

Lattice gauge ensembles and data management

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We summarize the status of lattice QCD ensemble generation efforts and their data management characteristics. Namely, this proceeding summarizes contributions to a dedicated parallel session during the 41st International Symposium on Lattice Field Theory (Lattice 2024), during which representatives of 16 lattice QCD collaborations provided details on their simulation program, with focus on plans for publication, data management, and storage requirements. The parallel session was organized by the International Lattice Data Grid (ILDG), following an open call to the lattice QCD for participation in the session.

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¹ For the TELOS collaboration ² For the FASTSUM collaboration ³ For the QCDSF collaboration ⁴ For the HAL QCD collaboration ⁵ For the MILC collaboration ⁶ For the Jlab/W&M/LANL/MIT/Marseille effort ⁷ For the CLS ⁸ For the JLQCD collaboration ⁹ For the TWEXT collaboration ¹⁰ For the ETM collaboration (ETMC) ¹¹ For the RBC-UKQCD collaboration ¹² For the RC* collaboration ¹³ For the OpenLat initiative ¹⁴ For the HotQCD collaboration ¹⁵ For the PACS collaboration ¹⁶ For the CLQCD collaboration # Conveners * Speaker

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1. Introduction

The simulation of Quantum Chromodynamics (QCD) via its Euclidean-time, discrete formulation on a lattice, has been one of the most compute-intensive applications in scientific computing, consuming substantial fractions of computer time at leadership HPC facilities internationally. In particular, the generation of ensembles of gauge configurations, for multiple values of the QCD parameters such as the QCD coupling, the quark masses, and the extent of the finite volume, requires multi-year simulation campaigns, coordinated by multi-member research collaborations. It is thus common that collaborations store and reuse the same gauge ensembles for multiple observables of interest, and in many cases also share the ensembles with researchers external to the collaboration that generated them.

The purpose of this proceeding is to summarize the available gauge ensembles generated by various lattice QCD collaborations internationally, with a focus on the data management practices each collaboration employs. It follows a parallel session at the 41st International Symposium on Lattice Field Theory (Lattice 2024), during which 16 collaborations provided status reports of

their simulation efforts, responding to an open call for participation addressed to the lattice QCD community prior to the conference. The first such session was during Lattice 2022 and a report of the contributions presented during that session can be found in Ref. [1].

These sessions are organized by the International Lattice Data Grid (ILDG) with the intention of obtaining, gathering, and summarizing the evolving needs of the lattice community in terms of data storage and management. The ILDG was setup in the early 2000s [2–5] by the lattice community, which realized early on the value in standardizing data management practices across the field. ILDG is organized as a federation of autonomous *regional grids*, within a single Virtual Organization [6]. It standardizes interfaces for the services, which are to be operated by each regional grid, such as storage and a searchable metadata catalog, so that the regional services are interoperable. Within ILDG, working groups specify community-wide agreed metadata schemas (QCDml) [7] to concisely mark-up the gauge configurations and develop relevant middleware tools for facilitating the use of ILDG services. The middleware and metadata specifications developed by ILDG adhere to most of the FAIR (Findable, Accessible, Interoperable, Reusable) principles [8]. A summary of recent developments in ILDG, referred to as ILDG 2.0, was presented during the same session and can be found in a separate proceeding [9].

In the remainder of this proceeding, we present the status of ensemble generation of each of the 16 collaborations that contributed to the parallel session. We restrict ourselves to simulations of QCD, and at present these are carried out using $N_f=2+1$, $N_f=2+1+1$, and $N_f=1+1+1+1$ sea quark flavors with various fermion discretizations. The contributors were asked to specify whether their data are public or if they plan on making them public, their interest in using ILDG services and tools for that purpose, as well as some overall information regarding storage requirements. This information is collected in a table and summary section that follows the individual contributions.

2. Contributions

The contributions from each collaboration follow, in the order presented during the parallel session. The original presentations can be found on the conference website [10].

2.1 CLQCD

The CLQCD collaboration focuses on the first principles QCD study of the spectrum of exotic hadrons, parton structure of the nucleon, and other traditional hadron, N-point correlation functions related to high accuracy tests of the standard model, and also QCD in extreme conditions. For this purpose, CLQCD generated a set of configurations of $N_f=2+1$ fermions using the tadpole improved clover fermion action with stout smearing and the tadpole improved Symanzik gauge action, at 5 values of the lattice spacing $\in [0.05, 0.11]$ fm, several pion masses $\in [120, 350]$ MeV, several volumes with $m_\pi L \in [2.6, 5.8]$ fm, and several temperatures $\in (0, 464]$ MeV. In addition, there are several anisotropic ensembles using clover fermions with $\xi = 5$ at different lattice spacing, pion mass, volume and flavors which concentrate on the high precision studies related to glueball properties. For the gauge generation the Chroma package [11] with QUDA [12–14] is used at present, and we plan to switch to PyQUDA [15] with QUDA in the near future. Currently the CLQCD collaboration has ~ 20 ensembles (one ensemble corresponds to one point in the space lattice spacing-spatial volume-pion mass-temperature), which occupy ~ 100 TB of disk space. Configurations are stored in the SCIDAC format. Possible collaborations on the analysis of the correlation functions are welcome and CLQCD is preparing the hardware and software for the data sharing service.

2.2 Jlab/W&M/LANL/MIT/Marseille

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2.3 HotQCD

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2.4 FASTSUM

The FASTSUM collaboration use $N_f=2+1$ flavor anisotropic gauge ensembles in the fixed-scale approach to study the behavior of QCD as a function of temperature in hadronic and plasma phases. Specifically we have considered the behavior of hadronic states including light, strange, charm and bottom quarks, the electrical conductivity of QCD matter, the interquark potential and properties of the chiral transition. FASTSUM gauge fields utilise an $O(a^2)$ improved Symanzik gauge action and an $O(a)$ improved spatially stout-smearred Wilson-Clover action following the parameter tuning and zero-temperature ensembles of the Hadron Spectrum collaboration [16, 17]. “Generation 2” ensembles were generated using the Chroma [11] software suite while the newer “Generation 2L” used a modification [18] of the openQCD [19, 20] package which introduces stout-link smearing and anisotropic actions. Generation 2 and 2L have an anisotropy $\xi = a_s/a_\tau \sim 3.5$ with $a_s \sim 0.12$ fm, $N_s = 24$ or 32 and a wide range of N_τ corresponding to $T \in [44, 760]$ MeV. Full details of these ensembles may be found in Refs. [21, 22]. We are in the process of production for “Generation 3” - a parameter set similar to “Generation 2” but with twice the anisotropy $\xi \sim 7$ - using openQCD-FASTSUM [18]. We maintain a centralised metadata repository detailing (among other information) who was responsible for each run, on which machine that run was produced and where copies may be found. The gauge fields are redundantly stored on two (well-separated) storage servers managed by Swansea University in the openQCD format. The “Generation 2” ensembles are publically available [23] while other ensembles will be available after an embargo period. We anticipate making ensembles available through the next incarnation of the ILDG with supplementary information also available on Zenodo [23, 24].

2.5 TELOS

The TELOS collaboration performs **T**heoretical **E**xplorations on the **L**attice with **O**rthogonal and **S**ymplectic groups. Problems of interest focus on physics beyond the Standard Model, in particular composite Higgs models. Our work to date has made use of the Wilson gauge action and Wilson fermion action. Our ensembles include studies of the $\text{Sp}(4)$ theory with two fundamental fermion flavours ($N_f=2$) [25] (five values of $\beta \in [6.9, 7.5]$, $V \leq 48 \times 42^3$, $m_{\text{PS}}/m_V \gtrsim 0.407(16)$), the $\text{Sp}(4)$ theory with three antisymmetric fermion flavours ($N_{\text{as}}=3$) [26], (six values of $\beta \in [6.6, 6.9]$, $V \leq 54 \times 36^3$, $m_{\text{PS}}/m_V \gtrsim 0.7954(44)$); and the $\text{Sp}(4)$ theory with $N_f=2$ and $N_{\text{as}}=3$ [27], (three values of $\beta \in [6.45, 6.5]$, $V \leq 56 \times 36^3$, $m_{\text{PS}}/m_V \gtrsim 0.8768(30)$, $m_{\text{PS}}/m_V \gtrsim 0.9022(27)$). In the latter two cases, the topological charge becomes slow running at small m_{as} , and at larger β . We do not retain our pure gauge ensembles, used for studies of the large- N limit of $\text{Sp}(2N)$ [28–30], since the costs of storage and data transfer are higher than those of regenerating the ensembles.

Additionally, we present ensembles generated by a subset of the collaboration, with applications to conformal and near-conformal dynamics, and to potential Walking Technicolor theories, again using the Wilson gauge and Wilson fermion actions [31]. Specifically, these are $\text{SU}(2)$ with one adjoint flavour ($N_{\text{adj}}=1$) (seven values of $\beta \in [2.05, 2.4]$, $V \leq 96 \times 48^3$, $m_{2_s^+} \gtrsim 0.28$), and $N_{\text{adj}}=2$ ($V \leq 128 \times 64^3$, $m_{2_s^+} \gtrsim 0.47$). In the former case, the majority of ensembles show ergodic topology, with $\beta = 2.4$ being marginal; in the latter, at large volumes we see significant topological freezing.

Ensembles are generated using HiRep [32, 33] and Grid [34, 35]. The above ensembles will be made available as soon as the infrastructure is in place to do so.

We are in the process of generating ensembles for $\text{Sp}(4)$ $N_f=2$ and for $\text{SU}(2)$ $N_f=1, 2$ with Möbius domain wall fermions, and for $\text{SU}(2)$ $N_f=1, 2$ with Wilson fermions and additional Pauli–Villars fields, which we aim to make available concurrently with the corresponding papers.

2.6 HAL QCD

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2.7 TWEXT

1 The TWEXT (Twisted Wilson @ Extreme conditions) collaboration studies the properties of
2 QCD at high temperature using Wilson Twisted Mass fermions. Problems under investigation in-
3 clude chiral properties of QCD, in particular the behavior of QCD around the chiral phase transition
4 and its scaling window [36], topological properties of QCD and QCD axion [37], hadron masses,
5 symmetries of QCD and others. For this purpose, TWEXT generated a set of configurations for
6 $N_f=2+1+1$ fermions at the physical pion mass and also uses older configurations with heavier pion
7 mass. Configurations with the physical pion mass have three lattice spacings $a \in (0.057, 0.080)$ fm
8 and cover a wide range of temperatures from ~ 120 MeV to ~ 900 MeV. It allows the TWEXT
9 collaboration to perform the continuum extrapolation for quantities of interest in this temperature
10 range. For the generation the tmLQCD software package [38–40] is used and the parameters of
11 the ensembles were taken from the zero temperature simulations of the ETM collaboration [41].
12 Currently, the TWEXT collaboration has 80 ensembles (one ensemble corresponds to one point in
13 the space temperature-pion mass-lattice spacing), which occupy ~ 80 TB of disk space. Configu-
14 rations are stored in the ILDG format. Possible collaborations are welcome and TWEXT plans to
15 make configurations public/use ILDG in the future, after performing the ongoing analysis.

2.8 QCDSF

 The main focus of the QCDSF collaboration is hadron spectrum and structure at zero tem-
perature. Our ensembles are generated using the Symanzik improved gauge action and Stout Link
Non-perturbative Clover (SLiNC) fermion action, for which the link variables appearing in the Dirac
term are stout smeared, while the links in the clover term are not [42]. The clover coefficient is
determined non-perturbatively. Our most recent set of ensembles are $2+1$ -flavour, which cover pion
masses ranging $m_\pi^{phys} \lesssim m_\pi \lesssim 470$ MeV, and 5 lattice spacings in between $a = 0.052 - 0.082$ fm
(inclusive). In total, there are 22 ensembles available and an additional 2 at almost-physical pion
mass still being generated. A recent listing of available ensembles can be found in [43]. Our
approach to the physical point follows the $\bar{m}^R = (2m_\ell^R + m_s^R)/3 = const$ line [44], i.e. we start
from the SU(3) symmetric point where the renormalized masses of strange (m_s^R) and light quarks
(m_ℓ^R) are equal to each other, $m_s^R = m_\ell^R = \bar{m}^R/3$ and we increase m_s^R as m_ℓ^R decreases. The BQCD
software suite [45] is used to generate the ensembles, utilising the hybrid Monte Carlo (HMC) and
rational HMC algorithms. All of our gauge configurations are stored in the ILDG format with
metadata compliant with the ILDG scheme. We have made use of ILDG storage systems before,
where the hub at the CSSM, Adelaide served as one of the regional grids, and some older configu-
rations are still stored in the ILDG servers. The QCDSF collaboration kindly asks any prospective
users to contact the collaboration before utilizing these configurations for their projects. We plan to
make newly generated ensembles available upon request through ILDG, pending the collaboration's
confirmation. QCDSF is open to new collaborative projects.

2.9 RBC-UKQCD

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2.10 OpenLat

The Open Lattice Initiative is generating QCD ensembles using Stabilized Wilson Fermions (SWF) with $N_f=2+1$ [46], based on the exponentially improved Dirac Wilson fermion formulation. The code used for the generation is OpenQCD [20]. These ensembles are intended to be general-purpose datasets, shared with the broader LQCD community in line with the principles of open science. Lines of constant physics are defined through the ϕ_4 parameter setting at a fixed trace M . The initial tuning was performed at the SU(3) flavor symmetric point. Additionally, a set of more chiral trajectories has been put into production, identified by four different pion masses, aiming to reach the physical point.

The generation has been divided into three stages, each to be released with a publication. The first stage, which is nearly complete, focuses on the flavor symmetric point; the second on $m_\pi = 300, 200$ MeV; and the third on the physical point. For each pion mass, there are four lattice spacings with periodic boundary conditions ($a = 0.12, 0.094, 0.077, 0.064$ fm) and one ensemble with open boundary conditions at a finer lattice spacing of $a = 0.054$ fm. The volumes have been chosen to always ensure that $m_\pi L > 4$. The collaboration is strongly in favor of uploading configurations to the ILDG, as it aligns with our core principles. The files are already stored in the ILDG format and a preliminary upload of a few configurations has been done before the LATTICE24 conference. Our ensembles are publicly available for use after the publication of the corresponding stage has been completed. The total storage required is expected to be 0.5 PB.

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2.12 ETMC

1 The ETM collaboration focuses on hadron spectroscopy, hadron structure, and flavor physics
2 at zero temperature. Ensembles employ the twisted mass formulation, realizing $O(a)$ -improvement
3 by tuning to maximal twist, and include a clover term to further reduce the size of lattice artifacts.
4 The Iwasaki gauge action is used. The main simulation effort is for the generation of ensembles
5 with degenerate up- and down-, strange- and charm-quarks ($N_f=2+1+1$) with lattice spacing ranging
6 between 0.049 and 0.091 fm. $M_\pi \cdot L$ varies from 2.5 up to ~ 5.5 . At the time of writing, 24 ensembles
7 are available or in the process of being generated, with 8 of these at approximately physical values of
8 the quark masses. For a recent listing of the ensembles, see [47]. Simulations are performed using
9 the Hybrid Monte Carlo (HMC) algorithm implemented in the tmLQCD software package [38–40].
10 See Ref. [41] for details on the simulation program, including the parameter tuning. The DD-
11 α AMG [48, 49] multigrid iterative solver is employed for the most poorly conditioned monomials
12 in the light sector while mixed-precision CG is used elsewhere. Multi-shift CG is used together
13 with shift-by-shift refinement using DD- α AMG [50] for a number of small shifts for the heavy
14 sector. tmLQCD has interfaces to QPhiX [51] and QUDA [12, 13]. tmLQCD automatically writes
15 gauge configurations in the ILDG format, with meta-data including creation date, target simulation
16 parameters, and the plaquette. ETMC policy is to make ensembles publicly available after a grace
17 period. Older $N_f=2$ and $N_f=2+1+1$ ensembles [52–54] have made use of ILDG storage elements.
18 The current ensembles are available upon request and the collaboration intends to use ILDG in the
19 near future. For these ensembles, we expect storage requirements to reach 3 PB.

2.13 JLQCD

The JLQCD collaboration uses a tree-level Symanzik improved gauge action and Möbius domain wall fermions with the scale factor 2 and 3 levels of stout smearing, of which details are found in the supplemental material of [55]. We have two targets of physics to study: one is $T = 0$ 2+1 flavor focusing on B-physics, and the other is finite T focusing chiral symmetry (2 and 2+1 flavors) and investigation of the Columbia plot (3 and 2+1 flavors).

The $T = 0$ ensembles are $32^3 \times 64 - 64^3 \times 128$ lattices with pion mass M_π between 230 – 500 MeV, and the lattice cutoff a^{-1} is 2.5, 3.6 and 4.5 GeV [55]. Together with 3 flavor ensembles with smaller volumes, we have more than 16 ensembles with 1.5k configurations.

Most of the finite- T 2+1 flavor sets are generated along the line of constant physics at $m_l = m_s^{\text{phys}}/27.4$ and $m_l = m_s^{\text{phys}}/10$ with $N_T = 12, 16$, and the remaining finite- T ensembles (2, 2+1 and 3 flavors) are generated with the fixed T approach. Some up-to-date details are found in [56] (2 flavor), [57] (3 flavor), and [58] (2+1 flavor). The recent configurations are generated mainly on the Fugaku supercomputer using Grid [59]. We employ the Hybrid Monte Carlo (HMC) and Rational Hybrid Monte Carlo algorithms. For the three flavor systems, we apply the same 2+1 flavor algorithm but with degenerate masses. The generated configurations are stored in a grid file system, the Japan Lattice Data Grid (JLDG). We are in close contact with ILDG members and plan to start uploading ensembles. The number of configurations and storage quoted in Table 1 assumes storing every 100 MD trajectories, which can be revised when uploading each ensemble.

2.14 MILC

The MILC Collaboration has been creating ensembles of gauge configurations for decades. Initial calculations used two dynamical flavors with naive staggered quarks. Second generation efforts used dynamical up, down, and strange quarks with the asqtad action, culminating with a review in Ref. [60]. Our third generation calculations use the highly improved staggered quark (HISQ) action [61]. They contain dynamical up, down, strange, and charm quarks. For most of the ensembles, up and down are degenerate. The charm quark is set near its physical value. For several ensembles, the light quarks are near their average physical value. There are also ensembles with $m_l = 0.1$, or $0.2m_s$, where m_l (m_s) is the light (strange) quark mass. We have generated a number of ensembles with m_s less than its physical value to explore low energy constants and the chiral Lagrangian. Lattice spacings are in the range of [0.15, 0.03] fm. In a few cases, multiple volumes are available. Details about the generation of configurations can be found in Refs. [62–64].

The sharing policy for the MILC ensembles is available on GitHub [65]. On that web page can be found links to 1) the sharing policy, 2) a Google Sheet detailing freely available ensembles, and 3) a document summarizing which papers to cite for the use of each ensemble. Anyone wishing to use an ensemble that is not listed in the Google Sheet, but has been used in a publication or noted in a talk, is welcome to contact a member of the Fermilab Lattice or MILC Collaborations to inquire as to whether the ensemble can be made available for a specific project. Many configurations are available on USQCD resources, making access relatively easy for USQCD members. The ILDG is not operational in the US, but should it become so, we would make an effort to use it. We have assisted transfer of configurations to other researchers both within and outside the US.

2.15 CLS

The CLS (Coordinated Lattice Simulations) effort uses the openQCD code [20] to generate ensembles [66, 67] with $N_f=2+1$ non-perturbatively $O(a)$ -improved Wilson quarks and tree-level improved Symanzik glue, mostly with open boundary conditions in time to avoid topological freezing [19, 68], but also with (anti-)periodic boundary conditions in time (on some ensembles at $a \gtrsim 0.06$ fm), at six fine lattice spacings $a \in [0.039, 0.1]$ fm and quark masses from the symmetric to the physical point on three chiral trajectories ($\text{Tr}[M] = \text{const.}$, $m_s \approx \text{const.}$, $m_s = m_l$) in large volumes satisfying $M_\pi L \geq 4$ throughout, with statistics typically $\gtrsim 2,000$ MDU.

Due to the algorithm used, two reweighting factors are needed to use CLS ensembles: one to correct for the twisted-mass stabilization of the light quarks, and one to correct for the rational approximation to $\sqrt{D^\dagger D}$ for the strange quark. In the latter case, $\det D < 0$ can occur [69], so that one also needs to correct for the wrong sign of the reweighting factor; fortunately, the fraction of configurations with a negative reweighting factor is very small (or zero) for most ensembles.

At the time of Lattice 2024, there were 149,766 configurations (1.384 PB) stored on tape in the openQCD (non-ILDG) data format. Metadata regarding data provenance, simulation setup, HMC stability, and data integrity are collected automatically via automated scripts, while reweighting factors and determinant minus signs are measured separately. A first batch of ensembles has been successfully uploaded to ILDG; several more ensembles will follow soon, and the remainder will follow after an embargo period. The XML metadata are generated automatically by extraction from the existing database; the (signed) reweighting factors are included in the Config XML.

2.16 PACS

In the past years, the PACS Collaboration has generated 2+1 flavor QCD configurations on very large lattices close to the physical point employing the stout-smear $O(a)$ -improved Wilson-clover quark action and Iwasaki gauge action. We use the stout smearing with six iterations. The improvement coefficient for the clover term is nonperturbatively determined using the Schrödinger functional scheme. These gauge configurations, which keep the space-time volumes larger than $(10 \text{ fm})^4$, are called “PACS10” configurations. We have finished generating three gauge ensembles of (lattice spacing, lattice size)=(0.085 fm, 128^4) [70], (0.064 fm, 160^4) [71], and (0.041 fm, 256^4), labeled as PACS10/L128, PACS10/L160, PACS10/L256, respectively. Using those PACS10 configurations, we have studied the hadron spectrum [70, 72], nucleon structure [73–76], and physics beyond the standard model [71, 77–79]. For more precise determinations of physical quantities, we have started to generate the PACS10_c configurations, which are 2+1+1 flavor QCD configurations satisfying the PACS10 conditions. In the configuration generation, the degenerate up- and down-quarks are simulated with the domain-decomposed HMC algorithm [80] and the strange quark with the rational HMC algorithm [81]. The up- and down-quark determinant is calculated by separating into UV and IR parts, with mass-preconditioning [82] for the IR part, even-odd preconditioning, and mixed precision nested BiCGStab [83] with the aid of the chronological inverter as a guess. Many gauge ensembles generated by the PACS Collaboration and its predecessors, the CP-PACS and PACS-CS Collaborations, are publicly available [84]. We plan to make the PACS10 and PACS10_c ensembles (roughly 200 TB) public through the new generation of ILDG via JLDG after some embargo time. Details of the data policies are under discussion within the collaboration.

3. Summary

Table 1: Public: (2 = currently public, 1 = after an embargo period, 0 = no); ILDG: (N = no interest, I = interest, P = planned, U = already using); #ens: Number of ensembles; #cfg: Total number of configurations; storage: Total storage needed in TBytes.

Collaboration	Public	ILDG	#ens	#cfg	Storage (TB)
CLQCD	1	I	20	10,000	100
Jlab/W&M/LANL/MIT/Marseille					
HotQCD					
FASTSUM	2,1	I	37	37,000	65
TELOS	1	P	250	800,000	120
HAL QCD					
TWEXT	1	P	80	70,000	80
QCDSF	1	P,U	24	20,000	55
RBC-UKQCD					
OpenLat	2	U	12	40,000	500
RC*					
ETMC	1	P,U	24	12,000	3,000
JLQCD	1	P	260	37,000	15
MILC	1	I	50	70,000	600
CLS	1	P,U	25	150,000	1,400
PACS	2,1	P,U	6	200	200

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SQUID at Osaka University,
Polarie and Grand Chariot at Hokkaido University,
LUMI-C and LUMI-G at CSC, Finland,
IBM Blue Gene system at KEK,
Irène Joliot-Curie at Très Grand Centre de Calcul (TGCC) in Bruyères-le-Châtel, France,
JUGENE, JUWELS and JUWELS-Booster at Jülich Supercomputing Centre (JSC),
HAWK at Höchstleistungsrechenzentrum Stuttgart (HLRS), Germany,
SuperMUC and SuperMUC-NG at Leibniz Rechenzentrum (LRZ) in Garching, Germany,
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References

- [1] G. Bali et al., *Lattice gauge ensembles and data management*, *PoS LATTICE2022* (2022) 203 [[2212.10138](#)].
- [2] UKQCD collaboration, C. T. H. Davies, A. C. Irving, R. D. Kenway and C. M. Maynard, *International lattice data grid*, *Nucl. Phys. B Proc. Suppl.* **119** (2003) 225 [[hep-lat/0209121](#)].

- [3] T. Yoshie, *Making use of the International Lattice Data Grid*, *PoS LATTICE2008* (2008) 019 [0812.0849].
- [4] C. M. Maynard, *International Lattice Data Grid: Turn On, Plug In, and Download*, *PoS LAT2009* (2009) 020 [1001.5207].
- [5] M. G. Beckett, B. Joo, C. M. Maynard, D. Pleiter, O. Tatebe et al., *Building the International Lattice Data Grid*, *Comput. Phys. Commun.* **182** (2011) 1208 [0910.1692].
- [6] I. L. D. Grid, “Organization of ildg activities.”
<https://hpc.desy.de/ildg/organization/>. Accessed 2024-08-06.
- [7] ILDG METADATA WORKING GROUP collaboration, P. Coddington, B. Joo, C. M. Maynard, D. Pleiter and T. Yoshie, *Marking up lattice QCD configurations and ensembles*, *PoS LATTICE2007* (2007) 048 [0710.0230].
- [8] M. D. Wilkinson, M. Dumontier, I. J. Aalbersberg, G. Appleton, M. Axton et al., *The fair guiding principles for scientific data management and stewardship*, *Scientific Data* **3** (2016) 160018.
- [9] H. Matsufuru, H. Simma and C. Urbach, *ILDG 2.0*, *PoS LATTICE2024* (2024) .
- [10] “Lattice data session.”
<https://conference.ippp.dur.ac.uk/event/1265/sessions/1744/#20240802>. Accessed 2024-08-02.
- [11] SciDAC, LHPC, UKQCD collaboration, R. G. Edwards and B. Joó, *The Chroma software system for lattice QCD*, *Nucl. Phys. Proc. Suppl.* **140** (2005) 832 [hep-lat/0409003].
- [12] QUDA collaboration, M. A. Clark, R. Babich, K. Barros, R. C. Brower and C. Rebbi, *Solving Lattice QCD systems of equations using mixed precision solvers on GPUs*, *Comput. Phys. Commun.* **181** (2010) 1517 [0911.3191].
- [13] QUDA collaboration, R. Babich, M. A. Clark, B. Joo, G. Shi, R. C. Brower et al., *Scaling lattice QCD beyond 100 GPUs*, in *International Conference for High Performance Computing, Networking, Storage and Analysis*, 9, 2011, 1109.2935, DOI.
- [14] QUDA collaboration, M. A. Clark, B. Joó, A. Strelchenko, M. Cheng, A. Gambhir et al., *Accelerating lattice QCD multigrid on GPUs using fine-grained parallelization*, in *International Conference for High Performance Computing, Networking, Storage and Analysis*, 12, 2016, 1612.07873, DOI.
- [15] X. Jiang, C. Shi, Y. Chen, M. Gong and Y.-B. Yang, *Use QUDA for lattice QCD calculation with Python*, 2411.08461.
- [16] R. G. Edwards, B. Joo and H.-W. Lin, *Tuning for Three-flavors of Anisotropic Clover Fermions with Stout-link Smearing*, *Phys. Rev. D* **78** (2008) 054501 [0803.3960].

- [17] HADRON SPECTRUM collaboration, H.-W. Lin et al., *First results from 2+1 dynamical quark flavors on an anisotropic lattice: Light-hadron spectroscopy and setting the strange-quark mass*, *Phys. Rev. D* **79** (2009) 034502 [0810.3588].
- [18] J. R. Glesaaen and B. Jäger, “openqcd-fastsum.” <https://gitlab.com/fastsum>, Apr., 2018. 10.5281/zenodo.2216355.
- [19] M. Lüscher and S. Schaefer, *Lattice QCD with open boundary conditions and twisted-mass reweighting*, *Comput. Phys. Commun.* **184** (2013) 519 [1206.2809].
- [20] “openQCD: Simulation programs for lattice QCD.” <https://luscher.web.cern.ch/luscher/openQCD/>.
- [21] G. Aarts, C. Allton, A. Amato, P. Giudice, S. Hands et al., *Electrical conductivity and charge diffusion in thermal QCD from the lattice*, *JHEP* **02** (2015) 186 [1412.6411].
- [22] G. Aarts et al., *Properties of the QCD thermal transition with $N_f=2+1$ flavors of Wilson quark*, *Phys. Rev. D* **105** (2022) 034504 [2007.04188].
- [23] G. Aarts, C. Allton, A. Amato, R. Bignell, T. J. Burns et al., *FASTSUM Generation 2 Anisotropic Thermal Lattice QCD Gauge Ensembles*, July, 2024. 10.5281/zenodo.8403827.
- [24] European Organization For Nuclear Research and OpenAIRE, *Zenodo*, 2013. 10.25495/7GXX-RD71.
- [25] E. Bennett, D. K. Hong, J.-W. Lee, C. J. D. Lin, B. Lucini et al., *$Sp(4)$ gauge theories on the lattice: $N_f = 2$ dynamical fundamental fermions*, *JHEP* **12** (2019) 053 [1909.12662].
- [26] H. Hsiao, E. Bennett, D. K. Hong, J.-W. Lee, C. J. D. Lin et al., *Spectroscopy of $Sp(4)$ lattice gauge theory with $n_f = 3$ antisymmetric fermions*, *PoS LATTICE2022* (2023) 211 [2210.08154].
- [27] E. Bennett, D. K. Hong, H. Hsiao, J.-W. Lee, C. J. D. Lin et al., *Lattice studies of the $Sp(4)$ gauge theory with two fundamental and three antisymmetric Dirac fermions*, *Phys. Rev. D* **106** (2022) 014501 [2202.05516].
- [28] E. Bennett, J. Holligan, D. K. Hong, J.-W. Lee, C. J. D. Lin et al., *Spectrum of mesons in quenched $Sp(2N)$ gauge theories*, *Phys. Rev. D* **109** (2024) 094517 [2312.08465].
- [29] E. Bennett, D. K. Hong, J.-W. Lee, C. J. D. Lin, B. Lucini et al., *Color dependence of the topological susceptibility in Yang-Mills theories*, *Phys. Lett. B* **835** (2022) 137504 [2205.09254].
- [30] E. Bennett, J. Holligan, D. K. Hong, J.-W. Lee, C. J. D. Lin et al., *Glueballs and strings in $Sp(2N)$ Yang-Mills theories*, *Phys. Rev. D* **103** (2021) 054509 [2010.15781].
- [31] A. Athenodorou, E. Bennett, G. Bergner, P. Butti, J. Lenz et al., *$SU(2)$ gauge theory with one and two adjoint fermions towards the continuum limit*, 2408.00171.

- [32] E. Bennett, D. K. Hong, J.-W. Lee, C.-J. D. Lin, B. Lucini et al., *Sp(4) gauge theories on the lattice: quenched fundamental and antisymmetric fermions*, *Phys. Rev. D* **101** (2020) 074516 [[1912.06505](#)].
- [33] L. Del Debbio, A. Patella and C. Pica, *Higher representations on the lattice: Numerical simulations. SU(2) with adjoint fermions*, *Phys. Rev. D* **81** (2010) 094503 [[0805.2058](#)].
- [34] E. Bennett et al., *Symplectic lattice gauge theories in the grid framework: Approaching the conformal window*, *Phys. Rev. D* **108** (2023) 094508 [[2306.11649](#)].
- [35] A. Yamaguchi, P. Boyle, G. Cossu, G. Filaci, C. Lehner et al., *Grid: OneCode and FourAPIs*, *PoS LATTICE2021* (2022) 035 [[2203.06777](#)].
- [36] A. Y. Kotov, M. P. Lombardo and A. Trunin, *QCD transition at the physical point, and its scaling window from twisted mass Wilson fermions*, *Phys. Lett. B* **823** (2021) 136749 [[2105.09842](#)].
- [37] A. Y. Kotov, A. Trunin and M. P. Lombardo, *QCD topology and axion's properties from Wilson twisted mass lattice simulations*, *PoS LATTICE2021* (2022) 032 [[2111.15421](#)].
- [38] K. Jansen and C. Urbach, *tmLQCD: A Program suite to simulate Wilson Twisted mass Lattice QCD*, *Comput. Phys. Commun.* **180** (2009) 2717 [[0905.3331](#)].
- [39] A. Deuzeman, K. Jansen, B. Kostrzewa and C. Urbach, *Experiences with OpenMP in tmLQCD*, *PoS LATTICE2013* (2014) 416 [[1311.4521](#)].
- [40] A. Abdel-Rehim, F. Burger, A. Deuzeman, K. Jansen, B. Kostrzewa et al., *Recent developments in the tmLQCD software suite*, *PoS LATTICE2013* (2014) 414 [[1311.5495](#)].
- [41] C. Alexandrou et al., *Simulating twisted mass fermions at physical light, strange and charm quark masses*, *Phys. Rev. D* **98** (2018) 054518 [[1807.00495](#)].
- [42] N. Cundy, M. Göckeler, R. Horsley, T. Kaltenbrunner, A. D. Kennedy et al., *Non-perturbative improvement of stout-smear three flavour clover fermions*, *Phys. Rev. D* **79** (2009) 094507 [[0901.3302](#)].
- [43] QCDSF/UKQCD/CSSM collaboration, R. E. Smail, M. Batelaan, R. Horsley, Y. Nakamura, H. Perlt et al., *Constraining beyond the standard model nucleon isovector charges*, *Phys. Rev. D* **108** (2023) 094511 [[2304.02866](#)].
- [44] W. Bietenholz, V. Bornyakov, M. Göckeler, R. Horsley, W. G. Lockhart et al., *Flavour blindness and patterns of flavour symmetry breaking in lattice simulations of up, down and strange quarks*, *Phys. Rev. D* **84** (2011) 054509 [[1102.5300](#)].
- [45] T. R. Haar, Y. Nakamura and H. Stuben, *An update on the BQCD Hybrid Monte Carlo program*, *EPJ Web Conf.* **175** (2018) 14011 [[1711.03836](#)].

- [46] A. S. Francis, F. Cuteri, P. Fritzsch, G. Pederiva, A. Rago et al., *Properties, ensembles and hadron spectra with Stabilised Wilson Fermions*, *PoS LATTICE2021* (2022) 118 [2201.03874].
- [47] B. Kostrzewa, “Status of the etmc ensemble generation effort.” <https://conference.ippp.dur.ac.uk/event/1265/contributions/7655/>.
- [48] A. Frommer, K. Kahl, S. Krieg, B. Leder and M. Rottmann, *Adaptive Aggregation-Based Domain Decomposition Multigrid for the Lattice Wilson–Dirac Operator*, *SIAM J. Sci. Comput.* **36** (2014) A1581 [1303.1377].
- [49] C. Alexandrou, S. Bacchio, J. Finkenrath, A. Frommer, K. Kahl et al., *Adaptive Aggregation-based Domain Decomposition Multigrid for Twisted Mass Fermions*, *Phys. Rev. D* **94** (2016) 114509 [1610.02370].
- [50] C. Alexandrou, S. Bacchio and J. Finkenrath, *Multigrid approach in shifted linear systems for the non-degenerated twisted mass operator*, *Comput. Phys. Commun.* **236** (2019) 51 [1805.09584].
- [51] B. Joó, D. D. Kalamkar, K. Vaidyanathan, M. Smelyanskiy, K. Pamnany et al., *Lattice QCD on Intel® Xeon Phi Coprocessors*, *Lect. Notes Comput. Sci.* **7905** (2013) 40.
- [52] R. Baron et al., *Light hadrons from lattice QCD with light (u,d), strange and charm dynamical quarks*, *JHEP* **06** (2010) 111 [1004.5284].
- [53] EUROPEAN TWISTED MASS collaboration, R. Baron et al., *Computing K and D meson masses with $N_f = 2+1+1$ twisted mass lattice QCD*, *Comput. Phys. Commun.* **182** (2011) 299 [1005.2042].
- [54] ETM collaboration, R. Baron et al., *Light Meson Physics from Maximally Twisted Mass Lattice QCD*, *JHEP* **08** (2010) 097 [0911.5061].
- [55] JLQCD collaboration, B. Colquhoun, S. Hashimoto, T. Kaneko and J. Koponen, *Form factors of $B \rightarrow \pi \ell \nu$ and a determination of $|V_{ub}|$ with Möbius domain-wall fermions*, *Phys. Rev. D* **106** (2022) 054502 [2203.04938].
- [56] JLQCD collaboration, S. Aoki, Y. Aoki, H. Fukaya, S. Hashimoto, C. Rohrhofer et al., *Role of the axial $U(1)$ anomaly in the chiral susceptibility of QCD at high temperature*, *PTEP* **2022** (2022) 023B05 [2103.05954].
- [57] Y. Zhang, Y. Aoki, S. Hashimoto, I. Kanamori, T. Kaneko et al., *Exploring the QCD phase diagram with three flavors of Möbius domain wall fermions*, *PoS LATTICE2023* (2024) 203 [2401.05066].
- [58] JLQCD collaboration, S. Aoki, Y. Aoki, H. Fukaya, S. Hashimoto, I. Kanamori et al., *Axial $U(1)$ symmetry near the pseudocritical temperature in $N_f = 2 + 1$ lattice QCD with chiral fermions*, *PoS LATTICE2023* (2024) 185 [2401.14022].

- [59] P. A. Boyle, G. Cossu, A. Yamaguchi and A. Portelli, *Grid: A next generation data parallel C++ QCD library*, *PoS LATTICE2015* (2016) 023.
- [60] MILC collaboration, A. Bazavov et al., *Nonperturbative QCD Simulations with 2+1 Flavors of Improved Staggered Quarks*, *Rev. Mod. Phys.* **82** (2010) 1349.
- [61] HPQCD collaboration, E. Follana, Q. Mason, C. Davies, K. Hornbostel, G. P. Lepage et al., *Highly improved staggered quarks on the lattice, with applications to charm physics*, *Phys. Rev. D* **75** (2007) 054502 [[hep-lat/0610092](#)].
- [62] MILC collaboration, A. Bazavov et al., *Scaling studies of QCD with the dynamical HISQ action*, *Phys. Rev. D* **82** (2010) 074501 [[1004.0342](#)].
- [63] MILC collaboration, A. Bazavov et al., *Lattice QCD Ensembles with Four Flavors of Highly Improved Staggered Quarks*, *Phys. Rev. D* **87** (2013) 054505.
- [64] FERMILAB LATTICE, MILC collaboration, A. Bazavov et al., *B- and D-meson leptonic decay constants from four-flavor lattice QCD*, *Phys. Rev. D* **98** (2018) 074512 [[1712.09262](#)].
- [65] “Milc and fermilab lattice collaboration policy for sharing lattice files.”
<https://github.com/milc-qcd/sharing/wiki/LatticeSharing>.
- [66] M. Bruno et al., *Simulation of QCD with $N_f = 2 + 1$ flavors of non-perturbatively improved Wilson fermions*, *JHEP* **02** (2015) 043 [[1411.3982](#)].
- [67] D. Mohler, S. Schaefer and J. Simeth, *CLS 2+1 flavor simulations at physical light- and strange-quark masses*, *EPJ Web Conf.* **175** (2018) 02010 [[1712.04884](#)].
- [68] ALPHA collaboration, S. Schaefer, R. Sommer and F. Virota, *Critical slowing down and error analysis in lattice QCD simulations*, *Nucl. Phys. B* **845** (2011) 93 [[1009.5228](#)].
- [69] D. Mohler and S. Schaefer, *Remarks on strange-quark simulations with Wilson fermions*, *Phys. Rev. D* **102** (2020) 074506 [[2003.13359](#)].
- [70] PACS collaboration, K. I. Ishikawa, N. Ishizuka, Y. Kuramashi, Y. Nakamura, Y. Namekawa et al., *Finite size effect on pseudoscalar meson sector in 2+1 flavor QCD at the physical point*, *Phys. Rev. D* **99** (2019) 014504 [[1807.06237](#)].
- [71] PACS collaboration, E. Shintani and Y. Kuramashi, *Hadronic vacuum polarization contribution to the muon $g - 2$ with 2+1 flavor lattice QCD on a larger than $(10\text{ fm})^4$ lattice at the physical point*, *Phys. Rev. D* **100** (2019) 034517 [[1902.00885](#)].
- [72] PACS collaboration, K. I. Ishikawa, N. Ishizuka, Y. Kuramashi, Y. Nakamura, Y. Namekawa et al., *Finite size effect on vector meson and baryon sectors in 2+1 flavor QCD at the physical point*, *Phys. Rev. D* **100** (2019) 094502 [[1907.10846](#)].
- [73] E. Shintani, K.-I. Ishikawa, Y. Kuramashi, S. Sasaki and T. Yamazaki, *Nucleon form factors and root-mean-square radii on a $(10.8\text{ fm})^4$ lattice at the physical point*, *Phys. Rev. D* **99** (2019) 014510 [[1811.07292](#)], [Erratum: *Phys.Rev.D* 102, 019902 (2020)].

- [74] PACS collaboration, R. Tsuji, N. Tsukamoto, Y. Aoki, K.-I. Ishikawa, Y. Kuramashi et al., *Nucleon isovector couplings in $N_f=2+1$ lattice QCD at the physical point*, *Phys. Rev. D* **106** (2022) 094505 [2207.11914].
- [75] PACS collaboration, R. Tsuji, Y. Aoki, K.-I. Ishikawa, Y. Kuramashi, S. Sasaki et al., *Nucleon form factors in $N_f=2+1$ lattice QCD at the physical point: Finite lattice spacing effect on the root-mean-square radii*, *Phys. Rev. D* **109** (2024) 094505 [2311.10345].
- [76] PACS collaboration, R. Tsuji, Y. Aoki, K.-I. Ishikawa, Y. Kuramashi, S. Sasaki et al., *Discretization effects on nucleon root-mean-square radii from lattice QCD at the physical point*, *PoS LATTICE2023* (2024) 323 [2401.05340].
- [77] PACS collaboration, J. Kakazu, K.-i. Ishikawa, N. Ishizuka, Y. Kuramashi, Y. Nakamura et al., *K_{l3} form factors at the physical point on a $(10.9\text{ fm})^3$ volume*, *Phys. Rev. D* **101** (2020) 094504 [1912.13127].
- [78] PACS collaboration, K.-i. Ishikawa, N. Ishizuka, Y. Kuramashi, Y. Namekawa, Y. Taniguchi et al., *$K\ell 3$ form factors at the physical point: Toward the continuum limit*, *Phys. Rev. D* **106** (2022) 094501 [2206.08654].
- [79] PACS collaboration, T. Yamazaki, K.-i. Ishikawa, N. Ishizuka, Y. Kuramashi, Y. Namekawa et al., *$|V_{us}|$ from kaon semileptonic form factor in $N_f = 2 + 1$ QCD at the physical point on $(10\text{ fm})^4$* , *PoS LATTICE2023* (2024) 276 [2311.16755].
- [80] M. Luscher, *Lattice QCD and the Schwarz alternating procedure*, *JHEP* **05** (2003) 052 [hep-lat/0304007].
- [81] M. A. Clark and A. D. Kennedy, *Accelerating dynamical fermion computations using the rational hybrid Monte Carlo (RHMC) algorithm with multiple pseudofermion fields*, *Phys. Rev. Lett.* **98** (2007) 051601 [hep-lat/0608015].
- [82] M. Hasenbusch, *Speeding up the hybrid Monte Carlo algorithm for dynamical fermions*, *Phys. Lett. B* **519** (2001) 177 [hep-lat/0107019].
- [83] PACS-CS collaboration, S. Aoki et al., *$2+1$ Flavor Lattice QCD toward the Physical Point*, *Phys. Rev. D* **79** (2009) 034503 [0807.1661].
- [84] “Japan lattice data grid.” <https://www.jldg.org>.