Self-supervised Classification of Clinical Multivariate Time Series using Time Series Dynamics

Data Mining presentation P14

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Presentation scheme

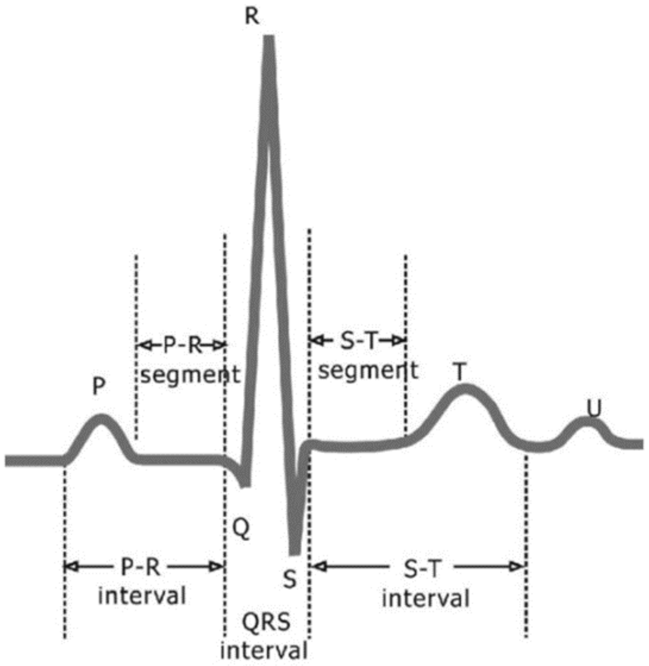
1. Introduction to the topic of the paper
2. Background of Koopman theory
3. Model description and implementation details
4. Experimental results

Introduction

- The purpose is to **improve the accuracy of multivariate time series classification** with a novel **self-supervised** paradigm that captures the dynamics of MTS.
- The study is conducted on **clinical data** and the time series are multivariate because they are obtained from a sequence of measurements coming from multiple sensors.
- **Theoretically grounded** approach: **Koopman theory** allows us to project the complex and nonlinear dynamics of our time series into a linear space.

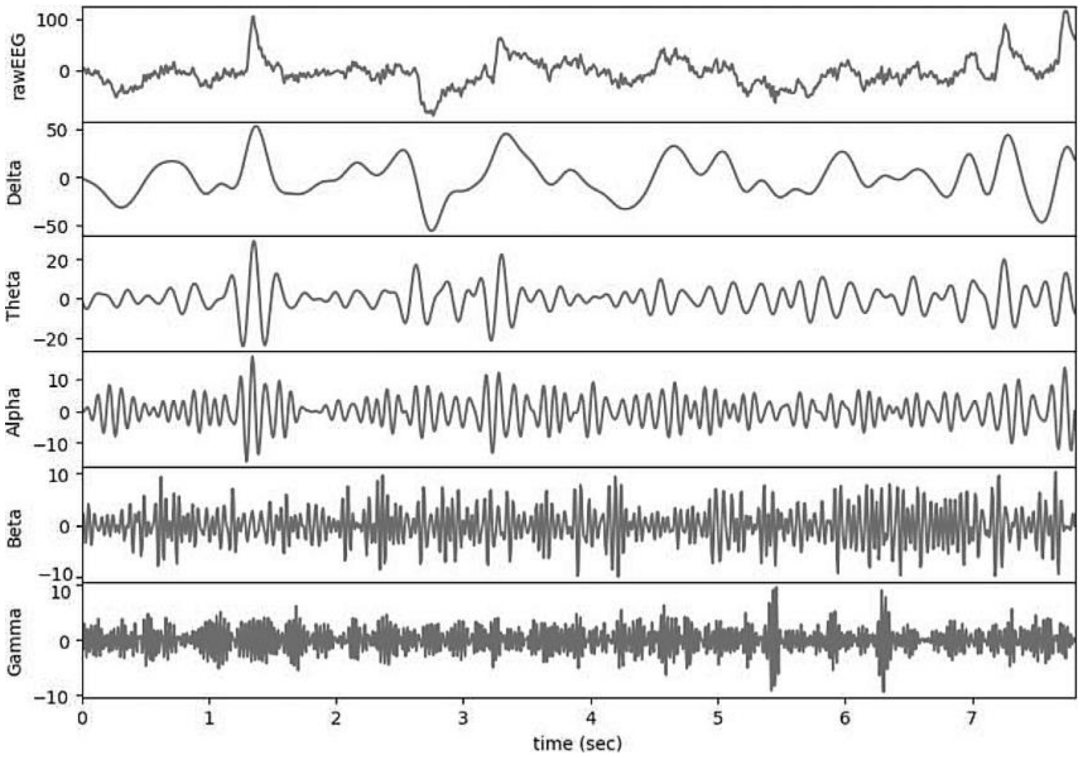
Electrocardiogram (ECG)

- An electrocardiogram (ECG) is a test that records the electrical activity of the heart.
- 10 electrodes (**12 leads**), each lead generates a time series of data.
- Duration of **10 seconds**.
- Sampling frequency of **500 Hz**.
- Combining the results of the different leads allows to infer cardiac activity and can help diagnose a variety of health conditions (arrhythmias, ventricular dysfunction, etc.)



Electroencephalogram (EEG)

- An electroencephalogram (EEG) is an electrical signal generated by the brain.
- **1-256 electrodes,** each electrode generates a time series of data called **channel**.
- Duration of **20-60 minutes**.
- From the raw EEG signal, specific frequency components are extracted:
  - Delta (< 4 Hz)
  - Theta (4-7 Hz)
  - Alpha (8-13 Hz)
  - Beta (14-30 Hz)
  - Gamma (> 30 Hz)
- The analysis of the different frequency components and waveforms allows to identify various neurological conditions (epilepsy, sleep disorders, brain tumor, etc.)



Method

- ECG and EEG signals can be modeled by **ODEs**.
- ODEs describe the evolution of a system over time, often in terms of nonlinear functions. In many cases, solving **nonlinear ODEs** analytically can be challenging.
- **Koopman theory** is a mathematical and dynamical systems framework used in the analysis of nonlinear systems.
- Koopman theory offers a method to **linearize** the dynamics of a nonlinear system.
- More specifically: we switch from a finite-dimensional nonlinear system to an infinite-dimensional linear one with the **Koopman operator**, which is an infinite-dimensional linear operator that acts on scalar observable functions by composition.

Koopman Theory

We focus on discrete-time dynamical systems since ECG and EEG data describes a continuous-time system sampled in discrete time:

$$x_{t+1} = F(x_t)$$

with $F : \mathcal{M} \to \mathcal{M},\ dim(\mathcal{M}) = L$

Def. Given $\mathcal{G} = \{g : \mathcal{M} \to \mathbb{R}\}$, the Koopman operator for our dynamical system is

$$\mathcal{K} : \mathcal{G} \to \mathcal{G} \mid g \mapsto g \circ F$$

We observe that:

- $g(x_{t+1}) = g \circ F(x_t) = \mathcal{K}g(x_t)$
- $\mathcal{K}$ is linear: $\mathcal{K}(\alpha g + \beta h) = \alpha \mathcal{K}g + \beta \mathcal{K}h$

Learning Koopman Embedding

We want to find $K : \mathbb{R}^D \to \mathbb{R}^D$ a finite-dimension approximation of $\mathcal{K}$ and $\Gamma : \mathbb{R}^D \to \mathcal{G}$ s.t.

$$\Gamma \circ K \circ \Gamma^{-1} \approx \mathcal{K}$$

Notice that: $\mathcal{G} \xrightarrow{\Gamma^{-1}} \mathbb{R}^D \xrightarrow{K} \mathbb{R}^D \xrightarrow{\Gamma} \mathcal{G}$

We can rewrite the equation of the system as:

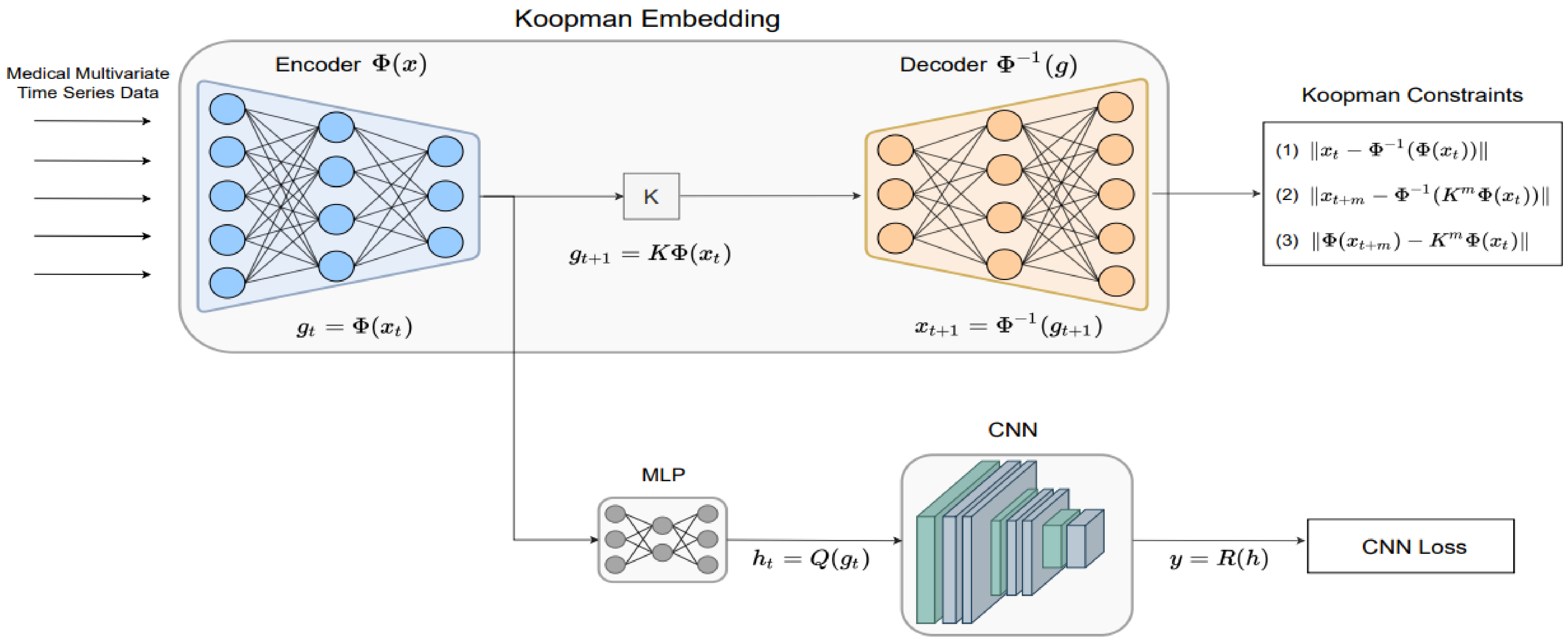
$$g(x_{t+1}) = \mathcal{K} g(x_t) = \Gamma K \Gamma^{-1} g(x_t)$$

$$\Gamma^{-1} g(x_{t+1}) = K \Gamma^{-1} g(x_t)$$

We define $\Phi = \Gamma^{-1} g : \mathbb{R}^L \to \mathbb{R}^D$ and we obtain: $\boxed{\Phi(x_{t+1}) = K \Phi(x_t)}$

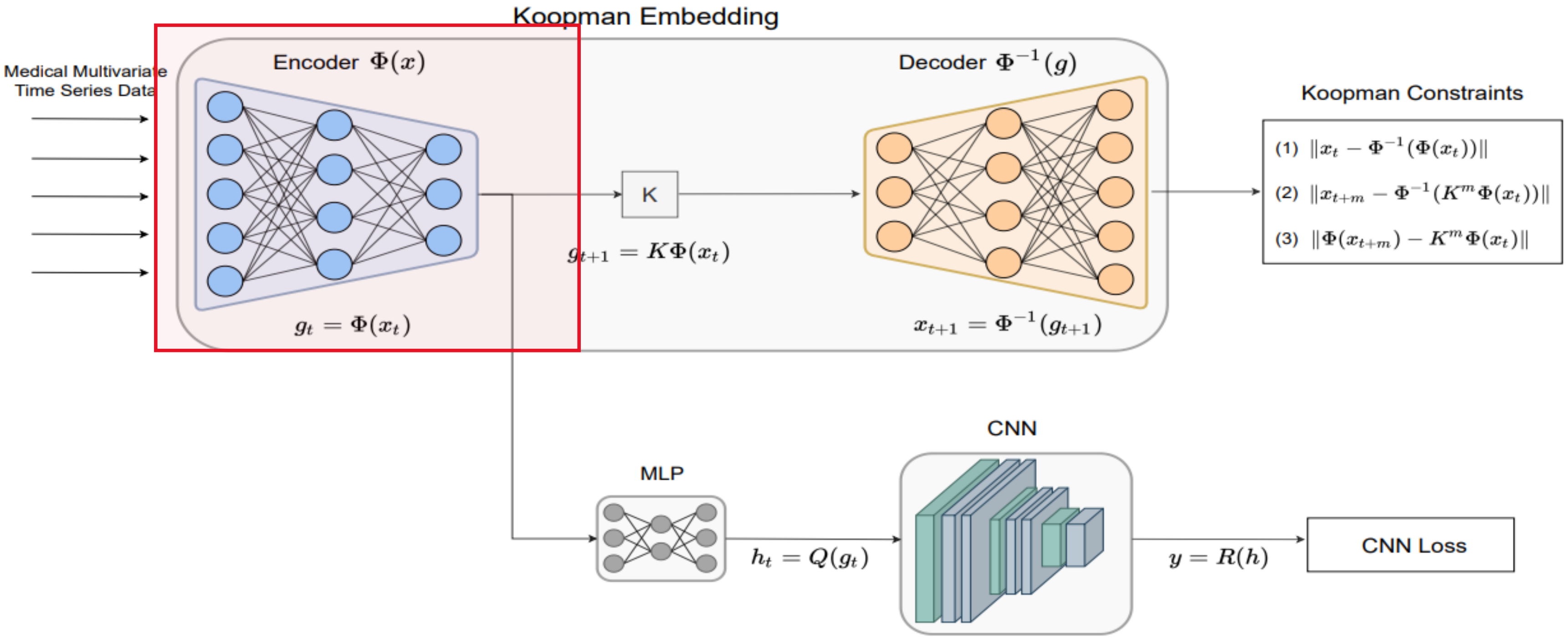
We use a neural model to find Φ and K.

Model



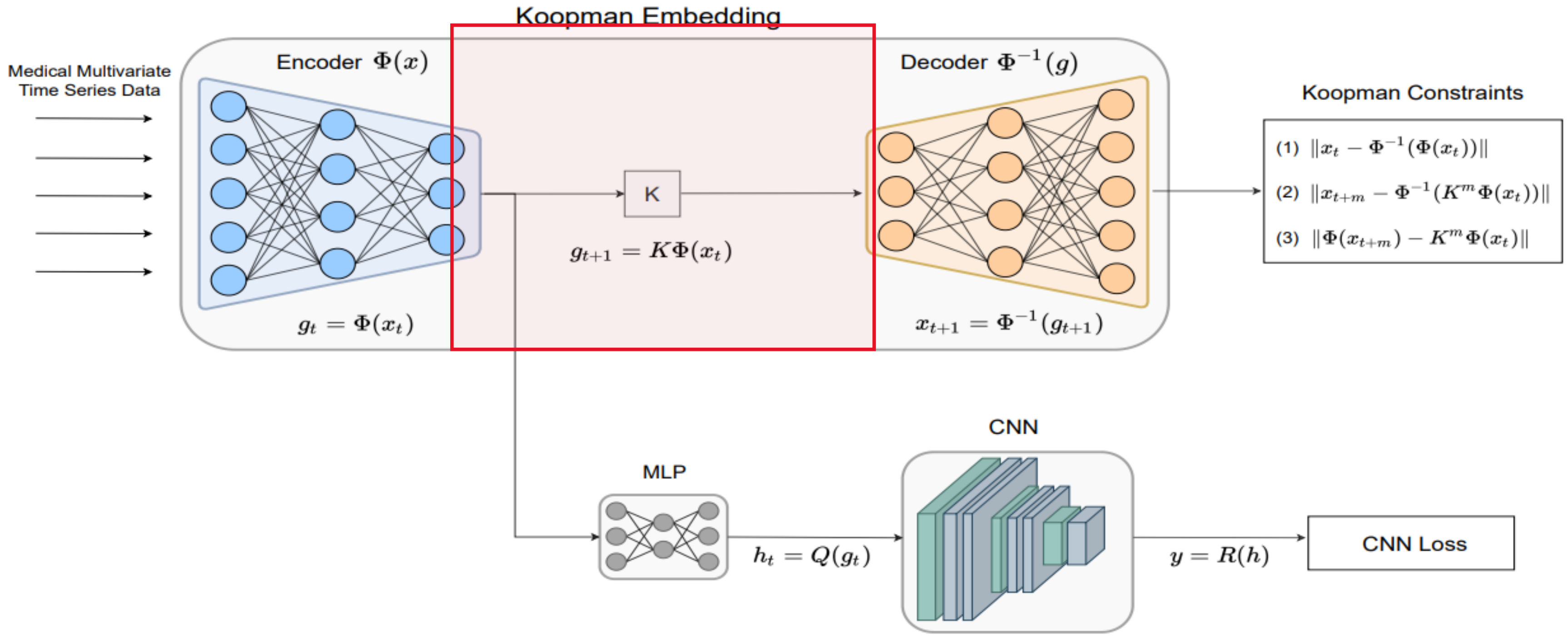
Koopman Embedding
Medical Multivariate Time Series Data
Encoder $\Phi(x)$
Decoder $\Phi^{-1}(g)$
K
$g_{t+1} = K\Phi(x_t)$
$g_t = \Phi(x_t)$
$x_{t+1} = \Phi^{-1}(g_{t+1})$
Koopman Constraints
(1) $\|x_t - \Phi^{-1}(\Phi(x_t))\|$
(2) $\|x_{t+m} - \Phi^{-1}(K^m\Phi(x_t))\|$
(3) $\|\Phi(x_{t+m}) - K^m\Phi(x_t)\|$
MLP
$h_t = Q(g_t)$
CNN
$y = R(h)$
CNN Loss

Model



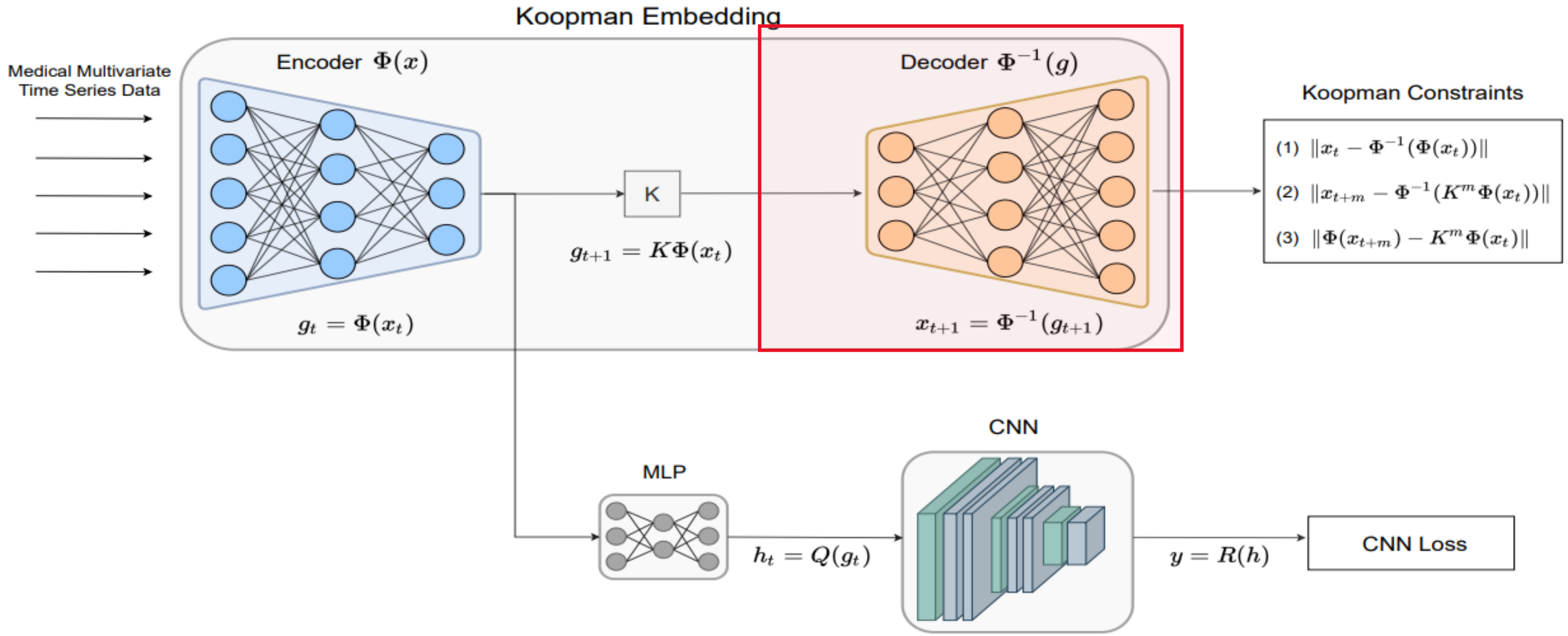
1. The encoder Φ encodes x_t into the Koopman embedding space G.
   In formula, $\Phi(x_t) = g(t)$.

Model



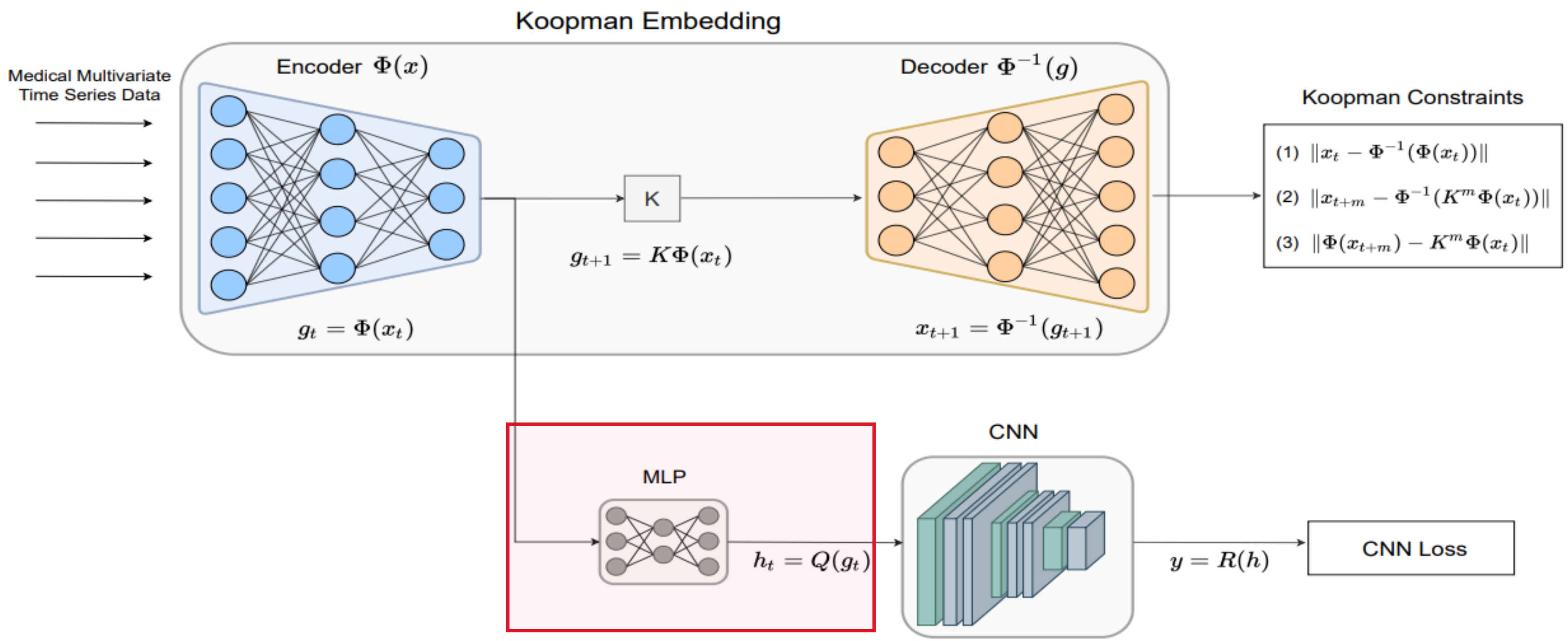
2a. In the Koopman space, the dynamic is linear: K is a singe matrix, and $g_{t+1} = Kg_t$.

Model



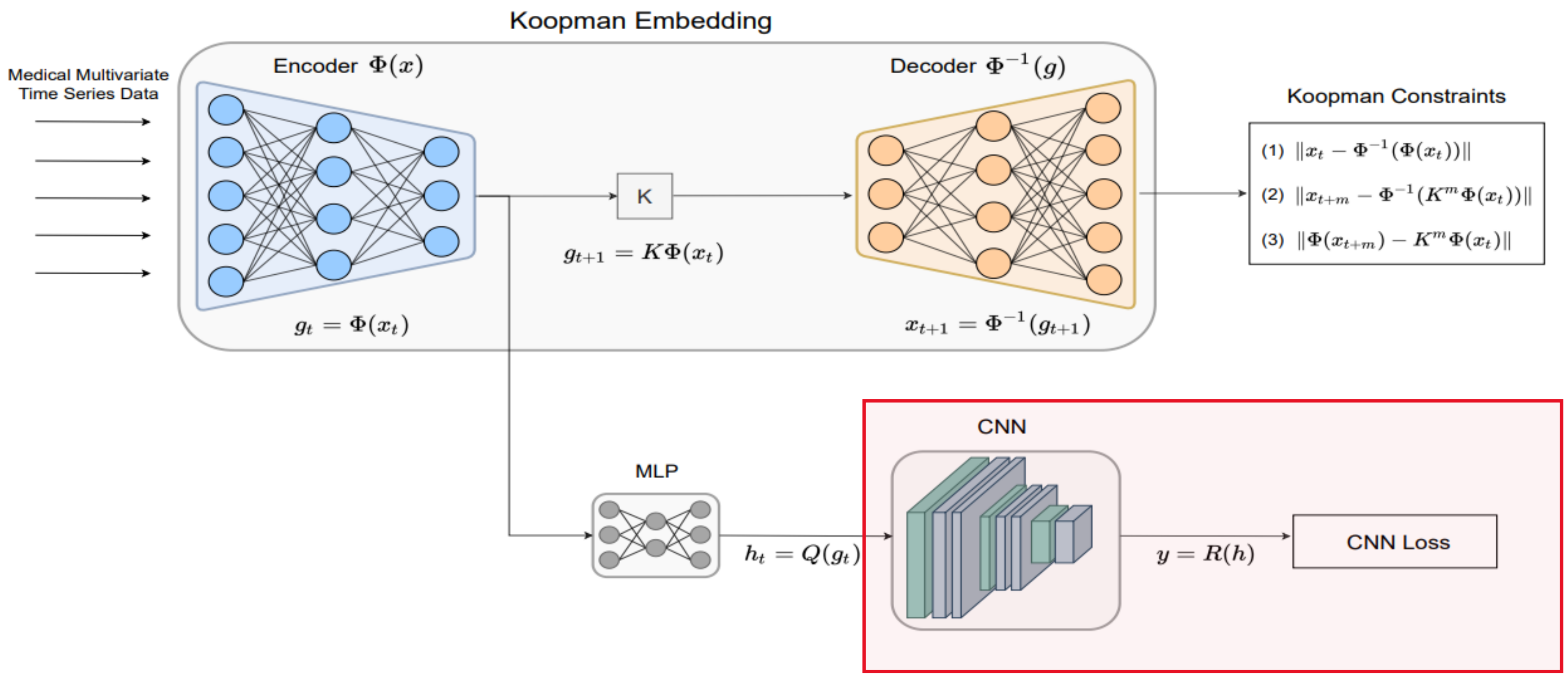
3a. The decoder Φ^{-1} brings g_{t+1} back to the original space.
In formula, $\Phi^{-1}(g_{t+1}) = x_{t+1}$.

Model



2b. The encoded input g_t also passes into a Feedforward Neural Network, which extracts useful features for classification $h_t = Q(g_t)$

Model



3b. Finally, the sequence of features $[h_1, \ldots, h_t, \ldots h_T]$ is passed through a Convolutional Neural Network, that outputs the classification results

Losses of the model

In order to learn the correct linearization of the dynamics, we need a combination of three loss functions.

- Reconstruction Loss:

$$L_{rec} = ||x_t - \Phi^{-1}\big(\Phi(x_t)\big)||$$

- Linear Dynamics:

$$L_{lin} = ||\, \Phi(x_{t+m}) - K^m \Phi(x_t)||, \forall\, m \geq 1$$

- Prediction of the Dynamics:

$$L_{pred} = ||x_{t+m} - \Phi^{-1}\big(K^m \Phi(x_t)\big)||, \forall\, m \geq 1$$

In addition, a standard supervised loss (e.g. binary cross-entropy) is used to train the CNN

Datasets

ECG: two datasets

- **G12EC:** 10344 12-lead ECGs from 7871 patients
- **PTB-XL:** 21837 12-lead ECGs from 18885 patients
- 10 seconds, 500 Hz sampling frequence → length of TS 5.000
- **Multi-Label** classification task, up to 27 different diagnoses
- We focus on the 6 most common anomalies

EEG: Temple University Hospital dataset

- 2,993 recordings,
- various lengths, 21 channels with different frequencies
- **Binary** classification task, normal or abnormal EEG

Results (ECG)

- Compared against SotA, a ResNet CNN
- Improvements in sensitivity and specificity of recognising almost every type of diagnosis.
- All statistically significative results, p-value < 0.05

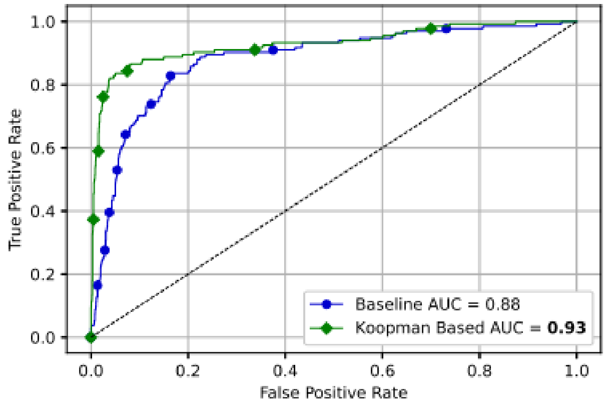
Table 1: Evaluation results - ECG (G12EC Dataset)

	Temporal Projection + ResNet		**ResNet Based**	
Abnormality	Sensitivity	Specificity	Sensitivity	Specificity
AF	0.88	**0.89**	0.88	0.83
TAb	**0.88**	**0.76**	0.85	0.72
QAb	**0.89**	**0.79**	0.84	0.72
VPB	**0.80**	**0.55**	0.77	0.54
SA	**0.68**	**0.65**	0.65	0.55
LAD	**0.97**	**0.90**	0.94	0.87

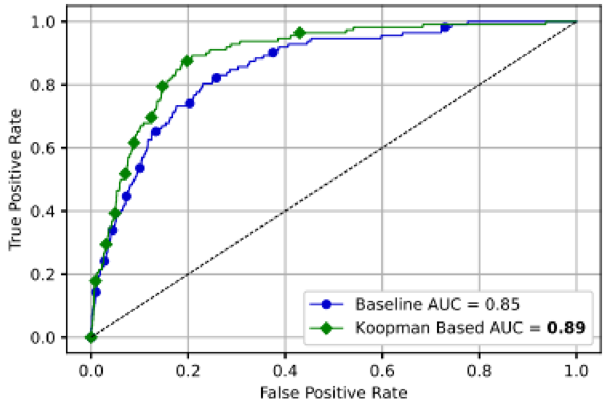
Table 2: Evaluation results - ECG (PTB-XL Dataset)

	Temporal Projection + ResNet		**ResNet Based**	
Abnormality	Sensitivity	Specificity	Sensitivity	Specificity
AF	0.98	**0.94**	0.98	0.93
TAb	0.89	**0.75**	0.89	0.70
QAb	**0.85**	**0.77**	0.84	0.74
SA	**0.67**	**0.59**	0.65	0.53
LAD	0.91	**0.90**	0.91	0.87

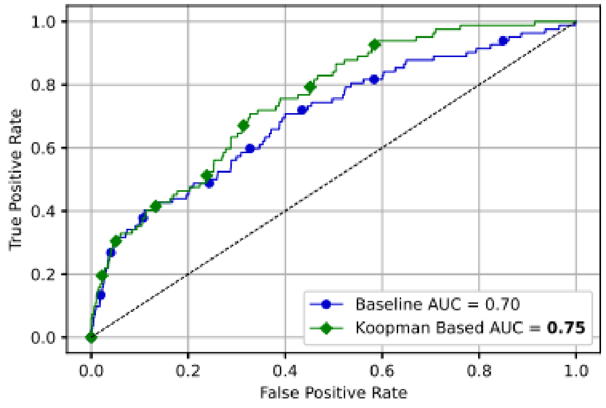
ROC Curves (ECG)



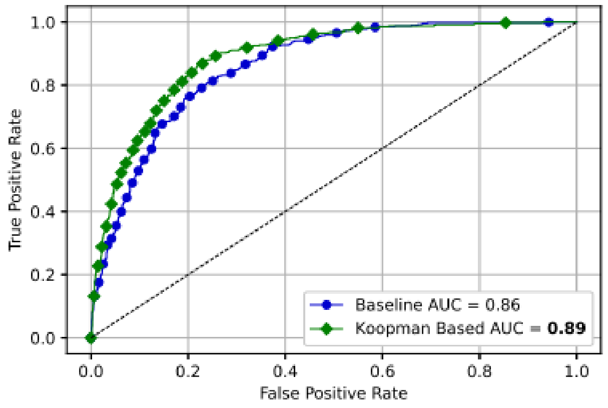
(a) Atrial fibrillation (AF)



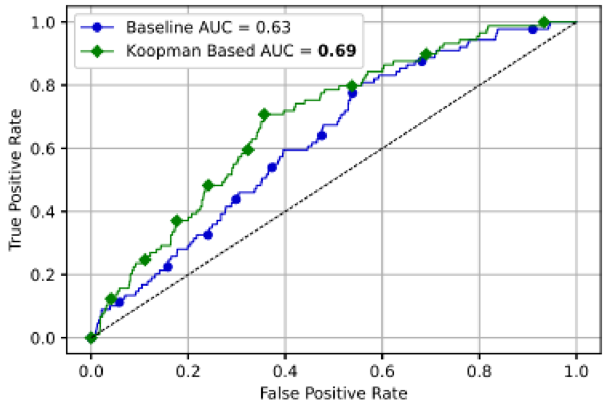
(b) Q wave abnormal (QAb)



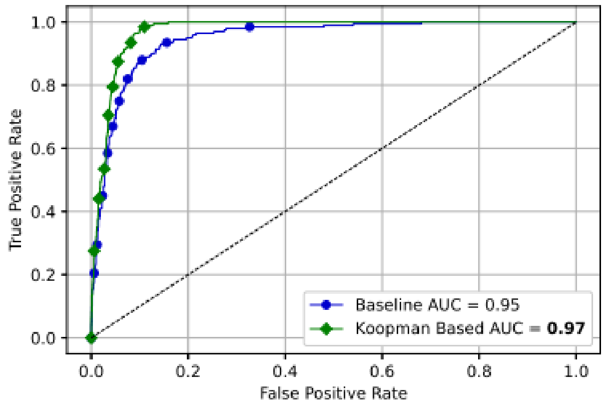
(c) Ventricular premature beats (VPB)



(d) T wave abnormal (TAb)



(e) Sinus arrhythmia (SA)

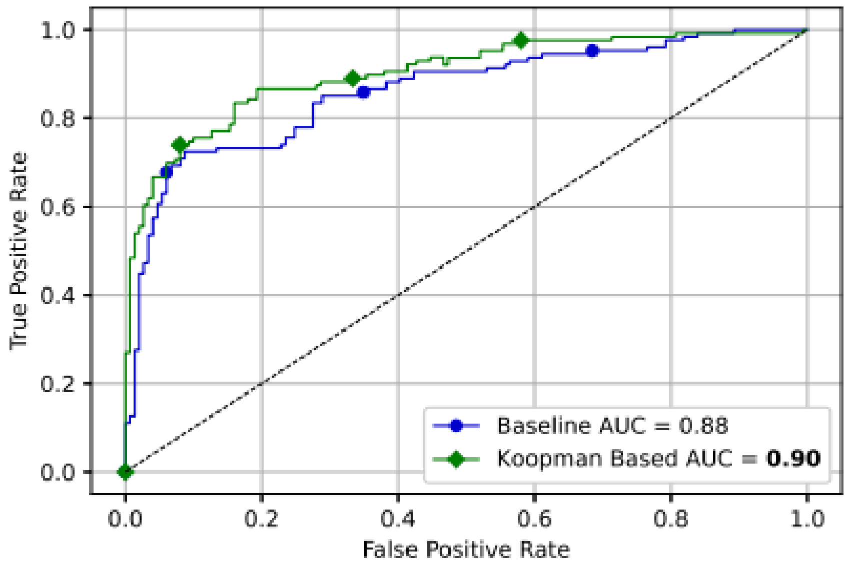


(f) Left axis deviation (LAD)

Results (EEG)

Table 3: Evaluation results - EEG

	Temporal Projection + CNN		CNN Based	
	Sensitivity	Specificity	Sensitivity	Specificity
Normal vs. Abnormal	**0.81**	**0.84**	0.80	0.76



- Previous SotA: Convolutional Neural Network
- Huge improvement in specificity

Dimension of the Embeddings

- Choosing the right dimension for the embedding space is fundamental.
- For the ECGs a dimension of 30 turns out to be the best, for the EEGs is 40.
- Manually tune the hyperparameter for each task

Table 4: ECG Task: Koopman Embedding size

	Temporal Projection + ResNet							
	Dim = 10		Dim = 20		Dim = 30		Dim = 40	
Abnormality	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
AF	**0.88**	0.86	**0.88**	0.88	**0.88**	**0.89**	0.87	0.87
TAb	0.85	0.74	0.87	0.74	**0.88**	**0.76**	0.86	0.75
QAb	0.84	0.72	**0.89**	0.78	**0.89**	**0.79**	0.87	0.75
VPB	0.77	0.54	**0.80**	**0.55**	0.79	**0.55**	0.75	**0.55**
SA	0.65	0.55	0.66	0.58	**0.68**	**0.65**	**0.68**	**0.65**
LAD	0.94	0.89	0.95	0.89	**0.97**	**0.90**	**0.97**	0.89

Table 5: EEG Task: Koopman Embedding size

	Temporal Projection +CNN							
	Dim = 20		Dim = 30		Dim = 40		Dim = 50	
Abnormality	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity	Sensitivity	Specificity
Normal vs. Abnormal	0.80	0.82	0.80	0.78	**0.81**	**0.84**	0.80	0.80

Importance of Linearity Constraint

- Experiment to evaluate the importance of the linearity loss

  $L_{lin} = || \Phi(x_{t+m}) - K^m \Phi(x_t) ||$

- Training with only the autoencoder loss, results are worse

Table 6: The importance of the linearity constraint - ECG

	Temporal Projection + ResNet		AutoEncoder + ResNet	
Abnormality	Sensitivity	Specificity	Sensitivity	Specificity
AF	0.88	**0.89**	0.88	0.86
TAb	**0.88**	**0.76**	0.85	0.74
QAb	**0.89**	**0.79**	0.86	0.75
VPB	**0.80**	0.55	0.77	0.55
SA	**0.68**	**0.65**	0.66	0.59
LAD	**0.97**	**0.90**	0.96	0.88

Table 7: The importance of the linearity constraint - EEG

	Temporal Projection + CNN		AutoEncoder + CNN + CNN	
	Sensitivity	Specificity	Sensitivity	Specificity
Normal vs. Abnormal	**0.81**	**0.84**	0.80	0.79

Conclusions

Major contributions:

- Self-supervised learning offers us additional information about multivariate time series dynamics, specifically ECGs and EEGs.
- Koopman embeddings are an approach that is grounded in theory of control of dynamical systems, and that proved to be very solid also experimentally.

Major limitations:

- Embedding size is an additional hyperparameter, not obvious from the context.
- Embedding size is also often bigger than the original size, which leads to more expensive computations.

Bibliography

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Thanks for
the attention!