

## Report of Entropy estimates based on NIST SP 800-90B non-IID track

2023-Aug-20 17:12:40.856579

## 1 Identification information

### 1.1 Identification of acquisition data from entropy source

Table 1 Identification information of acquisition data from entropy source

|  |   |
|--|---|
| URL of the acquisition data                      | <a href="https://github.com/usnistgov/SP800-90B_EntropyAssessment/blob/master/bin/biased-random-bytes.bin">https://github.com/usnistgov/SP800-90B_EntropyAssessment/blob/master/bin/biased-random-bytes.bin</a> |
| SHA-256 hash value of the acquisition data [hex] | 146bd749 7d8e2d61 a6e8559c 9342ee79 f6005a39 0ee4d776 ba43500d 00eb508d   |

- Name of the submitter of the acquisition data :
- Brief explanation of the acquisition data (or entropy source) :

## 1.2 Identification of analysis environment

Table 2 Identification information of analysis environment

|                      |                        |  |
|----------------------|------------------------|--|
| Analysis tool        | Name                   | Another entropy estimation tool with extensions        |
|                      | Versioning information | 1.0.50   |
|                      | built as               | 64-bit application                                     |
|                      | built by               | Intel C++ Compiler ( __INTEL_LLVM_COMPILER: 20230200 ) |
|                      | linked libraries       | Boost C++ 1.83.0                                       |
| Analysis environment | Hostname               | [REDACTED]   |
|                      | CPU information        | AMD Ryzen [REDACTED]                                   |
|                      | Physical memory size   | [REDACTED] MiB   |
|                      | OS information         | Windows 10 or greater 64-bit                           |
|                      | Username               | [REDACTED]   |

### 1.3 Identification of analysis conditions

Table 3 Identification information of analysis conditions

|                        |                                  |
|------------------------|----------------------------------|
| Number of samples      | 1000000                          |
| Bits per sample        | 8                                |
| Byte to bit conversion | Most Significant bit (MSb) first |

#### 1.4 Identification of analysis method

NIST SP 800-90B [1] 6.3 with corrections [2] is applied

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Executive summary

2.1

Numerical results of min-entropy estimates based on non-IID track

Table 4 Numerical results

| Estimator   | $H_{\text{original}}$ <sup>a</sup><br>[bit / 8 - bit] | Notes to $H_{\text{original}}$ | $H_{\text{bitstring}}$ <sup>b</sup><br>[bit / 1 - bit] | Notes to $H_{\text{bitstring}}$ |
|---|---|--------------------------------|--|---------------------------------|
| The Most Common Value Estimate  | 0.319651  | see 3.1                        | 0.151827   | see 4.1                         |
| The Collision Estimate  | —   | —                              | 0.0727058  | see 4.2                         |
| The Markov Estimate   | —   | —                              | 0.0916044  | see 4.3                         |
| The Compression Estimate  | —   | —                              | 0.0631355  | see 4.4                         |
| The t-Tuple Estimate  | 0.29116   | see 3.2                        | 0.0322176  | see 4.5                         |
| The Longest Repeated Substring (LRS) Estimate   | 0.519281  | see 3.3                        | 0.0648017  | see 4.6                         |
| Multi Most Common in Window Prediction Estimate   | 0.319646  | see 3.4                        | 0.0419265  | see 4.7                         |
| The Lag Prediction Estimate   | 0.466258  | see 3.5                        | 0.0420028  | see 4.8                         |
| The MultiMMC Prediction Estimate  | 0.320277  | see 3.6                        | 0.0419265  | see 4.9                         |
| The LZ78Y Prediction Estimate   | 0.330375  | see 3.7                        | 0.0419265  | see 4.10                        |
| The initial entropy source estimate [bit / 8 - bit]<br>$H_I = \min(H_{\text{original}}, 8 \times H_{\text{bitstring}})$       | 0.257741  |                                |  |                                 |
| <sup>a</sup> Entropy estimate of the sequential dataset [source: NIST SP 800-90B [1] 3.1.3]                                   |   |                                |  |                                 |
| <sup>b</sup> An additional entropy estimation (per bit) for the non-binary sequential dataset [see NIST SP 800-90B [1] 3.1.3] |   |                                |  |                                 |

## 2.2 Visual comparison of min-entropy estimates from original samples

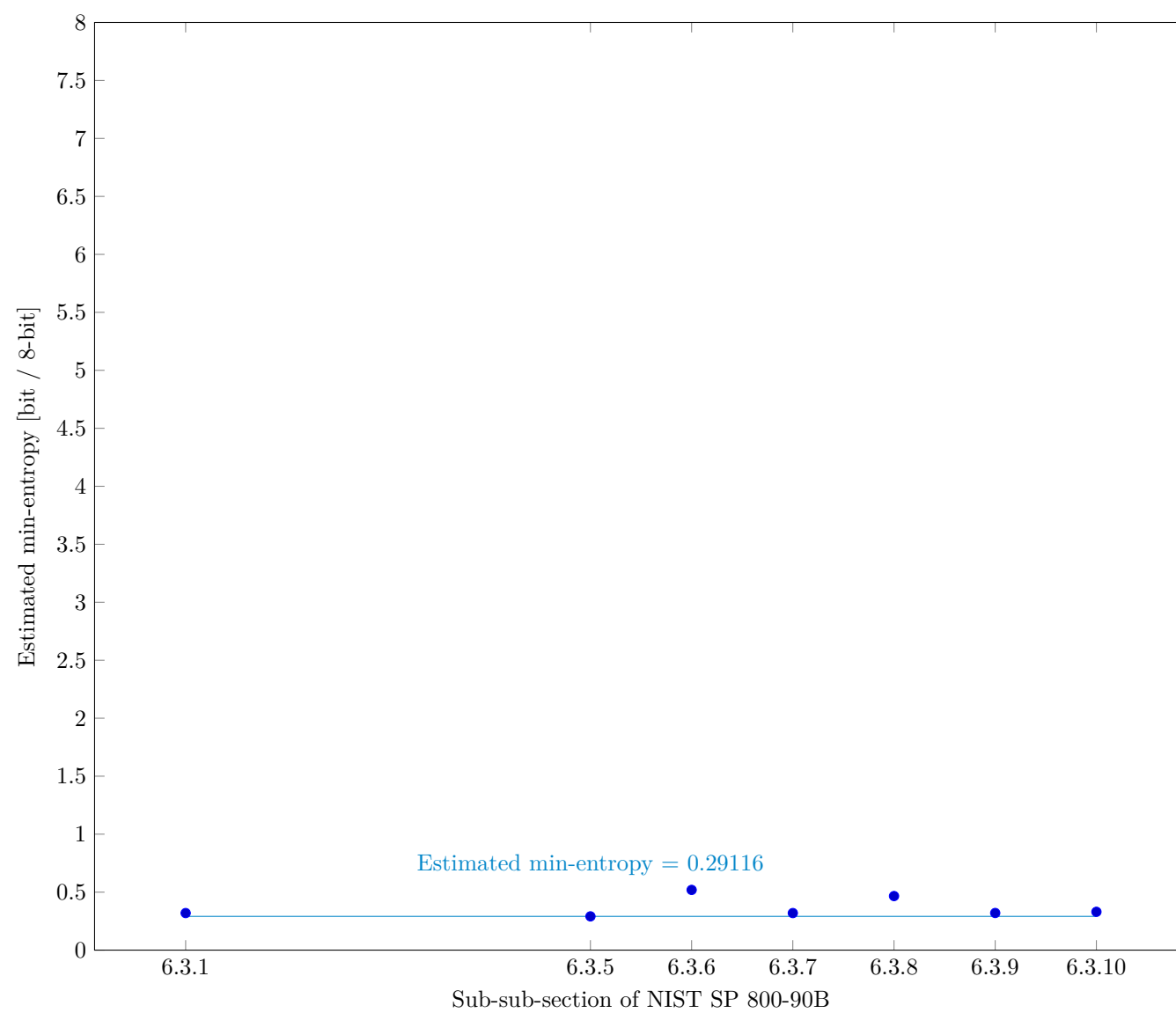


Fig. 1 Estimated Min-Entropy using §6.3 of NIST SP 800-90B

## 2.3 Visual comparison of min-entropy estimates by interpreting each sample as bitstring

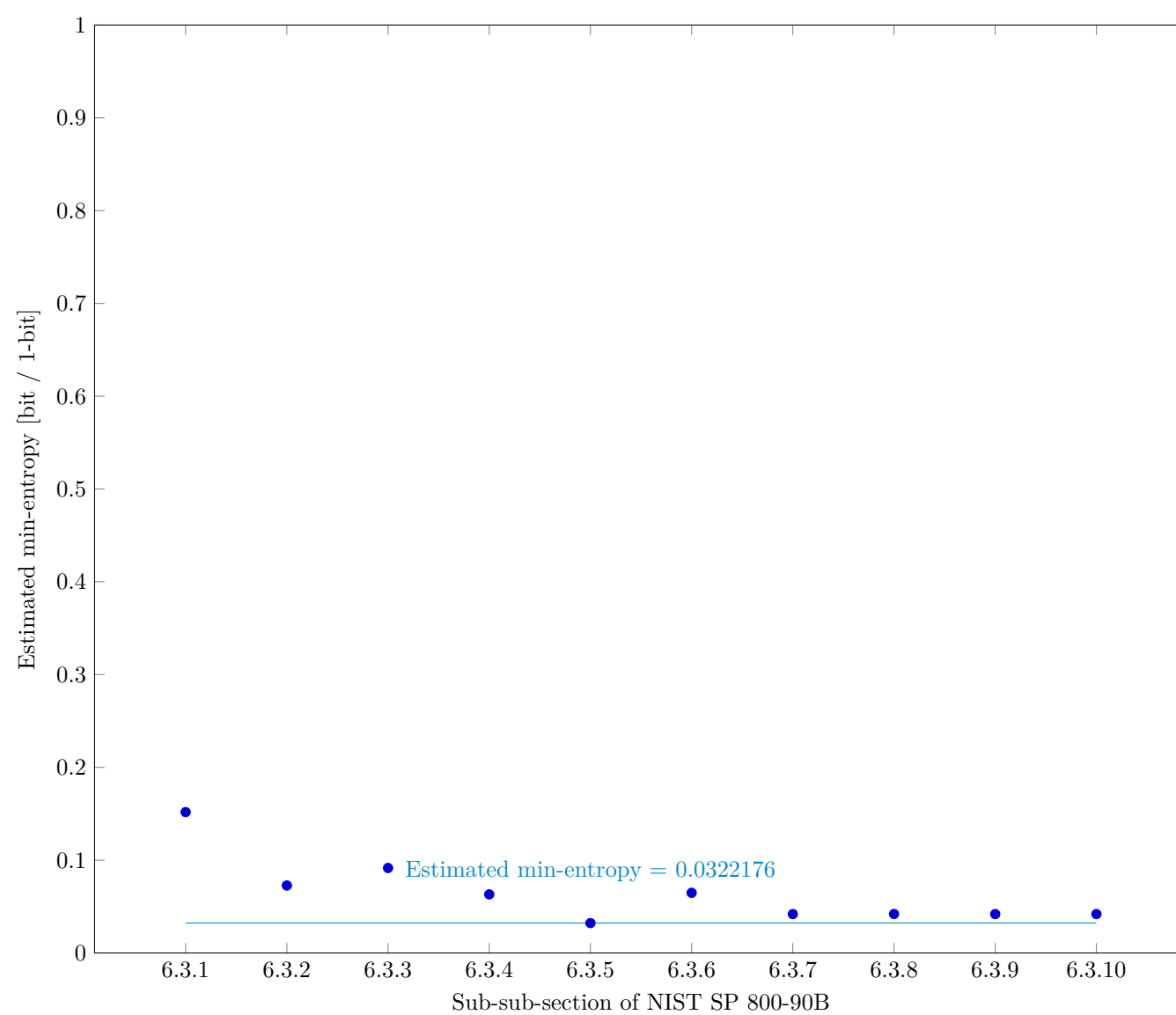


Fig. 2 Estimated Min-Entropy using §6.3 of NIST SP 800-90B

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Detailed results of analysis from original samples

3.1

The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)

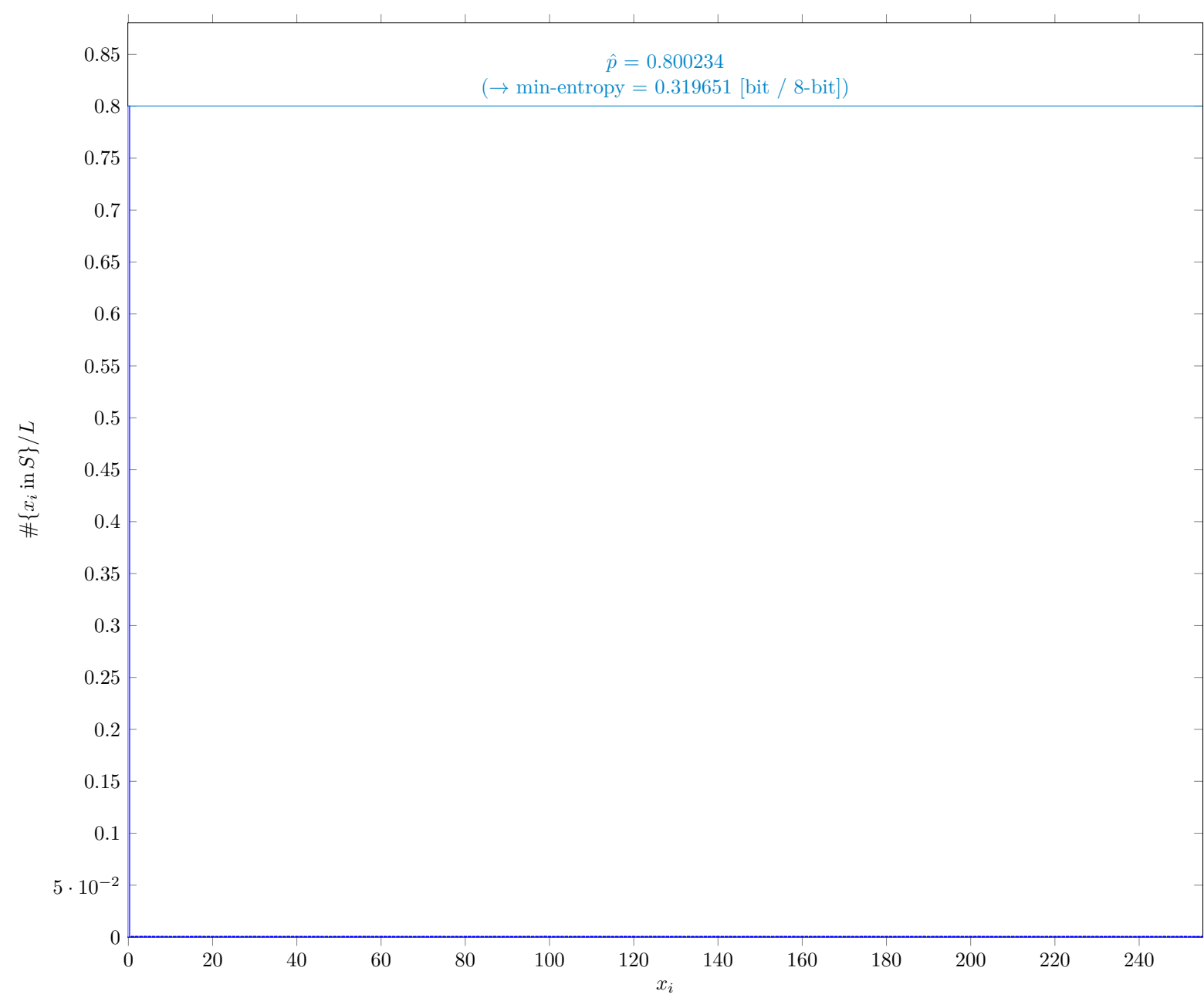


Fig. 3 Distribution of  $x_i$

3.1.1

Supplemental information for traceability

Table 5 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol    | Value    |
|-----------|----------|
| mode      | 800234   |
| $\hat{p}$ | 0.800234 |
| $p_u$     | 0.801264 |

### 3.2 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)

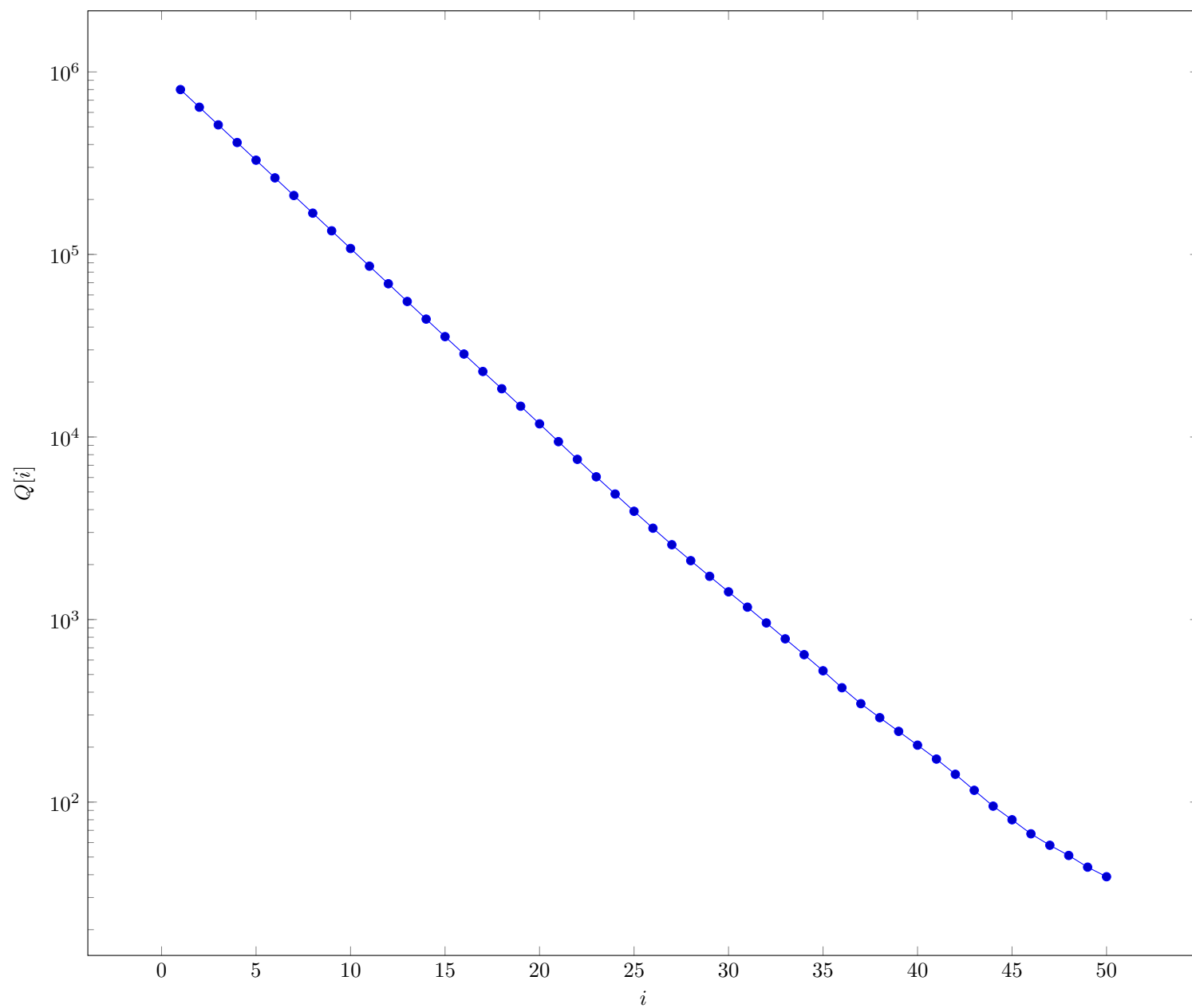


Fig. 4 Intermediate value  $Q[i]$  in §6.3.5 of NIST SP 800-90B

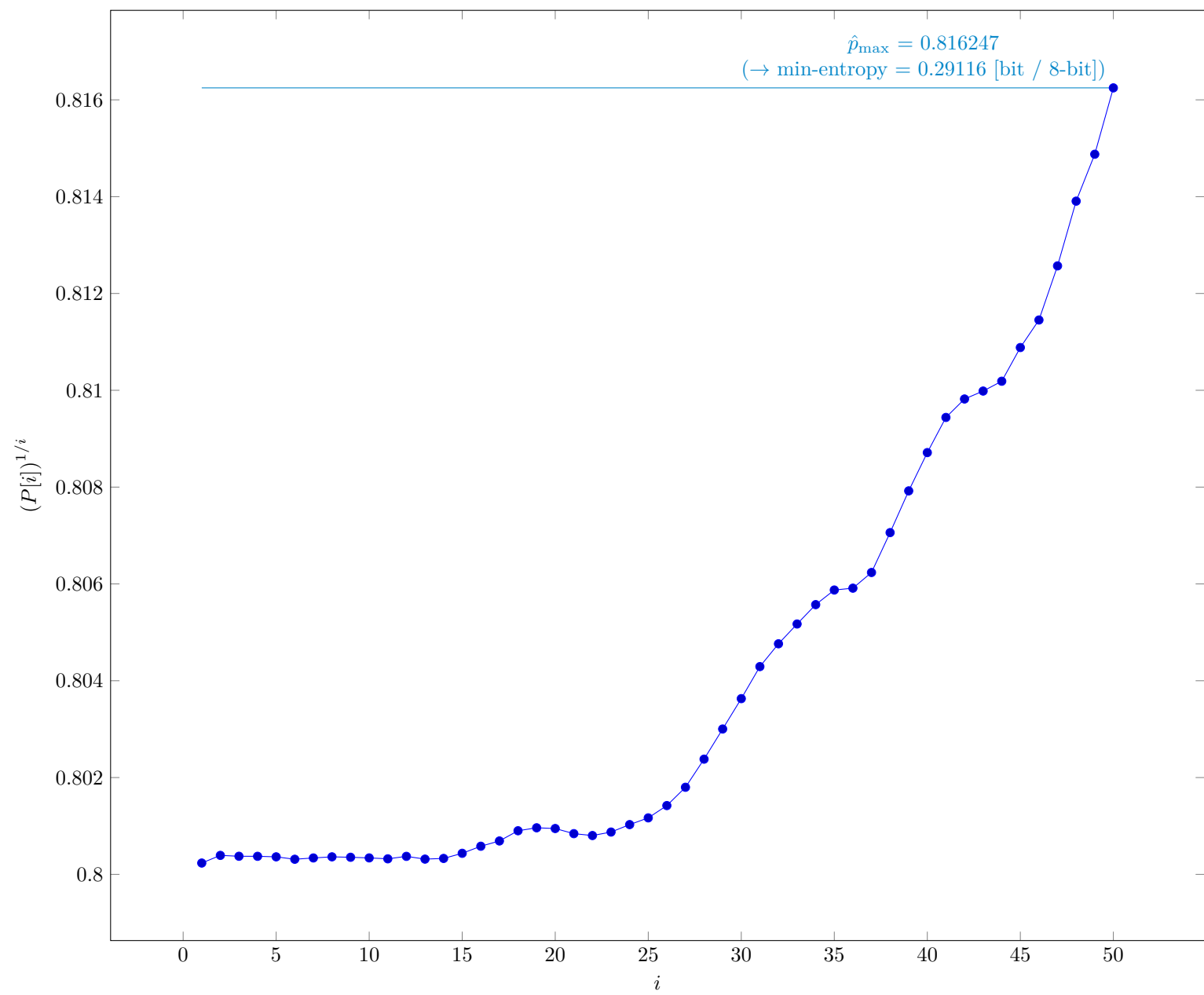


Fig. 5  $P[i]^{1/i}$  in §6.3.5 of NIST SP 800-90B

3.2.1 Supplemental information for traceability

Table 6 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol           | Value    |
|------------------|----------|
| $t$              | 50       |
| $\hat{p}_{\max}$ | 0.816247 |
| $p_u$            | 0.817245 |

### 3.3 The LRS Estimate (NIST SP 800-90B Section 6.3.6)

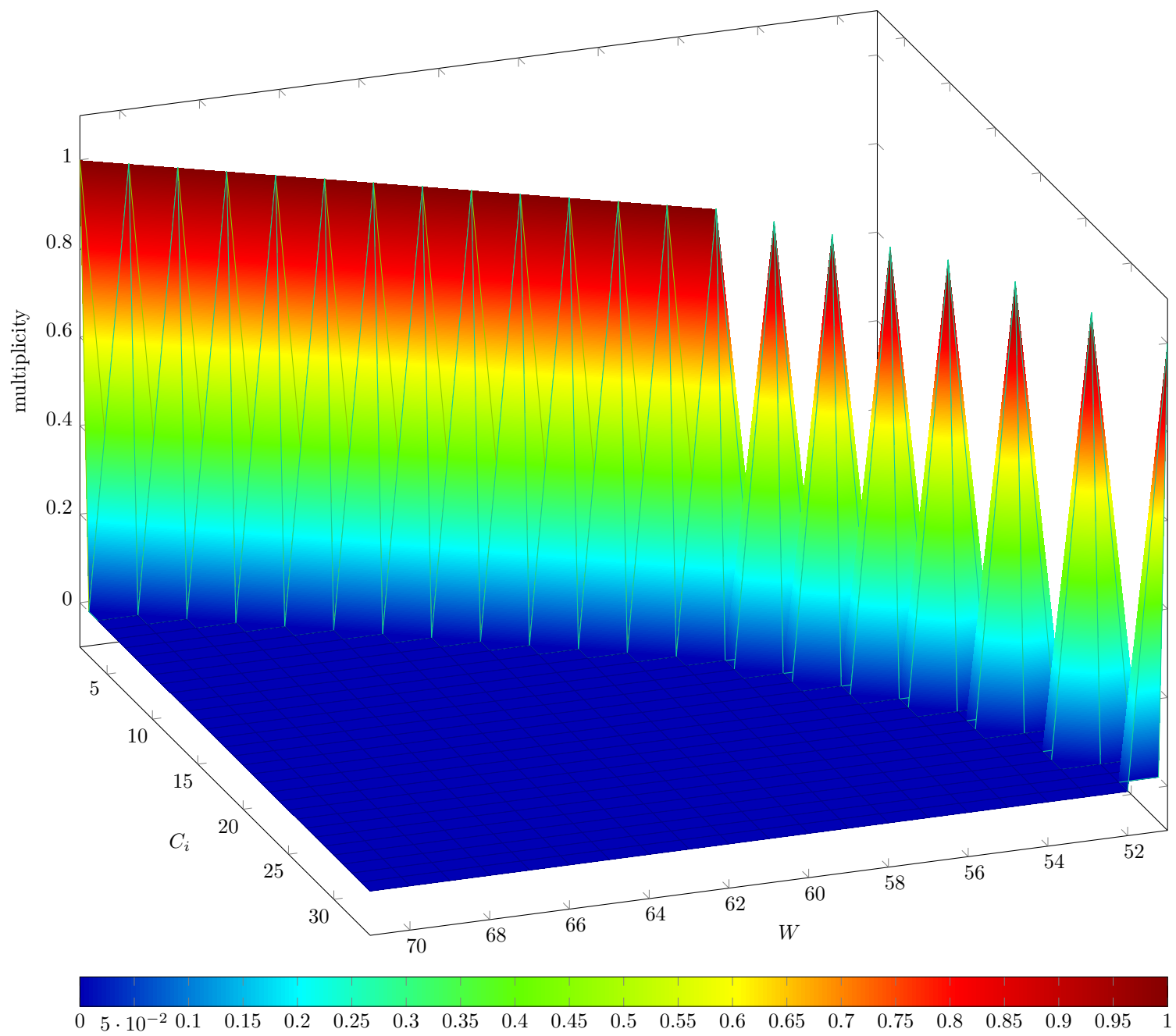


Fig. 6 Estimated  $W$ -tuple collision probability in Step 3 of §6.3.6 of NIST SP 800-90B



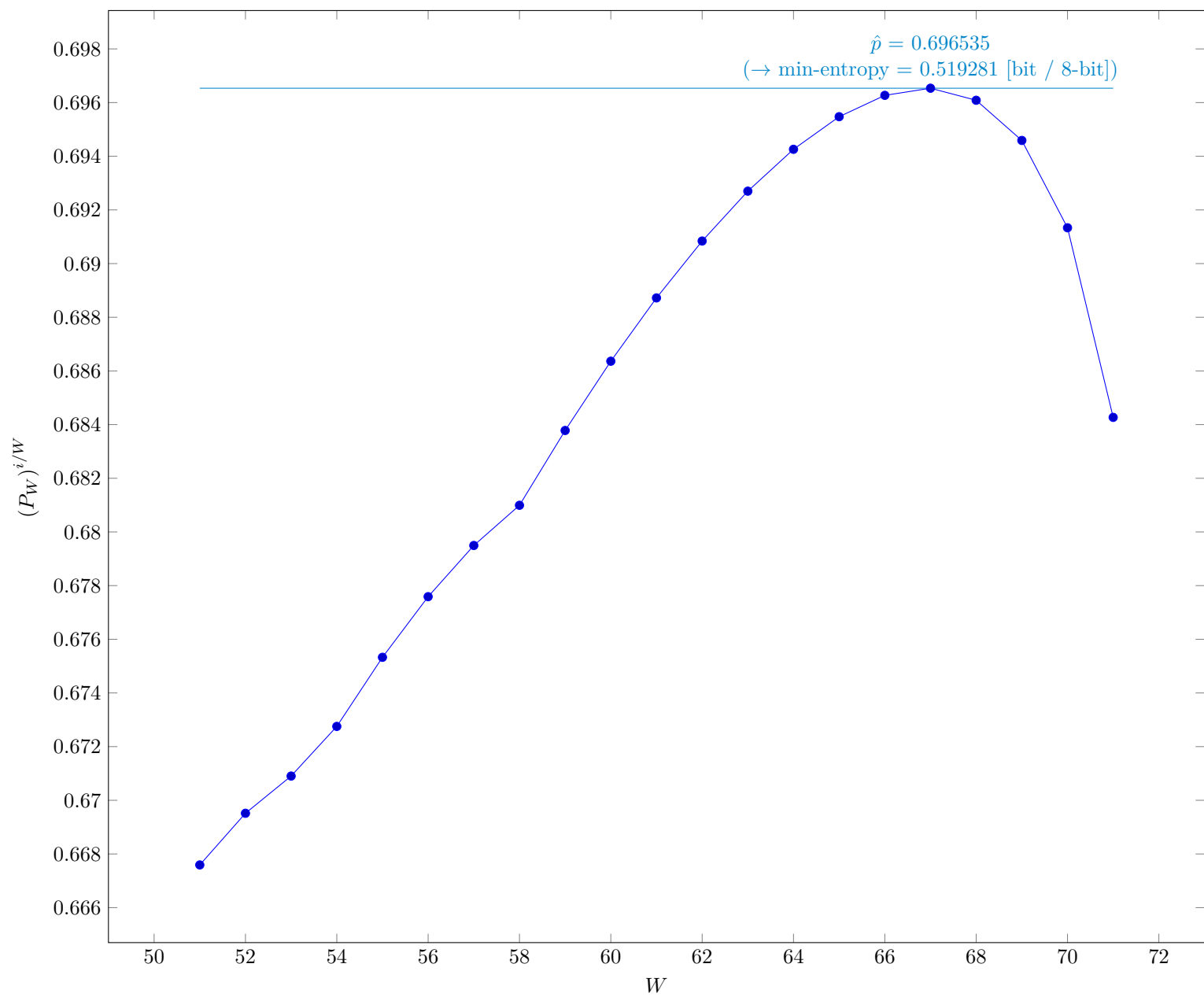


Fig. 7 Estimated average collision probability per string symbol in Step 3 of §6.3.6 of NIST SP 800-90B

3.3.1 Supplemental information for traceability

Table 7 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol    | Value    |
|-----------|----------|
| $u$       | 51       |
| $v$       | 71       |
| $\hat{p}$ | 0.696535 |
| $p_u$     | 0.697719 |

### 3.4 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

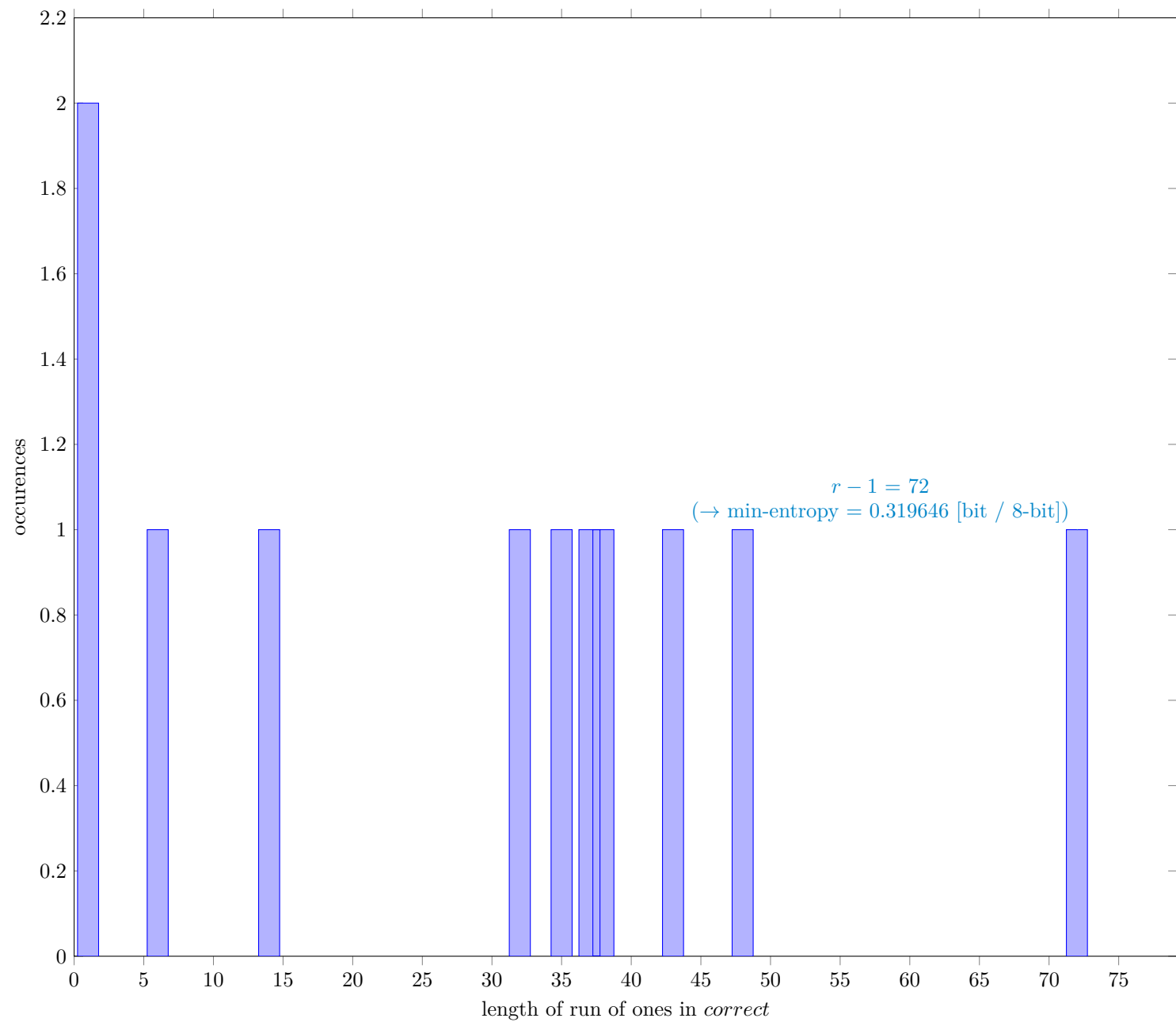


Fig. 8 Distribution of *correct*

#### 3.4.1 Supplemental information for traceability

Table 8 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol               | Value    |
|----------------------|----------|
| $N$                  | 999937   |
| $C$                  | 800186   |
| $P_{\text{global}}$  | 0.800236 |
| $P'_{\text{global}}$ | 0.801266 |
| $r$                  | 73       |
| $P_{\text{local}}$   | 0.794039 |

3.5 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

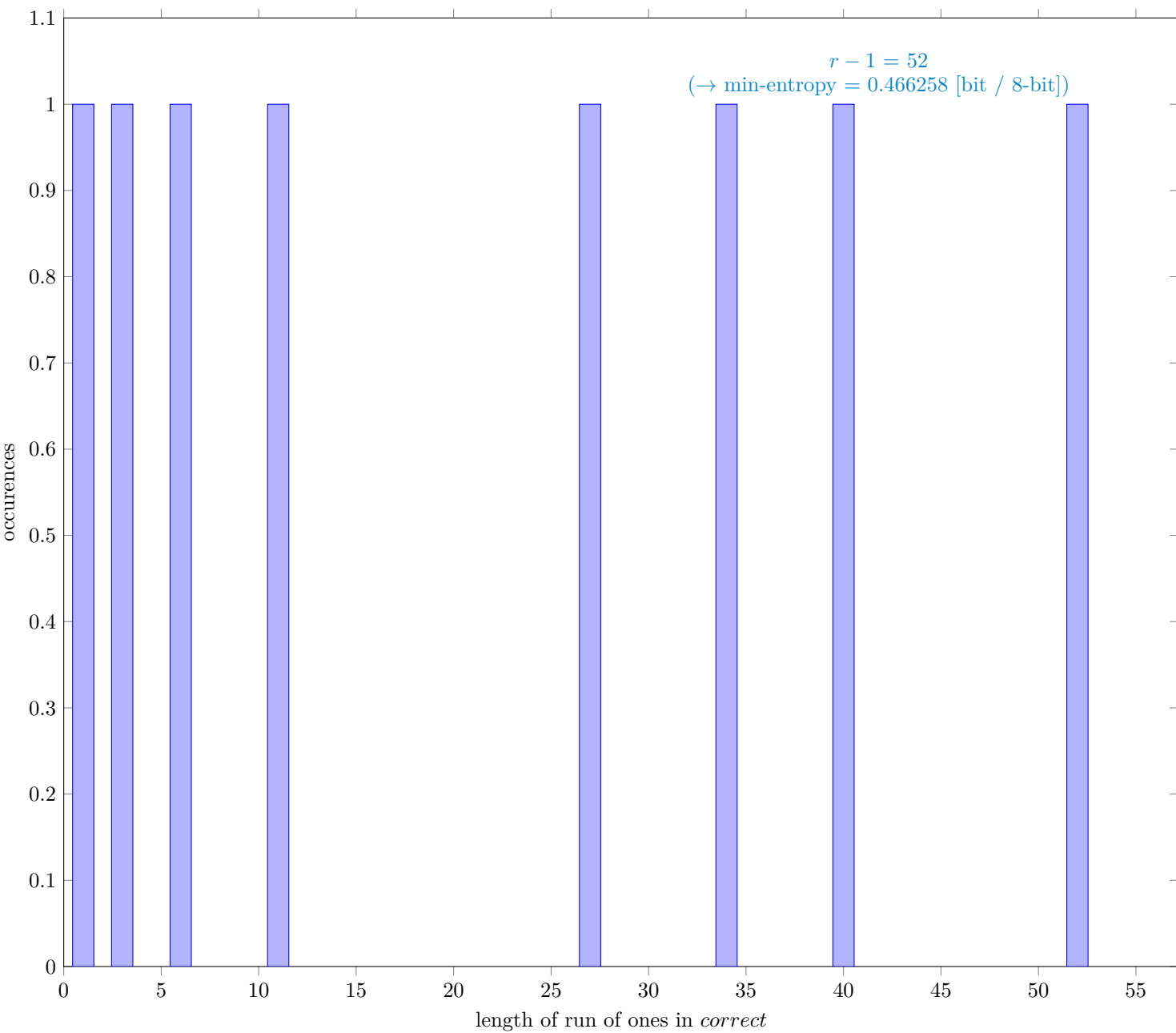


Fig. 9 Distribution of *correct*

3.5.1 Supplemental information for traceability

Table 9 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol               | Value    |
|----------------------|----------|
| $N$                  | 999999   |
| $C$                  | 639881   |
| $P_{\text{global}}$  | 0.639882 |
| $P'_{\text{global}}$ | 0.641118 |
| $r$                  | 53       |
| $P_{\text{local}}$   | 0.723839 |

### 3.6 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

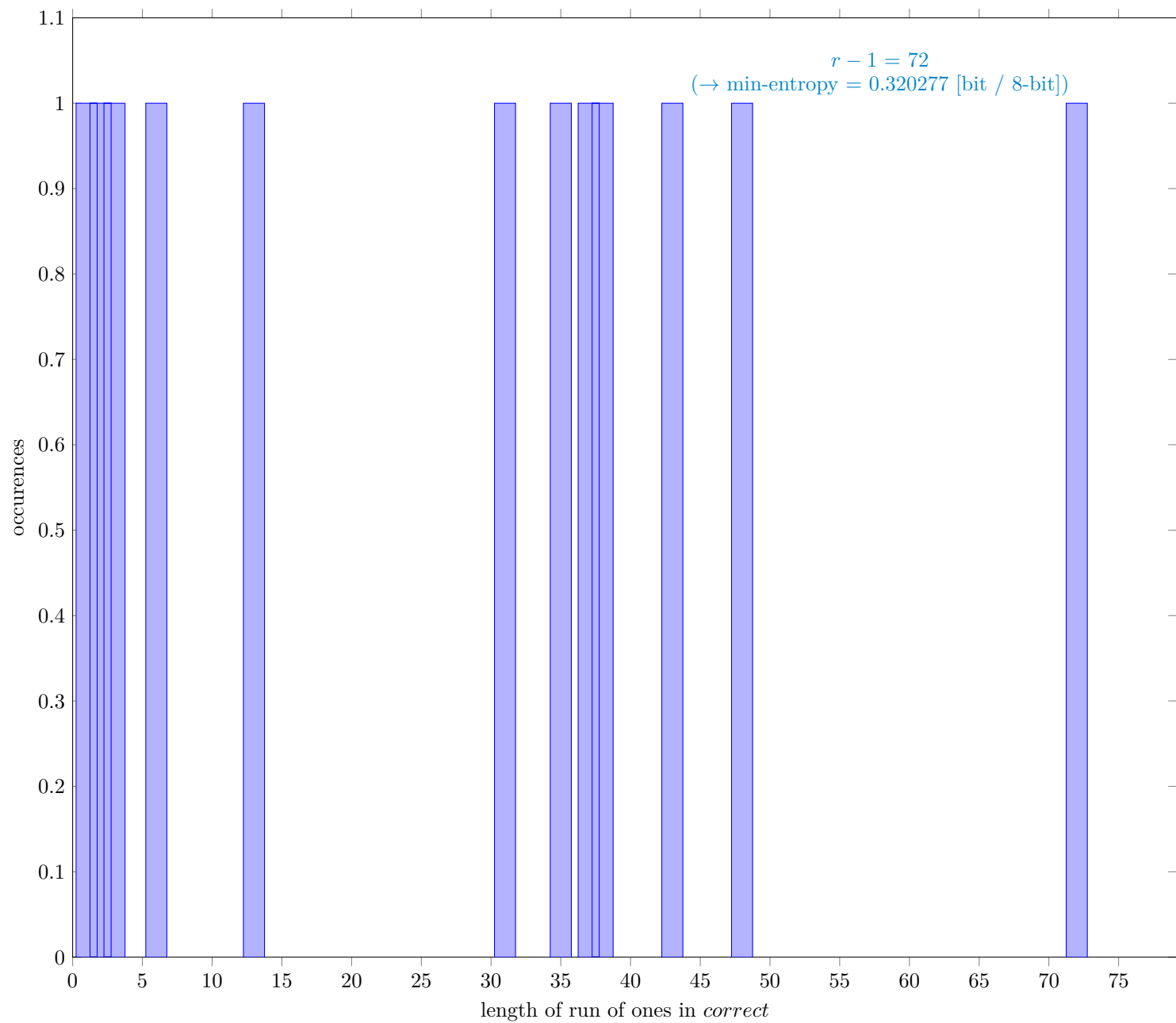


Fig. 10 Distribution of *correct*

#### 3.6.1 Supplemental information for traceability

Table 10 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol               | Value    |
|----------------------|----------|
| $N$                  | 999998   |
| $C$                  | 799884   |
| $P_{\text{global}}$  | 0.799886 |
| $P'_{\text{global}}$ | 0.800916 |
| $r$                  | 73       |
| $P_{\text{local}}$   | 0.794038 |

3.7 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

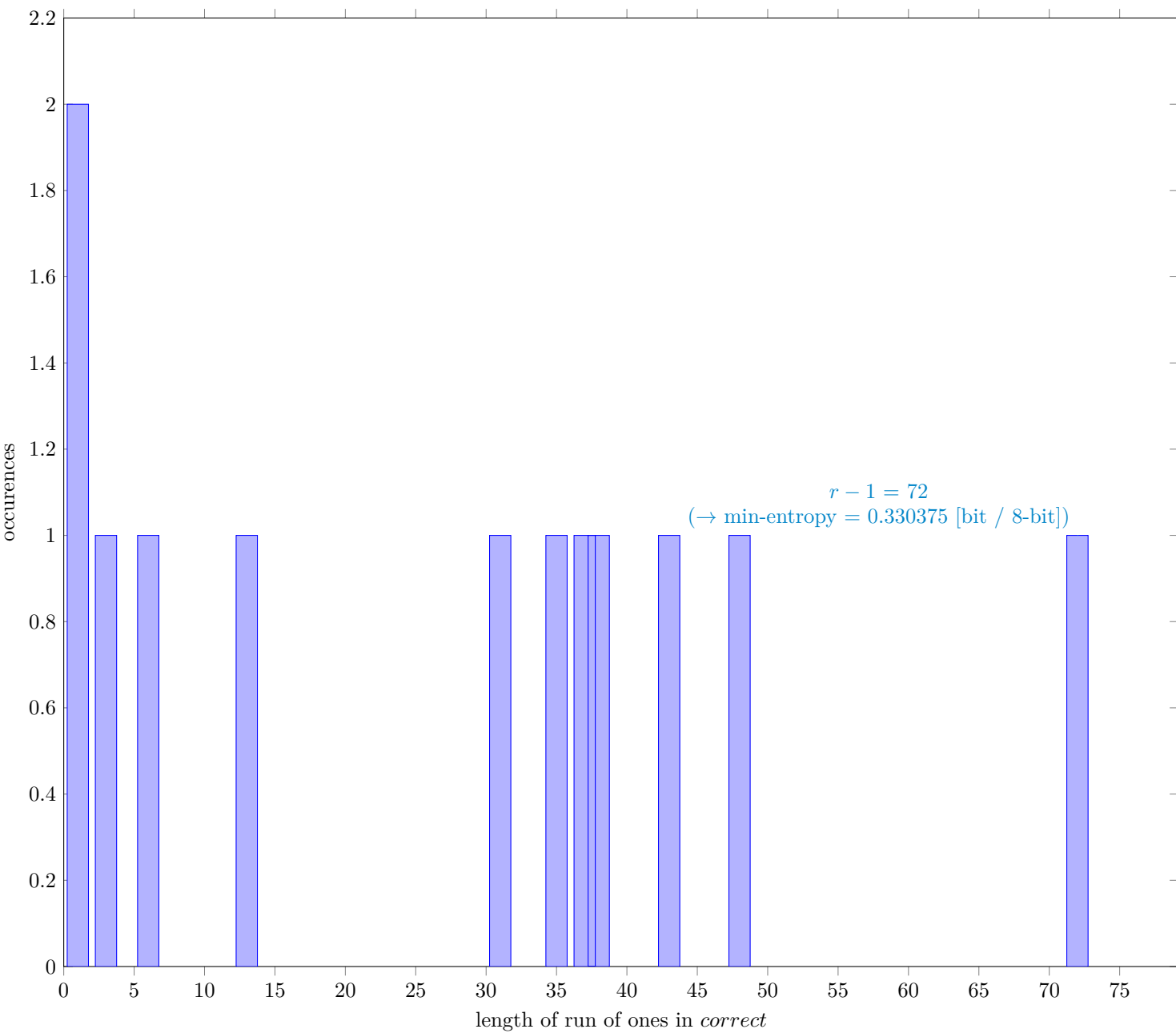


Fig. 11 Distribution of *correct*

3.7.1 Supplemental information for traceability

Table 11 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol               | Value    |
|----------------------|----------|
| $N$                  | 999983   |
| $C$                  | 794275   |
| $P_{\text{global}}$  | 0.794289 |
| $P'_{\text{global}}$ | 0.79533  |
| $r$                  | 73       |
| $P_{\text{local}}$   | 0.794038 |

4

Detailed results of analysis by interpreting each sample as bitstrings

4.1

The Most Common Value Estimate (NIST SP 800-90B Section 6.3.1)

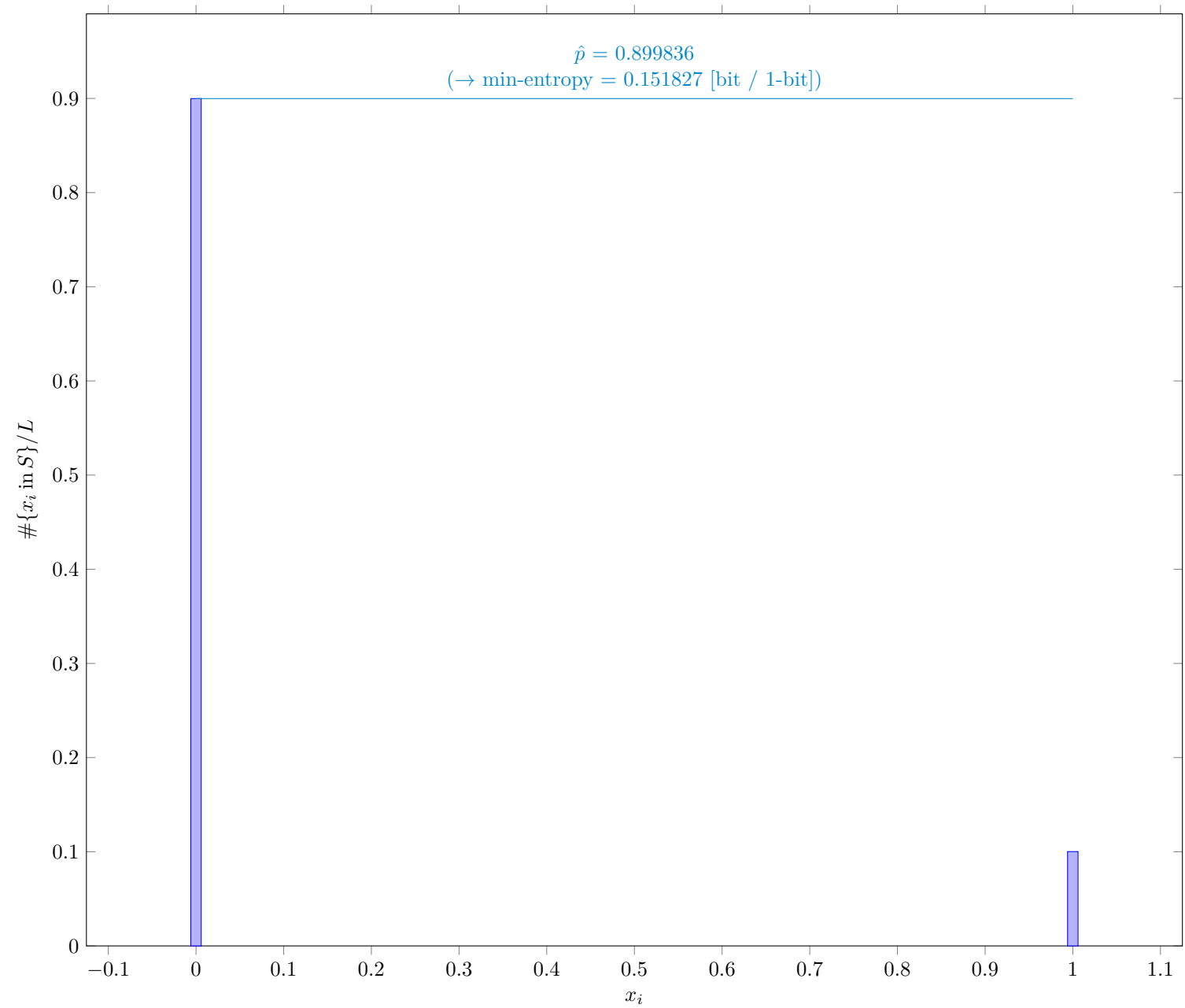


Fig. 12 Distribution of  $x_i$

4.1.1

Supplemental information for traceability

Table 12 Supplemental information for traceability (NIST SP 800-90B Section 6.3.1)

| Symbol    | Value    |
|-----------|----------|
| mode      | 7198690  |
| $\hat{p}$ | 0.899836 |
| $p_u$     | 0.90011  |

## 4.2 The Collision Estimate (NIST SP 800-90B Section 6.3.2)

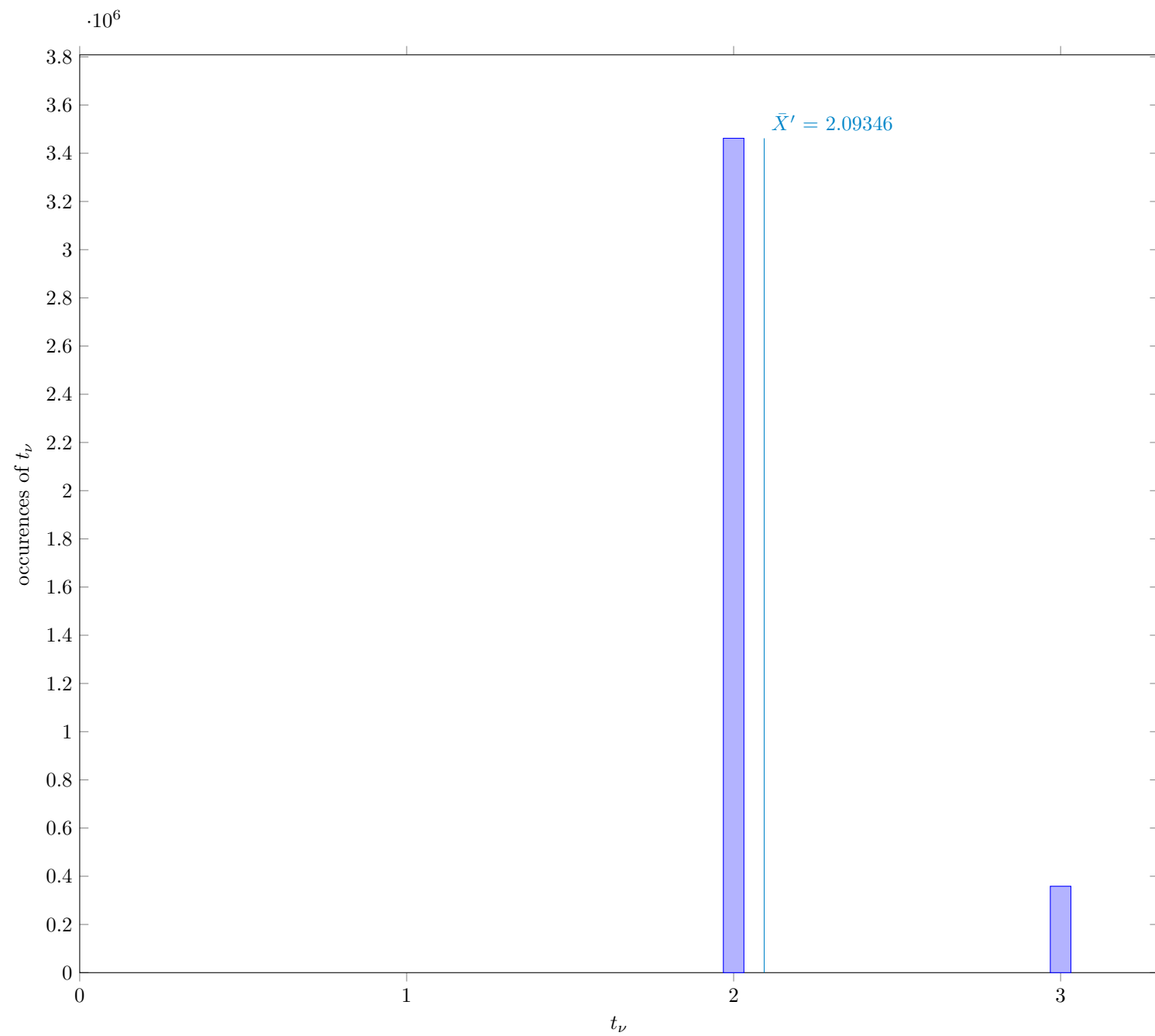


Fig. 13 Distribution of intermediate value  $t_\nu$



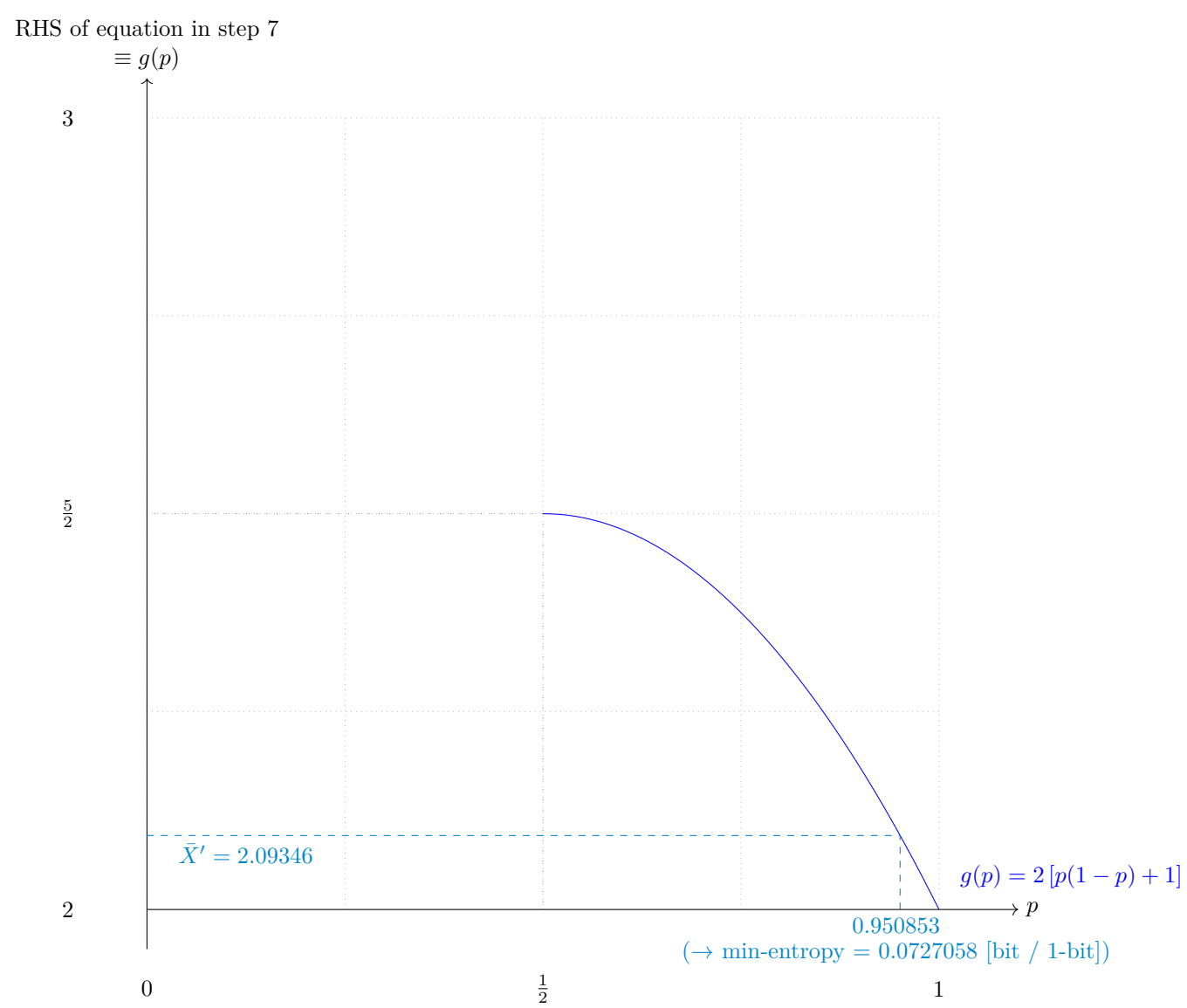


Fig. 14 Solution to the equation in step 7

4.2.1 Supplemental information for traceability

Table 13 Supplemental information for traceability (NIST SP 800-90B Section 6.3.2)

| Symbol         | Value    |
|----------------|----------|
| $p$            | 0.950853 |
| $\bar{X}$      | 2.09385  |
| $\bar{X}'$     | 2.09346  |
| $\hat{\sigma}$ | 0.291617 |

### 4.3 The Markov Estimate (NIST SP 800-90B Section 6.3.3)

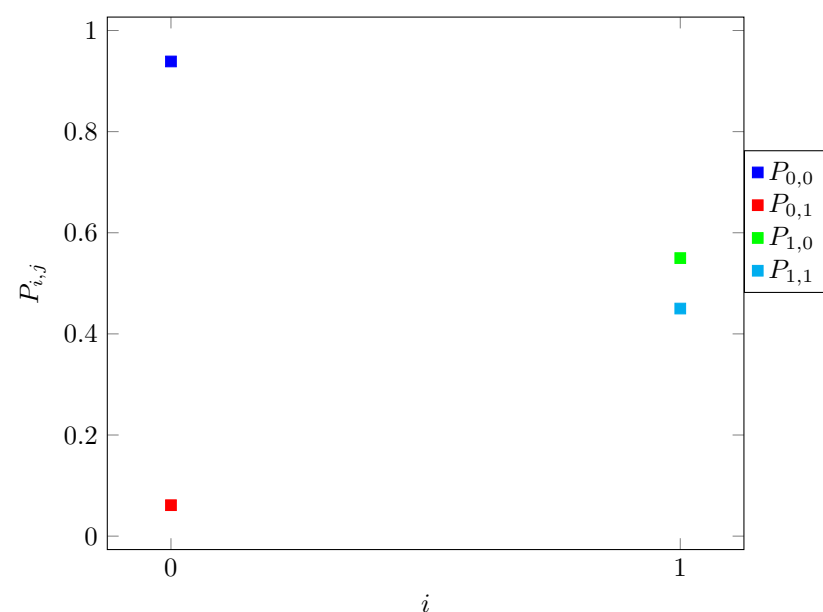


Fig. 15 Transition probability  $P_{i,j}$  of §6.3.3 of NIST SP 800-90B

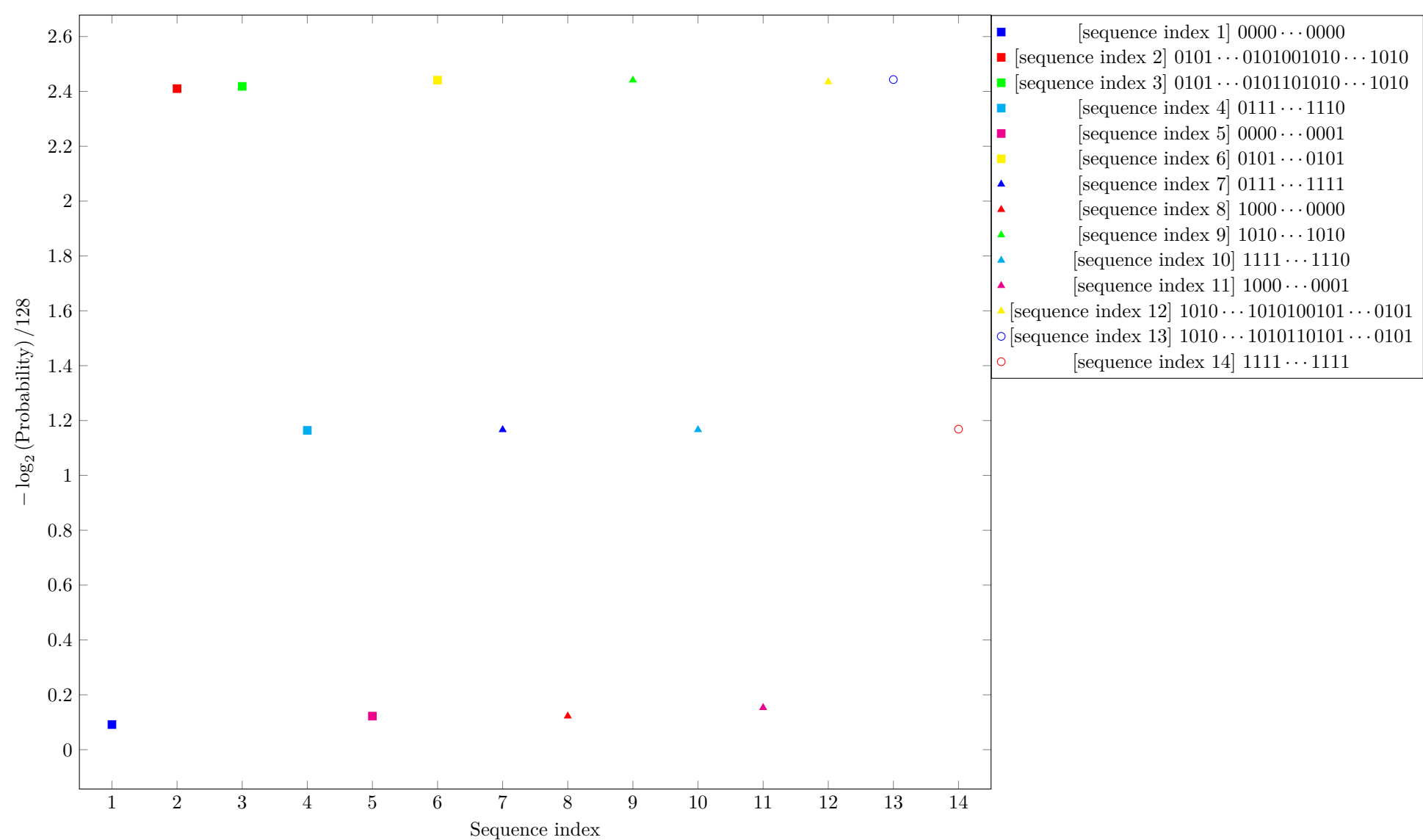


Fig. 16 Estimated Min-Entropy using §6.3.3 of NIST SP 800-90B

4.4 The Compression Estimate (NIST SP 800-90B Section 6.3.4)

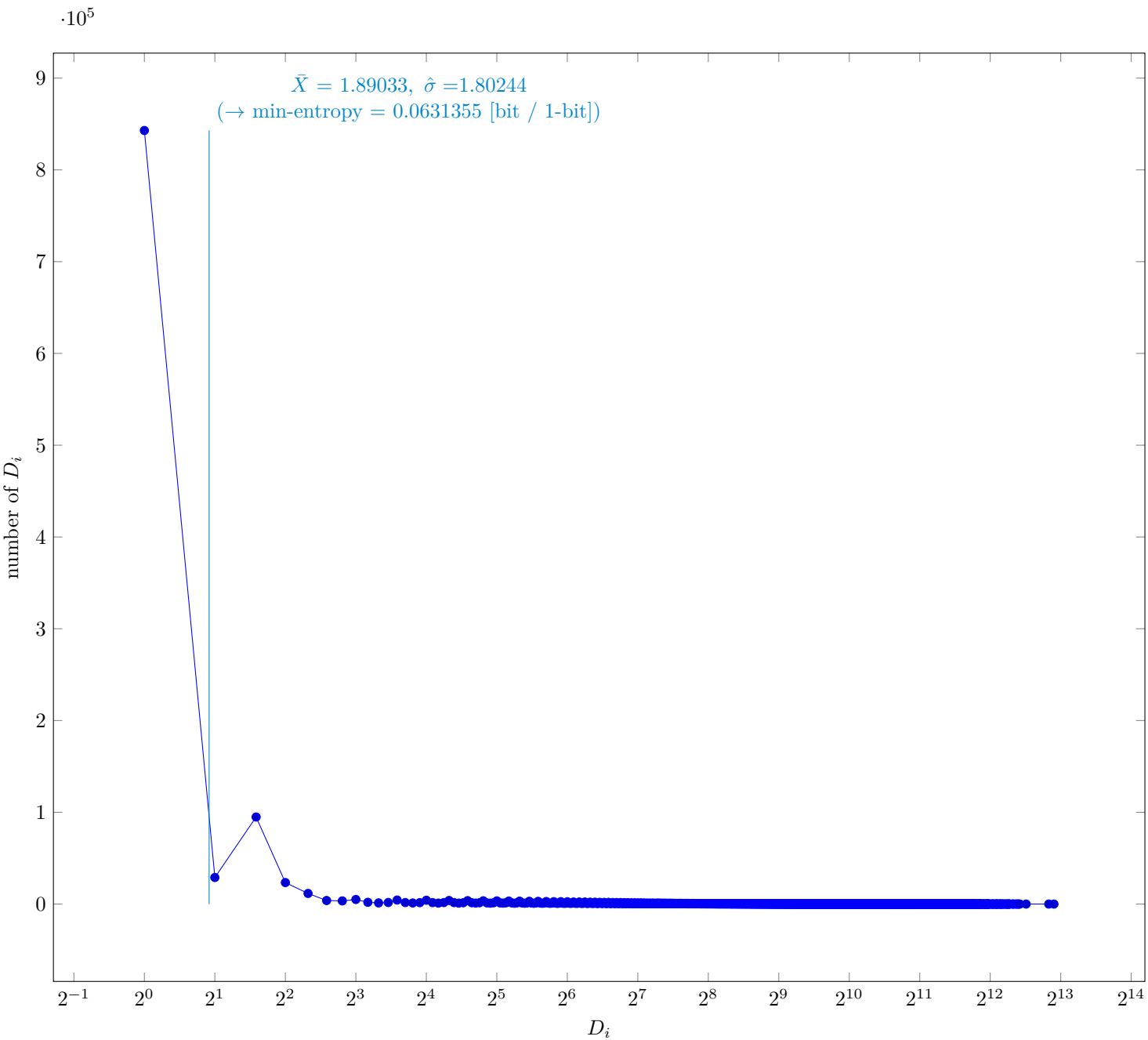


Fig. 17 Distribution of intermediate value  $D_i$

4.4.1 Supplemental information for traceability

Table 14 Supplemental information for traceability (NIST SP 800-90B Section 6.3.4)

| Symbol         | Value   |
|----------------|---------|
| $p$            | 0.76907 |
| $\bar{X}$      | 1.89033 |
| $\hat{\sigma}$ | 1.80244 |
| $\bar{X}'$     | 1.88631 |

#### 4.5 The t-tuple Estimate (NIST SP 800-90B Section 6.3.5)

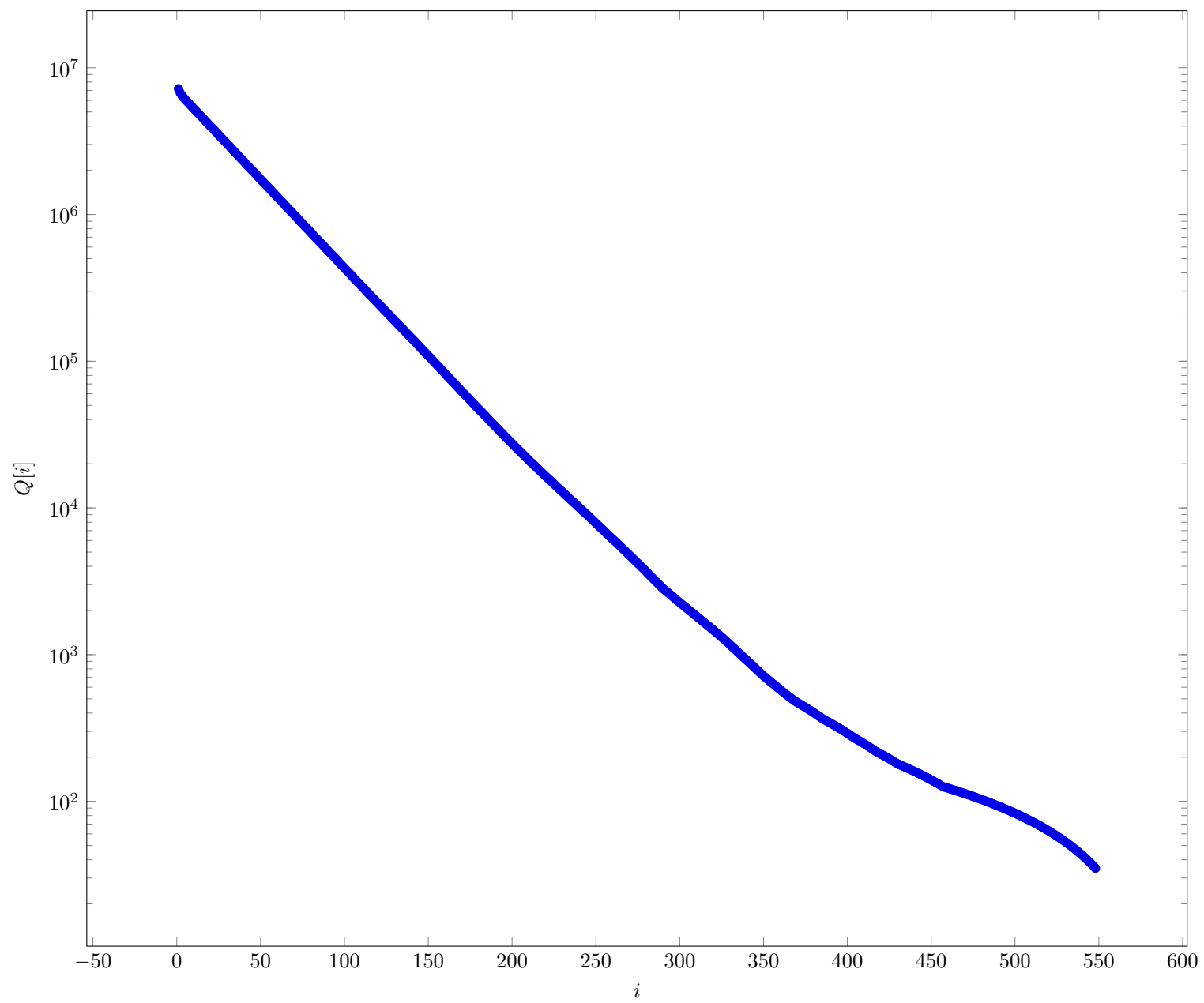


Fig. 18 Intermediate value  $Q[i]$  in §6.3.5 of NIST SP 800-90B

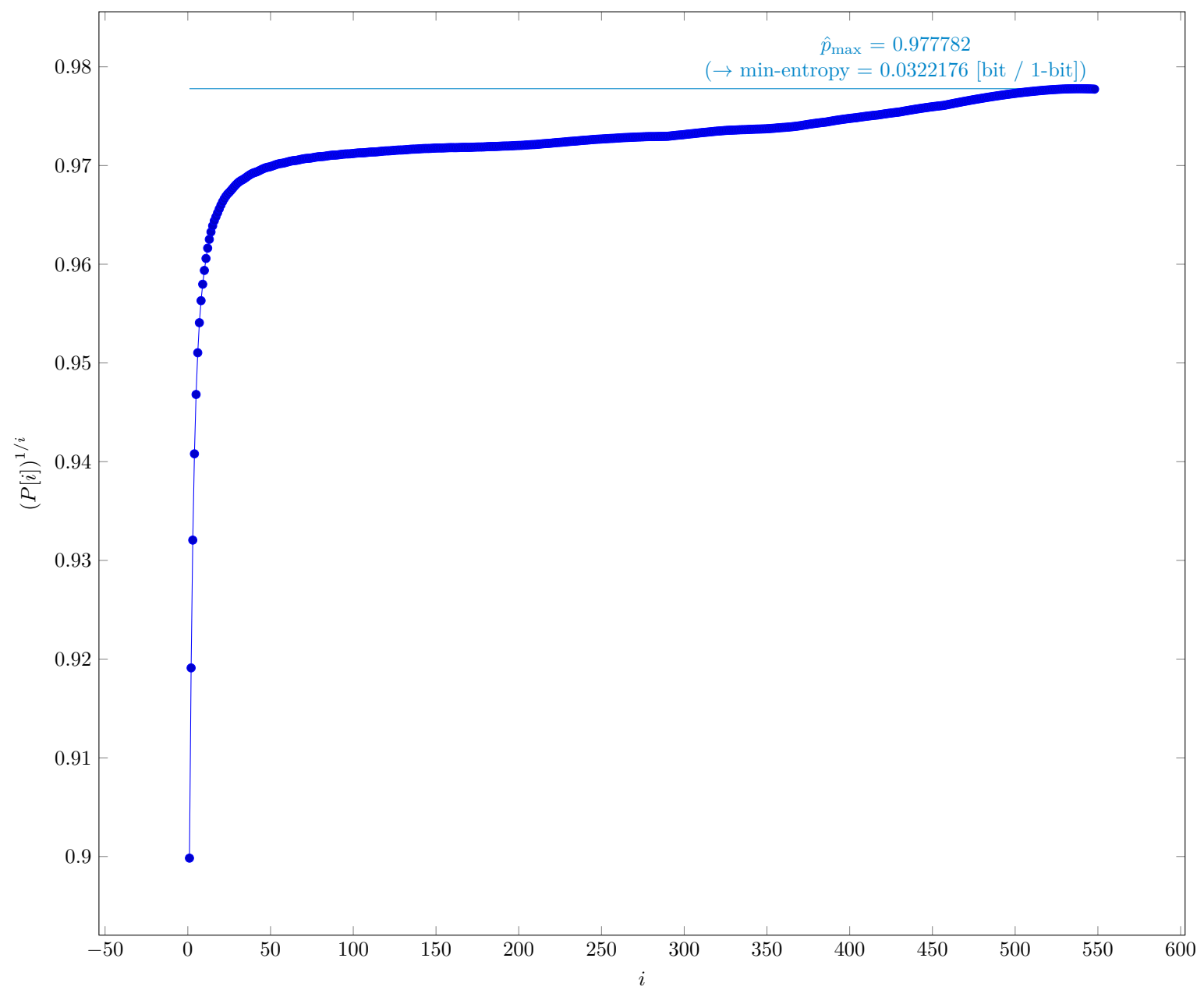


Fig. 19  $P[i]^{1/i}$  in §6.3.5 of NIST SP 800-90B

4.5.1 Supplemental information for traceability

Table 15 Supplemental information for traceability (NIST SP 800-90B Section 6.3.5)

| Symbol           | Value    |
|------------------|----------|
| $t$              | 548      |
| $\hat{p}_{\max}$ | 0.977782 |
| $p_u$            | 0.977916 |

#### 4.6 The LRS Estimate (NIST SP 800-90B Section 6.3.6)

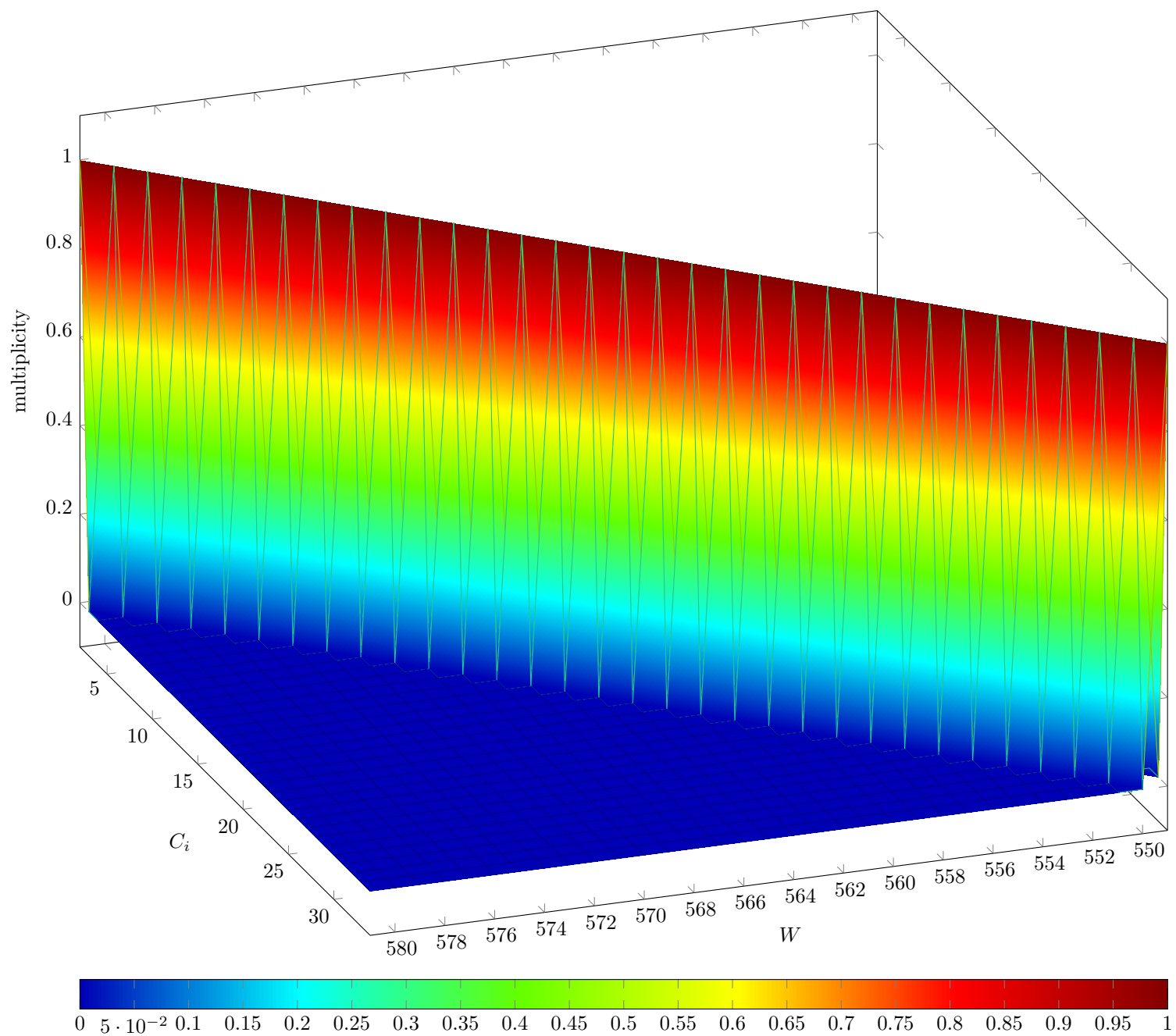


Fig. 20 Estimated  $W$ -tuple collision probability in Step 3 of §6.3.6 of NIST SP 800-90B



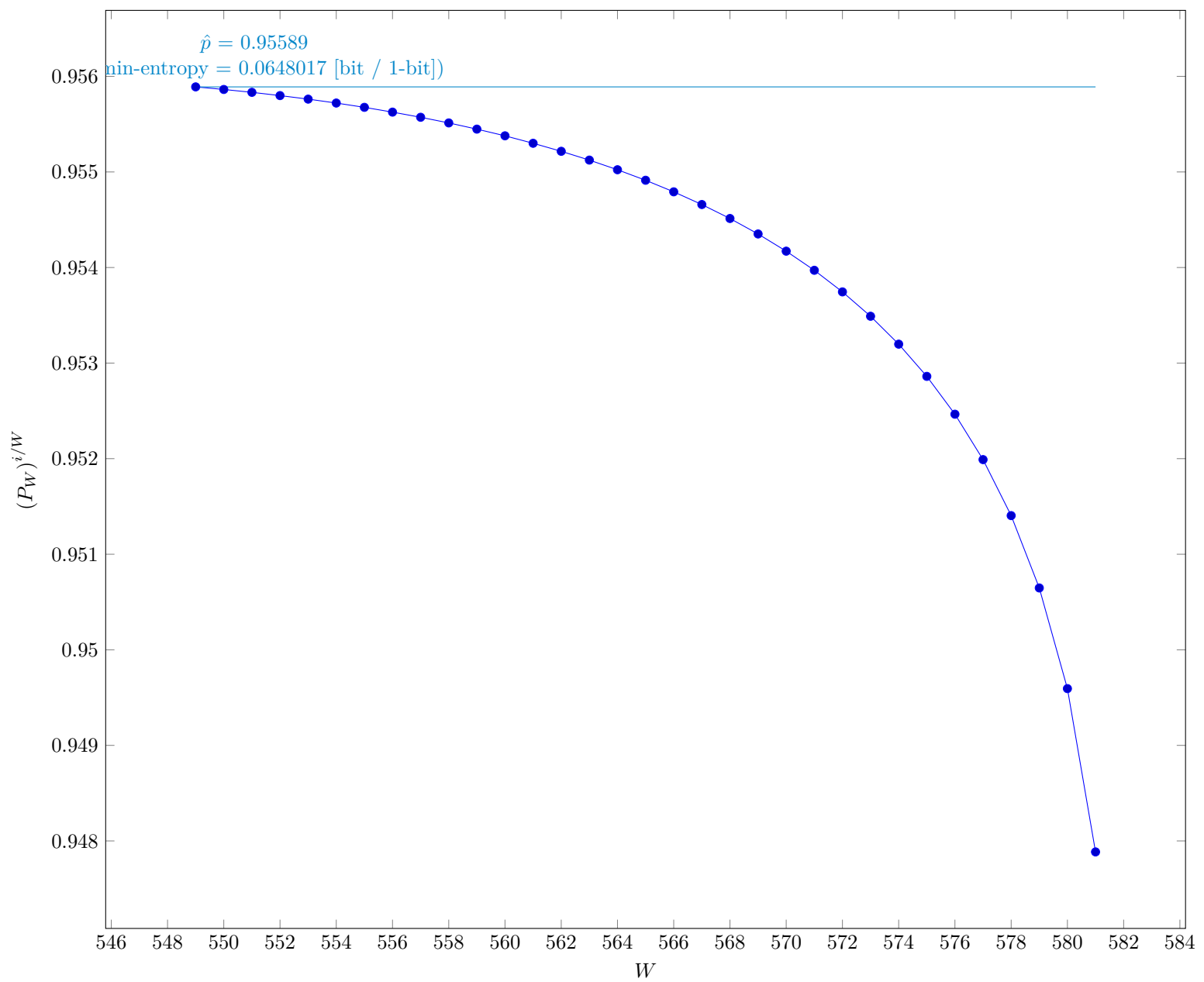


Fig. 21 Estimated average collision probability per string symbol in Step 3 of §6.3.6 of NIST SP 800-90B

4.6.1 Supplemental information for traceability

Table 16 Supplemental information for traceability (NIST SP 800-90B Section 6.3.6)

| Symbol    | Value    |
|-----------|----------|
| $u$       | 549      |
| $v$       | 581      |
| $\hat{p}$ | 0.95589  |
| $p_u$     | 0.956077 |

#### 4.7 Multi Most Common in Window Prediction Estimate (NIST SP 800-90B Section 6.3.7)

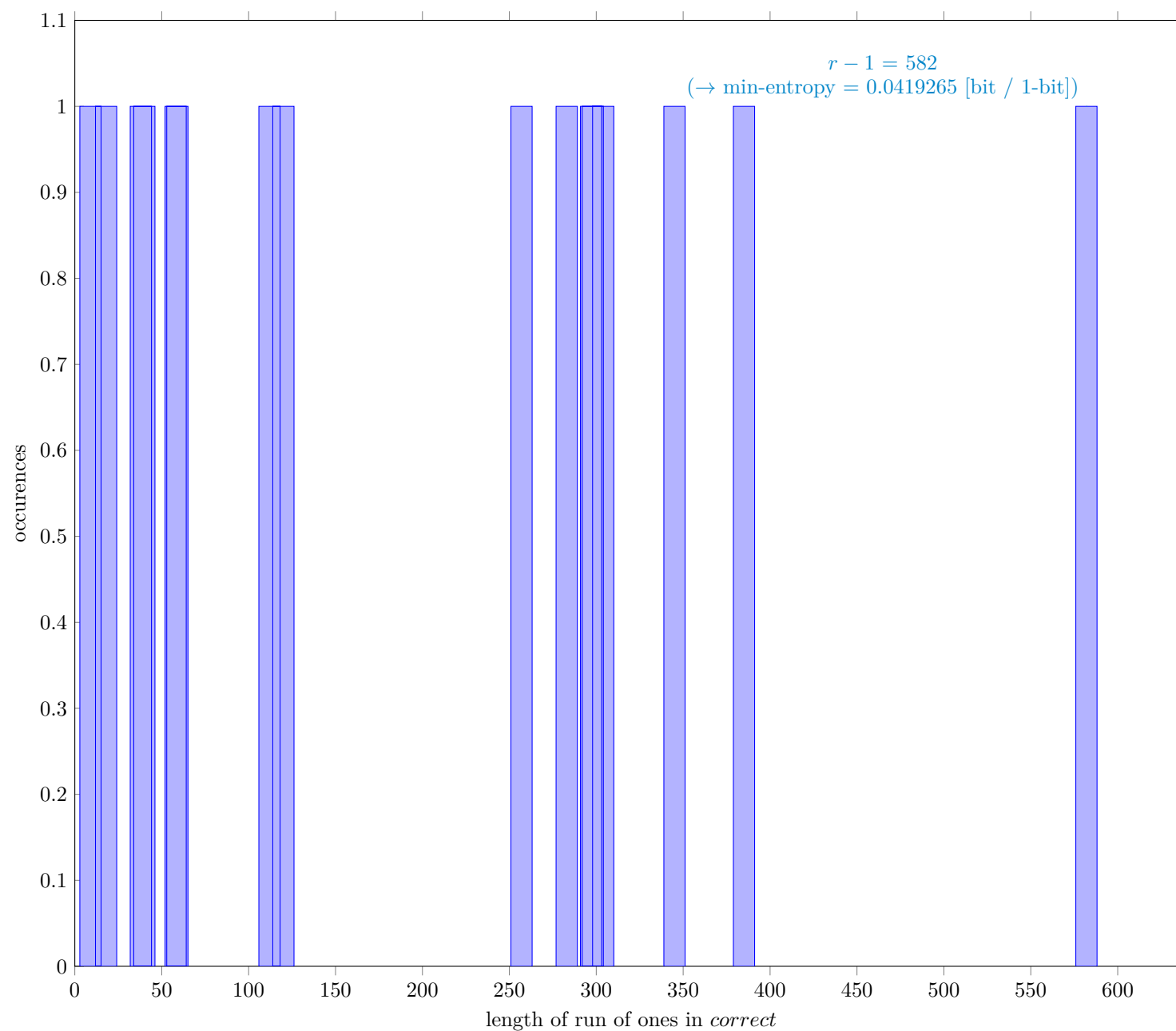


Fig. 22 Distribution of *correct*

##### 4.7.1 Supplemental information for traceability

Table 17 Supplemental information for traceability (NIST SP 800-90B Section 6.3.7)

| Symbol               | Value    |
|----------------------|----------|
| $N$                  | 7999937  |
| $C$                  | 7198538  |
| $P_{\text{global}}$  | 0.899824 |
| $P'_{\text{global}}$ | 0.900098 |
| $r$                  | 583      |
| $P_{\text{local}}$   | 0.971357 |

4.8 Lag Prediction Estimate (NIST SP 800-90B Section 6.3.8)

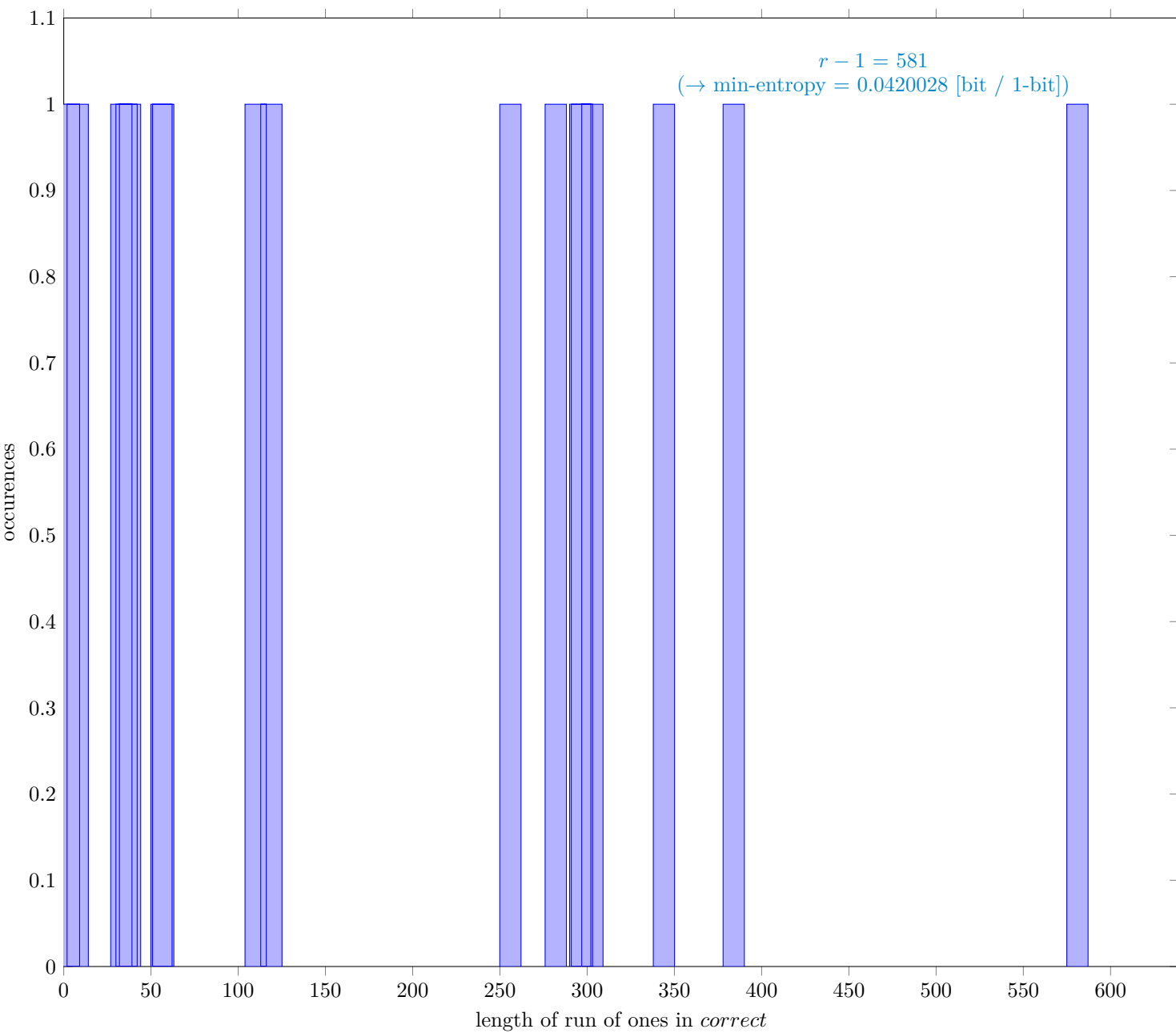


Fig. 23 Distribution of *correct*

4.8.1 Supplemental information for traceability

Table 18 Supplemental information for traceability (NIST SP 800-90B Section 6.3.8)

| Symbol               | Value    |
|----------------------|----------|
| $N$                  | 7999999  |
| $C$                  | 7118705  |
| $P_{\text{global}}$  | 0.889838 |
| $P'_{\text{global}}$ | 0.890123 |
| $r$                  | 582      |
| $P_{\text{local}}$   | 0.971306 |

#### 4.9 The MultiMMC Prediction Estimate (NIST SP 800-90B Section 6.3.9)

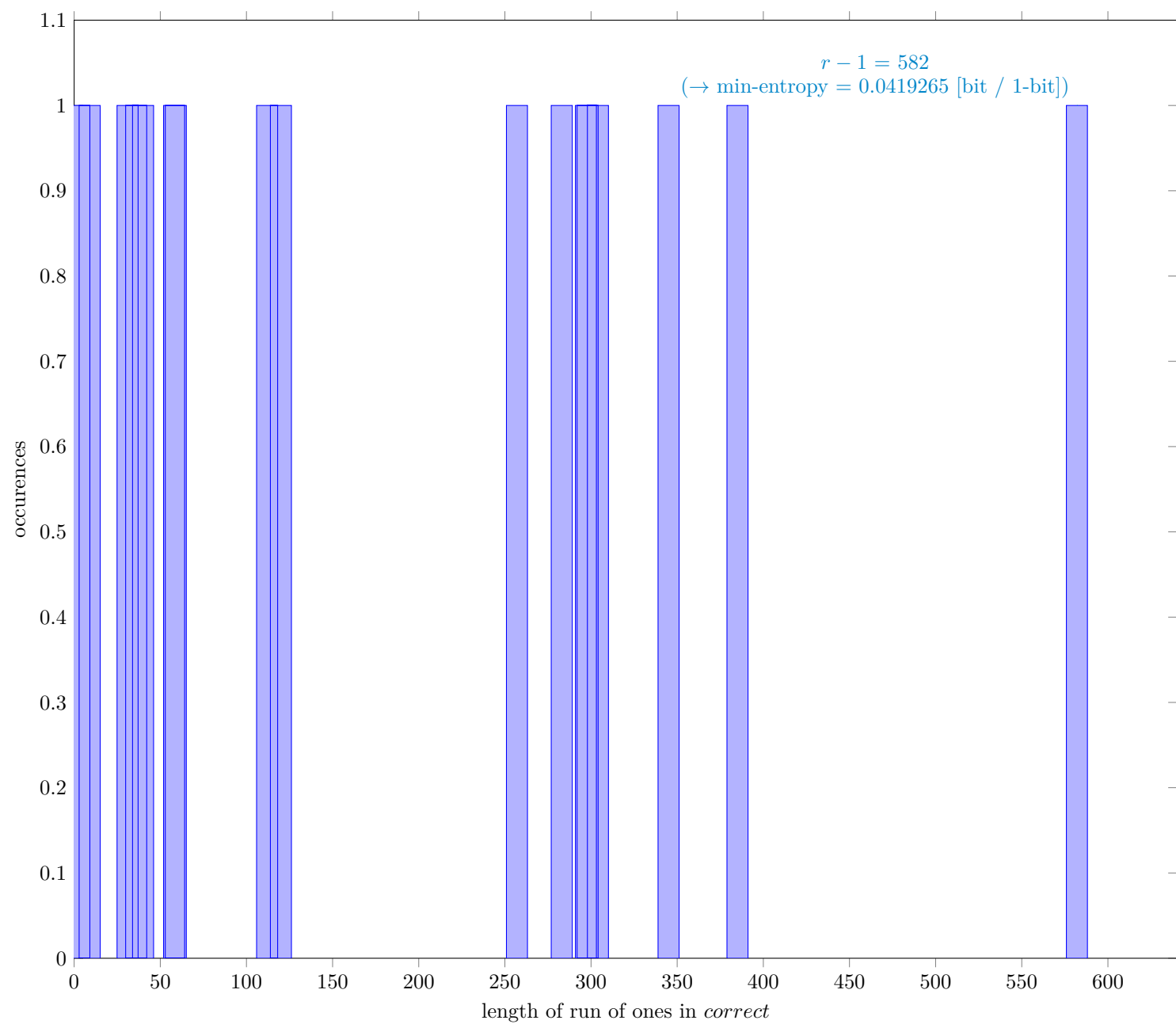


Fig. 24 Distribution of *correct*

#### 4.9.1 Supplemental information for traceability

Table 19 Supplemental information for traceability (NIST SP 800-90B Section 6.3.9)

| Symbol               | Value    |
|----------------------|----------|
| $N$                  | 7999998  |
| $C$                  | 7198679  |
| $P_{\text{global}}$  | 0.899835 |
| $P'_{\text{global}}$ | 0.900109 |
| $r$                  | 583      |
| $P_{\text{local}}$   | 0.971357 |

4.10 The LZ78Y Prediction Estimate (NIST SP 800-90B Section 6.3.10)

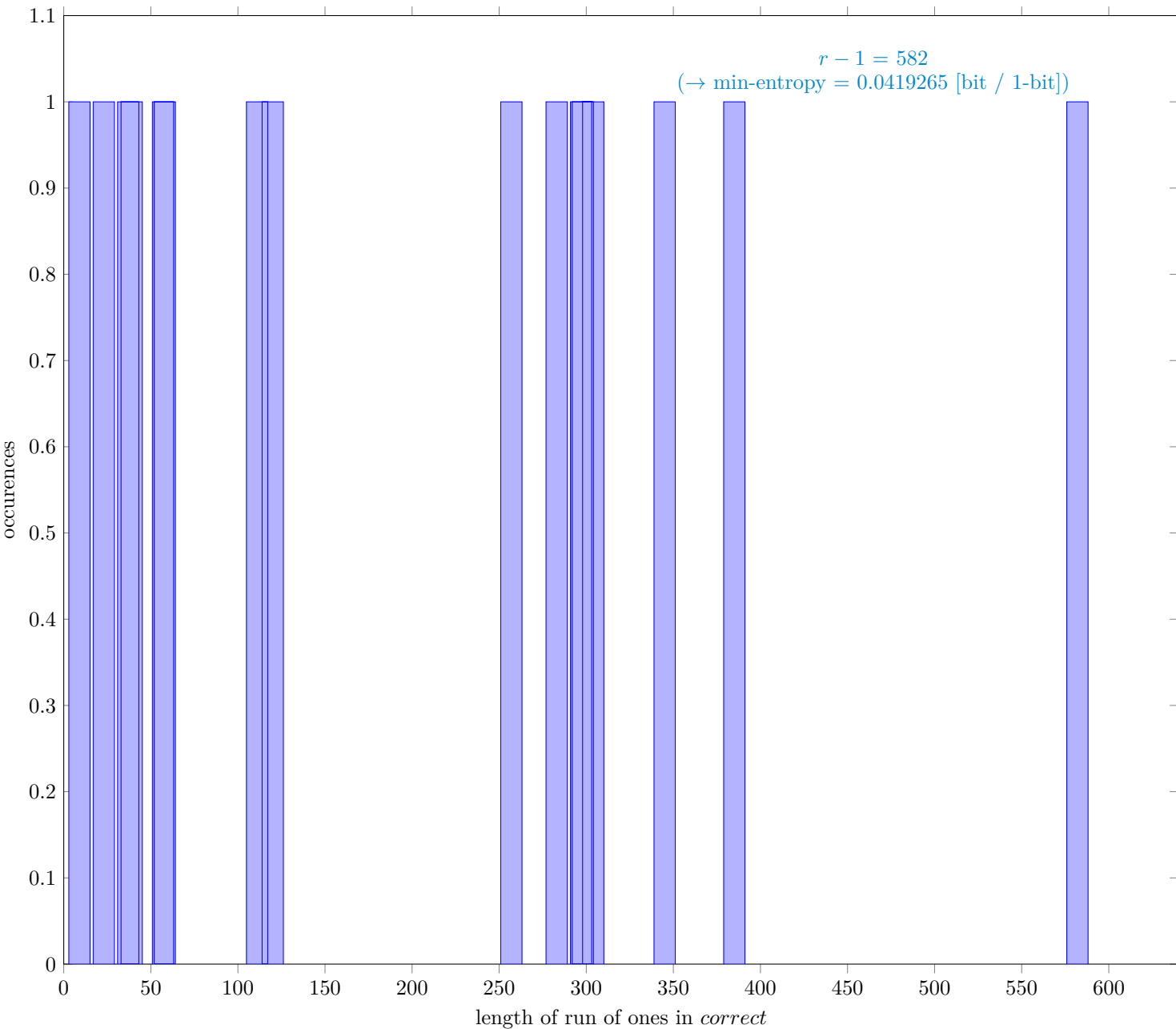


Fig. 25 Distribution of *correct*

4.10.1 Supplemental information for traceability

Table 20 Supplemental information for traceability (NIST SP 800-90B Section 6.3.10)

| Symbol               | Value    |
|----------------------|----------|
| $N$                  | 7999983  |
| $C$                  | 7198670  |
| $P_{\text{global}}$  | 0.899836 |
| $P'_{\text{global}}$ | 0.900109 |
| $r$                  | 583      |
| $P_{\text{local}}$   | 0.971357 |

4 References

[1] Meltem Sönmez Turan, Elaine Barker, John Kelsey, Kerry A. McKay, Mary L. Baish, Mike Boyle *Recommendation for the Entropy Sources Used for Random Bit Generation*, NIST Special Publication 800-90B, Jan. 2018 <https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-90B.pdf>

[2] G. Sakurai, *Proposed list of corrections for NIST SP 800-90B 6.3 Estimators*, Dec. 2022 [https://github.com/g-g-sakura/AnotherEntropyEstimationTool/blob/main/documentation/ProposedListOfCorrections\\_SP800-90B.pdf](https://github.com/g-g-sakura/AnotherEntropyEstimationTool/blob/main/documentation/ProposedListOfCorrections_SP800-90B.pdf)