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CMPT440 - Formal Languages and Computability
Midterm Practice

1. **Webber Chap. 11 Exercise 1**
   Prove that $\{a^n b^n c^n\}$ is not regular.
   `Let M = (Q, {a, b, c}, G, q0, F) be any DFA over the alphabet {a, b, c}; we will show that L(M)` $\neq$ `{`$a^n b^n c^n$`}. Consider the behavior of M on an arbitrarily long string of as. As it reads the string, M visits a sequence of states: first` δ`*(q0,` ϵ`), then` δ`*(q0, a), then` δ`*(q0, aa), and so on.`

   `Eventually, since M has only finitely many states, it must revisit a state; that is, there must be some i and j with i < j for which` δ`*(q0,` a^i`) =` δ`*(q0,` a^j`). Now by appending` b^j `and` c^j `to both strings, we see that` δ`*(q0,` $a^i b^j c^j$`) =` δ`*(q0,` $a^j b^j c^j$`).`

   `Thus M ends up in the same final state for both` $a^i b^j c^j$ `and` $a^j b^j c^j$`. If that is an accepting state, M accepts both; if it is a rejecting state, M rejects both. But this means that L(M)` $\neq$ `{`$a^n b^n c^n$`}, since` $a^j b^j c^j$ `is in {`$a^n b^n c^n$`} while` $a^i b^j c^j$ `is not. Since we have shown that L(M)` $\neq$ `{`$a^n b^n c^j$`} for any DFA M, we conclude that {`$a^n b^n c^n$`} is not regular.`

2. **Webber Chap. 11 Exercise 7**
   Let B = $\{a^n b^n c^n \mid n \geq 0\}$.
   Using the pumping lemma for regular languages, prove that B is not regular.

   (a) Assume that B is regular, so the pumping lemma holds for B. Let k be as given by the pumping lemma.

   (b) x = a^k
   y = b^k
   z = c^k
   xyz = $a^n b^n c^n \in$ B and $|y| \geq$ k as required.

   (c) Let u, v, and w be as given by the pumping lemma, so that uvw = y, $|v| > 0$, and for all i $\geq$ 0, $xuv^i wz \in$ B.

   (d) Choose i = 2. Since v contains at least one b and nothing but bs, uv^2w has more bs than uvw. So xuv^2wz has more bs than cs and as, and $xuv^2wz \notin$ L.

   (e) By contradiction, B = $\{a^n b^n c^n\}$ is not regular.