CS 3511: Algorithms Honors, Homework 5

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Please submit a LaTeX-formatted PDF containing your solutions on T-Square, by April 6, 3:00 pm.

1 Make America Strongly Connected Again!

The President of the United States needs your help again, only this time for a directed road network connecting cities in America!

(10 points) Given a directed graph $G = (V, E)$, devise a linear-time algorithm to find the greatest lower bound (that you can) on the smallest number K such that adding K directed edges to G would make it a strongly connected graph. Give a brief justification for the lower bound. You may use Kosaraju's algorithm (discussed in class) to find strongly connected components as a black-box.

2 Shortest Cycles?

The following is an approach to find the length of the shortest cycle in an undirected graph.

When a back edge, say (v, w), is encountered during a depth-first search, it forms a cycle with the tree edges from w to v. The length of the cycle is level$[v]$ − level$[w]$ + 1, where the level of a vertex is its distance in the DFS tree from the root vertex. This suggests the following algorithm:

1. Do a depth-first search, keeping track of the level of each vertex.
2. Each time a back edge is encountered, compute the cycle length and save it if it is smaller than the shortest one previously seen.

(10 points) Show that this strategy does not always work by providing a counterexample as well as a brief (one or two sentence) explanation.

3 Distinct Shortest Paths

(10 points) Given an undirected graph $G = (V, E)$ and two vertices $u, v \in V$, give a linear-time algorithm and pseudocode to find the number of distinct shortest paths from u to v.

4 Validation of Shortest Path Tree

(10 points) Given a directed graph $G = (V, E)$ with (possibly negative) edge weights, a vertex $s \in V$ and a tree $T = (V, E')$ where $E' \subseteq E$, give a linear-time algorithm to check whether T is a shortest-path tree for G with starting point s and prove that it is correct.