# Report of Entropy estimates based on NIST SP 800-90B non-IID track

#### 2024-Jul-10 14:45:26.216049

# 1 Identification information

#### 1.1 Identification of acquisition data from entropy source

Table 1 Identification information of acquisition data from entropy source

| URL of the acquisition data                        | https://github.com/usnistgov/SP800-90B_EntropyAssessment/blob/master/bin/truerand_8bit.bin |
|--|--|
| SHA-256 hash value of the acqui- sition data [hex] |  |

- $\bullet\,$  Name of the submitter of the acquisition data :
- $\bullet\,$  Brief explanation of the acquisition data (or entropy source) :

#### 1.2 Identification of analysis environment

Table 2 Identification information of analysis environment

| Analysis tool        | Name                   | Another entropy estimation tool with extensions     |
|----------------------|------------------------|---|
|                      | Versioning information | 1.0.55  |
|                      | built as               | 64-bit application                                  |
|                      | built by               | Intel C++ Compiler (INTEL_LLVM_COMPILER: 20240100 ) |
|                      | linked libraries       | Boost C++ 1.85.0                                    |
| Analysis environment | Hostname               |   |
|                      | CPU information        | AMD Ryzen   |
|                      | Physical memory size   | MiB   |
|                      | OS information         | Windows 10 or greater 64-bit                        |
|                      | Username               |   |

#### 1.3 Identification of analysis conditions

Table 3 Identification information of analysis conditions

| Number of samples      | 1000000                          |
|------------------------|----------------------------------|
| Bits per sample        | 8                                |
| Byte to bit conversion | Most Significant bit (MSb) first |

### 1.4 Identification of analysis method

NIST SP 800-90B [1] 6.3 with corrections [2] is applied

# 2 Executive summary

#### 2.1 Numerical results of min-entropy estimates based on non-IID track

Table 4 Numerical results

| Estimator  | $H_{ m original}{}^{ m a}$ | Notes to $H_{\text{original}}$ | $H_{ m bitstring}^{ m \ b}$ | Notes to $H_{\text{bitstring}}$ |
|--|----------------------------|--------------------------------|-----------------------------|---------------------------------|
|  | [bit / 8 - bit]            |                                | [bit / 1 - bit]             |                                 |
| The Most Common Value Estimate                                   | 7.86512                    | see 3.1                        | 0.998199                    | see 4.1                         |
| The Collision Estimate   | _                          | _                              | 0.95841                     | see 4.3                         |
| The Markov Estimate  | _                          | _                              | 0.999439                    | see 4.5                         |
| The Compression Estimate   | _                          | _                              | 0.904233                    | see 4.6                         |
| The t-Tuple Estimate   | 7.86512                    | see 3.3                        | 0.933569                    | see 4.8                         |
| The Longest Repeated Substring (LRS) Estimate                    | 7.9392                     | see 3.5                        | 0.998671                    | see 4.10                        |
| Multi Most Common in Window Prediction Estimate                  | 7.98858                    | see 3.7                        | 0.999563                    | see 4.12                        |
| The Lag Prediction Estimate                                      | 7.93976                    | see 3.9                        | 0.998402                    | see 4.14                        |
| The MultiMMC Prediction Estimate                                 | 7.92681                    | see 3.11                       | 0.99966                     | see 4.16                        |
| The LZ78Y Prediction Estimate                                    | 7.91928                    | see 3.13                       | 0.998465                    | see 4.18                        |
| The intial entropy source estimate [bit / 8 - bit]               |                            | 7.2                            | 3386                        |                                 |
| $H_I = \min(H_{\text{original}}, 8 \times H_{\text{bitstring}})$ |                            |                                |                             |                                 |

 $<sup>^</sup>a$   $\,$  Entropy estimate of the sequential dataset [source: NIST SP 800-90B [1] 3.1.3]

 $<sup>^</sup>b$  An additional entropy estimation (per bit) for the non-binary sequential dataset [see NIST SP 800-90B [1] 3.1.3]

#### 2.2 Visual comparison of min-entropy estimates from original samples

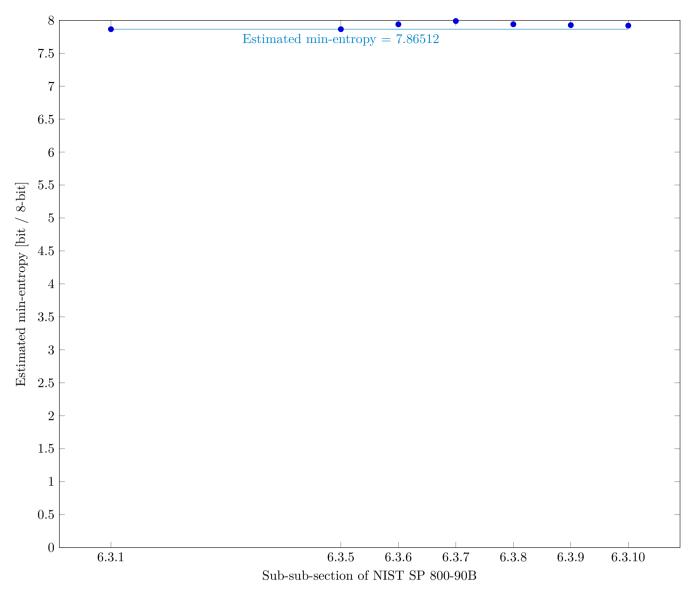


Fig. 1 Estimated Min-Entropy using  $\S 6.3$  of NIST SP 800-90B

#### 2.3 Visual comparison of min-entropy estimates by interpreting each sample as bitstring



Fig. 2  $\,$  Estimated Min-Entropy using  $\S 6.3$  of NIST SP 800-90B  $\,$ 

# 3 Detailed results of analysis from original samples

#### 3.1 The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)

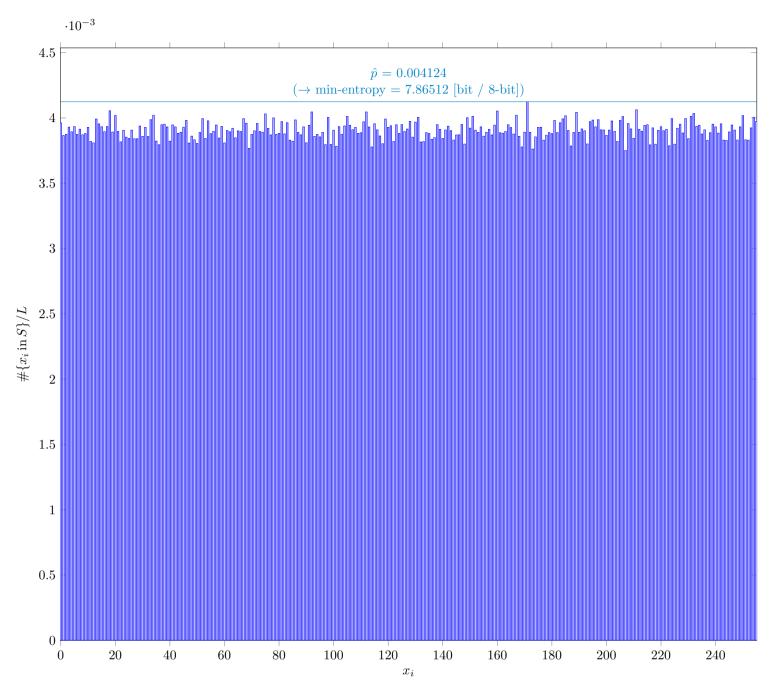


Fig. 3 Distribution of  $x_i$ 

#### 3.2 Supplemental information for traceability

Table 5 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol    | Value      |
|-----------|------------|
| mode      | 4124       |
| $\hat{p}$ | 0.004124   |
| $p_u$     | 0.00428907 |

# 3.3 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)



Fig. 4 Intermediate value Q[i] in §6.3.5 of NIST SP 800-90B



Fig. 5  $P[i]^{1/i}$  in §6.3.5 of NIST SP 800-90B

# 3.4 Supplemental information for traceability

Table 6 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol                   | Value      |
|--------------------------|------------|
| t                        | 1          |
| $\hat{p}_{\mathrm{max}}$ | 0.004124   |
| $p_u$                    | 0.00428907 |
|                          |            |

# 3.5 The LRS Estimate (NIST SP 800-90B Section 6.3.6)

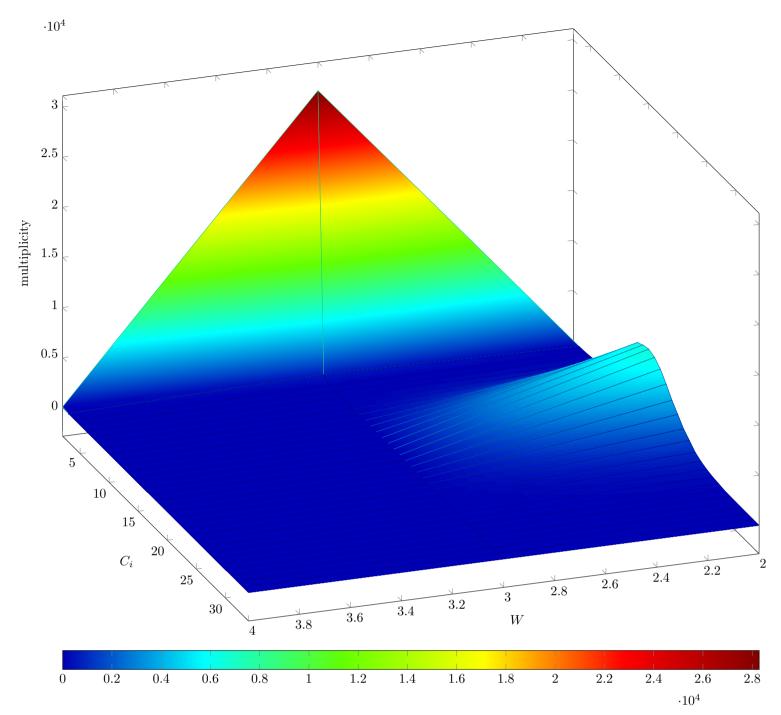
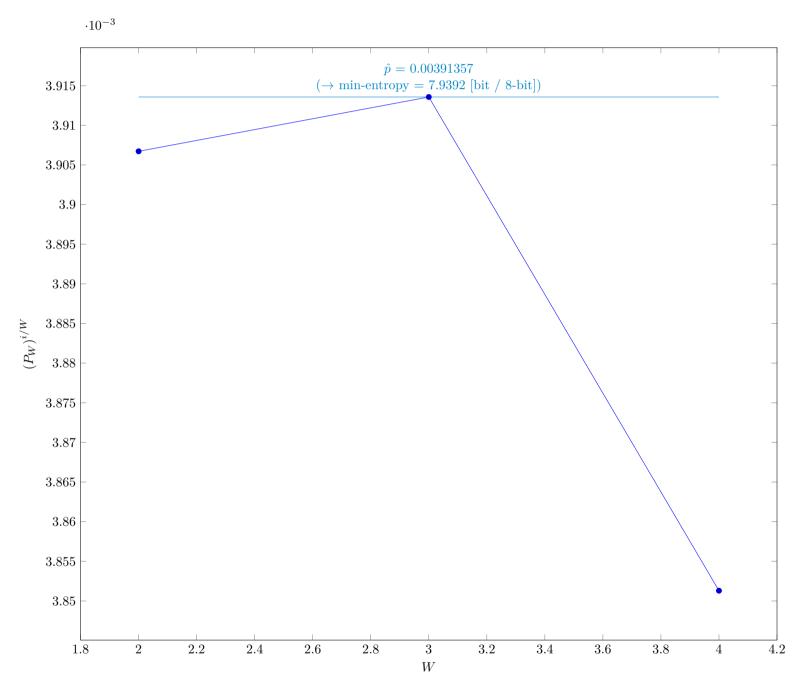


Fig. 6 Estimated W-tuple collision probability in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B



 $Fig.~7 \quad Estimated~average~collision~probability~per~string~symbol~in~Step~3~of~\S 6.3.6~of~NIST~SP~800-90B \\$ 

# 3.6 Supplemental information for traceability

Table 7 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol    | Value      |
|-----------|------------|
| u         | 2          |
| v         | 4          |
| $\hat{p}$ | 0.00391357 |
| $p_u$     | 0.00407439 |
|           |            |

#### 3.7 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

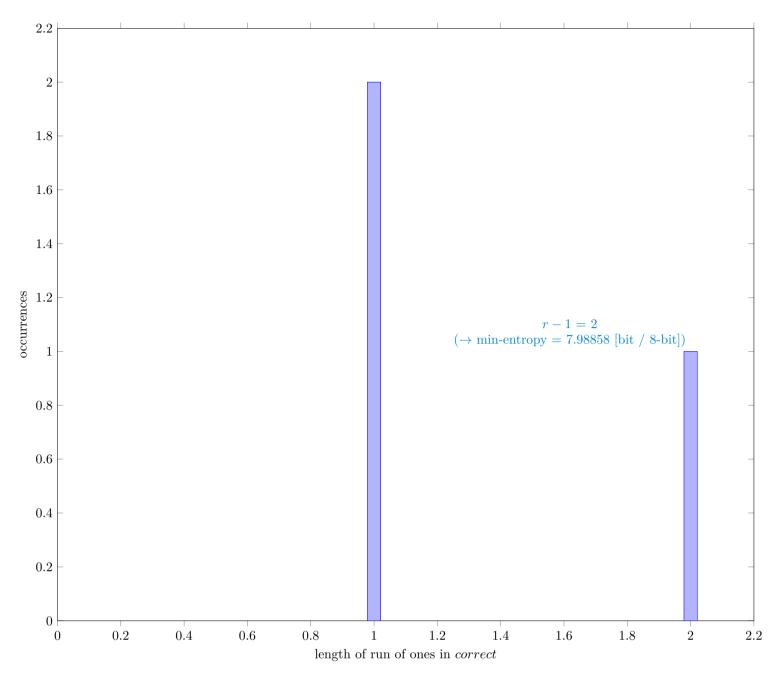


Fig. 8 Distribution of correct

#### 3.8 Supplemental information for traceability

Table 8 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol           | Value      |
|------------------|------------|
| N                | 999937     |
| C                | 3779       |
| $P_{ m global}$  | 0.00377924 |
| $P'_{ m global}$ | 0.00393729 |
| r                | 3          |
| $P_{ m local}$   | 0.00215965 |

### 3.9 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

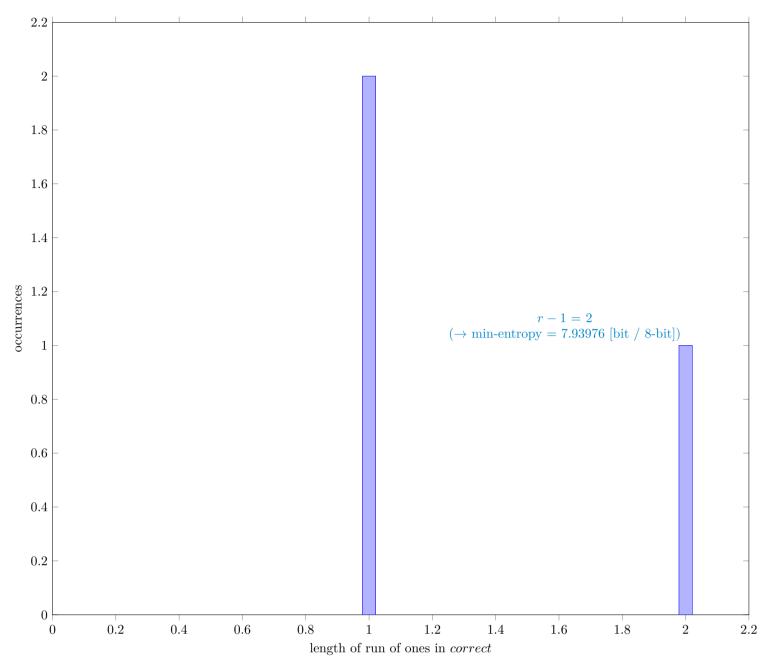


Fig. 9 Distribution of correct

### 3.10 Supplemental information for traceability

Table 9 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol                | Value     |
|-----------------------|-----------|
| N                     | 999999    |
| C                     | 3912      |
| $P_{\mathrm{global}}$ | 0.003912  |
| $P'_{ m global}$      | 0.0040728 |
| r                     | 3         |
| $P_{ m local}$        | 0.0021596 |

#### 3.11 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

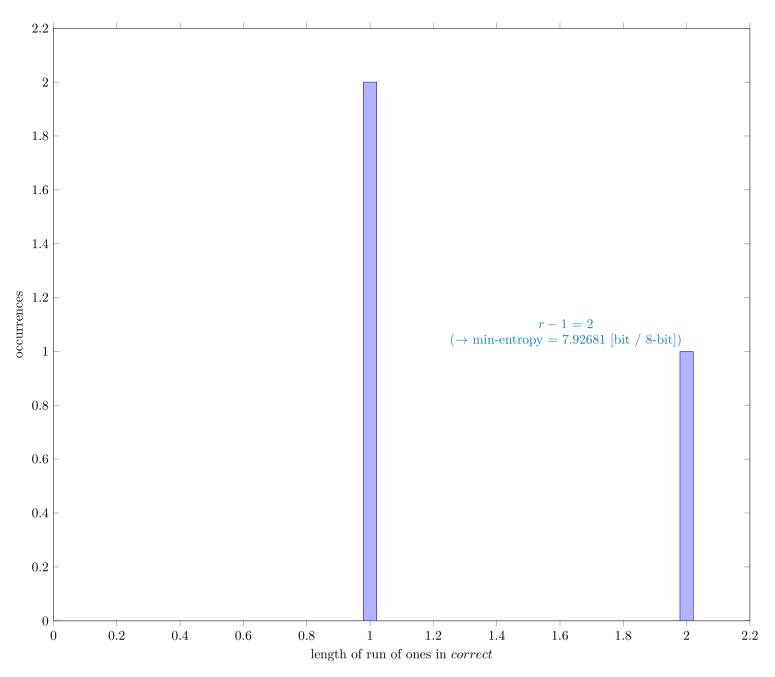


Fig. 10  $\,$  Distribution of correct

#### 3.12 Supplemental information for traceability

Table 10 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol             | Value      |
|--------------------|------------|
| N                  | 999998     |
| C                  | 3948       |
| $P_{\rm global}$   | 0.00394801 |
| $P'_{ m global}$   | 0.00410954 |
| r                  | 3          |
| $P_{\text{local}}$ | 0.0021596  |

#### 3.13 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

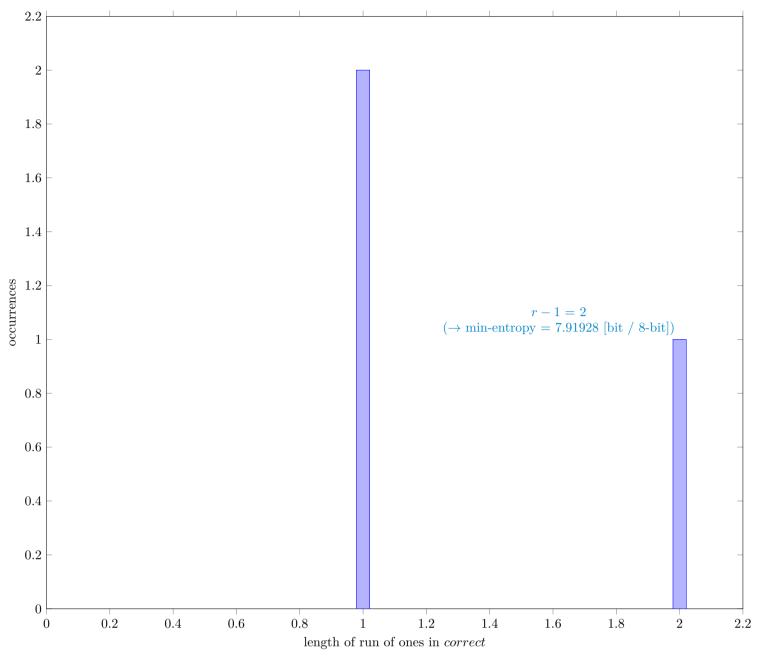


Fig. 11 Distribution of correct

#### 3.14 Supplemental information for traceability

Table 11 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol           | Value      |
|------------------|------------|
| N                | 999983     |
| C                | 3969       |
| $P_{ m global}$  | 0.00396907 |
| $P'_{ m global}$ | 0.00413103 |
| r                | 3          |
| $P_{ m local}$   | 0.00215961 |

# 4 Detailed results of analysis by interpreting each sample as bitstrings

### 4.1 The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)

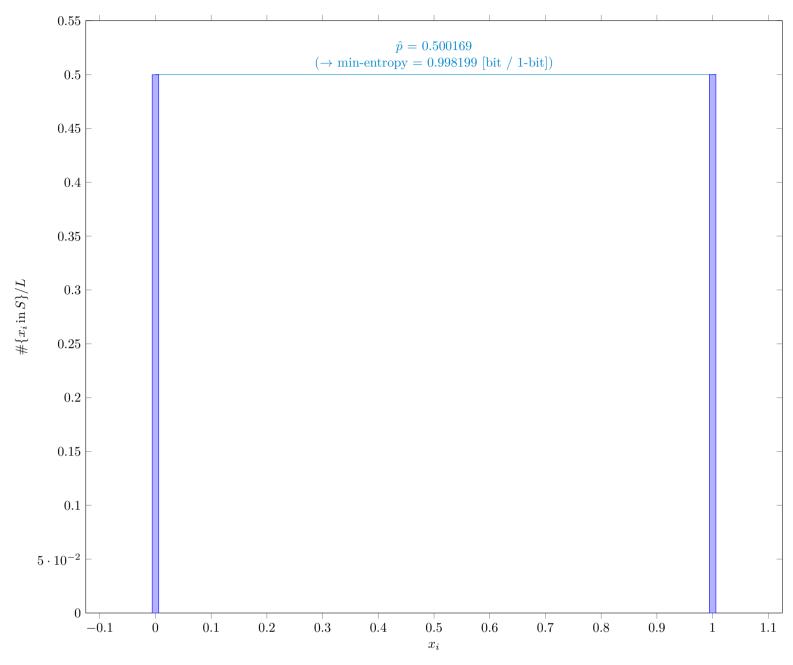


Fig. 12 Distribution of  $x_i$ 

#### 4.2 Supplemental information for traceability

Table 12 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol    | Value    |
|-----------|----------|
| mode      | 4001353  |
| $\hat{p}$ | 0.500169 |
| $p_u$     | 0.500624 |

# 4.3 The Collision Estimate (NIST SP 800-90B Section 6.3.2)

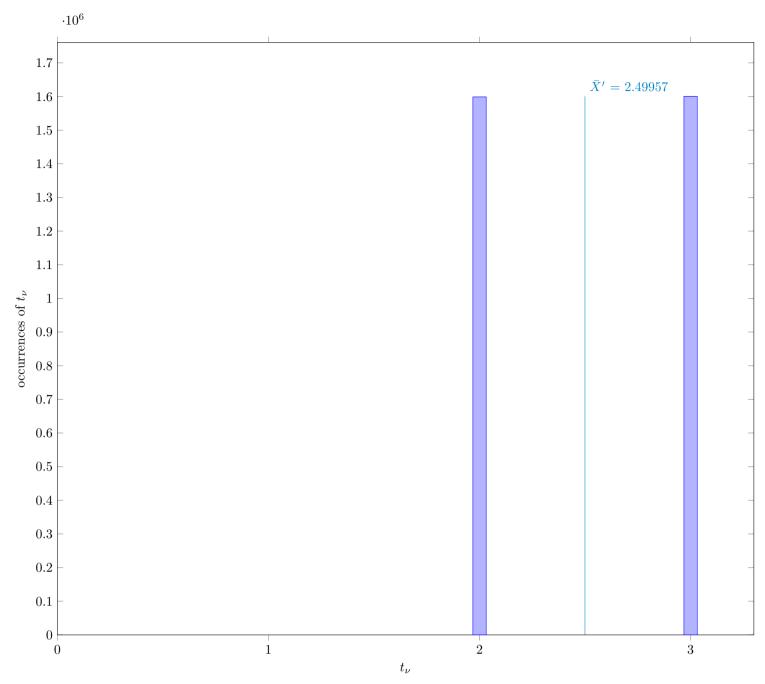


Fig. 13 Distribution of intermediate value  $t_{\nu}$ 

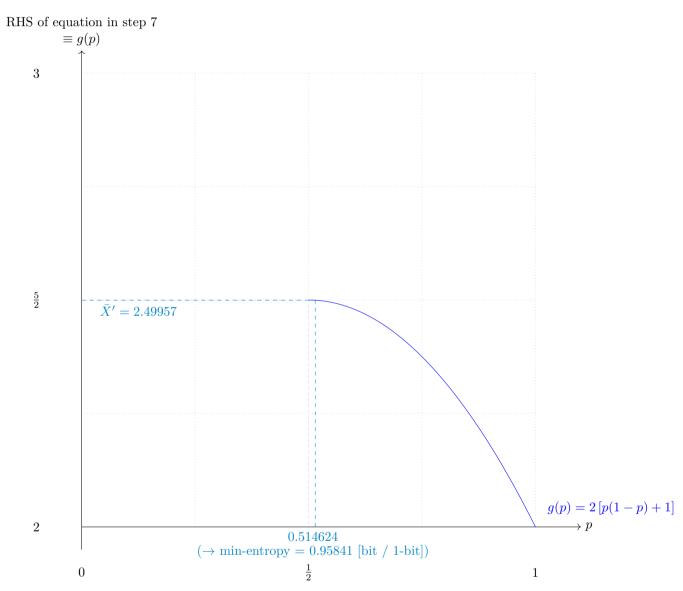


Fig. 14 Solution to the equation in step 7

# 4.4 Supplemental information for traceability

Table 13 Supplemental information for traceability (NIST SP 800-90B Section 6.3.2)

| Symbol         | Value    |
|----------------|----------|
| p              | 0.514624 |
| $\bar{X}$      | 2.50029  |
| $ar{X}'$       | 2.49957  |
| $\hat{\sigma}$ | 0.5      |
|                |          |

#### 4.5 The Markov Estimate (NIST SP 800-90B Section 6.3.3)

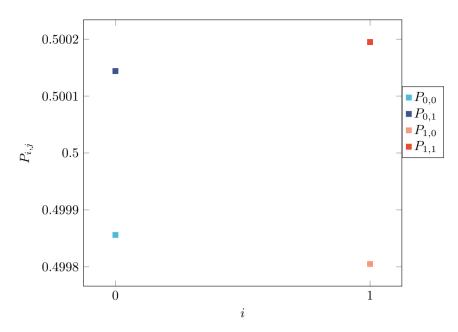


Fig. 15 Transition probability  $P_{i,j}$  of §6.3.3 of NIST SP 800-90B

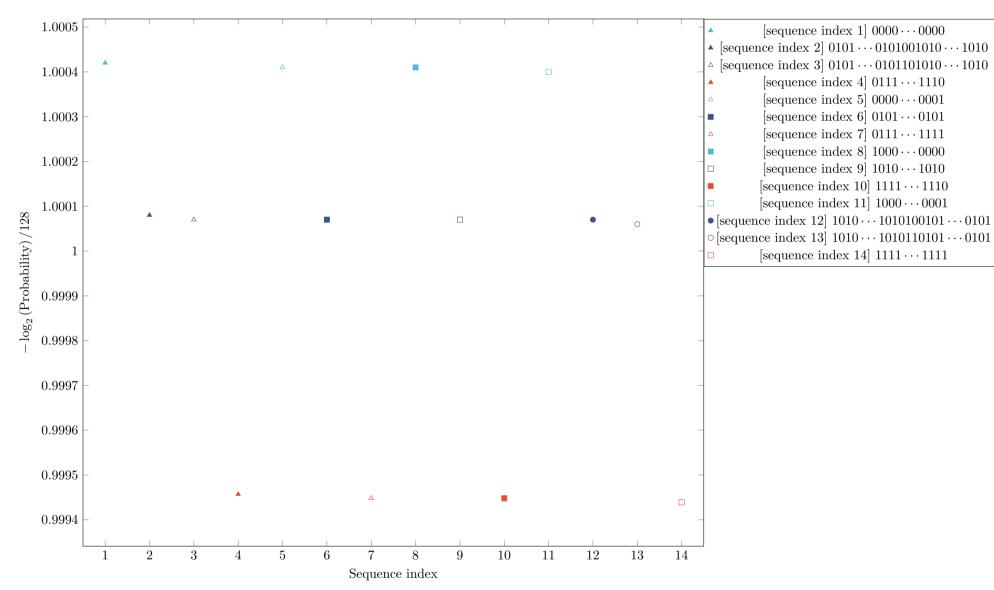


Fig. 16 Estimated Min-Entropy using  $\S 6.3.3$  of NIST SP 800-90B

#### 4.6 The Compression Estimate (NIST SP 800-90B Section 6.3.4)

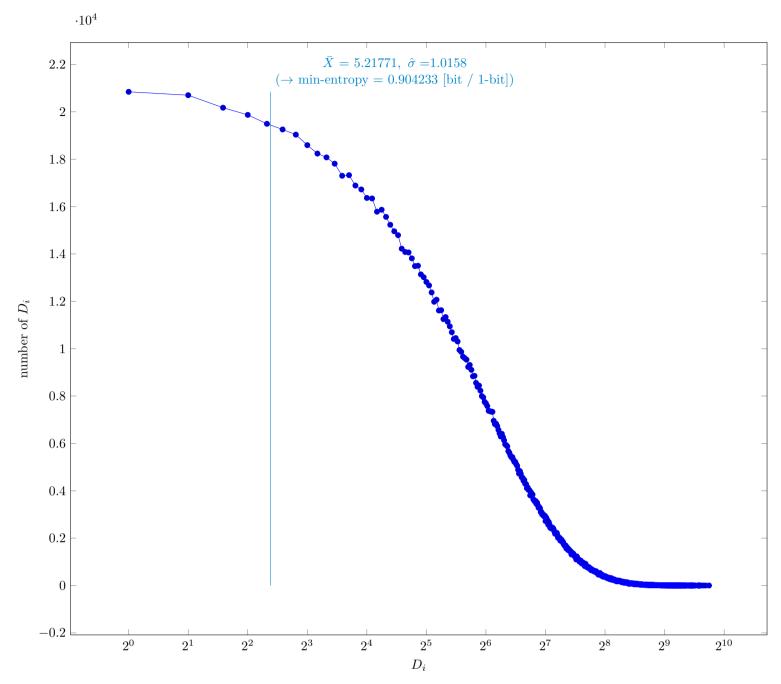


Fig. 17 Distribution of intermediate value  $D_i$ 

#### 4.7 Supplemental information for traceability

Table 14 Supplemental information for traceability (NIST SP 800-90B Section 6.3.4)

| Symbol         | Value     |
|----------------|-----------|
| p              | 0.0232698 |
| $\bar{X}$      | 5.21771   |
| $\hat{\sigma}$ | 1.0158    |
| $ar{X}'$       | 5.21545   |

# 4.8 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)

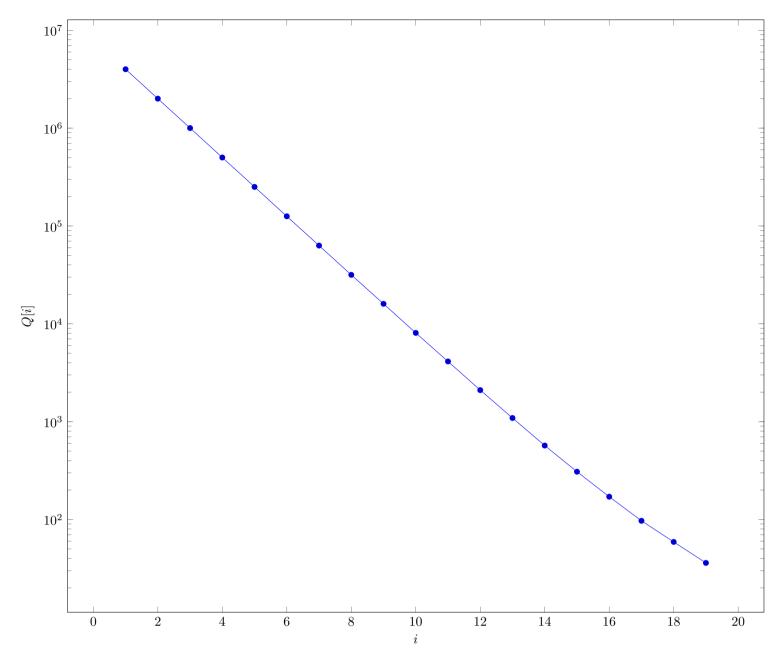


Fig. 18 Intermediate value Q[i] in  $\S 6.3.5$  of NIST SP 800-90B

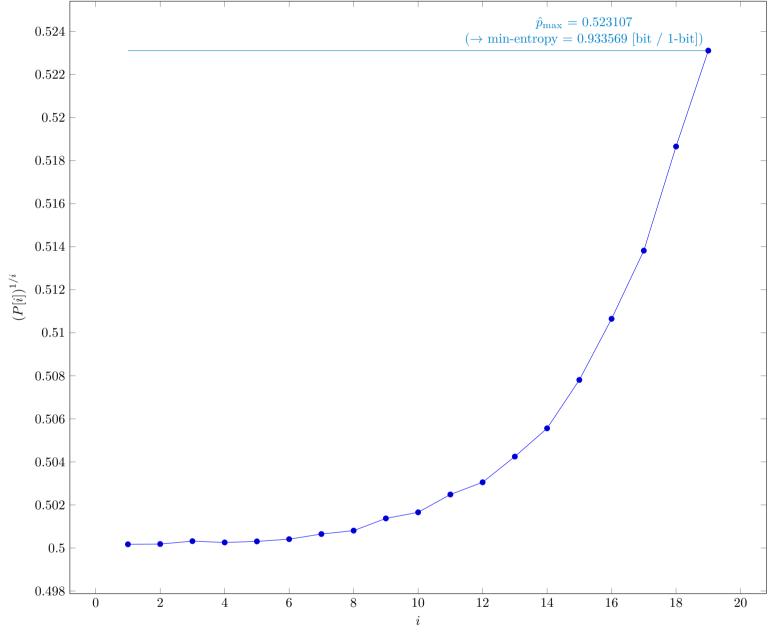


Fig. 19  $P[i]^{1/i}$  in §6.3.5 of NIST SP 800-90B

# 4.9 Supplemental information for traceability

Table 15 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol                   | Value    |
|--------------------------|----------|
| t                        | 19       |
| $\hat{p}_{\mathrm{max}}$ | 0.523107 |
| $p_u$                    | 0.523561 |

# 4.10 The LRS Estimate (NIST SP 800-90B Section 6.3.6)

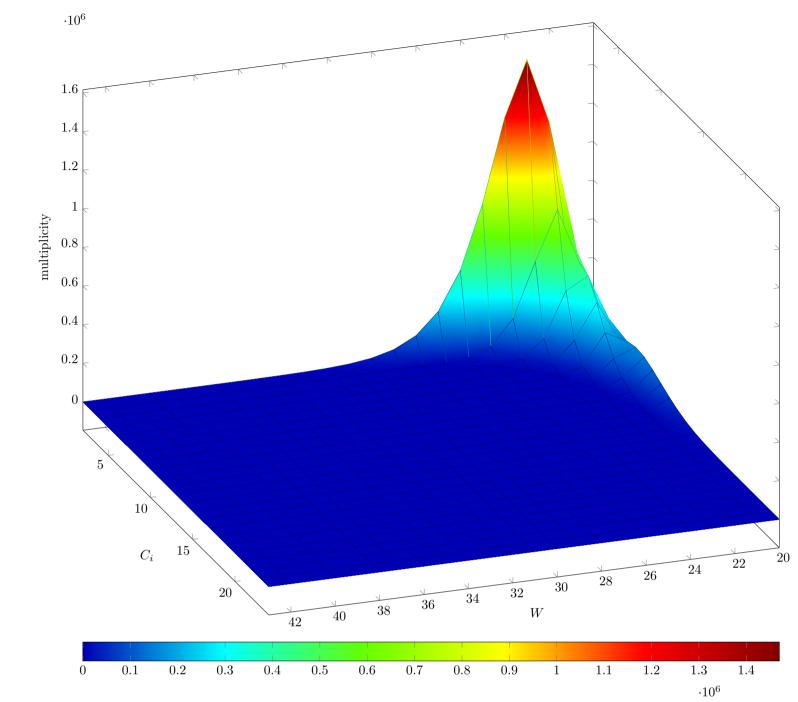


Fig. 20 Estimated W-tuple collision probability in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B

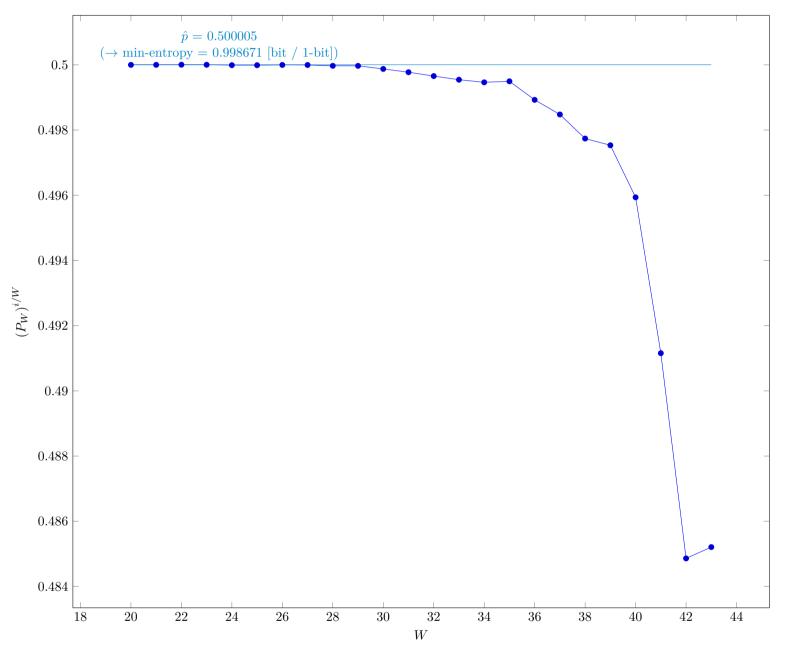


Fig. 21 Estimated average collision probability per string symbol in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B

### 4.11 Supplemental information for traceability

Table 16 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol    | Value    |
|-----------|----------|
| u         | 20       |
| v         | 43       |
| $\hat{p}$ | 0.500005 |
| $p_u$     | 0.500461 |
|           |          |

#### 4.12 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

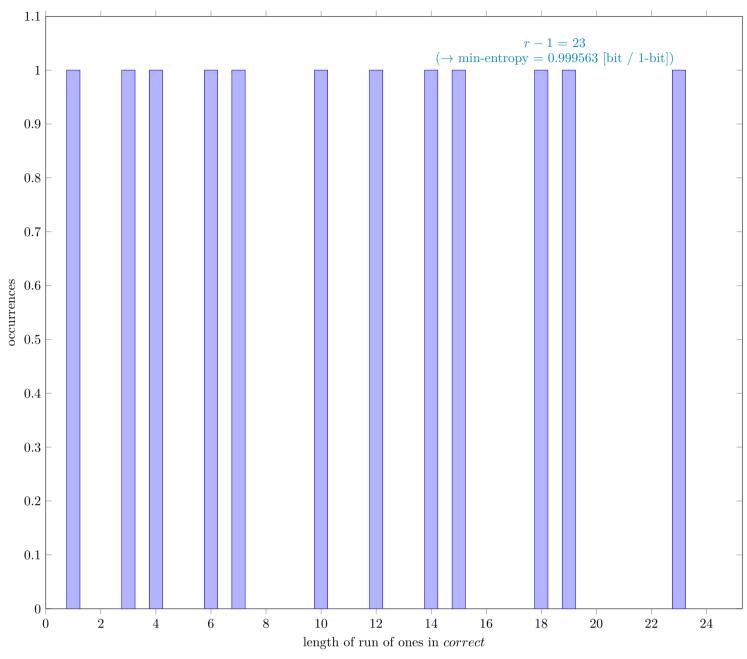


Fig. 22 Distribution of correct

#### 4.13 Supplemental information for traceability

Table 17 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol                | Value    |
|-----------------------|----------|
| N                     | 7999937  |
| C                     | 3997538  |
| $P_{\mathrm{global}}$ | 0.499696 |
| $P'_{ m global}$      | 0.500152 |
| r                     | 24       |
| $P_{ m local}$        | 0.436006 |

### 4.14 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

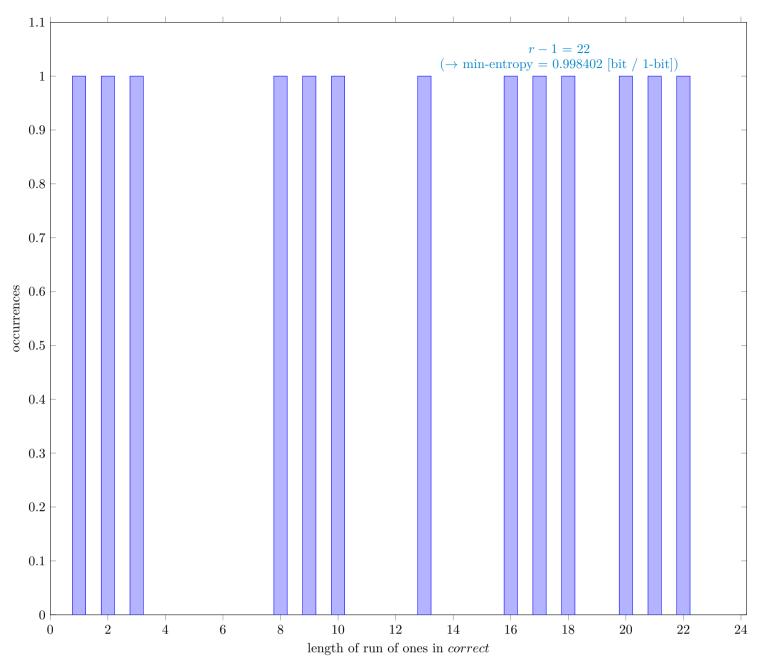


Fig. 23 Distribution of correct

### 4.15 Supplemental information for traceability

Table 18 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol           | Value    |
|------------------|----------|
| N                | 7999999  |
| C                | 4000791  |
| $P_{ m global}$  | 0.500099 |
| $P'_{ m global}$ | 0.500554 |
| r                | 23       |
| $P_{ m local}$   | 0.42004  |

#### 4.16 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

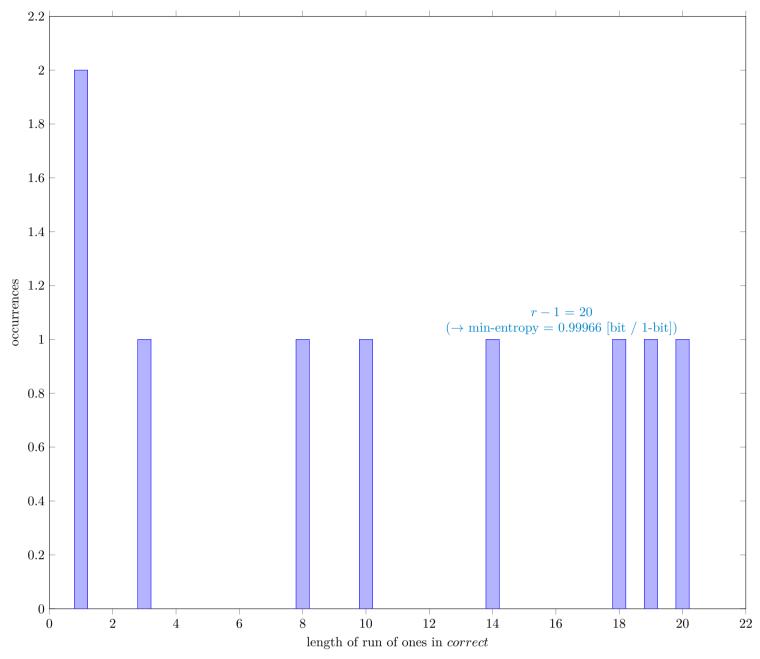


Fig. 24 Distribution of correct

### 4.17 Supplemental information for traceability

Table 19 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol           | Value    |
|------------------|----------|
| N                | 7999998  |
| C                | 3997298  |
| $P_{ m global}$  | 0.499662 |
| $P'_{ m global}$ | 0.500118 |
| r                | 21       |
| $P_{ m local}$   | 0.385677 |

#### 4.18 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

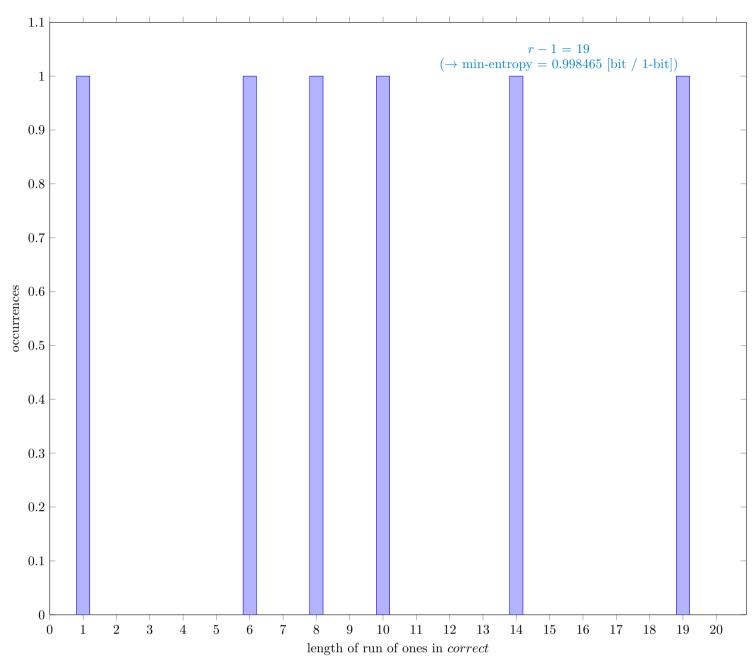


Fig. 25 Distribution of correct

#### 4.19 Supplemental information for traceability

Table 20 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol           | Value    |
|------------------|----------|
| N                | 7999983  |
| C                | 4000606  |
| $P_{ m global}$  | 0.500077 |
| $P'_{ m global}$ | 0.500532 |
| r                | 20       |
| $P_{ m local}$   | 0.36719  |

# 4 References

<sup>[1]</sup> Meltem Sönmez Turan, Elaine Barker, John Kelsey, Kerry A. McKay, Mary L. Baish, Mike Boyle Recommendation for the Entropy Sources Used for Random Bit Generation, NIST Special Publication 800-90B, Jan. 2018 https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-90B.pdf

<sup>[2]</sup> G. Sakurai, Proposed list of corrections for NIST SP 800-90B 6.3 Estimators, Dec. 2022 https://github.com/g-g-sakura/AnotherEntropyEstimationTool/blob/main/documentation/ProposedListOfCorrections\_SP800-90B.pdf