CS 455: Principles of Database Systems

Review Guide 3: Database Theory (Selected Soln)

1. Given: $\alpha, \beta, \gamma, \delta$ refer to distinct sets of attributes in R. For each of the following inference rules, show it is either sound through derivation using only **Armstrong's Axioms**, or unsound by providing a counterexample.

   (a) $\alpha \rightarrow \beta \stackrel{?}{\Longrightarrow} \alpha \cup \gamma \rightarrow \beta$

$$\alpha \rightarrow \beta \text{ Given} \quad (1)$$
$$\alpha \cup \gamma \rightarrow \beta \cup \gamma \text{ Aug (1) with } \gamma \quad (2)$$
$$\beta \cup \gamma \rightarrow \beta \text{ Trivial} \quad (3)$$
$$\alpha \cup \gamma \rightarrow \beta \text{ Transitivity} \quad (4)$$
$$\therefore \alpha \rightarrow \beta \implies \alpha \cup \gamma \rightarrow \beta \quad \square \quad (5)$$

   (b) $\alpha \rightarrow \beta \stackrel{?}{\Longrightarrow} \beta \subseteq \alpha$

   Consider a relation $Pres(ssn, name, leaning)$ and that $FD(Pres) = \{name, party \rightarrow isCorrupt\}$. You can observe that $FD(Pres)$ holds on the relational instance below. However, $isCorrupt \not\subseteq \{name, party\}$.

name	*party*	*isCorrupt*
Johnson	*R*	*y*
Johnson	*D*	*n*
Cleveland	*R*	*n*
Lincoln	*R*	*n*

   (c) $\alpha \rightarrow \beta, \beta \rightarrow \gamma \stackrel{?}{\Longrightarrow} \alpha \cup \delta \rightarrow \gamma \cup \delta$

$$\alpha \rightarrow \beta \text{ Given} \quad (1)$$
$$\beta \rightarrow \gamma \text{ Given} \quad (2)$$
$$\alpha \rightarrow \gamma \text{ Transitivity of (1) and (2).} \quad (3)$$
$$\alpha \cup \delta \rightarrow \gamma \cup \delta \text{ Augmentation of } \gamma \text{ on (3)} \quad (4)$$
$$\therefore \alpha \rightarrow \beta, \beta \rightarrow \gamma \implies \alpha \cup \delta \rightarrow \gamma \cup \delta \quad \square \quad (5)$$

(d) ** $\alpha \rightarrow \beta, \beta \cup \gamma \rightarrow \delta \overset{?}{\Longrightarrow} \alpha \cup \gamma \rightarrow \beta \cup \delta$

$$\alpha \rightarrow \beta \text{ Given} \tag{1}$$
$$\alpha \cup \gamma \rightarrow \beta \cup \gamma \text{ Aug (1) with } \gamma \tag{2}$$
$$\beta \cup \gamma \rightarrow \delta \text{ Given} \tag{3}$$
$$\beta \cup \gamma \rightarrow \beta \text{ Trivial} \tag{4}$$
$$\alpha \cup \gamma \rightarrow \delta \text{ Transitivity between (2) and (3)} \tag{5}$$
$$\alpha \cup \gamma \rightarrow \beta \text{ Transitivity between (2) and (4)} \tag{6}$$
$$(\alpha \cup \gamma) \cup (\alpha \cup \gamma) \rightarrow \delta \cup \alpha \cup \gamma \text{ Aug (5) with } \alpha \cup \gamma \tag{7}$$
$$\alpha \cup \gamma \rightarrow \alpha \cup \gamma \cup \delta \text{ Definition of } \cup \tag{8}$$
$$\alpha \cup \gamma \cup \delta \rightarrow \beta \cup \delta \text{ Aug (6) with } \delta \tag{9}$$
$$\alpha \cup \gamma \rightarrow \beta \cup \delta \text{ Transitivity between (8) and (9)} \tag{10}$$
$$\therefore \alpha \rightarrow \beta, \beta \cup \gamma \rightarrow \delta \Longrightarrow \alpha \cup \gamma \rightarrow \beta \cup \delta \quad \square \tag{11}$$

2. ** Consider the relation $U(W, X, Y, Z)$ with a set of functional dependencies

$$\begin{aligned} FD(U) = \{ & \\ & XZ \rightarrow YZ, \\ & Y \rightarrow Z\} \end{aligned}$$

(a) List all of U's superkeys with respect to $FD(U)$.

Sol: You could run the attribute closure algorithm for the power set of the set of attributes. By themselves, $\{X, Y\}$ and $\{X, Z\}$ can determine everything except W. Because no FDs can determine W, it must be included as a key attribute to trivially determine itself. Therefore, it follows that $\{X, W, Y\}$ and $\{X, W, Z\}$ are candidate keys, and $\{X, W, Y, Z\}$ is a super key.

(b) Is U in BCNF with respect to $FD(U)$? If so, show that every functional dependency $\alpha \rightarrow \beta$ is either trivial or that α is a superkey in U. Otherwise, decompose U into a set of BCNF relations with respect to $FD(U)$. Show your work.

Sol: I started by splitting on $XZ \rightarrow YZ$ to obtain $U_1(X, Y, Z)$ and $U_2(X, Z, W)$. U_2's candidate key is $\{X, W\}$, and it is in BCNF because the only FD preserved is $XZ \rightarrow Z$, which is trivial. Different story for U_1. The candidate key is $\{X, Z\}$ and $\{X, Y\}$ and both of the original FDs are preserved. $Y \rightarrow Z$ violates BCNF for U_1, so we split it into $U_{1a}(Y, Z)$ and $U_{1b}(Y, X)$. They are both in BCNF. The final set of normalized relations are as follows: $U_{1a}(Y, Z)$, $U_{1b}(Y, X)$, and $U_2(X, Z, W)$.

(c) Find $FD_c(U)$, the canonical cover of $FD(U)$.

Sol:

$$\begin{aligned} FD_c(U) = \{ & \\ & XZ \rightarrow Y, \\ & Y \rightarrow Z\} \end{aligned}$$

(d) List all of U's superkeys with respect to $FD_c(U)$.

Sol: The set of keys do not change from earlier since $FD(U)^+ = FD_c(U)^+$.

(e) Is U in BCNF with respect to $FD_c(U)$? If so, show that every functional dependency $\alpha \to \beta$ is either trivial or that α is a superkey in U. Otherwise, decompose U into a set of BCNF relations with respect to $FD_c(U)$. Show your work.

Sol: $U_1(Y, Z)$ where Y is key, $U_2(Y, X)$ where YX is key, and $U_3(X, Z, W)$ where XYW is key.