# Report of Entropy estimates based on NIST SP 800-90B non-IID track

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# 1 Identification information

#### 1.1 Identification of acquisition data from entropy source

Table 1 Identification information of acquisition data from entropy source

| URL of the acquisition data                        | https://github.com/usnistgov/SP800-90B_EntropyAssessment/blob/master/bin/biased-random-bytes.bin |
|--|--|
| SHA-256 hash value of the acqui- sition data [hex] | 146bd749 7d8e2d61 a6e8559c 9342ee79 f6005a39 0ee4d776 ba43500d 00eb508d                          |

- Name of the submitter of the acquisition data :
- Brief explanation of the acquisition data (or entropy source) :

### 1.2 Identification of analysis environment

Table 2 Identification information of analysis environment

| Analysis tool        | Name                   | Another entropy estimation tool with extensions     |
|----------------------|------------------------|---|
|                      | Versioning information | 1.0.56  |
|                      | built as               | 64-bit application                                  |
|                      | built by               | Intel C++ Compiler (INTEL_LLVM_COMPILER: 20240102 ) |
|                      | linked libraries       | Boost C++ 1.85.0                                    |
| Analysis environment | Hostname               |   |
|                      | CPU information        | Intel(R) Core(TM)                                   |
|                      | Physical memory size   | MiB   |
|                      | OS name                | Microsoft Windows 11 Pro                            |
|                      | OS version             | 10.0.22631 N/A Build 22631                          |
|                      | System type            | 64-bit  |
|                      | Username               |   |

### 1.3 Identification of analysis conditions

Table 3 Identification information of analysis conditions

| Number of samples      | 1000000                          |
|------------------------|----------------------------------|
| Bits per sample        | 8                                |
| Byte to bit conversion | Most Significant bit (MSb) first |

# 1.4 Identification of analysis method

NIST SP 800-90B [1] 6.3 with corrections [2] is applied

# 2 Executive summary

## 2.1 Numerical results of min-entropy estimates based on non-IID track

Table 4 Numerical results

| Estimator  | $H_{ m original}{}^{ m a}$ | Notes to $H_{\text{original}}$ | $H_{ m bitstring}^{ m \ b}$ | Notes to $H_{\text{bitstring}}$ |
|--|----------------------------|--------------------------------|-----------------------------|---------------------------------|
|  | [bit / 8 - bit]            |                                | [bit / 1 - bit]             |                                 |
| The Most Common Value Estimate                                   | 0.319651                   | see $3.1$                      | 0.151827                    | see 4.1                         |
| The Collision Estimate   | _                          | _                              | 0.0727058                   | see 4.2                         |
| The Markov Estimate  | _                          | _                              | 0.0916044                   | see 4.3                         |
| The Compression Estimate   | _                          | _                              | 0.0631355                   | see 4.4                         |
| The t-Tuple Estimate   | 0.29116                    | see $3.2$                      | 0.0322176                   | see 4.5                         |
| The Longest Repeated Substring (LRS) Estimate                    | 0.519281                   | see $3.3$                      | 0.0648017                   | see 4.6                         |
| Multi Most Common in Window Prediction Estimate                  | 0.319646                   | see $3.4$                      | 0.0419265                   | see 4.7                         |
| The Lag Prediction Estimate                                      | 0.466258                   | see $3.5$                      | 0.0420028                   | see 4.8                         |
| The MultiMMC Prediction Estimate                                 | 0.320277                   | see $3.6$                      | 0.0419265                   | see 4.9                         |
| The LZ78Y Prediction Estimate                                    | 0.330375                   | see $3.7$                      | 0.0419265                   | see 4.10                        |
| The intial entropy source estimate [bit / 8 - bit]               |                            | 0.25                           | 57741                       |                                 |
| $H_I = \min(H_{\text{original}}, 8 \times H_{\text{bitstring}})$ |                            |                                |                             |                                 |

 $<sup>^</sup>a$   $\,$  Entropy estimate of the sequential dataset [source: NIST SP 800-90B [1] 3.1.3]

 $<sup>^</sup>b$  An additional entropy estimation (per bit) for the non-binary sequential dataset [see NIST SP 800-90B [1] 3.1.3]

### 2.2 Visual comparison of min-entropy estimates from original samples

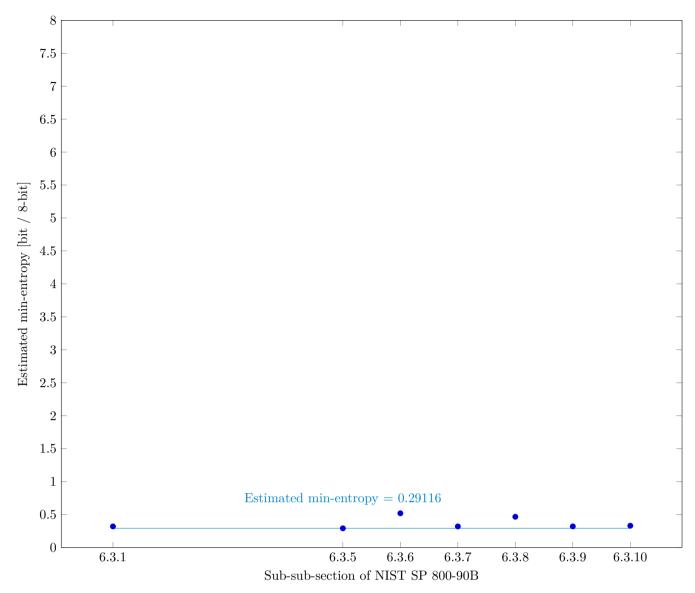


Fig. 1 Estimated Min-Entropy using  $\S 6.3$  of NIST SP 800-90B

### 2.3 Visual comparison of min-entropy estimates by interpreting each sample as bitstring

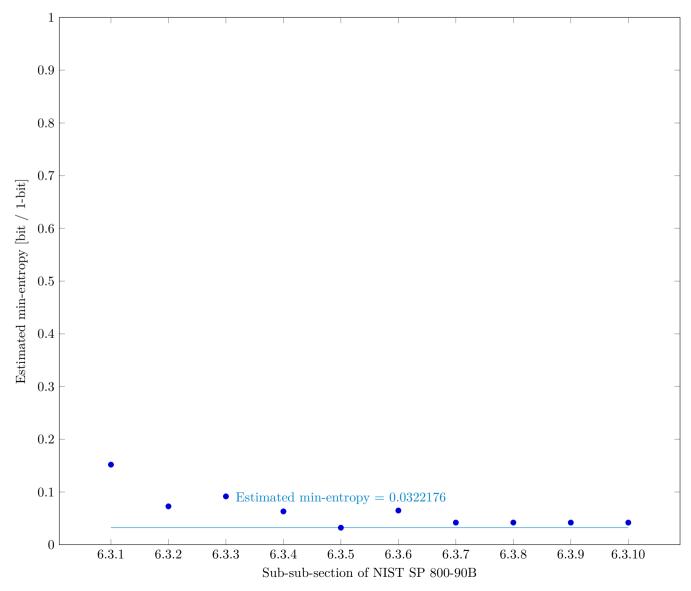


Fig. 2  $\,$  Estimated Min-Entropy using  $\S 6.3$  of NIST SP 800-90B  $\,$ 

# 3 Detailed results of analysis from original samples

# 3.1 The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)



Fig. 3 Distribution of  $x_i$ 

### 3.1.1 Supplemental information for traceability

Table 5 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol    | Value    |
|-----------|----------|
| mode      | 800234   |
| $\hat{p}$ | 0.800234 |
| $p_u$     | 0.801264 |

# 3.2 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)

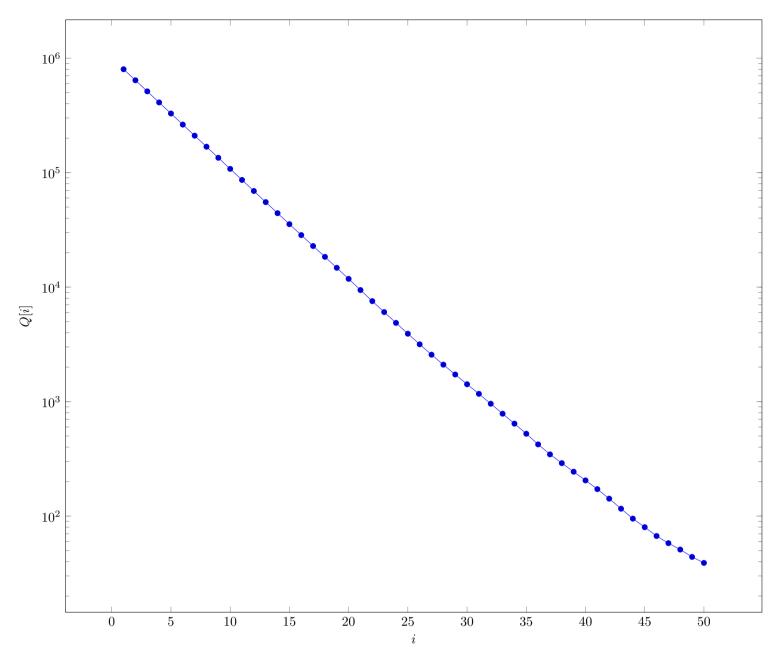


Fig. 4 Intermediate value Q[i] in  $\S 6.3.5$  of NIST SP 800-90B

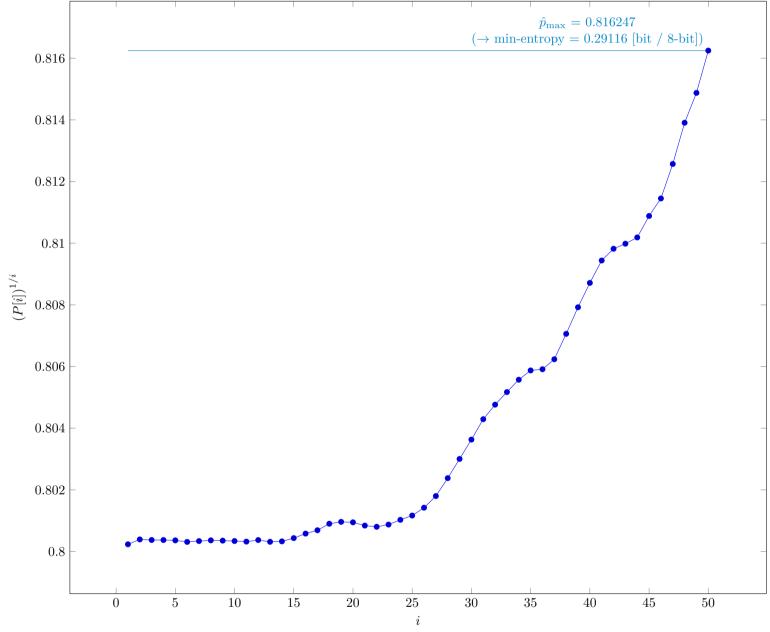


Fig. 5  $P[i]^{1/i}$  in  $\S 6.3.5$  of NIST SP 800-90B

## 3.2.1 Supplemental information for traceability

Table 6 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol                   | Value    |
|--------------------------|----------|
| t                        | 50       |
| $\hat{p}_{\mathrm{max}}$ | 0.816247 |
| $p_u$                    | 0.817245 |

# 3.3 The LRS Estimate (NIST SP 800-90B Section 6.3.6)

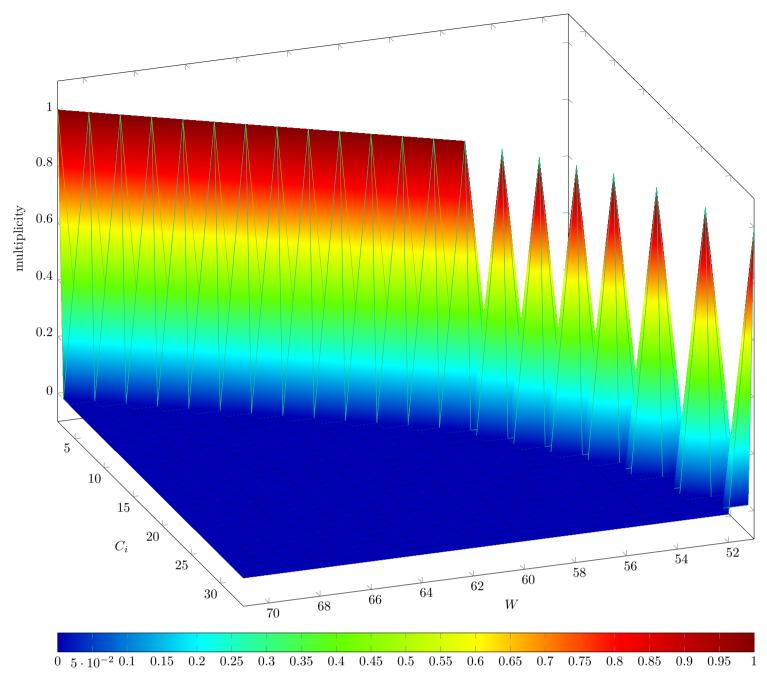


Fig. 6 Estimated W-tuple collision probability in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B



 $Fig. \ 7 \quad Estimated \ average \ collision \ probability \ per \ string \ symbol \ in \ Step \ 3 \ of \ \S 6.3.6 \ of \ NIST \ SP \ 800-90B$ 

### 3.3.1 Supplemental information for traceability

Table 7 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol    | Value    |
|-----------|----------|
| u         | 51       |
| v         | 71       |
| $\hat{p}$ | 0.696535 |
| $p_u$     | 0.697719 |

## 3.4 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

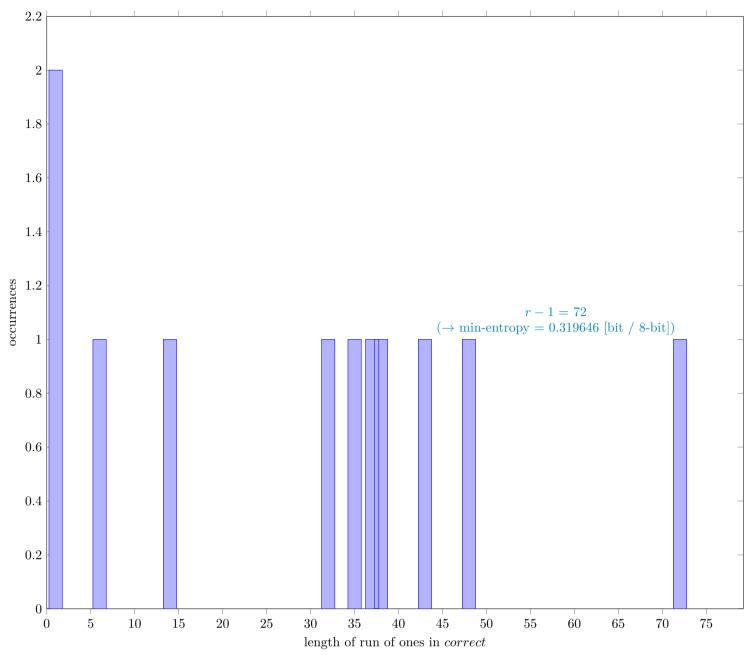


Fig. 8 Distribution of correct

### 3.4.1 Supplemental information for traceability

Table 8 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol           | Value    |
|------------------|----------|
| N                | 999937   |
| C                | 800186   |
| $P_{ m global}$  | 0.800236 |
| $P'_{ m global}$ | 0.801266 |
| r                | 73       |
| $P_{ m local}$   | 0.794039 |

# 3.5 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

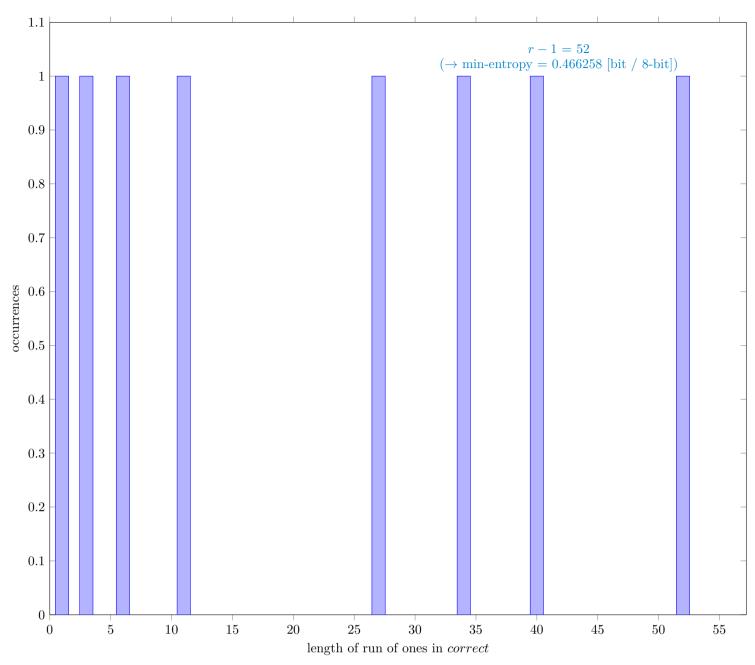


Fig. 9 Distribution of correct

### 3.5.1 Supplemental information for traceability

Table 9 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol           | Value    |
|------------------|----------|
| N                | 999999   |
| C                | 639881   |
| $P_{ m global}$  | 0.639882 |
| $P'_{ m global}$ | 0.641118 |
| r                | 53       |
| $P_{ m local}$   | 0.723839 |

## 3.6 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

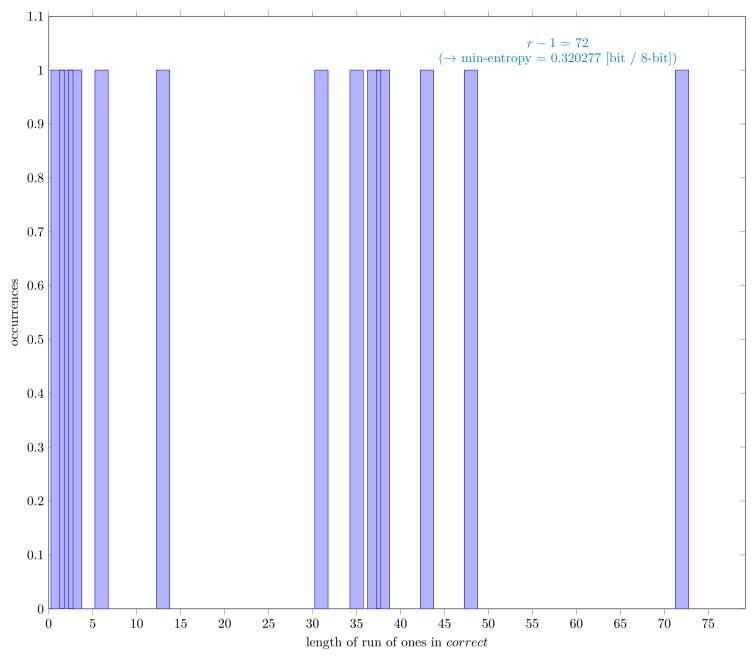


Fig. 10 Distribution of correct

### 3.6.1 Supplemental information for traceability

Table 10 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol               | Value    |
|----------------------|----------|
| N                    | 999998   |
| C                    | 799884   |
| $P_{ m global}$      | 0.799886 |
| $P'_{ m global}$     | 0.800916 |
| r                    | 73       |
| $P_{\mathrm{local}}$ | 0.794038 |

# 3.7 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

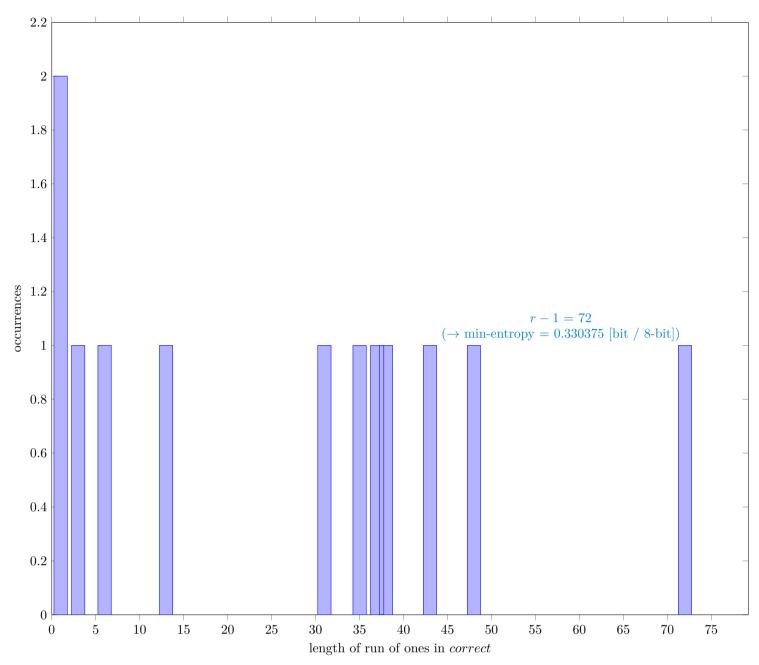


Fig. 11 Distribution of correct

### 3.7.1 Supplemental information for traceability

Table 11 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol           | Value    |
|------------------|----------|
| N                | 999983   |
| C                | 794275   |
| $P_{ m global}$  | 0.794289 |
| $P'_{ m global}$ | 0.79533  |
| r                | 73       |
| $P_{ m local}$   | 0.794038 |

# 4 Detailed results of analysis by interpreting each sample as bitstrings

# 4.1 The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)



Fig. 12 Distribution of  $x_i$ 

#### 4.1.1 Supplemental information for traceability

 $\begin{tabular}{ll} Table 12 & Supplemental information for traceability (NIST SP 800-90B Section 6.3.1) \\ \end{tabular}$ 

| Symbol    | Value    |
|-----------|----------|
| mode      | 7198690  |
| $\hat{p}$ | 0.899836 |
| $p_u$     | 0.90011  |

# 4.2 The Collision Estimate (NIST SP 800-90B Section 6.3.2)

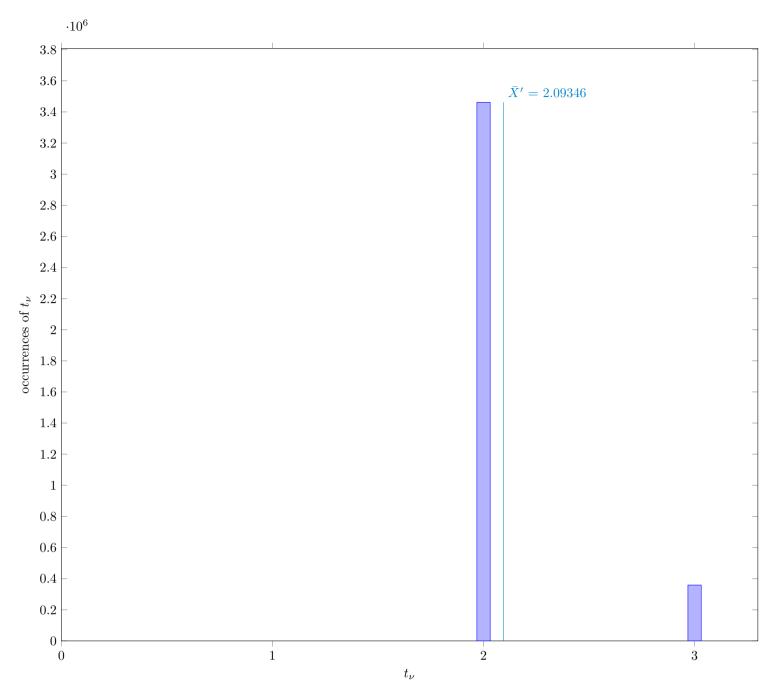


Fig. 13 Distribution of intermediate value  $t_{\nu}$ 



Fig. 14 Solution to the equation in step 7

## 4.2.1 Supplemental information for traceability

Table 13 Supplemental information for traceability (NIST SP 800-90B Section 6.3.2)

| Symbol         | Value    |
|----------------|----------|
| p              | 0.950853 |
| $\bar{X}$      | 2.09385  |
| $ar{X}'$       | 2.09346  |
| $\hat{\sigma}$ | 0.291617 |

## 4.3 The Markov Estimate (NIST SP 800-90B Section 6.3.3)

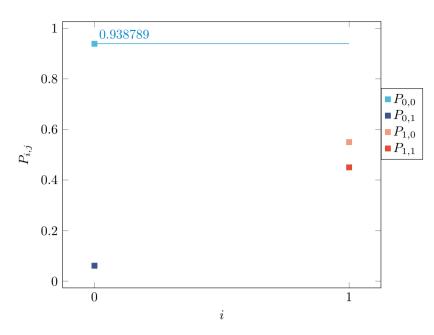


Fig. 15 Transition probability  $P_{i,j}$  of §6.3.3 of NIST SP 800-90B

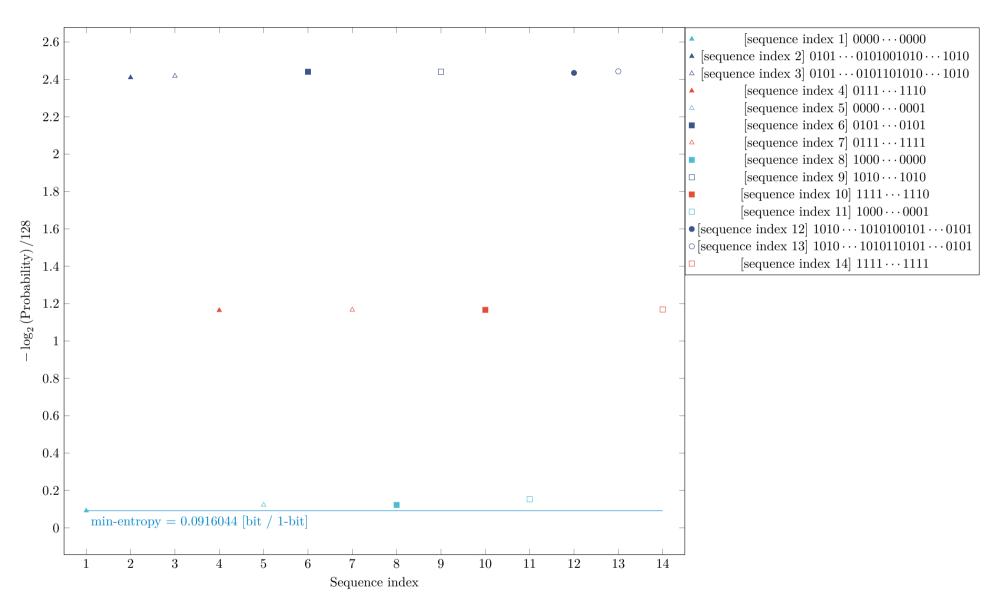


Fig. 16 Estimated Min-Entropy using §6.3.3 of NIST SP 800-90B

## 4.4 The Compression Estimate (NIST SP 800-90B Section 6.3.4)

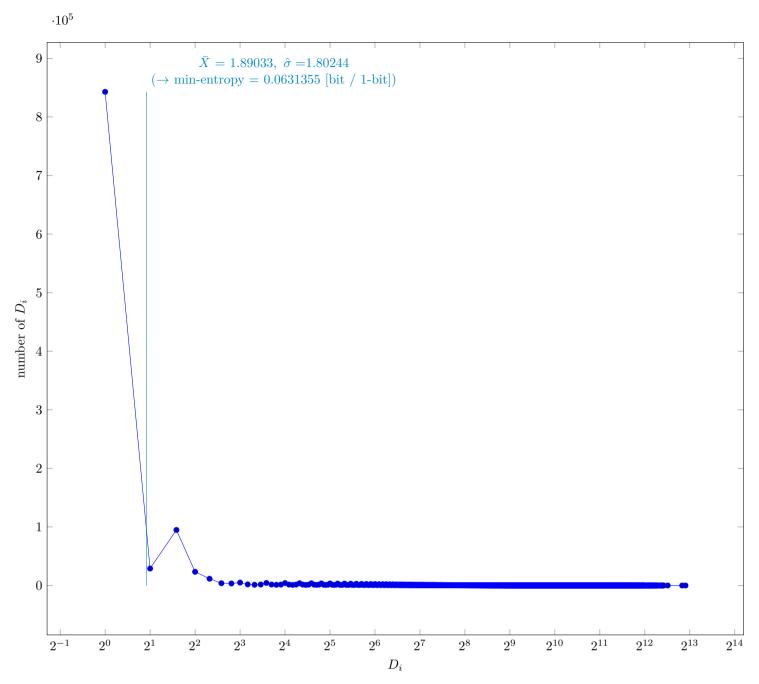


Fig. 17 Distribution of intermediate value  $D_i$ 

## 4.4.1 Supplemental information for traceability

Table 14 Supplemental information for traceability (NIST SP 800-90B Section 6.3.4)

| Symbol         | Value   |
|----------------|---------|
| p              | 0.76907 |
| $\bar{X}$      | 1.89033 |
| $\hat{\sigma}$ | 1.80244 |
| $\bar{X}'$     | 1.88631 |

# 4.5 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)

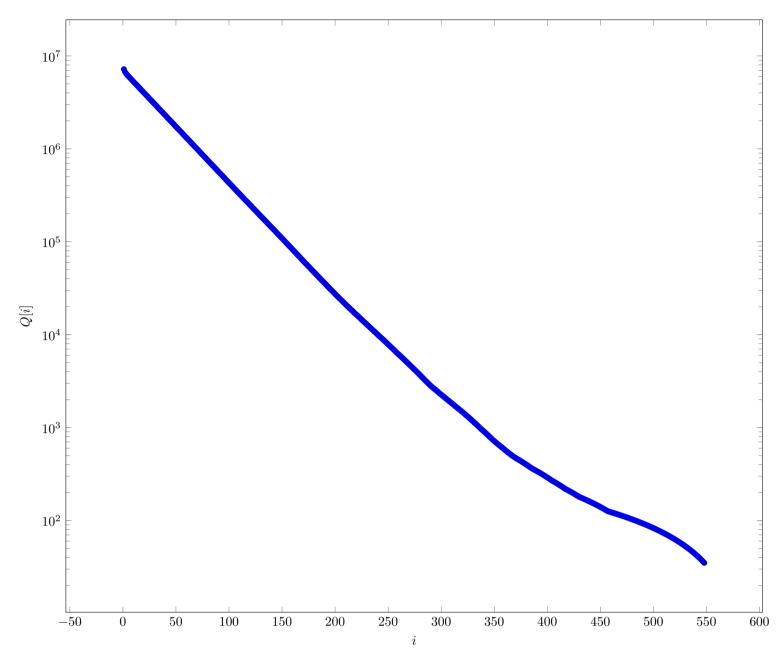


Fig. 18 Intermediate value Q[i] in  $\S 6.3.5$  of NIST SP 800-90B



Fig. 19  $P[i]^{1/i}$  in §6.3.5 of NIST SP 800-90B

## 4.5.1 Supplemental information for traceability

Table 15 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol                   | Value    |
|--------------------------|----------|
| t                        | 548      |
| $\hat{p}_{\mathrm{max}}$ | 0.977782 |
| $p_u$                    | 0.977916 |
|                          |          |

# 4.6 The LRS Estimate (NIST SP 800-90B Section 6.3.6)

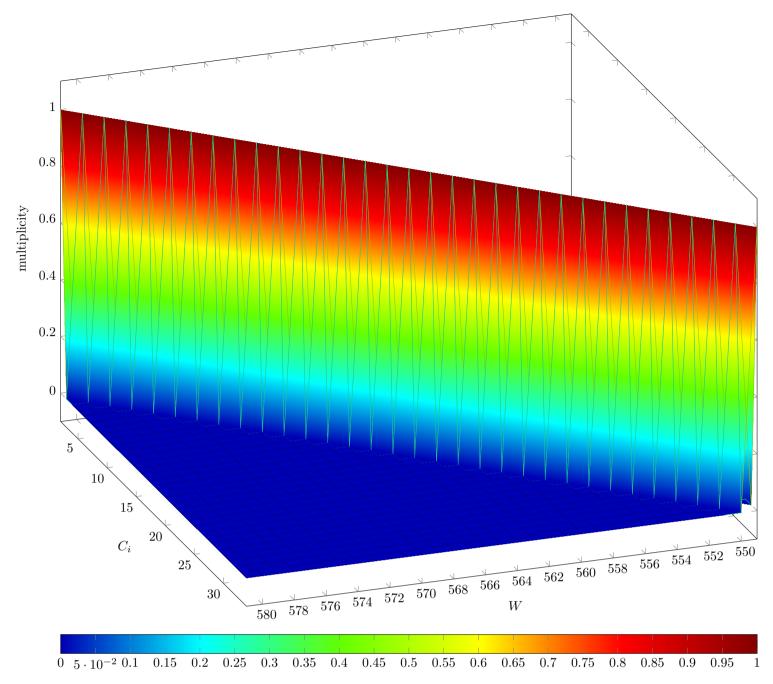
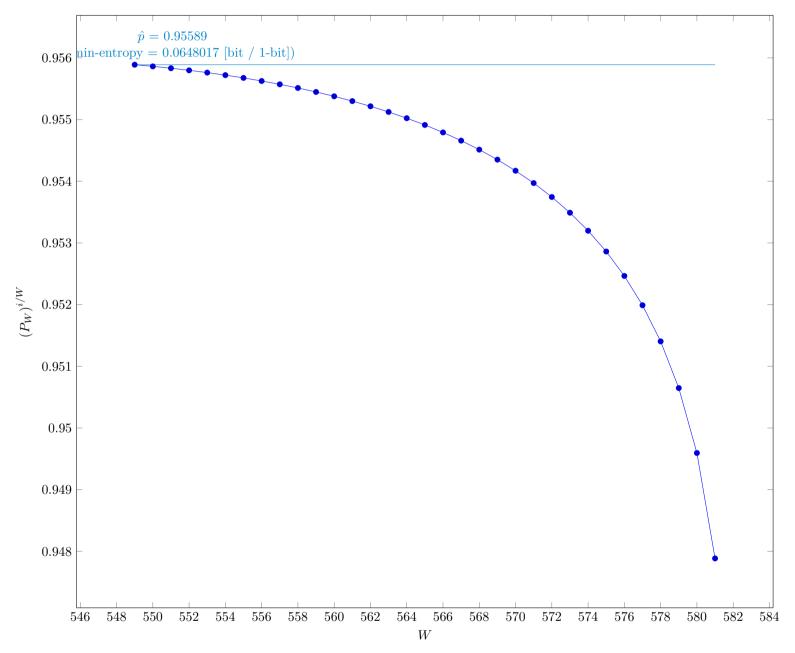


Fig. 20 Estimated W-tuple collision probability in Step 3 of  $\S 6.3.6$  of NIST SP 800-90B



 $Fig.~21~Estimated~average~collision~probability~per~string~symbol~in~Step~3~of~\S 6.3.6~of~NIST~SP~800-90B$ 

### 4.6.1 Supplemental information for traceability

Table 16 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol    | Value    |
|-----------|----------|
| u         | 549      |
| v         | 581      |
| $\hat{p}$ | 0.95589  |
| $p_u$     | 0.956077 |

## 4.7 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

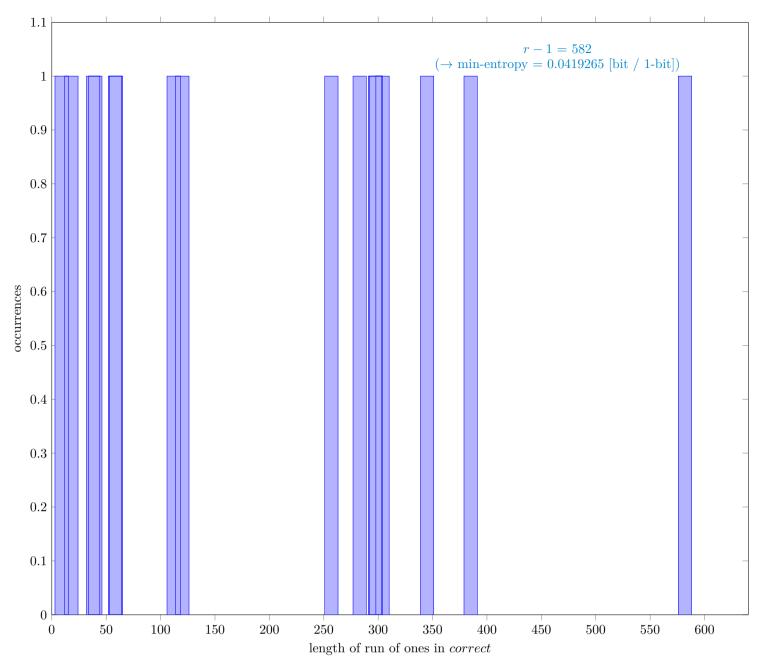


Fig. 22 Distribution of correct

### 4.7.1 Supplemental information for traceability

Table 17 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol                | Value    |
|-----------------------|----------|
| N                     | 7999937  |
| C                     | 7198538  |
| $P_{\mathrm{global}}$ | 0.899824 |
| $P'_{ m global}$      | 0.900098 |
| r                     | 583      |
| $P_{\text{local}}$    | 0.971357 |

# 4.8 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

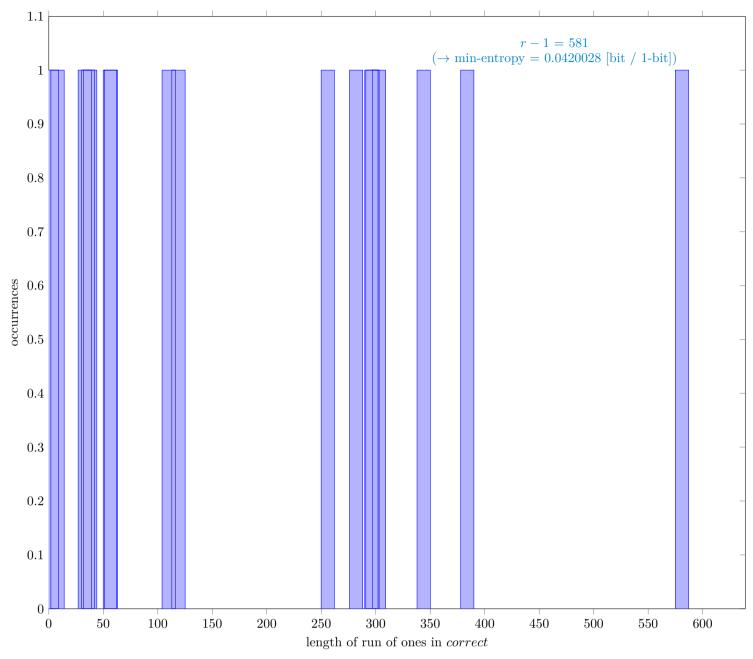


Fig. 23 Distribution of correct

### 4.8.1 Supplemental information for traceability

Table 18 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol                | Value    |
|-----------------------|----------|
| N                     | 7999999  |
| C                     | 7118705  |
| $P_{\mathrm{global}}$ | 0.889838 |
| $P'_{ m global}$      | 0.890123 |
| r                     | 582      |
| $P_{ m local}$        | 0.971306 |

## 4.9 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

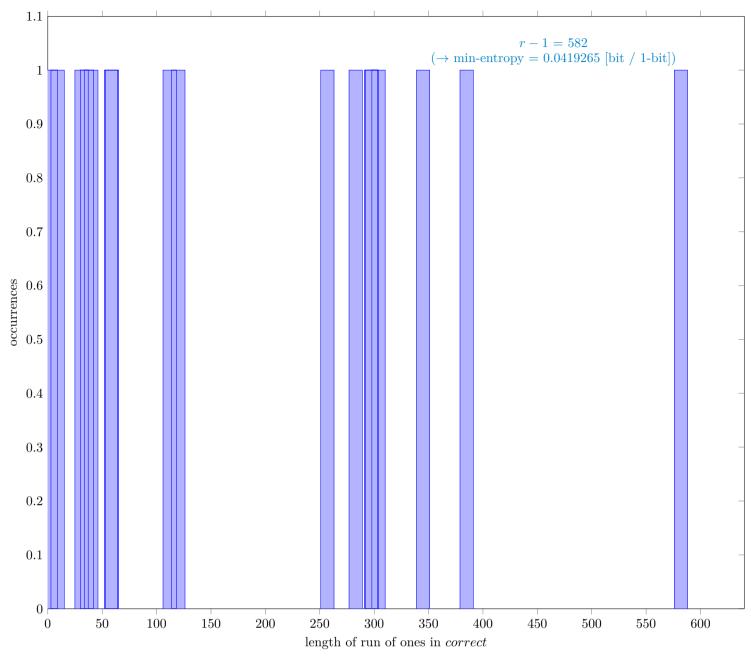


Fig. 24 Distribution of correct

### 4.9.1 Supplemental information for traceability

Table 19 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol                | Value    |
|-----------------------|----------|
| N                     | 7999998  |
| C                     | 7198679  |
| $P_{\mathrm{global}}$ | 0.899835 |
| $P'_{ m global}$      | 0.900109 |
| r                     | 583      |
| $P_{ m local}$        | 0.971357 |

## 4.10 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

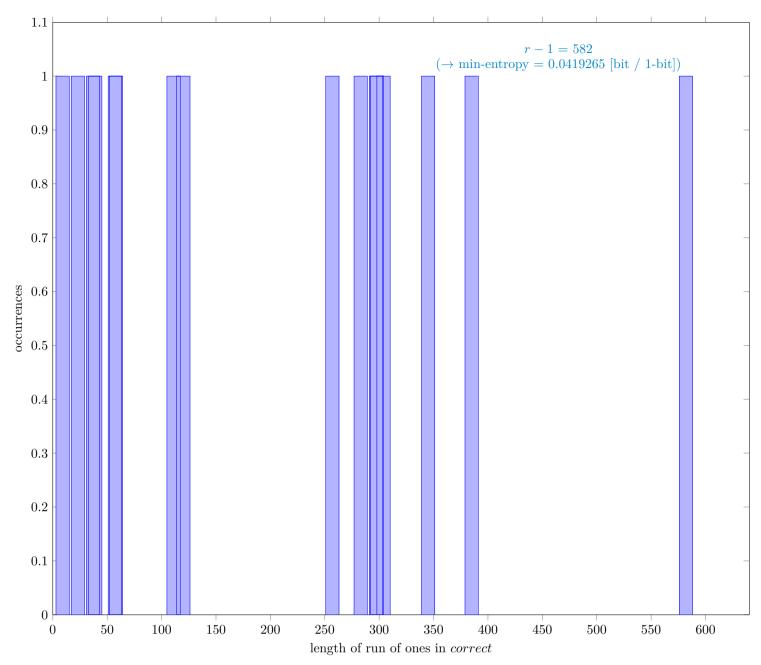


Fig. 25 Distribution of correct

#### 4.10.1 Supplemental information for traceability

Table 20 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol               | Value    |
|----------------------|----------|
| N                    | 7999983  |
| C                    | 7198670  |
| $P_{ m global}$      | 0.899836 |
| $P'_{ m global}$     | 0.900109 |
| r                    | 583      |
| $P_{\mathrm{local}}$ | 0.971357 |

# 4 References

<sup>[1]</sup> Meltem Sönmez Turan, Elaine Barker, John Kelsey, Kerry A. McKay, Mary L. Baish, Mike Boyle, Recommendation for the Entropy Sources Used for Random Bit Generation, NIST Special Publication 800-90B, Jan. 2018 https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-90B.pdf

<sup>[2]</sup> G. Sakurai, Proposed list of corrections for NIST SP 800-90B 6.3 Estimators, Dec. 2022 https://github.com/g-g-sakura/AnotherEntropyEstimationTool/blob/main/documentation/ProposedListOfCorrections\_SP800-90B.pdf