CS 3511: Algorithms Honors, Homework 4

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Please submit a LaTeX-formatted PDF containing your solutions on T-Square, by March 16, 3:00 pm.

1 Odd Reachability

(15 points) Given a directed graph $G = (V, E)$ and vertices $s, t \in V$, devise an algorithm to find if t is reachable from s using an odd number of edges, in $O(|V| + |E|)$ time. Provide a formal proof of correctness for your algorithm using induction.

2 Word Modification

You're given a dictionary D containing N words, each with up to K characters, and you're given two words s and t. Your task is simple: if possible, modify the word s into the word t by changing one letter at a time while ensuring that each intermediate word belongs to D. The number of modifications done must be minimized. For example, if we have

$$D = [\text{sire}, \text{size}, \text{then}, \text{wire}, \text{wore}, \text{word}, \text{when}, \dots],$$

one way to change **word** to **size** is:

$$\text{word} \to \text{wore} \to \text{wire} \to \text{sire} \to \text{size}.$$

(5 points) Model this problem using a graph. What would the vertices and edges of the graph be? State the time-complexity of constructing this graph and the space-complexity of storing it. Restate the original problem of changing the word s to the word t in terms of this graph.

(10 points) Devise an efficient algorithm to solve the problem, write down the pseudocode and analyze its run-time.

3 Make America Connected Again!

Today is International Women's Day, so we set this story a few years into the future: The President of the United States needs your help! She is considering enhancing the road network connecting all the cities in the country, but due to tight budgetary constraints, she needs to be very careful with her spending. So before embarking on the project, she wants to do some cost-analysis. Specifically, given a map of the road network connecting cities in America, she wants to know the minimum number of new roads that can be constructed to make the network connected.

(10 points) Given an undirected graph $G = (V, E)$, devise a linear-time algorithm to find the smallest number K such that adding K edges to G would make it a connected graph and argue briefly that the algorithm is correct.