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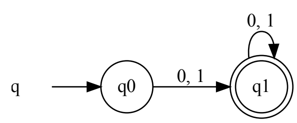
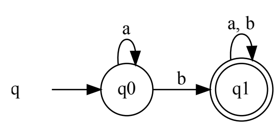
CMPT440 - Formal Languages and Computability
Assignment 6 - Grammars

1. **Webber Chap. 9 Exercise 1**
For each of the following DFAs, list the unreachable states if any, show L(M, q) for each q ϵ Q, and construct the minimized DFA using the procedure of Section 9.1.

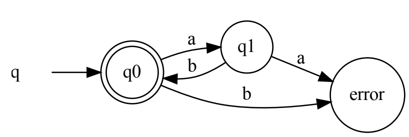
   (a) **Unreachable states:** q2
   L(M, q0) = {xy | x ϵ {0, 1} and y ϵ {0, 1}*}
   L(M, q1) = {x | x ϵ {0, 1}*}
   L(M, q2) = {xy | x ϵ {1, 00, 01} and y ϵ {0, 1}*}
   Minimized DFA:



   (b) **Unreachable states:** q0, q3
   L(M, q0) = {x | x ϵ {a}*}
   L(M, q1) = {x | x ϵ {a}*}
   L(M, q2) = {}
   L(M, q3) = {ax | x ϵ {a, b}*}
   Minimized DFA:



   (c) **Unreachable states:** q0, q3
   L(M, q0) = {$(a, b)^n$ | n ≥ 0}
   L(M, q1) = {$(a, b)^n$ | n ≥ 1}
   L(M, q2) = {$(a, b)^n$ | n ≥ 1}
   L(M, q3) = {}
   L(M, q4) = {$(a, b)^n$ | n ≥ 1}}
   L(M, q5) = {$(a, b)^n$ | n ≥ 1}
   Minimized DFA:



2. **Webber Chap. 10 Exercise 3**
   Grammar G_2:
   $S \rightarrow aS$
   $S \rightarrow X$
   $X \rightarrow bX$
   $X \rightarrow \epsilon$

   (a) Show a derivation from S for the string bbb.
   $S \rightarrow X \rightarrow bX \rightarrow bbX \rightarrow bbbX \rightarrow bbb\epsilon = bbb$

   (b) Show a derivation from S for the string aabb.
   $S \rightarrow aS \rightarrow aaS \rightarrow aabX \rightarrow aabbX \rightarrow aabb\epsilon = aabb$

   (c) Show a derivation from S for the empty string.
   $S \rightarrow X \rightarrow \epsilon$

3. **Webber Chap. 10 Exercise 4**
   Give a regular expression for the language generated by each of these grammars.

   (a) $S \rightarrow abS \mid \epsilon$
   = L((ab)*)

   (b) $S \rightarrow aS \mid aA$
   $A \rightarrow aS \mid aA$
   = L(a(a)*

   (c) $S \rightarrow smellA \mid fishA$
   $A \rightarrow y \mid \epsilon$
   = L((smell+fish)y*)

   (d) $S \rightarrow aaSa \mid \epsilon$
   = L((aaa)*)

4. **Webber Chap. 10 Exercise 5**
   Give a grammar for each of the following languages. In each case, use S as the start symbol.

   (a) L(a*)
   $S \rightarrow aS \mid \epsilon$

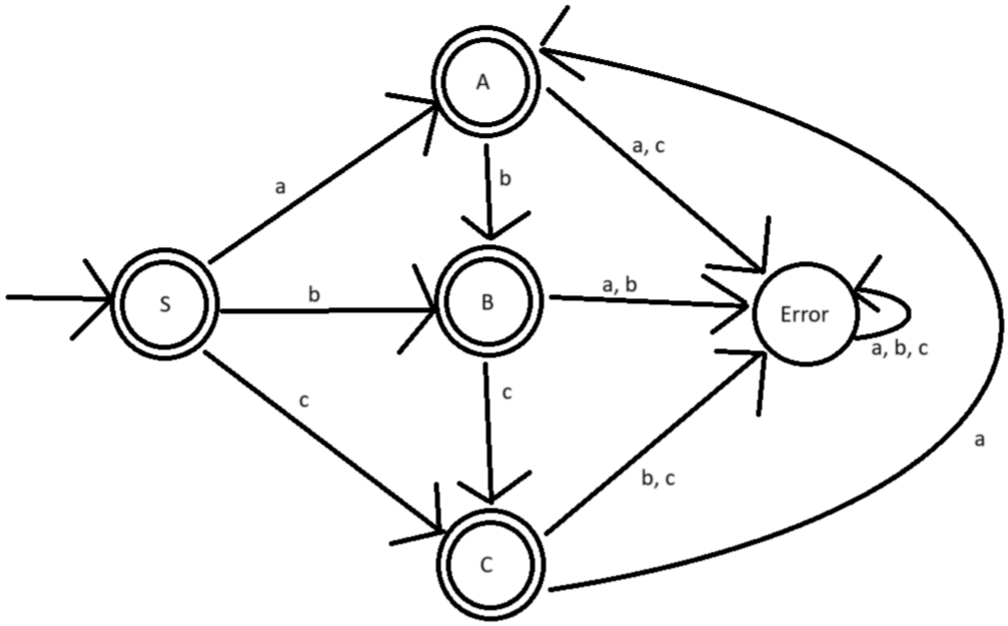
   (b) L(aa*)
   $S \rightarrow aX$
   $X \rightarrow aX \mid \epsilon$

   (c) L(a*b*c*)
   $S \rightarrow aS \mid X$
   $X \rightarrow bX \mid Y$
   $Y \rightarrow cY \mid \epsilon$

   (d) L((abc)*)
   $S \rightarrow abcS \mid \epsilon$

   (e) The set of all strings consisting of one or more digits, where each digit is one of the symbols 0 through 9.
   $S \rightarrow DS \mid DX$
   $D \rightarrow 0|1|2|3|4|5|6|7|8|9$
   $X \rightarrow \epsilon$

5. **Webber Chap. 10 Exercise 6**
   Give a DFA that accepts the language generated by this grammar:
   S $\rightarrow$ A | B | C
   A $\rightarrow$ aB | ϵ
   B $\rightarrow$ bC | ϵ
   C $\rightarrow$ cA | ϵ



6. **Webber Chap. 10 Exercise 12**
   Using the construction of Theorem 10.1, make a right-linear grammar that generates the language accepted by each of these NFAs.

   (a) S $\rightarrow$ aS | aX
       X $\rightarrow$ bX | ϵ

   (b) S $\rightarrow$ bS | aX
       X $\rightarrow$ aS | bX | ϵ

   (c) S $\rightarrow$ aX | ϵ
       X $\rightarrow$ bY
       Y $\rightarrow$ cS

   (d) S $\rightarrow$ 0S | 1S | 1X
       X $\rightarrow$ 0Y | 1Y
       Y $\rightarrow$ ϵ

   (e) S $\rightarrow$ A | B
       A $\rightarrow$ aX | ϵ
       X $\rightarrow$ aA
       B $\rightarrow$ bY | ϵ
       Y $\rightarrow$ bB