

### 4.7 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

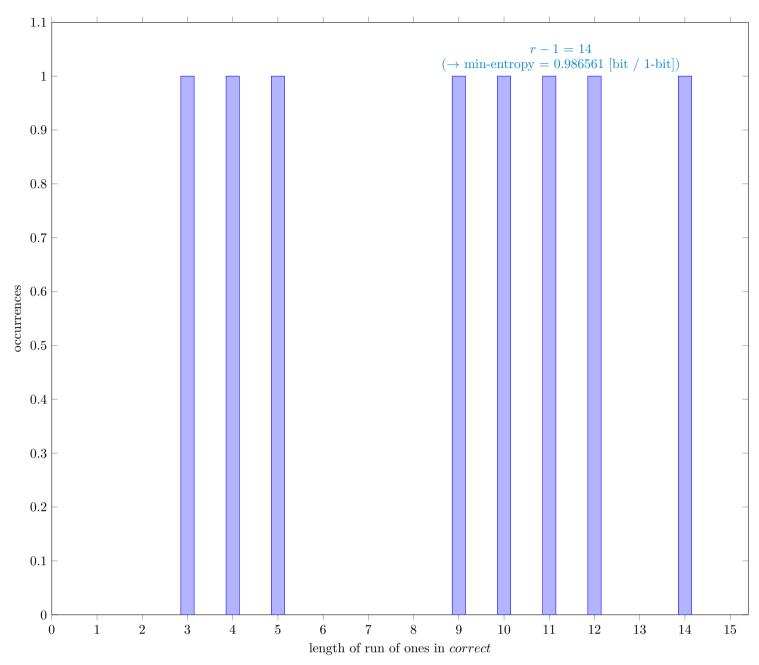


Fig. 22 Distribution of correct

### 4.7.1 Supplemental information for traceability

Table 17 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol                | Value    |
|-----------------------|----------|
| N                     | 39937    |
| C                     | 19898    |
| $P_{\mathrm{global}}$ | 0.498235 |
| $P'_{ m global}$      | 0.504679 |
| r                     | 15       |
| $P_{\mathrm{local}}$  | 0.374676 |

# 4.8 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

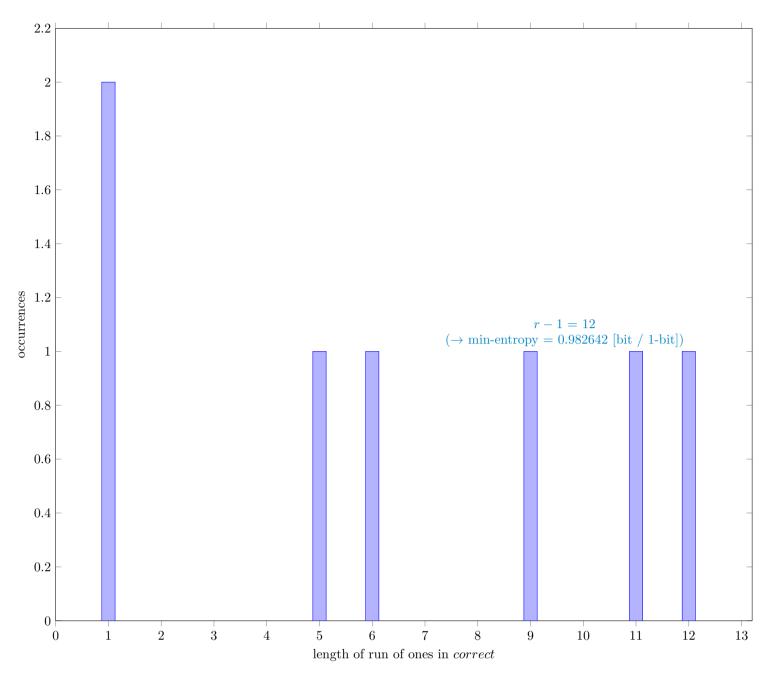


Fig. 23  $\,$  Distribution of  $\,$ 

### 4.8.1 Supplemental information for traceability

Table 18 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol           | Value    |
|------------------|----------|
| N                | 39999    |
| C                | 19984    |
| $P_{ m global}$  | 0.499612 |
| $P'_{ m global}$ | 0.506052 |
| r                | 13       |
| $P_{ m local}$   | 0.320047 |

### 4.9 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

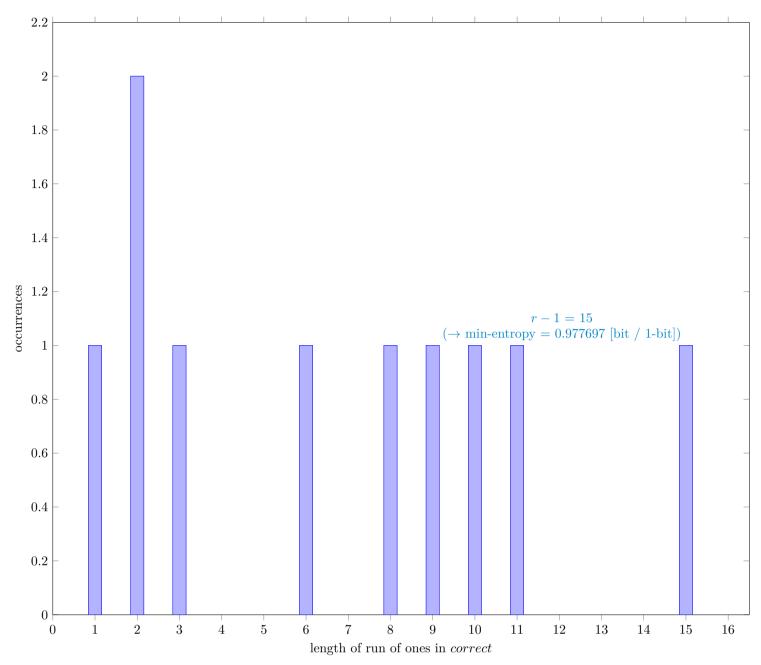


Fig. 24 Distribution of correct

### 4.9.1 Supplemental information for traceability

Table 19 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol                | Value    |
|-----------------------|----------|
| N                     | 39998    |
| C                     | 20053    |
| $P_{\mathrm{global}}$ | 0.50135  |
| $P'_{ m global}$      | 0.50779  |
| r                     | 16       |
| $P_{ m local}$        | 0.399351 |

### 4.10 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

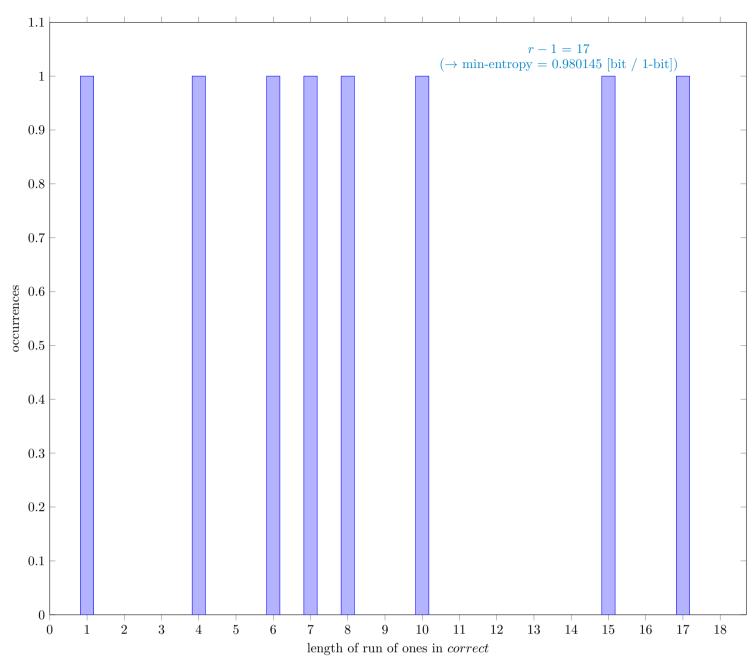


Fig. 25 Distribution of correct

#### 4.10.1 Supplemental information for traceability

Table 20 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol           | Value    |
|------------------|----------|
| N                | 39983    |
| C                | 20011    |
| $P_{ m global}$  | 0.500488 |
| $P'_{ m global}$ | 0.506929 |
| r                | 18       |
| $P_{ m local}$   | 0.444149 |

# 4 References

<sup>[1]</sup> Meltem Sönmez Turan, Elaine Barker, John Kelsey, Kerry A. McKay, Mary L. Baish, Mike Boyle, Recommendation for the Entropy Sources Used for Random Bit Generation, NIST Special Publication 800-90B, Jan. 2018 https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-90B.pdf

<sup>[2]</sup> G. Sakurai, Proposed list of corrections for NIST SP 800-90B 6.3 Estimators, Dec. 2022 https://github.com/g-g-sakura/AnotherEntropyEstimationTool/blob/main/documentation/ProposedListOfCorrections\_SP800-90B.pdf