CS 3511: Algorithms Honors, Homework 7

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Please submit a LaTeX-formatted PDF containing your solutions on T-Square, by April 22, 11:55 pm.

1 Cycle Removal

(10 points) Given an undirected graph $G = (V, E)$ with edge weights $w : E \to \mathbb{R}^+$, devise an efficient algorithm that finds a minimum weight subset $E^* \subseteq E$ such that removing the edges in E^* from G makes it acyclic.

2 Minimum Vertex Cover for a Tree

(10 points) A vertex cover for a graph $G = (V, E)$ is a subset of the vertices $S \subseteq V$ such that every edge in E is incident to at least one vertex in S, i.e., the set of vertices S "covers" all edges in E. Finding a minimum vertex cover S^* for any graph G is computationally hard and is classified as NP-complete, a property you will learn more about later. However, if G was a tree (a connected graph with no cycles), the problem becomes easy! Devise a linear-time algorithm to find the size of the minimum vertex cover for a tree T with n vertices. (Reading suggestion: section 6.7 in the DPV book.)

3 Uniqueness of Min-Cut

(10 points) Devise a polynomial-time algorithm to determine if an edge-weighted, directed graph G with source and sink vertices s and t has a unique min-cut. Prove that your algorithm is correct. (Reading suggestion: section 7.2 on flows in the DPV book.)

4 Linear Programming: Minimum Weight Vertex Cover

(10 points) Given a graph $G = (V, E)$ with vertex weights $w : V \to \mathbb{R}^+$, write down an Integer Linear Program (ILP) to find a vertex cover with minimum weight. Relax the integer constraint to obtain a general linear program (LP). Write the dual of the obtained LP and briefly provide a combinatorial interpretation of the dual LP. (Reading suggestion: section 7.4 on duality in the DPV book.)